

**THE DYNKIN DIAGRAMS PACKAGE**  
**VERSION 3.141592653589**

BEN MCKAY

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## 1. QUICK INTRODUCTION

Load the Dynkin diagram package (see options below)

```
\documentclass{amsart}
\usepackage{dynkin-diagrams}
\begin{document}
The Dynkin diagram of  $(B_3)$  is \dynkin B3.
\end{document}
```

Invoke it

The Dynkin diagram of  $(B_3)$  is \dynkin B3.

The Dynkin diagram of  $B_3$  is  $\bullet \rightarrow \bullet$ .

Inside a TikZ statement

The Dynkin diagram of  $(B_3)$  is  
 $\tikz \dynkin B3;$

The Dynkin diagram of  $B_3$  is  $\bullet \rightarrow \bullet$

Inside a Dynkin diagram environment

The Dynkin diagram of  $(B_3)$  is  
 $\begin{tikzpicture}$   
 $\begin{dynkinDiagram} B3$   
 $\draw[very thick,red] (root 1) to [out=-45, in=-135] (root 3);$   
 $\end{dynkinDiagram}$   
 $\end{tikzpicture}$

The Dynkin diagram of  $B_3$  is  $\bullet \rightarrow \bullet$

Inside a TikZ environment

Baseline controls vertical alignment:  
the Dynkin diagram of  $(B_3)$  is  
 $\begin{tikzpicture}[baseline=(origin.base)]$   
 $\dynkin B3$   
 $\draw[very thick,red] (root 1) to [out=-45, in=-135] (root 3);$   
 $\end{tikzpicture}$

Baseline controls vertical alignment: the Dynkin diagram of  $B_3$  is  $\bullet \rightarrow \bullet$

In a TikZ picture, you might need to kill the default vertical shift (needed to allow inline Dynkin diagrams):

## Inside TikZ pictures

```
\begin{tikzpicture}
\draw (0,0) -- (.5,1) -- (1,0);
\dynkin[vertical shift=0,edge length=1cm]G2
\end{tikzpicture}
```



## Indefinite rank Dynkin diagrams

```
\dynkin B{}
```



Table 1: The Dynkin diagrams of the reduced simple root systems  
[3] pp. 265–290, plates I–IX

$A_n$		<code>\dynkin A{}</code>
$C_n$		<code>\dynkin C{}</code>
$D_n$		<code>\dynkin D{}</code>
$E_6$		<code>\dynkin E6</code>
$E_7$		<code>\dynkin E7</code>
$E_8$		<code>\dynkin E8</code>
$F_4$		<code>\dynkin F4</code>
$G_2$		<code>\dynkin G2</code>

## 2. SET OPTIONS GLOBALLY

Most options set globally ...

```
\pgfkeys{/Dynkin diagram,
edge length=.5cm,
fold radius=.5cm,
indefinite edge/.style={
draw=black,fill=white,thin,densely dashed}}
```

You can also pass options to the package in `\usepackage`. *Danger*: spaces in option names are replaced with hyphens: `edge length=1cm` is `edge-length=1cm` as

a global option; moreover you should drop the extension `/style` on any option with spaces in its name (but not otherwise). For example,

...or pass global options to the package

```
\usepackage[
  ordering=Kac,
  edge/.style=blue,
  indefinite-edge={draw=green,fill=white,densely dashed},
  indefinite-edge-ratio=5,
  mark=o,
  root-radius=.06cm]
{dynkin-diagrams}
```

### 3. COXETER DIAGRAMS

Coxeter diagram option

```
\dynkin[Coxeter]{F}{4}
```



gonality option for  $G_2$  and  $I_n$  Coxeter diagrams

```
\(G_2=\dynkin[Coxeter,gonality=n]G2\), \
\ (I_n=\dynkin[Coxeter,gonality=n]I{\})\
```

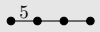

$G_2 = \bullet \overset{n}{\bullet} \bullet$ ,  $I_n = \bullet \overset{n}{\bullet}$

Table 2: The Coxeter diagrams of the simple reflection groups

$A_n$		<code>\dynkin[Coxeter]A{}</code>
$B_n$		<code>\dynkin[Coxeter]B{}</code>
$C_n$		<code>\dynkin[Coxeter]C{}</code>
$E_6$		<code>\dynkin[Coxeter]E6</code>
$E_7$		<code>\dynkin[Coxeter]E7</code>
$E_8$		<code>\dynkin[Coxeter]E8</code>
$F_4$		<code>\dynkin[Coxeter]F4</code>
$G_2$		<code>\dynkin[Coxeter,gonality=n]G2</code>
$H_3$		<code>\dynkin[Coxeter]H3</code>

continued ...


Table 2: ...continued

$H_4$		<code>\dynkin[Coxeter]H4</code>
$I_n$		<code>\dynkin[Coxeter,gonality=n]I{}</code>

## 4. SATAKE DIAGRAMS

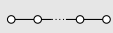
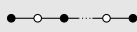
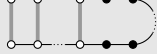
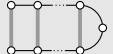

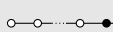

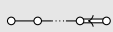
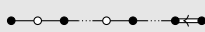
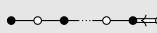
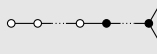


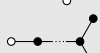
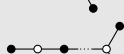
Satake diagrams use the standard name instead of a rank

`\(A_{IIIb}=\dynkin A{IIIb})\)`

$$A_{IIIb} = \text{Diagram}$$


We use a solid gray bar to denote the folding of a Dynkin diagram, rather than the usual double arrow, since the diagrams turn out simpler and easier to read.

Table 3: The Satake diagrams of the real simple Lie algebras [13] p. 532–534

$A_I$		<code>\dynkin AI</code>
$A_{II}$		<code>\dynkin A{II}</code>
$A_{IIIa}$		<code>\dynkin A{IIIa}</code>
$A_{IIIb}$		<code>\dynkin A{IIIb}</code>
$A_{IV}$		<code>\dynkin A{IV}</code>
$B_I$		<code>\dynkin BI</code>
$B_{II}$		<code>\dynkin B{II}</code>
$C_I$		<code>\dynkin CI</code>
$C_{IIa}$		<code>\dynkin C{IIa}</code>
$C_{IIb}$		<code>\dynkin C{IIb}</code>
$D_{Ia}$		<code>\dynkin D{Ia}</code>
$D_{Ib}$		<code>\dynkin D{Ib}</code>
$D_{Ic}$		<code>\dynkin D{Ic}</code>
$D_{II}$		<code>\dynkin D{II}</code>
$D_{IIIa}$		<code>\dynkin D{IIIa}</code>

continued ...

