THE DYNKIN DIAGRAMS PACKAGE
VERSION 3.14

BEN MCKAY

Contents

1. Quick introduction

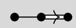
Load the Dynkin diagram package (see options below)

```
\documentclass{amsart}
\usepackage{dynkin-diagrams}
\begin{document}
The Dynkin diagram of \(B_3\) is \dynkin{B}{3}.
\end{document}
```

Date: 24 July 2018.

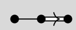
Invoke it

```
The Dynkin diagram of \(B_3\) is \dynkin{B}{3}.
```

The Dynkin diagram of B_3 is .

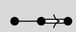
Inside a TikZ statement

```
The Dynkin diagram of \(B_3\) is
\tikz[baseline=-0.5ex] \dynkin{B}{3};
```

The Dynkin diagram of B_3 is 

Inside a TikZ environment

```
The Dynkin diagram of \(B_3\) is
\begin{tikzpicture}[baseline=-0.5ex]
        \dynkin{B}{3}
\end{tikzpicture}
```

The Dynkin diagram of B_3 is 

Indefinite rank Dynkin diagrams

```
\dynkin{B}{}
```

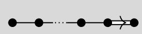
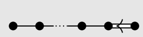
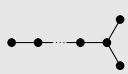
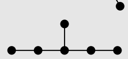
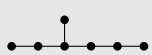
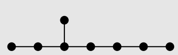
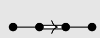


Table 1: The Dynkin diagrams of the reduced simple root systems [3] pp. 265–290, plates I–IX

A_n		`\dynkin{A}{}`
C_n		`\dynkin{C}{}`
D_n		`\dynkin{D}{}`
E_6		`\dynkin{E}{6}`
E_7		`\dynkin{E}{7}`
E_8		`\dynkin{E}{8}`
F_4		`\dynkin{F}{4}`
G_2		`\dynkin{G}{2}`

2. Set options globally

Most options set globally . . .

```
\pgfkeys{/Dynkin diagram,edgeLength=.5cm,foldradius=.5cm}
```

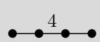
. . . or pass to the package

```
\usepackage[
     ordering=Kac,
     edge/.style=blue,
     mark=o,
     radius=.06cm]
     {dynkin-diagrams}
```

3. Coxeter diagrams

Coxeter diagram option

```
\dynkin[Coxeter]{F}{4}
```

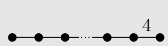
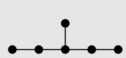
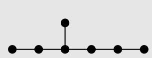
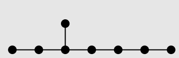


gonality option for G_2 and I_n Coxeter diagrams

```
\(G_2=\dynkin[Coxeter,gonality=n]{G}{2}\), \
\(I_n=\dynkin[Coxeter,gonality=n]{I}{}\)
```

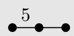
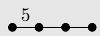
$G_2 =$ , $I_n =$ 

Table 2: The Coxeter diagrams of the simple reflection groups

A_n		`\dynkin[Coxeter]{A}{}`
B_n		`\dynkin[Coxeter]{B}{}`
C_n		`\dynkin[Coxeter]{C}{}`
E_6		`\dynkin[Coxeter]{E}{6}`
E_7		`\dynkin[Coxeter]{E}{7}`
E_8		`\dynkin[Coxeter]{E}{8}`
F_4		`\dynkin[Coxeter]{F}{4}`

continued . . .

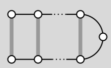
Table 2: . . . continued

G_2		`\dynkin[Coxeter,gonality=n]{G}{2}`
H_3		`\dynkin[Coxeter]{H}{3}`
H_4		`\dynkin[Coxeter]{H}{4}`
I_n		`\dynkin[Coxeter,gonality=n]{I}{}`

4. Satake diagrams

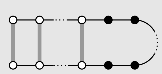
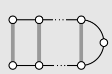
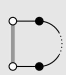
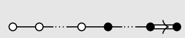
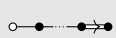
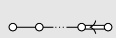
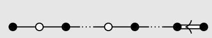
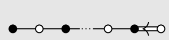
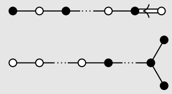
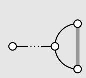
Satake diagrams use the standard name instead of a rank

```
\(A_{IIIb}=\dynkin{A}{IIIb}\)
```

$A_{IIIb} =$ 

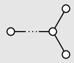
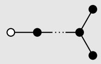
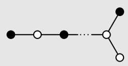
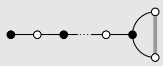
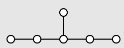
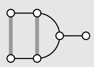
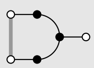
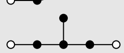
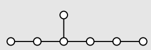
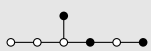
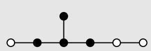
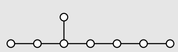
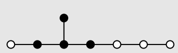
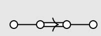
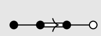
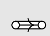
We use a solid gray bar to denote the folding of a Dynkin diagram, rather than the usual double arrow, since the diagrams turn out simpler and easier to read.

Table 3: The Satake diagrams of the real simple Lie algebras [12] p. 532–534

A_I		`\dynkin{A}{I}`
A_{II}		`\dynkin{A}{II}`
A_{IIIa}		`\dynkin{A}{IIIa}`
A_{IIIb}		`\dynkin{A}{IIIb}`
A_{IV}		`\dynkin{A}{IV}`
B_I		`\dynkin{B}{I}`
B_{II}		`\dynkin{B}{II}`
C_I		`\dynkin{C}{I}`
C_{IIa}		`\dynkin{C}{IIa}`
C_{IIb}		`\dynkin{C}{IIb}`
D_{Ia}		`\dynkin{D}{Ia}`
D_{Ib}		`\dynkin{D}{Ib}`

continued . . .

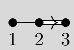
Table 3: ... continued

D_{Ic}		`\dynkin{D}{Ic}`
D_{II}		`\dynkin{D}{II}`
D_{IIIa}		`\dynkin{D}{IIIa}`
D_{IIIb}		`\dynkin{D}{IIIb}`
E_{I}		`\dynkin{E}{I}`
E_{II}		`\dynkin{E}{II}`
E_{III}		`\dynkin{E}{III}`
E_{IV}		`\dynkin{E}{IV}`
E_{V}		`\dynkin{E}{V}`
E_{VI}		`\dynkin{E}{VI}`
E_{VII}		`\dynkin{E}{VII}`
E_{VIII}		`\dynkin{E}{VIII}`
E_{IX}		`\dynkin{E}{IX}`
F_{I}		`\dynkin{F}{I}`
F_{II}		`\dynkin{F}{II}`
G_{I}		`\dynkin{G}{I}`

5. Labels for the roots

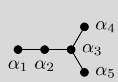
Label the roots by root number

```
\dynkin[label]{B}{3}
```



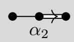
Make a macro to assign labels to roots

```
\dynkin[label,labelMacro/.code={\alpha_{#1}}]{D}{5}
```



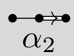
Label a single root

```
\begin{tikzpicture}
        \dynkin{B}{3}
        \dynkinLabelRoot{2}{\alpha_2}
\end{tikzpicture}
```



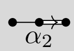
Use a text style

```
\begin{tikzpicture}
        \dynkin[text/.style={scale=1.2}]{B}{3};
        \dynkinLabelRoot{2}{\alpha_2}
\end{tikzpicture}
```



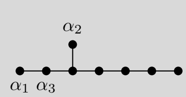
Access root labels via TikZ

```
\begin{tikzpicture}
        \dynkin{B}{3};
        \node[below] at (root 2) {\(\alpha_2\)};
\end{tikzpicture}
```



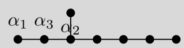
The labels have default locations

```
\begin{tikzpicture}
        \dynkin{E}{8};
        \dynkinLabelRoot{1}{\alpha_1}
        \dynkinLabelRoot{2}{\alpha_2}
        \dynkinLabelRoot{3}{\alpha_3}
\end{tikzpicture}
```



