Differentiate the following functions:

1. $f(x) = g(x)^{h(x)}$.

   Solution:

$$\begin{aligned} f(x) &= e^{\ln g(x)^{h(x)}} \\ &= e^{h(x)\ln g(x)} \\ f'(x) &= e^{h(x)\ln g(x)}(h'(x)\ln g(x) + h(x)\frac{g'(x)}{g(x)}) \\ &= g(x)^{h(x)}(h'(x)\ln g(x) + \frac{h(x)g'(x)}{g(x)}) \end{aligned}$$

2. $y = \arcsin(x)$

   Solution:

$$\sin(y) = x$$

   diff. w.r.t. x:

$$\begin{aligned} \cos y\frac{dy}{dx} &= 1 \\ \frac{dy}{dx} &= \frac{1}{\cos y} \\ &= \frac{1}{\sqrt{1-\sin^2 y}} \\ &= \frac{1}{\sqrt{1-x^2}}. \end{aligned}$$

3. $y = \arccos x$.

   Solution: $\cos y = x$ diff. w.r.t. x:

$$\begin{aligned} -\sin y\frac{dy}{dx} &= 1 \\ \frac{dy}{dx} &= \frac{-1}{\sin y} \\ &= \frac{-1}{\sqrt{1-\cos^2 y}} \\ &= \frac{-1}{\sqrt{1-x^2}} \end{aligned}$$

4. $y = \tan x$

   Solution:

$$\begin{aligned} y &= \tan x \\ &= \frac{\sin x}{\cos x} \\ \frac{dy}{dx} &= \frac{\cos x}{\cos x} + \sin x \times \frac{-1}{\cos^2 x} \times -\sin x \\ &= 1 + \tan^2 x \\ &= \sec^2 x. \end{aligned}$$

5. $y = \arctan x = \tan^{-1} x$

   Solution:

   $$\tan y = x$$

   diff w.r.t. x:

   $$\begin{aligned} \sec^2 y \frac{dy}{dx} &= 1 \\ \frac{dy}{dx} &= \frac{1}{\sec^2 y} \\ &= \frac{1}{1+\tan^2 y} \\ &= \frac{1}{1+x^2} \end{aligned}$$

6. $y = (\tan x)^{-1} = \cot x$

   Solution:

   $$\begin{aligned} \frac{dy}{dx} &= -(\tan x)^{-2} \sec^2 x \\ &= -\frac{\cos^2 x}{\sin^2 x} \cdot \frac{1}{\cos^2 x} \\ &= \frac{-1}{\sin^2 x} \\ &= -\csc^2 x. \end{aligned}$$

7. $y = \cos(x^2) \sin x.$

   Solution:

   $$\frac{dy}{dx} = -\sin(x^2) 2x \sin x + \cos(x^2) \cos x$$

8. $y = (x+1)\ln(x+1).$

   Solution:

   $$\begin{aligned} \frac{dy}{dx} &= \ln(x+1) + \frac{x+1}{x+1} \\ &= 1 + \ln(x+1). \end{aligned}$$

9. $f(x) = g(x)\ln(g(x)).$

   Solution:

   $$\begin{aligned} f'(x) &= g'(x)\ln(g(x)) + \frac{g(x)}{g(x)} g'(x) \\ &= g'(x)(1+\ln(g(x))). \end{aligned}$$

10. $y = \frac{\sin x}{x}.$

    Solution:

    $$\frac{dy}{dx} = \frac{\cos x}{x} - \frac{\sin x}{x^2}$$