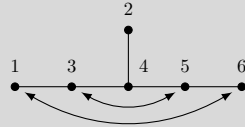
Table 3: ...continued

$D_{IIIb}$		<code>\dynkin D{IIIb}</code>
$E_I$		<code>\dynkin EI</code>
$E_{II}$		<code>\dynkin E{II}</code>
$E_{III}$		<code>\dynkin E{III}</code>
$E_{IV}$		<code>\dynkin E{IV}</code>
$E_V$		<code>\dynkin EV</code>
$E_{VI}$		<code>\dynkin E{VI}</code>
$E_{VII}$		<code>\dynkin E{VII}</code>
$E_{VIII}$		<code>\dynkin E{VIII}</code>
$E_{IX}$		<code>\dynkin E{IX}</code>
$F_I$		<code>\dynkin FI</code>
$F_{II}$		<code>\dynkin F{II}</code>
$G_I$		<code>\dynkin GI</code>

## 5. HOW TO FOLD

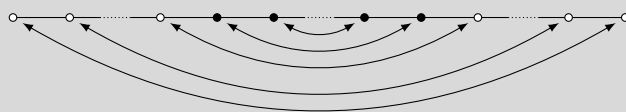
If you don't like the solid gray "folding bar", most people use arrows. Here is  $E_{II}$

```
\dynkin[%
  edge length=.75cm,
  labels*={1,...,6},
  involutions={16;35}]E6
```



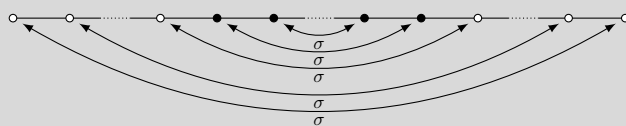
The double arrows for  $A_{IIIa}$  are big

```
\dynkin[edge length=.75cm,
  involutions={1{10};29;38;47;56}]{A}{oo.o**.**.oo}
```



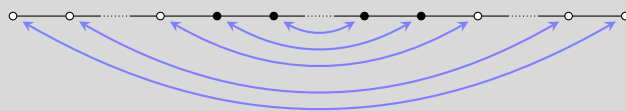
We can add labels

```
\dynkin[edge length=.75cm,
  involutions={
    1<below>[\sigma]{10};
    2<below>[\sigma]9;
    3<below>[\sigma]8;
    4<below>[\sigma]7;
    5<below>[\sigma]6}
]{A}{oo.o**.**.oo}
```



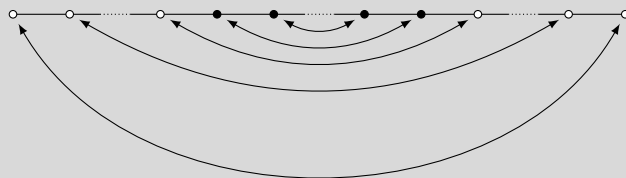
Style options

```
\dynkin[%
  edge length=.75cm,
  involution/.style={blue!50,stealth-stealth,thick},
  involutions={1{10};29;38;47;56}
]{A}{oo.o**.**.oo}
```



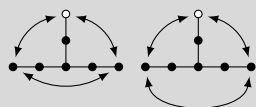
## Arrow angles

```
\dynkin[%
  edge length=.75cm,
  involutions={\in=-120,out=-60,relative]1{10};29;38;47;56}
]{A}{oo.o**.**o.oo}
```



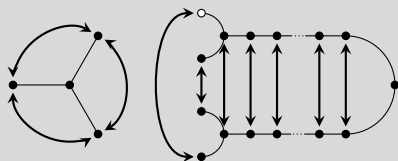
## Control over arrow angles

```
\dynkin[involutions={16;60;01}]E[1]{6}
\dynkin[involutions={\out=-80,in=-100,relative]16;60;01}]E[1]{6}
```



If you don't like the solid gray "folding bar", most people use arrows ...

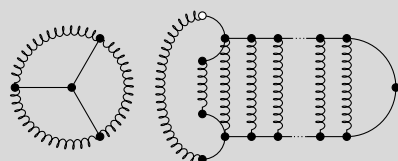
```
\tikzset{/Dynkin diagram/fold style/.style={stealth-stealth,thick,
shorten <=1mm,shorten >=1mm,}}
\dynkin[ply=3,edge length=.75cm]D4
\begin{dynkinDiagram}[ply=4]D[1]%
{****.*****.*****}
\dynkinFold 1{13}
\dynkinFold[bend right=90] 0{14}
\end{dynkinDiagram}
```





...but you could try springs pulling roots together

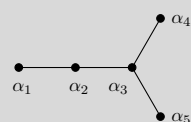
```
\tikzset{/Dynkin diagram/fold style/.style=
{decorate,decoration={name=coil,aspect=0.5,
segment length=1mm,amplitude=.6mm}}}
\dynkin[ply=3,edge length=.75cm]D4
\begin{dynkinDiagram}[ply=4]D[1]%
{****.*****.*****}
\dynkinFold 1{13}
\dynkinFold[bend right=90]0{14}
\end{dynkinDiagram}
```



## 6. LABELS FOR THE ROOTS

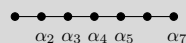
Make a macro to assign labels to roots

```
\dynkin[label,label macro/.code={\alpha_{\drlap{#1}}},edge
length=.75cm]D5
```



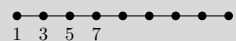
Labelling several roots

```
\dynkin[labels={2,...,5,,7},label macro/.code={\alpha_{\drlap{#1}}}]A7
```



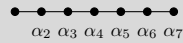
The foreach notation I

```
\dynkin[labels={1,3,...,7},]A9
```



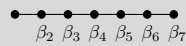
## The foreach notation II

```
\dynkin[labels={,\alpha_2,\alpha_....,\alpha_7},]A7
```



## The foreach notation III

```
\dynkin[label macro/.code={\beta_{\drlap{#1}}},labels={,2,...,7},]A7
```



## Label the roots individually by root number

```
\dynkin[label]B3
```



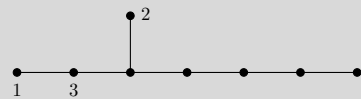
## Access root labels via TikZ

```
\begin{dynkinDiagram}B3
\node[below] at (root 2) {\(\alpha_{\drlap{2}}\)};
\end{dynkinDiagram}
```



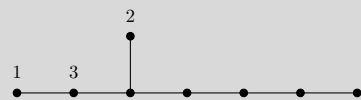
## The labels have default locations, mostly below roots

```
\dynkin[edge length=.75cm,labels={1,2,3}]E8
```



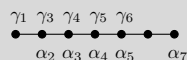
## The starred form flips labels to alternate locations, mostly above roots

```
\dynkin[edge length=.75cm,labels*={1,2,3}]E8
```



## Labelling several roots and alternates

```
\dynkin[%
label macro/.code={\alpha_{\drlap{#1}}},
label macro*/.code={\gamma_{\drlap{#1}}},
labels={,2,...,5,,7},
labels*={1,3,4,5,6}]A7
```

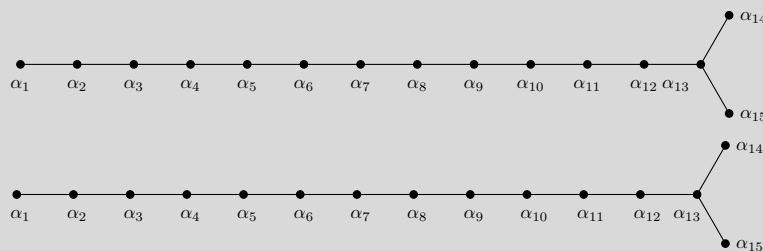


## 7. LABEL SUBSCRIPTS

Note the slight improvement that `\drlap` makes: the labels are centered on the middle of the letter  $\alpha$ , ignoring the space taken up by the subscripts, using the `mathtools` command `\mathrlap`, but only for labels which are *not* placed to the left or right of a root.