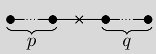
The starred form flips labels to alternate locations

```
\begin{tikzpicture}
        \dynkin{E}{8};
        \dynkinLabelRoot*{1}{\alpha_1}
        \dynkinLabelRoot*{2}{\alpha_2}
        \dynkinLabelRoot*{3}{\alpha_3}
\end{tikzpicture}
```



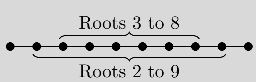
Labelling several roots

```
\begin{tikzpicture}
\dynkin{A}{*.*x*.*}
\dynkinBrace[p]{1}{2}
\dynkinBrace[q]{4}{5}
\end{tikzpicture}
```



Labelling several roots, and a starred form

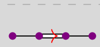
```
\begin{tikzpicture}
\dynkin{A}{10}
\dynkinBrace[\text{Roots 2 to 9}]{2}{9}
\dynkinBrace*[\text{Roots 3 to 8}]{3}{8}
\end{tikzpicture}
```



6. Style

Colours

```
\dynkin[
        edge/.style={blue!50,thick},
        */.style=blue!50!red,
        arrowColor=red]{F}{4}
```

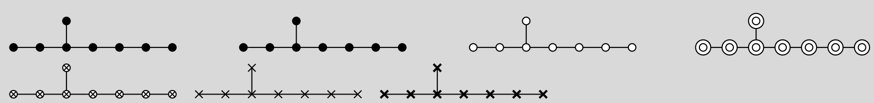


Edge lengths

```
\dynkin[edgeLength=1.2,parabolic=3]{A}{3}
```


Root marks

```
\dynkin{E}{8}
\dynkin[mark=*]{E}{8}
\dynkin[mark=o]{E}{8}
\dynkin[mark=O]{E}{8}
\dynkin[mark=t]{E}{8}
\dynkin[mark=x]{E}{8}
\dynkin[mark=X]{E}{8}
```



At the moment, you can only use:

`*` solid dot

`o` hollow circle

`O` double hollow circle

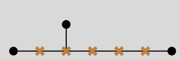
`t` tensor root

`x` crossed root

`X` thickly crossed root

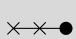
Mark styles

```
\dynkin[parabolic=124,x/.style={brown,very thick}]{E}{8}
```



Sizes of root marks

```
\dynkin[radius=.08cm,parabolic=3]{A}{3}
```



7. Suppress or reverse arrows

Some diagrams have double or triple edges

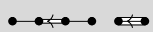
```
\dynkin{F}{4}
\dynkin{G}{2}
```

Suppress arrows

```
\dynkin[arrows=false]{F}{4}
\dynkin[arrows=false]{G}{2}
```


Reverse arrows

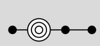
```
\dynkin[reverseArrows]{F}{4}
\dynkin[reverseArrows]{G}{2}
```



8. Drawing on top of a Dynkin diagram

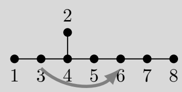
TikZ can access the roots themselves

```
\begin{tikzpicture}
        \dynkin{A}{4};
        \fill[white,draw=black] (root 2) circle (.15cm);
        \fill[white,draw=black] (root 2) circle (.1cm);
        \draw[black] (root 2) circle (.05cm);
\end{tikzpicture}
```



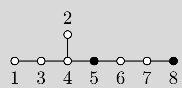
Draw curves between the roots

```
\begin{tikzpicture}
        \dynkin[label]{E}{8}
        \draw[very thick, black!50,-latex]
                (root 3.south) to [out=-45, in=-135] (root 6.south);
\end{tikzpicture}
```



Change marks

```
\begin{tikzpicture}
        \dynkin[mark=o,label]{E}{8};
        \dynkinRootMark{*}{5}
        \dynkinRootMark{*}{8}
\end{tikzpicture}
```

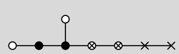


9. Mark lists

The package allows a list of root marks instead of a rank:

A mark list

```
\dynkin{E}{oo**ttxx}
```



The mark list `oo**ttxx` has one mark for each root: `o`, `o`, ..., `x`. Roots are listed in the current default ordering. (Careful: in an affine root system, a mark list will *not* contain a mark for root zero.)

If you need to repeat a mark, you can give a *single digit* positive integer to indicate how many times to repeat it.

A mark list with repetitions

```
\dynkin{A}{x4o3t4}
```


Table 4: Classical Lie superalgebras [10]. We need a slightly larger radius parameter to distinguish the tensor product symbols from the solid dots.

```
\tikzset{/Dynkin
diagram,radius=.07cm}
```

continued ...

Table 4: ... continued

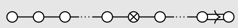
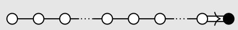
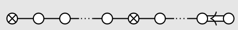
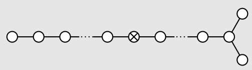
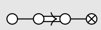
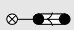
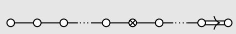
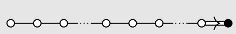
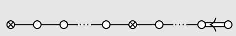
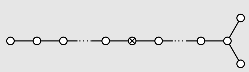
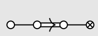
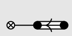
A_{mn}		`\dynkin{A}{ooo.oto.oo}`
B_{mn}		`\dynkin{B}{ooo.oto.oo}`
B_{0n}		`\dynkin{B}{ooo.ooo.o*}`
C_n		`\dynkin{C}{too.oto.oo}`
D_{mn}		`\dynkin{D}{ooo.oto.oooo}`
$D_{21\alpha}$		`\dynkin{A}{oto}`
F_4		`\dynkin{F}{ooot}`
G_3		`\dynkin[extended,affineMark=t, reverseArrows]{G}{2}`

Table 5: Classical Lie superalgebras [10]. Here we see the problem with using the default radius parameter, which is too small for tensor product symbols.

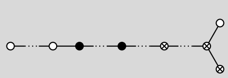
A_{mn}		`\dynkin{A}{ooo.oto.oo}`
B_{mn}		`\dynkin{B}{ooo.oto.oo}`
B_{0n}		`\dynkin{B}{ooo.ooo.o*}`
C_n		`\dynkin{C}{too.oto.oo}`
D_{mn}		`\dynkin{D}{ooo.oto.oooo}`
$D_{21\alpha}$		`\dynkin{A}{oto}`
F_4		`\dynkin{F}{ooot}`
G_3		`\dynkin[extended,affineMark=t, reverseArrows]{G}{2}`

10. Indefinite edges

An *indefinite edge* is a dashed edge between two roots, indicating that an indefinite number of roots have been omitted from the Dynkin diagram. In between any two entries in a mark list, place a period to indicate an indefinite edge:

Indefinite edges

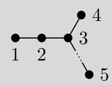
```
\dynkin{D}{o.o*.*.t.to.t}
```



In certain diagrams, roots may have an edge between them even though they are not subsequent in the ordering. For such rare situations, there is an option:

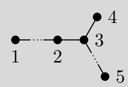
Indefinite edge option

```
\dynkin[makeIndefiniteEdge={3-5},label]{D}{5}
```



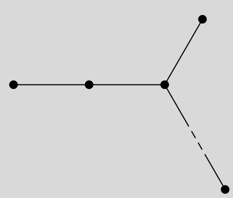
Give a list of edges to become indefinite

```
\dynkin[makeIndefiniteEdge/.list={1-2,3-5},label]{D}{5}
```



Indefinite edge style

```
\dynkin[indefiniteEdge/.style={draw=black,fill=white,thin,densely
    dashed},%
        edgeLength=1cm,%
        makeIndefiniteEdge={3-5}]
        {D}{5}
```



The ratio of the lengths of indefinite edges to those of other edges

```
\dynkin[edgeLength = .5cm,%
        indefiniteEdgeRatio=3,%
        makeIndefiniteEdge={3-5}]
        {D}{5}
```

