THE DYNKIN DIAGRAMS PACKAGE

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Contents

1. Quick introduction

This is a test of the Dynkin diagram package. Load the package via

```
\usepackage{dynkin-diagrams}
```

and invoke it directly:

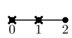
The flag variety of pointed lines in projective 3-space is associated to the Dykin diagram .

```
The flag variety of pointed lines in
projective 3−space is associated to
the Dykin diagram \dynk[parabolic=3]{A}{3}.
```

or use the long form inside a `\tikz` statement or environment:

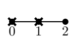

```
\tikz \dynkin[parabolic=3]{A}{3};
```

With labels for the roots:



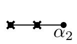

```
\tikz \dynkin[parabolic=3,label=true]{A}{3};
```

Inside an environment:




```
\begin{tikzpicture}
\dynkin[parabolic=3,label=true]{A}{3}
\end{tikzpicture}
```

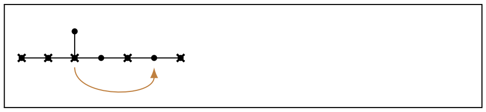
Make up your own labels for the roots:




```
\begin{tikzpicture}
\dynkin[parabolic=3]{A}{3};
\node at (root label 2) {\scalebox{.7}{\(\alpha_2\)}};
\end{tikzpicture}
```

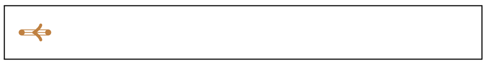
Date: September 12, 2017.

Drawing curves between the roots:

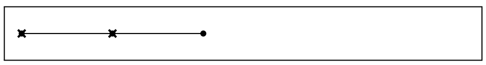


```
\begin{tikzpicture}
\dynkin[parabolic=429]{E}{8}
\draw[brown,-latex]
  (root 3.south)
  to [out=-90, in=-90]
  (root 6.south);
\end{tikzpicture}
```

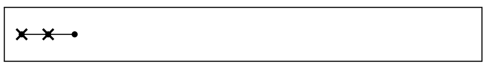
Various options:



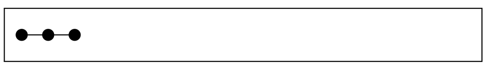
```
\tikz \dynkin[color=brown]{G}{2};
```



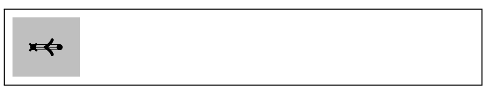
```
\tikz \dynkin[edgelength=1.2,parabolic=3]{A}{3};
```



```
\tikz \dynkin[crosssize=.1cm,parabolic=3]{A}{3};
```



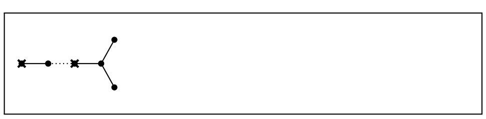
```
\tikz \dynkin[dotradius=.08cm,parabolic=3]{A}{3};
```



```
\begin{tikzpicture}[
    show background rectangle,
    background rectangle/.style={fill=lightgray}]
\dynkin[parabolic=1,background color=lightgray]{G}{2}
\end{tikzpicture}
```

2. Syntax

Inside a `\tikz` environment, the syntax is `\dynkin[<options>]{<letter>}{<rank>}` where `<letter>` is A, B, C, D, E, F or G, the family of root system for the Dynkin diagram, and `<rank>` is an integer representing the rank, or is the symbol `*` to represent an indefinite rank:



```
\begin{tikzpicture}
\dynkin[parabolic=5]{D}{*}
\end{tikzpicture}
```

Outside a `\tikz` environment, use `\dynk` instead of `\dynkin`.

3. Options

`parabolic` = ⟨`integer`⟩, `default` = `0`

A parabolic subgroup with specified integer, where the integer is computed as $n = \sum 2^i a_i$, $a_i = 0$ or 1, to say that root i is crossed, i.e. a noncompact root.

`color` = ⟨`color name`⟩, `default` = `black`

`background color` = ⟨`color name`⟩, `default` = `white`

This only says what color you have already set for the background rectangle. It is needed precisely for the G_2 root system, to draw the triple line correctly, and only when your background color is not white.

`dotradius` = ⟨`number`⟩`cm`, `default` = `.04cm`

size of the dots in the Dynkin diagram

`edgelength` = ⟨`number`⟩`cm`, `default` = `.35cm`

distance between nodes in the Dynkin diagram

`crosssize` = ⟨`number`⟩, `default` = `1.5`

size of the crosses, for parabolic subgroup diagrams.

`label` = `true or false`, `default` = `false`

whether to label the roots by their root numbers.

4. Finding the roots

The roots are labelled in the Bourbaki labelling, but from 0 to $r-1$, where r is the rank. The command sets up nodes `(root 0)`, `(root 1)`, and so on. Use these tikz nodes to draw on the Dynkin diagram. It also sets up nodes `(root label 0)`, `(root label 1)`, and so on for the labels.

5. Example: some parabolic subgroups

$A_{1,0}$

$A_{1,1}$

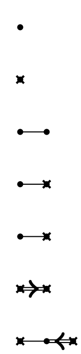
$A_{2,0}$

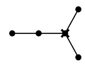
$A_{2,2}$

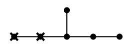
$A_{2,2}$

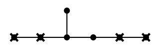
$B_{2,3}$

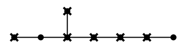
$C_{3,5}$

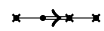


$D_{5,4}$ 

$E_{6,5}$ 

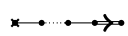
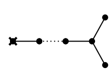
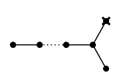
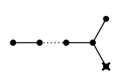
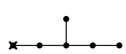
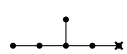
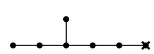
$E_{7,101}$ 

$E_{8,123}$ 

$F_{4,13}$ 

$G_{2,2}$ 

6. Example: the Hermitian symmetric spaces

A_n		Grassmannian of k-planes in $\mathbb{C}^{n+1}$
B_n		$(2n-1)$-dimensional hyperquadric, i.e. the variety of null lines in $\mathbb{C}^{2n+1}$
C_n		space of Lagrangian n-planes in $\mathbb{C}^{2n}$
D_n		$(2n-2)$-dimensional hyperquadric, i.e. the variety of null lines in $\mathbb{C}^{2n}$
D_n		one component of the variety of maximal dimension null subspaces of $\mathbb{C}^{2n}$
D_n		the other component
E_6		complexified octave projective plane
E_6		its dual plane
E_7		the space of null octave 3-planes in octave 6-space