## Label subscript spacing

```
\dynkin[label,label macro/.code={\alpha_{#1}},
edge length=.75cm]D{15}
\par\noindent{%
\dynkin[label,label macro/.code={\alpha_{\drlap{#1}}},
edge length=.75cm]D{15}
```

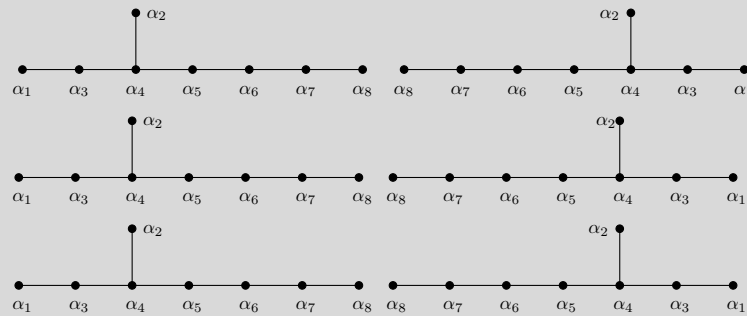


## Label subscript spacing

```

\dynkin[label,label macro/.code={\alpha_{#1}},
  edge length=.75cm]E8
\dynkin[label,label macro/.code={\alpha_{#1}},backwards,
  edge length=.75cm]E8
\par\noindent{}%
\dynkin[label,label macro/.code={\alpha_{\mathrlap{#1}}}},
  edge length=.75cm]E8
\dynkin[label,label macro/.code={\alpha_{\mathrlap{#1}}}},backwards,
  edge length=.75cm]E8
\par\noindent{}%
\dynkin[label,label macro/.code={\alpha_{\drlap{#1}}}},
  edge length=.75cm]E8
\dynkin[label,label macro/.code={\alpha_{\drlap{#1}}}},backwards,
  edge length=.75cm]E8

```

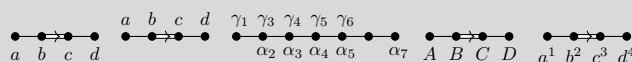


## 8. HEIGHT AND DEPTH OF LABELS

Labels are set with default maximum height the height of the character  $b$ , and default maximum depth the depth of the character  $g$ . To change these, set label height and label depth:

Change height and depth of characters

```
\dynkin[labels={a,b,c,d},label height=d,label depth=d]F4
\dynkin[labels*={a,b,c,d},label height=d,label depth=d]F4
\dynkin[%
label macro/.code={\alpha_{\drlap{#1}}},
label macro*/.code={\gamma_{\drlap{#1}}},
label height=$\alpha_1$,
label depth=$\alpha_1$,
labels={,2,...,5,,7},
labels*={1,3,4,5,6}]A7
\dynkin[labels={A,B,C,D},label height=$A$,label depth=$A$]F4
\dynkin[labels={a^1,b^2,c^3,d^4},label height=$X^X$]F4
```



## 9. TEXT STYLE FOR THE LABELS

Use a text style: big and blue

```
\begin{dynkinDiagram}[text style={scale=1.2,blue},
edge length=.75cm,
labels={1,2,n-1,n},
label macro/.code={\alpha_{\drlap{#1}}}
]A{}
\end{dynkinDiagram}
```



Use a text style; font selection is in the label macro

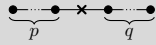
```
\begin{dynkinDiagram}[text style={scale=1.2,blue},
edge length=.75cm,
labels={1,2,n-1,n},
label macro/.code={\mathbb{A}_{\drlap{#1}}}A{}
\end{dynkinDiagram}
```



## 10. BRACING ROOTS

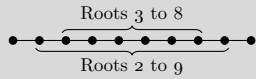
Bracing roots

```
\begin{dynkinDiagram}A{*.x*.*}
\dynkinBrace[p]12
\dynkinBrace[q]45
\end{dynkinDiagram}
```



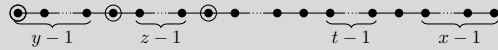
Bracing roots, and a starred form

```
\begin{dynkinDiagram}A{10}
\dynkinBrace[\text{Roots 2 to 9}]29
\dynkinBrace*[\text{Roots 3 to 8}]38
\end{dynkinDiagram}
```



Bracing roots

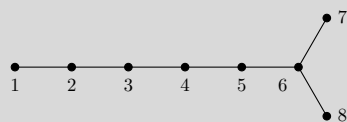
```
\newcommand\circleRoot[1]{\draw (root #1) circle (3pt);}
\begin{dynkinDiagram}A{**,***,***,***,***,**}
\circleRoot 4\circleRoot 7\circleRoot 10\circleRoot 13
\dynkinBrace[y-1]13
\dynkinBrace[z-1]56
\dynkinBrace[t-1]{11}{12}
\dynkinBrace[x-1]{14}{16}
\end{dynkinDiagram}
```



## 11. LABEL PLACEMENT

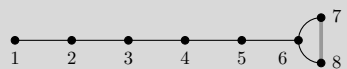
Take a  $D_8$ :

```
\dynkin[label,edge length=.75cm]D8
```



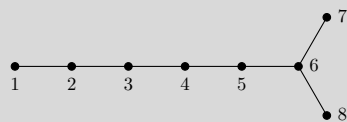
If you want to fold this diagram,

```
\dynkin[fold right=true,label,edge length=.75cm]D8
```



you will be glad that the 6 sits where it does, under and to the left. If you don't want to fold, you might prefer instead to put the 6 on the right side.

```
\dynkin[label,edge length=.75cm,label directions={,,,,,right,,}]D8
```



The default locations are overridden by the `label directions`. For extended diagrams, this list starts at 0-offset.

```
\dynkin[%
  label,
  label directions={above,,,,,,},
  involutions={ [out=-60,in=-120,relative]16;60;01}
]E[1]{6}
```