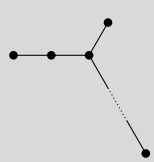
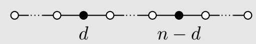
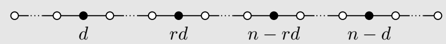
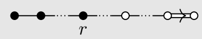
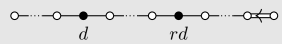
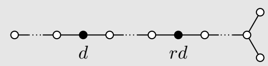
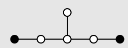
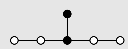
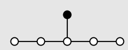
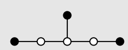
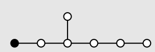
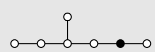
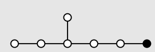
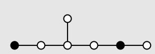
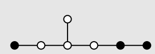
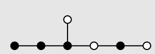
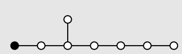
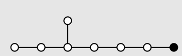
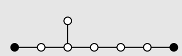
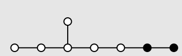
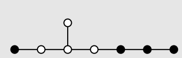
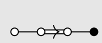
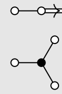


Table 6: Springer's table of indices [22], pp. 320-321, with one form of E_7 corrected

A_n		
A_n		
B_n		
C_n		
D_n		
E_6		`\dynkin{E}{*oooo*}`
E_6		`\dynkin{E}{o*o*oo}`
E_6		`\dynkin{E}{o*oooo}`
E_6		`\dynkin{E}{**ooo*}`
E_7		`\dynkin{E}{*oooooo}`
E_7		`\dynkin{E}{ooooo*o}`
E_7		`\dynkin{E}{oooooo*}`
E_7		`\dynkin{E}{*oooo*o}`
E_7		`\dynkin{E}{*oooo**}`
E_7		`\dynkin{E}{*o**o*o}`
E_8		`\dynkin{E}{*ooooooo}`
E_8		`\dynkin{E}{ooooooo*}`
E_8		`\dynkin{E}{*oooooo*}`
E_8		`\dynkin{E}{oooooo**}`
E_8		`\dynkin{E}{*oooo***}`
F_4		`\dynkin{F}{ooo*}`
D_4		`\dynkin{D}{o*oo}`

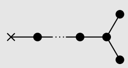
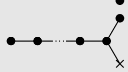
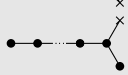
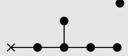
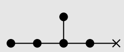
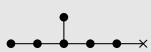
11. Parabolic subgroups

Each set of roots is assigned a number, with each binary digit zero or one to say whether the corresponding root is crossed or not:

```
The flag variety of pointed lines in
projective 3-space is associated to
the Dynkin diagram \dynkin[parabolic=3]{A}{3}.
```

The flag variety of pointed lines in projective 3-space is associated to the Dynkin diagram .

Table 7: The Hermitian symmetric spaces

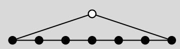
A_n		Grassmannian of k-planes in $\mathbb{C}^{n+1}$
B_n		$(2n-1)$-dimensional hyperquadric, i.e. the variety of null lines in $\mathbb{C}^{2n+1}$
C_n		space of Lagrangian n-planes in $\mathbb{C}^{2n}$
D_n		$(2n-2)$-dimensional hyperquadric, i.e. the variety of null lines in $\mathbb{C}^{2n}$
D_n		one component of the variety of maximal dimension null subspaces of $\mathbb{C}^{2n}$
D_n		the other component
E_6		complexified octave projective plane
E_6		its dual plane
E_7		the space of null octave 3-planes in octave 6-space

```
\NewDocumentCommand\HSS{mommm}
{#1&\IfNoValueTF{#2}{\dynkin{#3}{#4}}{\dynkin[parabolic=#2]{#3}{#4}}&#5\\}
\renewcommand*{\arraystretch}{1.5}
\begin{longtable}
{>{\columncolor[gray]{.9}}>$l<$>{\columncolor[gray]{.9}}>$l<$>{\columncolor[gray]{.9}}l}
\caption{The Hermitian symmetric spaces}\endfirsthead
\caption{\dots continued}\\ \endhead
\caption{continued \dots}\\ \endfoot
\endlastfoot
\HSS{A_n}{A}{**.*x*.**}{Grassmannian of $k$-planes in $\C{n+1}$}
\HSS{B_n}[1]{B}{}{$(2n-1)$-dimensional hyperquadric, i.e. the variety of null lines in $\C{2n+1}$}
\HSS{C_n}[16]{C}{}{space of Lagrangian $n$-planes in $\C{2n}$}
\HSS{D_n}[1]{D}{}{$(2n-2)$-dimensional hyperquadric, i.e. the variety of null lines in $\C{2n}$}
\HSS{D_n}[32]{D}{}{one component of the variety of maximal dimension null subspaces of $\C{2n}$}
\HSS{D_n}[16]{D}{}{the other component}
\HSS{E_6}[1]{E}{6}{complexified octave projective plane}
\HSS{E_6}[32]{E}{6}{its dual plane}
\HSS{E_7}[64]{E}{7}{the space of null octave 3-planes in octave 6-space}
\end{longtable}
```

12. Extended Dynkin diagrams

Extended Dynkin diagrams

```
\dynkin[extended]{A}{7}
```



The extended Dynkin diagrams are also described in the notation of Kac [14] p. 55 as affine untwisted Dynkin diagrams: we extend `\dynkin{A}{7}` to become `\dynkin{A}[1]{7}`:

Extended Dynkin diagrams

```
\dynkin{A}[1]{7}
```

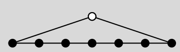
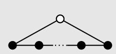
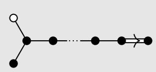
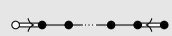
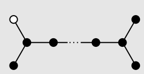
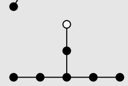
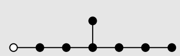
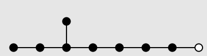
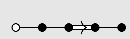
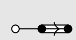


Table 8: The Dynkin diagrams of the extended simple root systems

A_1^1		`\dynkin[extended]{A}{1}`
A_n^1		`\dynkin[extended]{A}{}`
B_n^1		`\dynkin[extended]{B}{}`
C_n^1		`\dynkin[extended]{C}{}`
D_n^1		`\dynkin[extended]{D}{}`
E_6^1		`\dynkin[extended]{E}{6}`
E_7^1		`\dynkin[extended]{E}{7}`
E_8^1		`\dynkin[extended]{E}{8}`
F_4^1		`\dynkin[extended]{F}{4}`
G_2^1		`\dynkin[extended]{G}{2}`

13. Affine twisted and untwisted Dynkin diagrams

The affine Dynkin diagrams are described in the notation of Kac [14] p. 55:

Affine Dynkin diagrams

```
\(A^{(1)}_7=\dynkin{A}[1]{7}, \
E^{(2)}_6=\dynkin{E}[2]{6}, \
D^{(3)}_4=\dynkin{D}[3]{4}\)
```

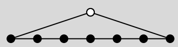
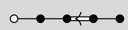
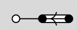
$A^{(1)}_7 =$ , $E^{(2)}_6 =$ , $D^{(3)}_4 =$ 

Table 9: The affine Dynkin diagrams

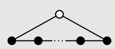
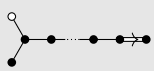
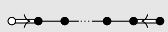
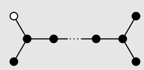
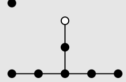
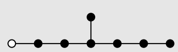
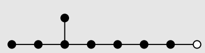
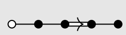
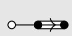
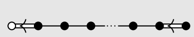
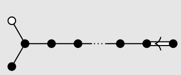
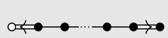
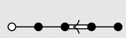
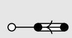
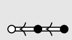
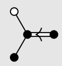
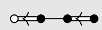
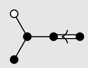
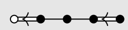
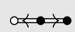
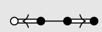
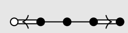
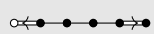
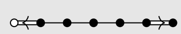
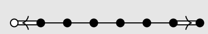
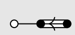
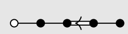
A^1_1		`\dynkin{A}[1]{1}`
A^1_n		`\dynkin{A}[1]{}`
B^1_n		`\dynkin{B}[1]{}`
C^1_n		`\dynkin{C}[1]{}`
D^1_n		`\dynkin{D}[1]{}`
E^1_6		`\dynkin{E}[1]{6}`
E^1_7		`\dynkin{E}[1]{7}`
E^1_8		`\dynkin{E}[1]{8}`
F^1_4		`\dynkin{F}[1]{4}`
G^1_2		`\dynkin{G}[1]{2}`
A^2_2		`\dynkin{A}[2]{2}`
A^2_{ev}		`\dynkin{A}[2]{even}`
A^2_{od}		`\dynkin{A}[2]{odd}`
D^2_n		`\dynkin{D}[2]{}`
E^2_6		`\dynkin{E}[2]{6}`
D^3_4		`\dynkin{D}[3]{4}`

Table 10: Some more affine Dynkin diagrams

A^2_4		`\dynkin{A}[2]{4}`

continued . . .

Table 10: . . . continued

A_5^2		`\dynkin{A}[2]{5}`
A_6^2		`\dynkin{A}[2]{6}`
A_7^2		`\dynkin{A}[2]{7}`
A_8^2		`\dynkin{A}[2]{8}`
D_3^2		`\dynkin{D}[2]{3}`
D_4^2		`\dynkin{D}[2]{4}`
D_5^2		`\dynkin{D}[2]{5}`
D_6^2		`\dynkin{D}[2]{6}`
D_7^2		`\dynkin{D}[2]{7}`
D_8^2		`\dynkin{D}[2]{8}`
D_4^3		`\dynkin{D}[3]{4}`
E_6^2		`\dynkin{E}[2]{6}`

14. Extended Coxeter diagrams

Extended and Coxeter options together

```
\dynkin[extended,Coxeter]{F}{4}
```

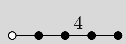
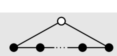
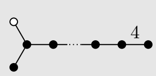
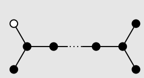
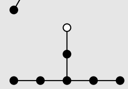
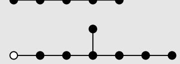
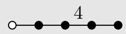
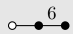
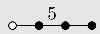
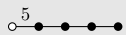


Table 11: The extended (affine) Coxeter diagrams

A_n		`\dynkin[extended,Coxeter]{A}{}`
B_n		`\dynkin[extended,Coxeter]{B}{}`
C_n		`\dynkin[extended,Coxeter]{C}{}`
D_n		`\dynkin[extended,Coxeter]{D}{}`
E_6		`\dynkin[extended,Coxeter]{E}{6}`
E_7		`\dynkin[extended,Coxeter]{E}{7}`

continued . . .

Table 11: . . . continued

E_8		`\dynkin[extended,Coxeter]{E}{8}`
F_4		`\dynkin[extended,Coxeter]{F}{4}`
G_2		`\dynkin[extended,Coxeter]{G}{2}`
H_3		`\dynkin[extended,Coxeter]{H}{3}`
H_4		`\dynkin[extended,Coxeter]{H}{4}`
I_1		`\dynkin[extended,Coxeter]{I}{1}`

15. Kac style

We include a style called `Kac` which tries to imitate the style of [14].

Kac style

```
\dynkin[Kac]{F}{4}
```

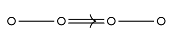
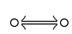
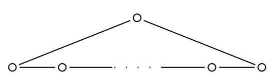
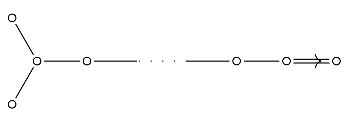
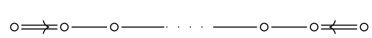
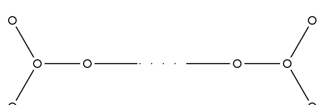
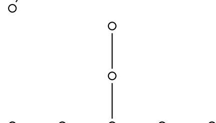
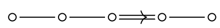
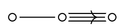


Table 12: The Dynkin diagrams of the extended simple root systems in Kac style. At the moment, it only works on a white background.

A_1^1		`\dynkin[extended]{A}{1}`
A_n^1		`\dynkin[extended]{A}{}`
B_n^1		`\dynkin[extended]{B}{}`
C_n^1		`\dynkin[extended]{C}{}`
D_n^1		`\dynkin[extended]{D}{}`
E_6^1		`\dynkin[extended]{E}{6}`
E_7^1		`\dynkin[extended]{E}{7}`
E_8^1		`\dynkin[extended]{E}{8}`

continued . . .

Table 12: ... continued

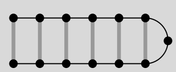
F_4^1		`\dynkin[extended]{F}{4}`
G_2^1		`\dynkin[extended]{G}{2}`

16. Folded Dynkin diagrams

The Dynkin diagrams package has limited support for folding Dynkin diagrams.

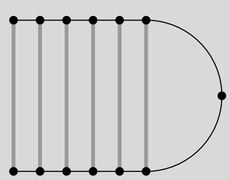
Folding

```
\dynkin[fold]{A}{13}
```



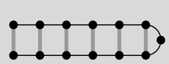
Big fold radius

```
\dynkin[fold,foldradius=1cm]{A}{13}
```



Small fold radius

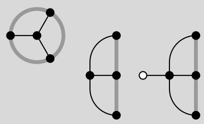
```
\dynkin[fold,foldradius=.2cm]{A}{13}
```



Some Dynkin diagrams have multiple foldings, which we attempt to distinguish (not entirely successfully) by their *ply*: the maximum number of roots folded together. Most diagrams can only allow a 2-ply folding, so `fold` is a synonym for `ply=2`.

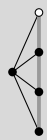
3-ply

```
\dynkin[ply=3]{D}{4}
\dynkin[ply=3,foldright]{D}{4}
\dynkin[ply=3]{D}[1]{4}
```



4-ply

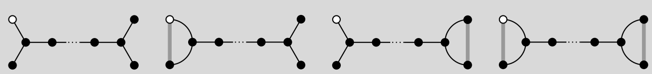
```
\dynkin[ply=4]{D}[1]{4}
```



The $D_\ell^{(1)}$ diagrams can be folded on their left end and separately on their right end:

Left, right and both

```
\dynkin{D}[1]{} \
\dynkin[foldleft]{D}[1]{} \
\dynkin[foldright]{D}[1]{} \
\dynkin[fold]{D}[1]{}
```



We have to be careful about the 4-ply foldings of $D_{2\ell}^{(1)}$, for which we can have two different patterns, so by default, the package only draws as much as it can without distinguishing the two:

Default $D_{2\ell}^{(1)}$ and the two ways to finish it

```
\begin{tikzpicture}
        \dynkin[ply=4]{D}[1]{****.*****.*****}%
\end{tikzpicture} \
\begin{tikzpicture}
        \dynkin[ply=4]{D}[1]{****.*****.*****}%
        \dynkinFold[bend right=65]{1}{13}%
        \dynkinFold[bend right=65]{0}{14}%
\end{tikzpicture} \
\begin{tikzpicture}
        \dynkin[ply=4]{D}[1]{****.*****.*****}%
        \dynkinFold{0}{1}%
        \dynkinFold{1}{13}%
        \dynkinFold{13}{14}%
```

```
\end{tikzpicture}
```

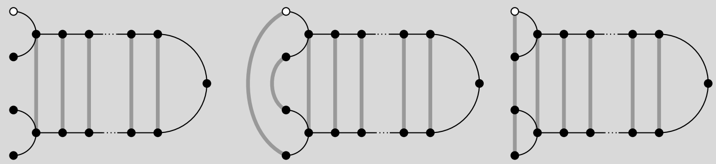
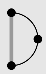
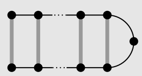
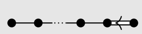
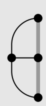
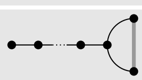
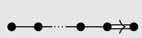
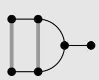
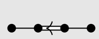
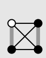
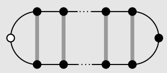
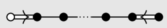
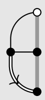
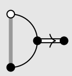
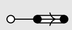
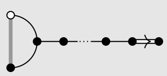
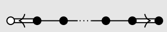
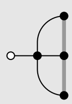
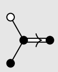
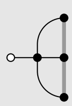
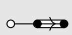
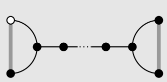
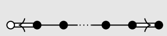
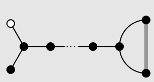
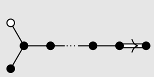


Table 13: Some foldings of Dynkin diagrams. For these diagrams, we want to compare a folding diagram with the diagram that results when we fold it, so it looks best to set `foldradius` and `edgeLength` to equal lengths.