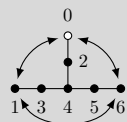
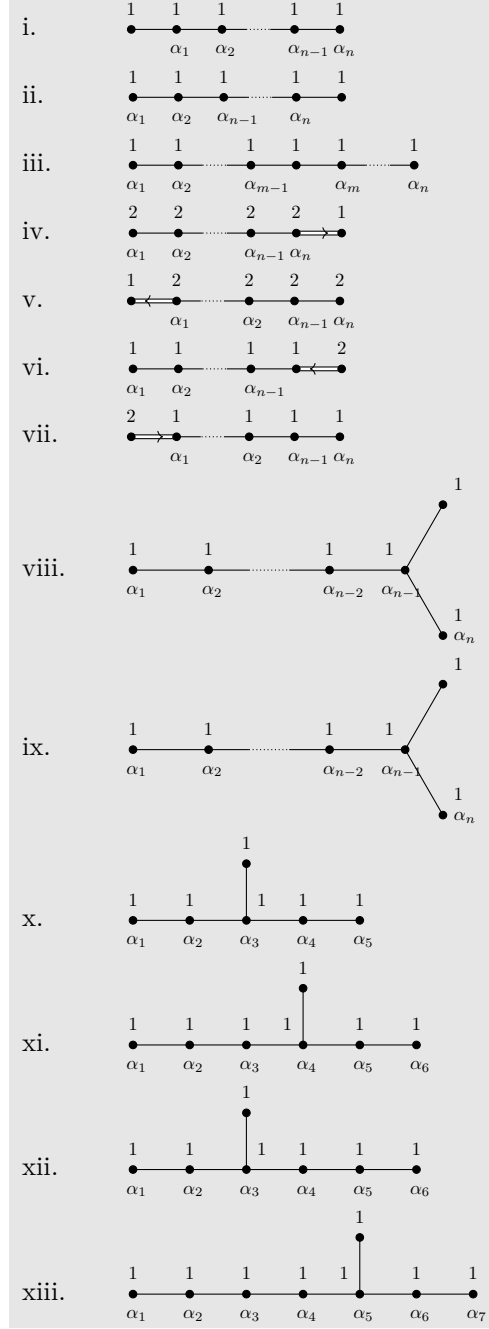


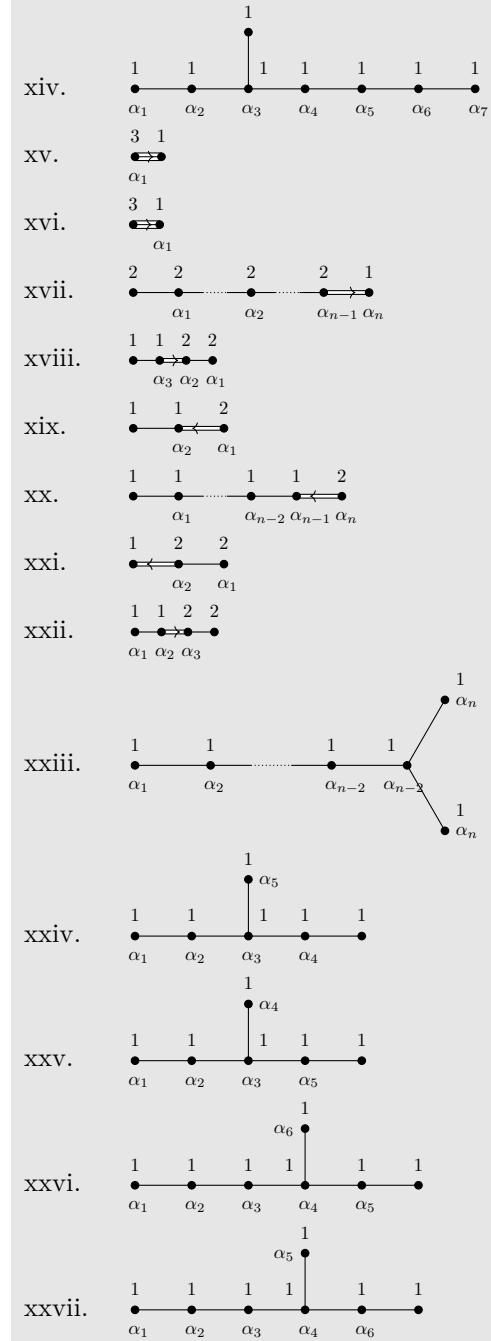
Table 4: Dynkin diagrams from Euler products [17]



continued ...

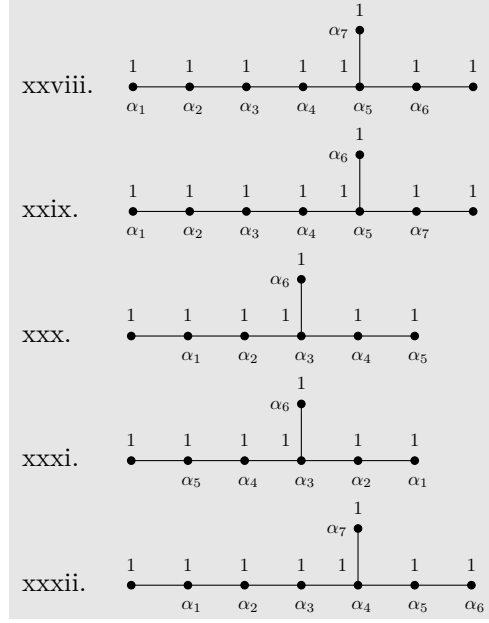


Table 4: ...continued



continued ...

Table 4: ...continued



```

\tikzset{/Dynkin diagram,ordering=Dynkin,label macro/.code={\alpha_{\drlap{#1}}}}
\newcounter{EPNo}
\setcounter{EPNo}{0}
\NewDocumentCommand\EP{smmmm}%
{%
\stepcounter{EPNo}\roman{EPNo}. &%
\def\eL{.6cm}%
\IfStrEqCase{#2}%
{%
D{\gdef\eL{1cm}}%
E{\gdef\eL{.75cm}}%
F{\gdef\eL{.35cm}}%
G{\gdef\eL{.35cm}}%
}%
\IfBooleanTF{#1}%
{\dynkin[edge length=\eL,backwards,labels*={#4},labels={#5}]{#2}{#3}}
{\dynkin[edge length=\eL,labels*={#4},labels={#5}]{#2}{#3}}
\\
}%
\renewcommand*\do[1]{\EP{#1}}%
\begin{longtable}{MM}
\caption{Dynkin diagrams from Euler products \cite{Langlands:1967}}\\
\endfirsthead
\caption{\dots continued}\\
\endhead
\multicolumn{2}{c}{continued \dots}\\
\endfoot
\endlastfoot