A_3		`\dynkin[fold]{A}[0]{3}`
C_2		`\dynkin{C}[0]{2}`
$A_{2\ell}$		`\dynkin[fold]{A}{**.*****.**}`
C_ℓ		`\dynkin{C}{}`
B_3		`\dynkin[fold]{B}[0]{3}`
G_2		`\dynkin[reverseArrows]{G}[0]{2}`
D_4		`\dynkin[ply=3,foldright]{D}{4}`
G_2		`\dynkin{G}{2}`
$D_{\ell+1}$		`\dynkin[fold]{D}{}`
B_ℓ		`\dynkin{B}{}`
E_6		`\dynkin[fold]{E}[0]{6}`
F_4		`\dynkin[reverseArrows]{F}[0]{4}`
A_3^1		`\dynkin[ply=4]{A}[1]{3}`
A_1^1		`\dynkin{A}[1]{1}`

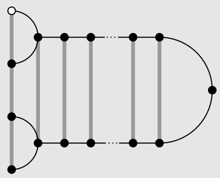
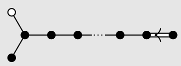
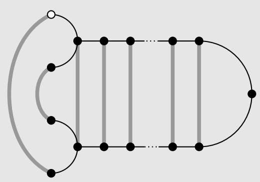
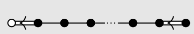
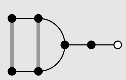
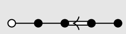
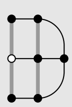
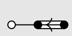
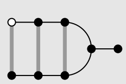
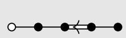
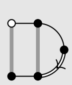
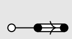
continued ...

Table 13: ...continued

$A^1_{2\ell}$		`\dynkin[fold]{A}[1]{**.*****.**}`
C^1_ℓ		`\dynkin{C}[1]{}`
B^1_3		`\dynkin[ply=3]{B}[1]{3}`
A^2_2		`\dynkin{A}[2]{2}`
B^1_3		`\dynkin[ply=2]{B}[1]{3}`
G^1_2		`\dynkin{G}[1]{2}`
B^1_ℓ		`\dynkin[fold]{B}[1]{}`
D^2_ℓ		`\dynkin{D}[2]{}`
D^1_4		`\dynkin[ply=3]{D}[1]{4}`
B^1_3		`\dynkin{B}[1]{3}`
D^1_4		`\dynkin[ply=3]{D}[1]{4}`
G^1_2		`\dynkin{G}[1]{2}`
$D^1_{\ell+1}$		`\dynkin[fold]{D}[1]{}`
D^2_ℓ		`\dynkin{D}[2]{}`
$D^1_{\ell+1}$		`\dynkin[foldright]{D}[1]{}`
B^1_ℓ		`\dynkin{B}[1]{}`

continued ...

Table 13: . . . continued

$D^1_{2\ell}$		\begin{tikzpicture}[baseline=0pt] \dynkin[ply=4]{D}[1]{****.*****.*****} \dynkinFold{0}{1} \dynkinFold{1}{13} \dynkinFold{13}{14} \end{tikzpicture}
A^2_{od}		\dynkin{A}[2]{odd}
$D^1_{2\ell}$		\begin{tikzpicture}[baseline=0pt] \dynkin[ply=4]{D}[1]{****.*****.*****} \dynkinFold[bend right=65]{1}{13} \dynkinFold[bend right=65]{0}{14} \end{tikzpicture}
A^2_{ev}		\dynkin{A}[2]{even}
E^1_6		\dynkin[fold]{E}[1]{6}
F^1_4		\dynkin[reverseArrows]{F}[1]{4}
E^1_6		\dynkin[ply=3]{E}[1]{6}
D^3_4		\dynkin{D}[3]{4}
E^1_7		\dynkin[fold]{E}[1]{7}
E^2_6		\dynkin{E}[2]{6}
F^1_4		\dynkin[fold]{F}[1]{4}
G^1_2		\dynkin{G}[1]{2}

continued . . .

Table 13: . . . continued

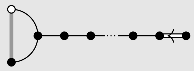
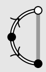
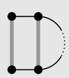
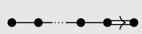
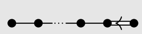
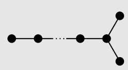
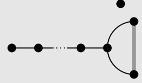
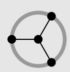
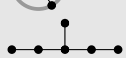
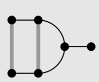
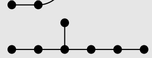
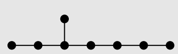
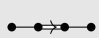
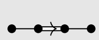
A^2_{od}		`\dynkin[odd,fold]{A}[2]{****.***}`
A^2_{ev}		`\dynkin{A}[2]{even}`
D^2_3		`\dynkin[fold]{D}[2]{3}`
A^2_2		`\dynkin{A}[2]{2}`

Table 14: Frobenius fixed point subgroups of finite simple groups of Lie type [4] p. 15

$A_{\ell\ge1}$		`\dynkin{A}{}`
${}^2A_{\ell\ge2}$		`\dynkin[fold]{A}{}`
$B_{\ell\ge2}$		`\dynkin{B}{}`
${}^2B_2$		`\dynkin[fold]{B}{2}`
$C_{\ell\ge3}$		`\dynkin{C}{}`
$D_{\ell\ge4}$		`\dynkin{D}{}`
${}^2D_{\ell\ge4}$		`\dynkin[fold]{D}{}`
${}^3D_4$		`\dynkin[ply=3]{D}{4}`
E_6		`\dynkin{E}{6}`
${}^2E_6$		`\dynkin[fold]{E}{6}`
E_7		`\dynkin{E}{7}`
E_8		`\dynkin{E}{8}`
F_4		`\dynkin{F}{4}`
${}^2F_4$		`\dynkin[fold]{F}{4}`
G_2		`\dynkin{G}{2}`

continued . . .

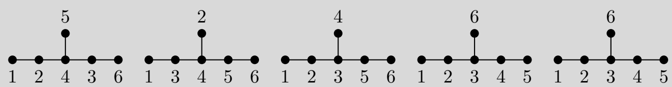
Table 14: ...continued

$^2G_2$		`\dynkin[fold]{G}{2}`

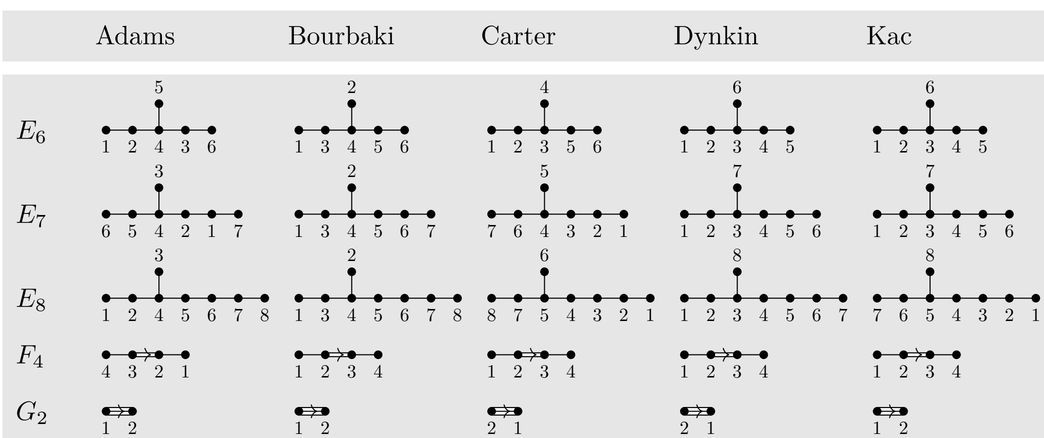
17. Root ordering

Root ordering

```
\dynkin[label,ordering=Adams]{E}{6}
\dynkin[label,ordering=Bourbaki]{E}{6}
\dynkin[label,ordering=Carter]{E}{6}
\dynkin[label,ordering=Dynkin]{E}{6}
\dynkin[label,ordering=Kac]{E}{6}
```

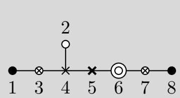


Default is Bourbaki. Sources are Adams [1] p. 56–57, Bourbaki [3] p. pp. 265–290 plates I-IX, Carter [5] p. 540–609, Dynkin [8], Kac [14] p. 43.