```

```

\docsvlist{
A{***.**}{1,1,1,1,1}{1,2,n-1,n},
A{***.**}{1,1,1,1,1}{1,2,n-1,n},
A{**.***.*}{1,1,1,1,1,1}{1,2,m-1,,m,n},
B{**.***}{2,2,2,2,1}{1,2,n-1,n},
*B{***.**}{2,2,2,2,1}{n,n-1,2,1},
C{**.***}{1,1,1,1,2}{1,2,n-1},
*C{***.**}{1,1,1,1,2}{n,n-1,2,1},
D{**.****}{1,1,1,1,1,1}{1,2,n-2,n-1,n},
D{**.****}{1,1,1,1,1,1}{1,2,n-2,n-1,n},
E6{1,1,1,1,1,1}{1,...,5},
*E7{1,1,1,1,1,1}{6,...,1},
E7{1,1,1,1,1,1}{1,...,6},
*E8{1,1,1,1,1,1,1}{7,...,1},
E8{1,1,1,1,1,1,1}{1,...,7},
G2{1,3}{1},
G2{1,3}{1},
B{**.***}{2,2,2,2,1}{1,2,n-1,n},
F4{1,1,2,2}{1,3,2,1},
C3{1,1,2}{1,2,1},
C{**.***}{1,1,1,1,2}{1,n-2,n-1,n},
*B3{2,2,1}{1,2},
F4{1,1,2,2}{1,2,3},
D{**.****}{1,1,1,1,1,1}{1,2,n-2,n-2,n,n},
E6{1,1,1,1,1,1}{1,2,3,4,5},
E6{1,1,1,1,1,1}{1,2,3,5,4},
*E7{1,1,1,1,1,1}{5,...,1,6},
*E7{1,1,1,1,1,1}{6,4,3,2,1,5},
*E8{1,1,1,1,1,1,1}{6,...,1,7},
*E8{1,1,1,1,1,1,1}{7,5,4,3,2,1,6},
*E7{1,1,1,1,1,1}{5,...,1,6},
*E7{1,1,1,1,1,1}{1,...,5,6},
*E8{1,1,1,1,1,1,1}{6,...,1,7}}
\end{longtable}

```

## 12. STYLE

## Colours

```
\dynkin[
  edge/.style={blue!50,thick},
  */.style=blue!50!red,
  arrow style={red,width=2mm,line width=1pt}]{F}{4}
```



## Arrow shapes

```
\dynkin[arrow shape/-.style={-Stealth[blue,width=3mm]}}] F4
```



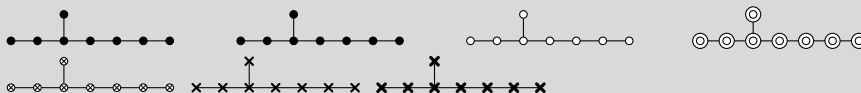
## Edge lengths

The Dynkin diagram of  $(A_3)$  is `\dynkin[edge length=1.2]A3`

The Dynkin diagram of  $A_3$  is

## Root marks

```
\dynkin E8
\dynkin[mark=*]E8
\dynkin[mark=o]E8
\dynkin[mark=O]E8
\dynkin[mark=t]E8
\dynkin[mark=x]E8
\dynkin[mark=X]E8
```

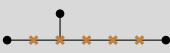


At the moment, you can only use:

- \* • solid dot
- o ○ hollow circle
- O ⊙ double hollow circle
- t ⊗ tensor root
- x × crossed root
- X ✕ thickly crossed root

#### Mark styles

The parabolic subgroup  $\backslash(E_{8,124})\backslash$  is  
`\dynkin[parabolic=124,x/.style={brown,very thick}]E8`

The parabolic subgroup  $E_{8,124}$  is 

#### Sizes of root marks

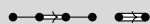
$\backslash(A_{3,3})\backslash$  with big root marks is `\dynkin[root  
radius=.08cm,parabolic=3]A3`

$A_{3,3}$  with big root marks is 

### 13. SUPPRESS OR REVERSE ARROWS

Some diagrams have double or triple edges

`\dynkin F4`  
`\dynkin G2`



Suppress arrows

`\dynkin[arrows=false]F4`  
`\dynkin[arrows=false]G2`



## Reverse arrows

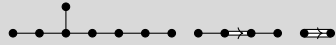
```
\dynkin[reverse arrows]F4
\dynkin[reverse arrows]G2
```



## 14. BACKWARDS AND UPSIDE DOWN

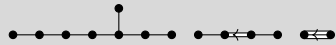
## Default

```
\dynkin E8
\dynkin F4
\dynkin G2
```



## Backwards

```
\dynkin[backwards]E8
\dynkin[backwards]F4
\dynkin[backwards]G2
```



## Reverse arrows

```
\dynkin[reverse arrows]F4
\dynkin[reverse arrows]G2
```



## Backwards, reverse arrows

```
\dynkin[backwards,reverse arrows]F4
\dynkin[backwards,reverse arrows]G2
```

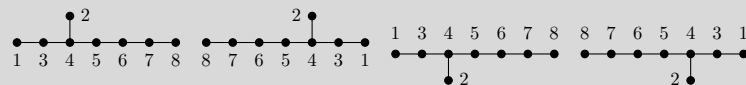


## Backwards versus upside down

```

\dynkin[label]E8
\dynkin[label,backwards]E8
\dynkin[label,upside down]E8
\dynkin[label,backwards,upside down]E8

```



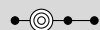
## 15. DRAWING ON TOP OF A DYNKIN DIAGRAM

## TikZ can access the roots themselves

```

\begin{dynkinDiagram}A4
  \fill[white,draw=black] (root 2) circle (.15cm);
  \fill[white,draw=black] (root 2) circle (.1cm);
  \draw[black] (root 2) circle (.05cm);
\end{dynkinDiagram}

```

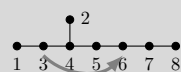


## Draw curves between the roots

```

\begin{dynkinDiagram}[label]E8
  \draw[very thick, black!50,-latex]
    (root 3.south) to [out=-45, in=-135] (root 6.south);
\end{dynkinDiagram}

```

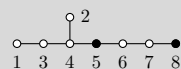


## Change marks

```

\begin{dynkinDiagram}[mark=o,label]E8
  \dynkinRootMark{*}5
  \dynkinRootMark{*}8
\end{dynkinDiagram}

```

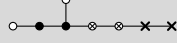


## 16. MARK LISTS

The package allows a list of root marks instead of a rank:

A mark list

```
\dynkin E{oo*ttxx}
```



The mark list `oo*ttxx` has one mark for each root: `o`, `o`,  $\dots$ , `x`. Roots are listed in the current default ordering. (Careful: in an affine root system, a mark list will *not* contain a mark for root zero.)

If you need to repeat a mark, you can give a *single digit* positive integer to indicate how many times to repeat it.

A mark list with repetitions

```
\dynkin A{x4o3t4}
```

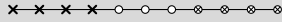


Table 5: Classical Lie superalgebras [10]. We need a slightly larger root radius parameter to distinguish the tensor product symbols from the solid dots.

		<code>\tikzset{/Dynkin diagram,root radius=.07cm}</code>
$A_{mn}$		<code>\dynkin A{o3.oto.oo}</code>
$B_{mn}$		<code>\dynkin B{o3.oto.oo}</code>
$B_{0n}$		<code>\dynkin B{o3.o3.o*}</code>
$C_n$		<code>\dynkin C{too.oto.oo}</code>
$D_{mn}$		<code>\dynkin D{o3.oto.o4}</code>
$D_{21\alpha}$		<code>\dynkin A{oto}</code>
$F_4$		<code>\dynkin F{ooot}</code>
$G_3$		<code>\dynkin[extended,affine mark=t, reverse arrows]G2</code>



Table 6: Classical Lie superalgebras [10]. Here we see the problem with using the default root radius parameter, which is too small for tensor product symbols.

$A_{mn}$		<code>\dynkin A{o3.oto.oo}</code>
$B_{mn}$		<code>\dynkin B{o3.oto.oo}</code>
$B_{0n}$		<code>\dynkin B{o3.o3.o*}</code>
$C_n$		<code>\dynkin C{too.oto.oo}</code>
$D_{mn}$		<code>\dynkin D{o3.oto.o4}</code>
$D_{21\alpha}$		<code>\dynkin A{oto}</code>
$F_4$		<code>\dynkin F{oooot}</code>
$G_3$		<code>\dynkin[extended,affine mark=t, reverse arrows]G2</code>

## 17. INDEFINITE EDGES

An *indefinite edge* is a dashed edge between two roots,  $\bullet\cdots\bullet$  indicating that an indefinite number of roots have been omitted from the Dynkin diagram. In between any two entries in a mark list, place a period to indicate an indefinite edge:

Indefinite edges

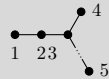
`\dynkin D{o.o*.t.to.t}`



In certain diagrams, roots may have an edge between them even though they are not subsequent in the ordering. For such rare situations, there is an option:

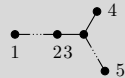
Indefinite edge option

`\dynkin[make indefinite edge={3-5},label]D5`



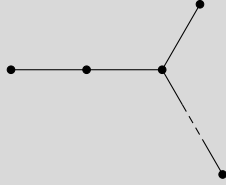
Give a list of edges to become indefinite

`\dynkin[make indefinite edge/.list={1-2,3-5},label]D5`



## Indefinite edge style

```
\dynkin[indefinite edge/.style={
    draw=black,fill=white,thin,densely dashed},
    edge length=1cm,
    make indefinite edge={3-5}]D5
```



## The ratio of the lengths of indefinite edges to those of other edges

```
\dynkin[edge length = .5cm,
    indefinite edge ratio=3,
    make indefinite edge={3-5}]D5
```

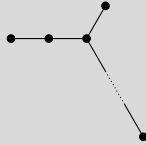


Table 7: Springer’s table of indices [24], pp. 320-321, with one form of  $E_7$  corrected

$A_n$		
$A_n$		
$B_n$		
$C_n$		
$D_n$		
$E_6$		<code>\dynkin E{*oooo*}</code>
$E_6$		<code>\dynkin E{o*o*oo}</code>
$E_6$		<code>\dynkin E{o*oooo}</code>

continued ...

Table 7: ...continued

$E_6$		<code>\dynkin E{**ooo*}</code>
$E_7$		<code>\dynkin E{*oooooo}</code>
$E_7$		<code>\dynkin E{ooooo*o}</code>
$E_7$		<code>\dynkin E{oooooo*}</code>
$E_7$		<code>\dynkin E{*oooo*o}</code>
$E_7$		<code>\dynkin E{*oooo**}</code>
$E_7$		<code>\dynkin E{*o**o*o}</code>
$E_8$		<code>\dynkin E{*ooooooo}</code>
$E_8$		<code>\dynkin E{ooooooo*}</code>
$E_8$		<code>\dynkin E{*oooooo*}</code>
$E_8$		<code>\dynkin E{oooooo**}</code>
$E_8$		<code>\dynkin E{*oooo***}</code>
$F_4$		<code>\dynkin F{ooo*}</code>
$D_4$		<code>\dynkin D{o*oo}</code>

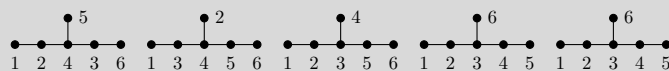
## 18. ROOT ORDERING

## Root ordering

```

\dynkin[label,ordering=Adams]E6
\dynkin[label,ordering=Bourbaki]E6
\dynkin[label,ordering=Carter]E6
\dynkin[label,ordering=Dynkin]E6
\dynkin[label,ordering=Kac]E6

```

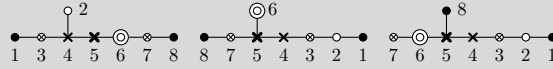


Default is Bourbaki. Sources are Adams [1] p. 56–57, Bourbaki [3] p. pp. 265–290 plates I–IX, Carter [5] p. 540–609, Dynkin [8], Kac [15] p. 43.

	Adams	Bourbaki	Carter	Dynkin	Kac
$E_6$					
$E_7$					
$E_8$					
$F_4$					
$G_2$					

The marks are set down in order according to the current root ordering:

```
\dynkin[label]E{*otxXOt*}
\dynkin[label,ordering=Carter]E{*otxXOt*}
\dynkin[label,ordering=Kac]E{*otxXOt*}
```



Convert between orderings

```
\newcount\r
\dynkinOrder E8.Carter::6->Bourbaki.{\r}
In \langle E_8 \rangle, root 6 in Carter's ordering is root \the\r{} in Bourbaki's
ordering.
```

In  $E_8$ , root 6 in Carter's ordering is root 2 in Bourbaki's ordering.

## 19. PARABOLIC SUBGROUPS

Each set of roots is assigned a number, with each binary digit zero or one to say whether the corresponding root is crossed or not:

The flag variety of pointed lines in projective 3-space is associated to the Dynkin diagram  $\text{\dynkin[parabolic=3]A3}$ .

The flag variety of pointed lines in projective 3-space is associated to the Dynkin diagram  $\times \rightarrow \times \rightarrow \bullet$ .

Table 9: The Hermitian symmetric spaces

$A_n$		Grassmannian of $k$ -planes in $\mathbb{C}^{n+1}$
$B_n$		$(2n-1)$ -dimensional hyperquadric, i.e. the variety of null lines in $\mathbb{C}^{2n+1}$
$C_n$		space of Lagrangian $n$ -planes in $\mathbb{C}^{2n}$
$D_n$		$(2n-2)$ -dimensional hyperquadric, i.e. the variety of null lines in $\mathbb{C}^{2n}$
$D_n$		one component of the variety of maximal dimension null subspaces of $\mathbb{C}^{2n}$
$D_n$		the other component
$E_6$		complexified octave projective plane
$E_6$		its dual plane
$E_7$		the space of null octave 3-planes in octave 6-space

```

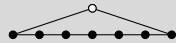
\NewDocumentCommand\HSS{mommm}
{#1&\IfNoValueTF{#2}{\dynkin{#3}{#4}}{\dynkin[parabolic=#2]{#3}{#4}}&#5\\}
\renewcommand*{\arraystretch}{1.5}
\begin{longtable}
{>\columncolor[gray]{.9}}>$1<$>\columncolor[gray]{.9}}>$1<$>\columncolor[gray]{.9}}1}
\caption{The Hermitian symmetric spaces}\endfirsthead
\caption{\dots continued}\endhead
\caption{continued \dots}\endfoot
\endlastfoot
\HSS{A_n}A{**.*x**}{Grassmannian of $k$-planes in $\mathbb{C}^{n+1}$}
\HSS{B_n}[1]B{\{$(2n-1)$-dimensional hyperquadric, i.e. the variety of null lines in $\mathbb{C}^{2n+1}$\}}
\HSS{C_n}[16]C{\{space of Lagrangian $n$-planes in $\mathbb{C}^{2n}$\}}
\HSS{D_n}[1]D{\{$(2n-2)$-dimensional hyperquadric, i.e. the variety of null lines in $\mathbb{C}^{2n}$\}}
\HSS{D_n}[32]D{\{one component of the variety of maximal dimension null subspaces of $\mathbb{C}^{2n}$\}}
\HSS{D_n}[16]D{\{the other component\}}
\HSS{E_6}[1]E6{complexified octave projective plane}
\HSS{E_6}[32]E6{its dual plane}
\HSS{E_7}[64]E7{the space of null octave 3-planes in octave 6-space}
\end{longtable}