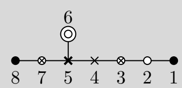


The marks are set down in order according to the current root ordering:

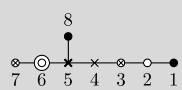
```
\begin{tikzpicture}
\dynkin[label]{E}{*otxXOt*}
\end{tikzpicture}
```



```
\begin{tikzpicture}
\dynkin[label,ordering=Carter]{E}{*otxXOt*}
\end{tikzpicture}
```




```
\begin{tikzpicture}
\dynkin[label,ordering=Kac]{E}{*otxXOt*}
\end{tikzpicture}
```

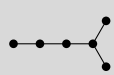


18. Connecting Dynkin diagrams

We can make some sophisticated folded diagrams by drawing multiple diagrams, each with a name:

Name a diagram

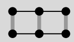
```
\dynkin[name=Bob]{D}{6}
```



We can then connect the two with folding edges:

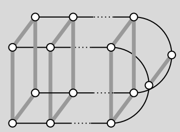
Connect diagrams

```
\begin{tikzpicture}
        \dynkin[name=upper]{A}{3}
        \node (current) at ($(upper root 1)+(0,-.3cm)$) {};
        \dynkin[at=(current),name=lower]{A}{3}
        \begin{scope}[on background layer]
                \foreach \i in {1,...,3}%
                {%
                        \draw[/Dynkin diagram/foldStyle]
                                ($(upper root \i)$) -- ($(lower
    root \i)$);%
                }%
        \end{scope}
\end{tikzpicture}
```



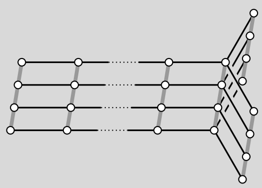
The following diagrams arise in the Satake diagrams of the pseudo-Riemannian symmetric spaces [2].

```
\pgfkeys{/Dynkin diagram,edgeLength=.5cm,foldradius=.5cm}
\begin{tikzpicture}
        \dynkin[name=1]{A}{IIIb}
        \node (a) at (.3,.4){};
        \dynkin[name=2,at=(a)]{A}{IIIb}
        \begin{scope}[on background layer]
                \foreach \i in {1,...,7}%
                {%
                        \draw[/Dynkin diagram/foldStyle]
                                ($(1 root \i)$)
                                --
                                ($(2 root \i)$);%
                }%
        \end{scope}
\end{tikzpicture}
```



```
\pgfkeys{/Dynkin diagram/edgeLength=.75cm,/Dynkin
    diagram/edge/.style={draw=example-color,double=black,very
    thick},
}
\begin{tikzpicture}
        \foreach \d in {1,...,4}
        {
                \node (current) at ($(\d*.05,\d*.3)$){};
                \dynkin[name=\d,at=(current)]{D}{oo.oooo}
        }
        \begin{scope}[on background layer]
                \foreach \i in {1,...,6}%
                {%
                        \draw[/Dynkin diagram/foldStyle] ($(1 root
    \i)$) -- ($(2 root \i)$);%
                        \draw[/Dynkin diagram/foldStyle] ($(2 root
    \i)$) -- ($(3 root \i)$);%
                        \draw[/Dynkin diagram/foldStyle] ($(3 root
    \i)$) -- ($(4 root \i)$);%
```

```
                }%
        \end{scope}
\end{tikzpicture}
```

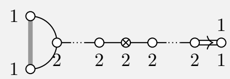


19. Other examples

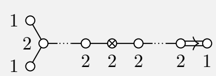
Below we draw the Vogan diagrams of some affine Lie superalgebras [19, 18].

$\mathfrak{sl}\,(2m|2n)^{(2)}$

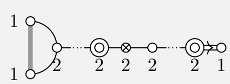
```
\begin{tikzpicture}
        \dynkin[ply=2,label]{B}[1]{oo.oto.oo}
        \dynkinLabelRoot*{7}{1}
\end{tikzpicture}
```



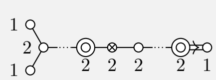

```
\dynkin[label]{B}[1]{oo.oto.oo}
```



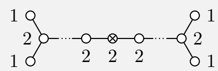

```
\dynkin[ply=2,label]{B}[1]{oo.Oto.Oo}
```



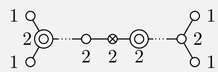

```
\dynkin[label]{B}[1]{oo.Oto.Oo}
```



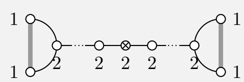
```
\dynkin[label]{D}[1]{oo.oto.ooo}
```



```
\dynkin[label]{D}[1]{oO.otO.ooo}
```

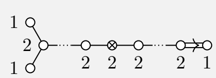


```
\dynkin[label,fold]{D}[1]{oo.oto.ooo}
```

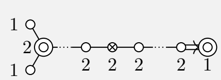


$\mathfrak{sl}\left(2m+1|2n\right)^2$

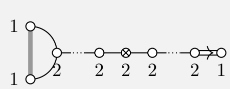
```
\dynkin[label]{B}[1]{oo.oto.oo}
```



```
\dynkin[label]{B}[1]{oO.oto.oO}
```

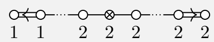


```
\dynkin[label,fold]{B}[1]{oo.oto.oo}
```

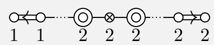


$\mathfrak{sl}\,(2m+1|2n+1)^2$

```
\dynkin[label]{D}[2]{o.oto.oo}
```

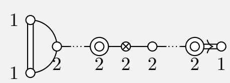



```
\dynkin[label]{D}[2]{o.OtO.oo}
```

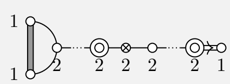


$\mathfrak{sl}\,(2|2n+1)^{(2)}$

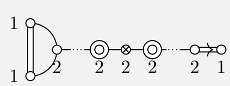
```
\dynkin[ply=2,label,doubleEdges]{B}[1]{oo.Oto.Oo}
```



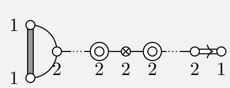

```
\dynkin[ply=2,label,doubleFold]{B}[1]{oo.Oto.Oo}
```




```
\dynkin[ply=2,label,doubleEdges]{B}[1]{oo.OtO.oo}
```

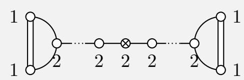



```
\dynkin[ply=2,label,doubleFold]{B}[1]{oo.OtO.oo}
```

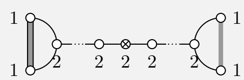


$\mathfrak{sl}\,(2|2n)^{(2)}$

```
\dynkin[ply=2,label,doubleEdges]{D}[1]{oo.oto.ooo}
```

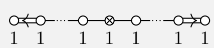



```
\dynkin[ply=2,label,doubleFoldLeft]{D}[1]{oo.oto.ooo}
```

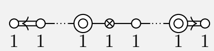


$\mathfrak{osp}\,(2m|2n)^{(2)}$

```
\dynkin[label,labelMacro/.code={1}]{D}[2]{o.oto.oo}
```

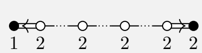



```
\dynkin[label,labelMacro/.code={1}]{D}[2]{o.Oto.Oo}
```

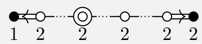


$\mathfrak{osp}\,(2|2n)^{(2)}$

```
\dynkin[label,labelMacro/.code=\lablIt{#1},
        affineMark=*]
        {D}[2]{o.o.o.o*}
```