```

## 20. EXTENDED DYNKIN DIAGRAMS

## Extended Dynkin diagrams

```
\dynkin[extended]A7
```



The extended Dynkin diagrams are also described in the notation of Kac [15] p. 55 as affine untwisted Dynkin diagrams: we extend `\dynkin{A}{7}` to become `\dynkin A[1]7`:

## Extended Dynkin diagrams

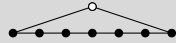
`\dynkin A[1]7`

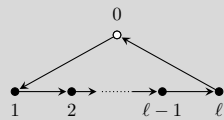
Table 10: The Dynkin diagrams of the extended simple root systems

$A_1^1$		<code>\dynkin[extended]A1</code>
$A_n^1$		<code>\dynkin[extended]A{}</code>
$B_n^1$		<code>\dynkin[extended]B{}</code>
$C_n^1$		<code>\dynkin[extended]C{}</code>
$D_n^1$		<code>\dynkin[extended]D{}</code>
$E_6^1$		<code>\dynkin[extended]E6</code>
$E_7^1$		<code>\dynkin[extended]E7</code>
$E_8^1$		<code>\dynkin[extended]E8</code>
$F_4^1$		<code>\dynkin[extended]F4</code>
$G_2^1$		<code>\dynkin[extended]G2</code>

## Directed edges

```

\dynkin[%
  edge length=.75cm,
  edge/.style={- stealth[sep=2pt]}},
  labels={1,2,\ell-1,\ell},
  labels*={0}]
A[1]{}
```



## 21. AFFINE TWISTED AND UNTWISTED DYNKIN DIAGRAMS

The affine Dynkin diagrams are described in the notation of Kac [15] p. 55:

Affine Dynkin diagrams

$\backslash(A^{(1)})_7=\backslash\text{dynkin A}[1]7, \backslash$   
 $E^{(2)}_6=\backslash\text{dynkin E}[2]6, \backslash$   
 $D^{(3)}_4=\backslash\text{dynkin D}[3]4\backslash$

$$A_7^{(1)} = \text{diagram}, E_6^{(2)} = \text{diagram}, D_4^{(3)} = \text{diagram}$$

Table 11: The affine Dynkin diagrams

$A_1^1$		$\backslash\text{dynkin A}[1]1$
$A_n^1$		$\backslash\text{dynkin A}[1]\{\}$
$B_n^1$		$\backslash\text{dynkin B}[1]\{\}$
$C_n^1$		$\backslash\text{dynkin C}[1]\{\}$
$D_n^1$		$\backslash\text{dynkin D}[1]\{\}$
$E_6^1$		$\backslash\text{dynkin E}[1]6$
$E_7^1$		$\backslash\text{dynkin E}[1]7$
$E_8^1$		$\backslash\text{dynkin E}[1]8$
$F_4^1$		$\backslash\text{dynkin F}[1]4$
$G_2^1$		$\backslash\text{dynkin G}[1]2$
$A_2^2$		$\backslash\text{dynkin A}[2]2$
$A_{ev}^2$		$\backslash\text{dynkin A}[2]\{\text{even}\}$
$A_{od}^2$		$\backslash\text{dynkin A}[2]\{\text{odd}\}$
$D_n^2$		$\backslash\text{dynkin D}[2]\{\}$
$E_6^2$		$\backslash\text{dynkin E}[2]6$
$D_4^3$		$\backslash\text{dynkin D}[3]4$

Table 12: Some more affine Dynkin diagrams

$A_4^2$		<code>\dynkin A[2]4</code>
$A_5^2$		<code>\dynkin A[2]5</code>
$A_6^2$		<code>\dynkin A[2]6</code>
$A_7^2$		<code>\dynkin A[2]7</code>
$A_8^2$		<code>\dynkin A[2]8</code>
$D_3^2$		<code>\dynkin D[2]3</code>
$D_4^2$		<code>\dynkin D[2]4</code>
$D_5^2$		<code>\dynkin D[2]5</code>
$D_6^2$		<code>\dynkin D[2]6</code>
$D_7^2$		<code>\dynkin D[2]7</code>
$D_8^2$		<code>\dynkin D[2]8</code>
$D_4^3$		<code>\dynkin D[3]4</code>
$E_6^2$		<code>\dynkin E[2]6</code>

Table 13: Some more Kac–Moody Dynkin diagrams, only allowed in Kac ordering

$E_6$		<code>\dynkin[ordering=Kac,label]E6</code>
$E_7$		<code>\dynkin[ordering=Kac,label]E7</code>
$E_8$		<code>\dynkin[ordering=Kac,label]E8</code>
$E_9$		<code>\dynkin[ordering=Kac,label]E9</code>
$E_{10}$		<code>\dynkin[ordering=Kac,label]E{10}</code>
$E_{11}$		<code>\dynkin[ordering=Kac,label]E{11}</code>



## 22. EXTENDED COXETER DIAGRAMS

Extended and Coxeter options together

`\dynkin[extended,Coxeter]F4`

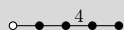


Table 14: The extended (affine) Coxeter diagrams

$A_n$		<code>\dynkin[extended,Coxeter]A{}</code>
$B_n$		<code>\dynkin[extended,Coxeter]B{}</code>
$C_n$		<code>\dynkin[extended,Coxeter]C{}</code>
$D_n$		<code>\dynkin[extended,Coxeter]D{}</code>
$E_6$		<code>\dynkin[extended,Coxeter]E6</code>
$E_7$		<code>\dynkin[extended,Coxeter]E7</code>
$E_8$		<code>\dynkin[extended,Coxeter]E8</code>
$F_4$		<code>\dynkin[extended,Coxeter]F4</code>
$G_2$		<code>\dynkin[extended,Coxeter]G2</code>
$H_3$		<code>\dynkin[extended,Coxeter]H3</code>
$H_4$		<code>\dynkin[extended,Coxeter]H4</code>
$I_1$		<code>\dynkin[extended,Coxeter]I1</code>

## 23. KAC STYLE

We include a style called `Kac` which tries to imitate the style of [15].