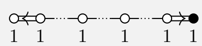


```
\dynkin[label,labelMacro/.code=\lablIt{#1},
        affineMark=*]
        {D}[2]{o.O.o.o*}
```

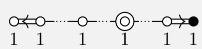


$\mathfrak{sl}(1|2n+1)^4$

```
\dynkin[label,labelMacro/.code={1}]{D}[2]{o.o.o.o*}
```

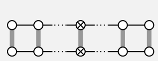



```
\dynkin[label,labelMacro/.code={1}]{D}[2]{o.o.O.o*}
```

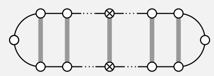


A^1

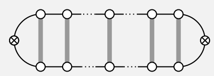
```
\begin{tikzpicture}
        \dynkin[name=upper]{A}{oo.t.oo}
        \node (Dynkin current) at (upper root 1){};
        \dynkinSouth
        \dynkin[at=(Dynkin
    current),name=lower]{A}{oo.t.oo}
        \begin{scope}[on background layer]
        \foreach \i in {1,...,5}{
                \draw[/Dynkin diagram/foldStyle]
                        ($(upper root \i)$) --
    ($(lower root \i)$);
        }
        \end{scope}
\end{tikzpicture}
```



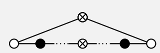
```
\dynkin[fold]{A}[1]{oo.t.ooooo.t.oo}
```



```
\dynkin[fold,affineMark=t]{A}[1]{oo.o.ootoo.o.oo}
```

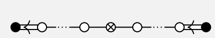


```
\dynkin[affineMark=t]{A}[1]{o*.t.*o}
```

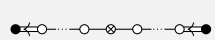


B^1

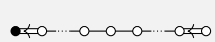
```
\dynkin[affineMark=*]{A}[2]{o.oto.o*}
```



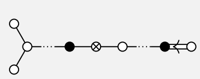
```
\dynkin[affineMark=*]{A}[2]{o.oto.o*}
```



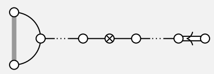
```
\dynkin[affineMark=*]{A}[2]{o.ooo.oo}
```



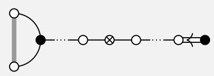
```
\dynkin[odd]{A}[2]{oo.*to.*o}
```



```
\dynkin[odd,fold]{A}[2]{oo.oto.oo}
```

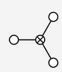


```
\dynkin[odd,fold]{A}[2]{o*.oto.o*}
```



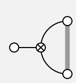
D^1

```
\dynkin{D}{otoo}
```



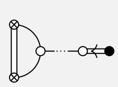
```
\dynkin{D}{ot*o}
```

```
\dynkin[fold]{D}{otoo}
```

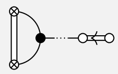


C^1

```
\dynkin[doubleEdges,fold,affineMark=t,odd]{A}[2]{to.o*}
```

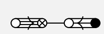


```
\dynkin[doubleEdges,fold,affineMark=t,odd]{A}[2]{t*.oo}
```

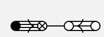


F^1

```
\begin{tikzpicture}%
        \dynkin{A}{oto*}%
        \dynkinQuadrupleEdge{1}{2}%
        \dynkinTripleEdge{4}{3}%
\end{tikzpicture}%
```

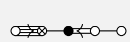


```
\begin{tikzpicture}%
        \dynkin{A}{*too}%
        \dynkinQuadrupleEdge{1}{2}%
        \dynkinTripleEdge{4}{3}%
\end{tikzpicture}%
```

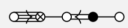


G^1

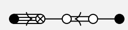
```
\begin{tikzpicture}%
        \dynkin{A}{ot*oo}%
        \dynkinQuadrupleEdge{1}{2}%
        \dynkinDefiniteDoubleEdge{4}{3}%
\end{tikzpicture}%
```



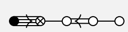
```
\begin{tikzpicture}%
        \dynkin{A}{oto*o}%
        \dynkinQuadrupleEdge{1}{2}%
        \dynkinDefiniteDoubleEdge{4}{3}%
\end{tikzpicture}%
```

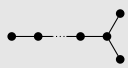
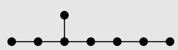
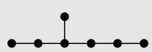
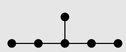
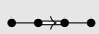


```
\begin{tikzpicture}%
        \dynkin{A}{*too*}%
        \dynkinQuadrupleEdge{1}{2}%
        \dynkinDefiniteDoubleEdge{4}{3}%
\end{tikzpicture}%
```



```
\begin{tikzpicture}%
        \dynkin{A}{*tooo}%
        \dynkinQuadrupleEdge{1}{2}%
        \dynkinDefiniteDoubleEdge{4}{3}%
\end{tikzpicture}%
```



$\mathfrak{g}$	Diagram	Weights	Roots	Simple roots
A_n		$\mathbb{Z}^{n+1}/\left\langle\sum e_j\right\rangle$	$e_i - e_j$	$e_i - e_{i+1}$
B_n		$\mathbb{Z}^n$	$\pm e_i, \pm e_i \pm e_j, i \neq j$	$e_i - e_{i+1}, e_n$
C_n		$\mathbb{Z}^n$	$\pm 2e_i, \pm e_i \pm e_j, i \neq j$	$e_i - e_{i+1}, 2e_n$
D_n		$\mathbb{Z}^n$	$\pm e_i \pm e_j, i \neq j$	$e_i - e_{i+1}, \quad i \leq n-1$ $e_{n-1} + e_n$
E_8		$\mathbb{Z}^8$	$\pm 2e_i \pm 2e_j, \quad i \neq j,$ $\sum_i (-1)^{m_i} e_i, \quad \sum m_i$ even	$2e_1 - 2e_2,$ $2e_2 - 2e_3,$ $2e_3 - 2e_4,$ $2e_4 - 2e_5,$ $2e_5 - 2e_6,$ $2e_6 + 2e_7,$ $-\sum e_j,$ $2e_6 - 2e_7$
E_7		$\mathbb{Z}^8/\langle e_1 - e_2\rangle$	quotient of E_8	quotient of E_8
E_6		$\mathbb{Z}^8/\langle e_1 - e_2, e_2 - e_3\rangle$	quotient of E_8	quotient of E_8
F_4		$\mathbb{Z}^4$	$\pm 2e_i,$ $\pm 2e_i \pm 2e_j, \quad i \neq j,$ $\pm e_1 \pm e_2 \pm e_3 \pm e_4$	$2e_2 - 2e_3,$ $2e_3 - 2e_4,$ $2e_4,$ $e_1 - e_2 - e_3 - e_4$
G_2		$\mathbb{Z}^3/\left\langle\sum e_j\right\rangle$	$\pm(1,-1,0),$ $\pm(-1,0,1),$ $\pm(0,-1,1),$ $\pm(2,-1,-1),$ $\pm(1,-2,1),$ $\pm(-1,-1,2)$	$(-1,0,1),$ $(2,-1,-1)$

```
\NewDocumentEnvironment{bunch}{}%
{\renewcommand*{\arraystretch}{1}\begin{array}{@{}ll@{}}\\ \midrule}{\\ \midrule\end{array}}
\small
\NewDocumentCommand\nct{mm}{\newcolumntype{#1}{>{\columncolor[gray]{.9}}>{$}m{#2cm}<{$}}}
\nct{G}{.3}\nct{D}{2.1}\nct{W}{2.8}\nct{R}{3.7}\nct{S}{3}
\NewDocumentCommand\LieG{}{\mathfrak{g}}
\NewDocumentCommand\W{om}{\ensuremath{\mathbb{Z}^{#2}\IfValueT{#1}{/\left<#1\right>}}}
\renewcommand*{\arraystretch}{1.5}
\NewDocumentCommand\quo{}{\text{quotient of } E_8}
\begin{longtable}{@{}GDWRS@{}}
\LieG&\text{Diagram}&\text{Weights}&\text{Roots}&\text{Simple roots}\\ \midrule\endfirsthead
\LieG&\text{Diagram}&\text{Weights}&\text{Roots}&\text{Simple roots}\\ \midrule\endhead
A_n&\dynkin{A}{}&\W[\sum e_j]{n+1}&e_i-e_j&e_i-e_{i+1}\\
B_n&\dynkin{B}{}&\W{n}& \pm e_i, \pm e_i \pm e_j, i\ne j&e_i-e_{i+1}, e_n\\
C_n&\dynkin{C}{}&\W{n}& \pm 2 e_i, \pm e_i \pm e_j, i\ne j&e_i-e_{i+1}, 2e_n\\
D_n&\dynkin{D}{}&\W{n}& \pm e_i \pm e_j, i\ne j &
\begin{bunch}e_i-e_{i+1},&i\le n-1\\e_{n-1}+e_n\end{bunch}\\
E_8&\dynkin{E}{8}&\W{8}&
\begin{bunch}\pm2e_i\pm2e_j,&i\ne j,\\ \sum_i(-1)^{m_i}e_i,&\sum m_i \text{ even}\end{bunch}&
\begin{bunch}
2e_1-2e_2,\\2e_2-2e_3,\\2e_3-2e_4,\\2e_4-2e_5,\\2e_5-2e_6,\\2e_6+2e_7,\\
-\sum e_j,\\2e_6-2e_7
\end{bunch}\\
E_7&\dynkin{E}{7}&\W[e_1-e_2]{8}&\quo&\quo\\
E_6&\dynkin{E}{6}&\W[e_1-e_2,e_2-e_3]{8}&\quo&\quo\\
F_4& \dynkin{F}{4}&\W{4}&
\begin{bunch}\pm 2e_i,\\ \pm 2e_i \pm 2e_j, \quad i \ne j,\\ \pm e_1 \pm e_2 \pm e_3 \pm e_4
\end{bunch}&
\begin{bunch}2e_2-2e_3,\\2e_3-2e_4,\\2e_4,\\e_1-e_2-e_3-e_4\end{bunch}\\
G_2&\dynkin{G}{2}&\W[\sum e_j]{3}&
\begin{bunch}
\pm(1,-1,0),\\ \pm(-1,0,1),\\ \pm(0,-1,1),\\ \pm(2,-1,-1),\\ \pm(1,-2,1),\\ \pm(-1,-1,2)
\end{bunch}&
\begin{bunch}(-1,0,1),\\(2,-1,-1)\end{bunch}
\end{longtable}
```

20. Syntax

The syntax is \dynkin[<options>]{<letter>}[<twisted rank>]{<rank>} where <letter> is A, B, C, D, E, F or G, the family of root system for the Dynkin diagram, <twisted rank> is 0, 1, 2, 3 (default is 0) representing:

0	finite root system
1	affine extended root system, i.e. of type $^{(1)}$
2	affine twisted root system of type $^{(2)}$
3	affine twisted root system of type $^{(3)}$

and <rank> is

(1) an integer representing the rank or
(2) blank to represent an indefinite rank or
(3) the name of a Satake diagram as in section 4.

21. Options

text/.style = ⟨TikZ style data⟩,
default : scale=.7
  Style for any labels on the roots.

name = ⟨string⟩,
default : anonymous
  A name for the Dynkin diagram, with anonymous treated as a blank; see section 18.

parabolic = ⟨integer⟩,
default : 0
  A parabolic subgroup with specified integer, where the integer is computed as $n = \sum 2^{i-1} a_i$, $a_i = 0$ or 1, to say that root i is crossed, i.e. a noncompact root.

radius = ⟨number⟩cm,
default : .05cm
  size of the dots and of the crosses in the Dynkin diagram

edgeLength = ⟨number⟩cm,
default : .35cm
  distance between nodes in the Dynkin diagram

edge/.style = TikZ style data,
default : thin
  style of edges in the Dynkin diagram

mark = ⟨o,O,t,x,X,*⟩,
default : *
  default root mark

affineMark = o,O,t,x,X,*,
default : *
  default root mark for root zero in an affine Dynkin diagram

label = true or false,
default : false
  whether to label the roots according to the current labelling scheme.

labelMacro = ⟨1-parameter TeX macro⟩,

continued ...

Table 17: ... continued

default : `#1`
    the current labelling scheme.
`makeIndefiniteEdge` = ⟨`edge pair` i–j `or list of such`⟩,
default : `{}`
    edge pair or list of edge pairs to treat as having indefinitely many roots on them.
`indefiniteEdgeRatio` = ⟨`float`⟩,
default : `1.6`
    ratio of indefinite edge lengths to other edge lengths.
`indefiniteEdge/.style` = ⟨`TikZ style data`⟩,
default : `draw=black,fill=white,thin,densely dotted`
    style of the dotted or dashed middle third of each indefinite edge.
`arrows` = ⟨`true or false`⟩,
default : `true`
    whether to draw the arrows that arise along the edges.
`reverseArrows` = ⟨`true or false`⟩,
default : `true`
    whether to reverse the direction of the arrows that arise along the edges.
`fold` = ⟨`true or false`⟩,
default : `true`
    whether, when drawing Dynkin diagrams, to draw them 2-ply.
`ply` = ⟨`0,1,2,3,4`⟩,
default : `0`
    how many roots get folded together, at most.
`foldleft` = ⟨`true or false`⟩,
default : `true`
    whether to fold the roots on the left side of a Dynkin diagram.
`foldright` = ⟨`true or false`⟩,
default : `true`
    whether to fold the roots on the right side of a Dynkin diagram.
`foldradius` = ⟨`length`⟩,
default : `.3cm`
    the radius of circular arcs used in curved edges of folded Dynkin diagrams.
`foldStyle` = ⟨`TikZ style data`⟩,
default : `draw=black!40,fill=none,line width=radius`
    when drawing folded diagrams, style for the fold indicators.
`*/.style` = ⟨`TikZ style data`⟩,
default : `draw=black,fill=black`
    style for roots like •
`o/.style` = ⟨`TikZ style data`⟩,
default : `draw=black,fill=black`
    style for roots like ∘
`O/.style` = ⟨`TikZ style data`⟩,
default : `draw=black,fill=black`

continued ...

Table 17: . . . continued

style for roots like ◎

`t/.style` = ⟨`TikZ style data`⟩,
default : `draw=black,fill=black`

style for roots like ⊗

`x/.style` = ⟨`TikZ style data`⟩,
default : `draw=black`

style for roots like ×

`X/.style` = ⟨`TikZ style data`⟩,
default : `draw=black,thick`

style for roots like **×**

`leftFold/.style` = ⟨`TikZ style data`⟩,
default :

style to override the `fold` style when folding roots together on the left half of a Dynkin diagram

`rightFold/.style` = ⟨`TikZ style data`⟩,
default :

style to override the `fold` style when folding roots together on the right half of a Dynkin diagram

`doubleEdges` = ⟨⟩,
default : `not set`

set to override the `fold` style when folding roots together in a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows).

`doubleFold` = ⟨⟩,
default : `not set`

set to override the `fold` style when folding roots together in a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows), but filled in solidly.

`doubleLeft` = ⟨⟩,
default : `not set`

set to override the `fold` style when folding roots together at the left side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows).

`doubleFoldLeft` = ⟨⟩,
default : `not set`

set to override the `fold` style when folding roots together at the left side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows), but filled in solidly.

`doubleRight` = ⟨⟩,
default : `not set`

continued . . .

Table 17: ...continued

set to override the `fold` style when folding roots together at the right side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows).

`doubleFoldRight` = ⟨⟩,
default : `not set`

set to override the `fold` style when folding roots together at the right side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows), but filled in solidly.

`arrowColor` = ⟨⟩,
default : `black`

set to override the default color for the arrows in nonsimply laced Dynkin diagrams.

`Coxeter` = ⟨`true or false`⟩,
default : `false`

whether to draw a Coxeter diagram, rather than a Dynkin diagram.

`ordering` = ⟨`Adams, Bourbaki, Carter, Dynkin, Kac`⟩,
default : `Bourbaki`

which ordering of the roots to use in exceptional root systems as in section 17.

All other options are passed to TikZ.

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