Kac style

`\dynkin[Kac]F4`



Table 15: The Dynkin diagrams of the simple root systems in Kac style

$A_n$	$\circ - \circ - \cdots - \circ - \circ$	<code>\dynkin A{}</code>
$B_n$	$\circ - \circ - \cdots - \circ - \circ \Rightarrow \circ$	<code>\dynkin B{}</code>
$C_n$	$\circ - \circ - \cdots - \circ - \circ \Leftarrow \circ$	<code>\dynkin C{}</code>
$D_n$	$\circ - \circ - \cdots - \circ - \circ \begin{array}{l} \nearrow \circ \\ \searrow \circ \end{array}$	<code>\dynkin D{}</code>
$E_6$	$\begin{array}{c} \circ \\   \\ \circ - \circ - \cdots - \circ - \circ \end{array}$	<code>\dynkin E6</code>
$E_7$	$\begin{array}{c} \circ \\   \\ \circ - \circ - \cdots - \circ - \circ - \circ \end{array}$	<code>\dynkin E7</code>
$E_8$	$\begin{array}{c} \circ \\   \\ \circ - \circ - \cdots - \circ - \circ - \circ - \circ \end{array}$	<code>\dynkin E8</code>
$F_4$	$\circ - \circ \Rightarrow \circ - \circ$	<code>\dynkin F4</code>
$G_2$	$\circ \Rightarrow \circ$	<code>\dynkin G2</code>

Table 16: The Dynkin diagrams of the extended simple root systems in Kac style

$A_1^1$	$\circ \Leftarrow \Rightarrow \circ$	<code>\dynkin[extended]A1</code>
$A_n^1$	$\begin{array}{c} \circ \\ \diagup \quad \diagdown \\ \circ - \circ - \cdots - \circ - \circ \end{array}$	<code>\dynkin[extended]A{}</code>
$B_n^1$	$\begin{array}{c} \circ \\   \\ \circ - \circ - \cdots - \circ - \circ \Rightarrow \circ \\   \\ \circ \end{array}$	<code>\dynkin[extended]B{}</code>
$C_n^1$	$\circ \Rightarrow \circ - \circ - \cdots - \circ - \circ \Leftarrow \circ$	<code>\dynkin[extended]C{}</code>
$D_n^1$	$\begin{array}{c} \circ \\   \\ \circ - \circ - \cdots - \circ - \circ \begin{array}{l} \nearrow \circ \\ \searrow \circ \end{array} \\   \\ \circ \end{array}$	<code>\dynkin[extended]D{}</code>
$E_6^1$	$\begin{array}{c} \circ \\   \\ \circ \\   \\ \circ - \circ - \cdots - \circ - \circ \end{array}$	<code>\dynkin[extended]E6</code>
$E_7^1$	$\begin{array}{c} \circ \\   \\ \circ \\   \\ \circ - \circ - \cdots - \circ - \circ - \circ \end{array}$	<code>\dynkin[extended]E7</code>
$E_8^1$	$\begin{array}{c} \circ \\   \\ \circ \\   \\ \circ - \circ - \cdots - \circ - \circ - \circ - \circ \end{array}$	<code>\dynkin[extended]E8</code>

continued ...

Table 16: ...continued

$F_4^1$	$\circ - \circ - \circ \Rightarrow \circ - \circ$	<code>\dynkin[extended]F4</code>
$G_2^1$	$\circ - \circ \Rightarrow \circ$	<code>\dynkin[extended]G2</code>

Table 17: The Dynkin diagrams of the twisted simple root systems in Kac style

$A_2^2$	$\circ \Leftarrow \circ$	<code>\dynkin[extended]A[2]2</code>
$A_{ev}^2$	$\circ \Leftarrow \circ - \circ - \circ - \cdots - \circ - \circ \Leftarrow \circ$	<code>\dynkin[extended]A[2]{even}</code>
$A_{od}^2$	$\begin{array}{c} \circ \\ \diagdown \\ \circ - \circ - \circ - \cdots - \circ - \circ \Leftarrow \circ \end{array}$	<code>\dynkin[extended]A[2]{odd}</code>
$D_n^2$	$\circ \Leftarrow \circ - \circ - \circ - \cdots - \circ - \circ - \circ \Rightarrow \circ$	<code>\dynkin[extended]D[2]{}</code>
$E_6^2$	$\circ - \circ - \circ \Leftarrow \circ - \circ$	<code>\dynkin[extended]E[2]6</code>
$D_4^3$	$\circ - \circ \Leftarrow \circ$	<code>\dynkin[extended]D[3]4</code>

## 24. CEREF STYLE

We include a style called `ceref` which paints oblong root markers with shadows. The word “ceref” is an old form of the word “serif”.

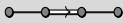


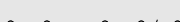
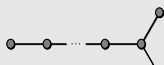


Ceref style
<code>\dynkin[ceref]F4</code>


Table 18: The Dynkin diagrams of the simple root systems in ceref style

$A_n$		<code>\dynkin A{}</code>
$B_n$		<code>\dynkin B{}</code>
$C_n$		<code>\dynkin C{}</code>
$D_n$		<code>\dynkin D{}</code>
$E_6$		<code>\dynkin E6</code>
$E_7$		<code>\dynkin E7</code>

continued ...

Table 18: ...continued

$E_8$		<code>\dynkin E8</code>
$F_4$		<code>\dynkin F4</code>
$G_2$		<code>\dynkin G2</code>

Table 19: The Dynkin diagrams of the extended simple root systems in cref style

$A_1^1$		<code>\dynkin[extended]A1</code>
$A_n^1$		<code>\dynkin[extended]A{}</code>
$B_n^1$		<code>\dynkin[extended]B{}</code>
$C_n^1$		<code>\dynkin[extended]C{}</code>
$D_n^1$		<code>\dynkin[extended]D{}</code>
$E_6^1$		<code>\dynkin[extended]E6</code>
$E_7^1$		<code>\dynkin[extended]E7</code>
$E_8^1$		<code>\dynkin[extended]E8</code>
$F_4^1$		<code>\dynkin[extended]F4</code>
$G_2^1$		<code>\dynkin[extended]G2</code>

Table 20: The Dynkin diagrams of the twisted simple root systems in cref style

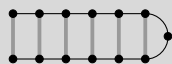
$A_2^2$		<code>\dynkin[extended]A[2]2</code>
$A_{ev}^2$		<code>\dynkin[extended]A[2]{even}</code>
$A_{od}^2$		<code>\dynkin[extended]A[2]{odd}</code>
$D_n^2$		<code>\dynkin[extended]D[2]{}</code>
$E_6^2$		<code>\dynkin[extended]E[2]6</code>
$D_4^3$		<code>\dynkin[extended]D[3]4</code>

## 25. MORE ON FOLDED DYNKIN DIAGRAMS

The Dynkin diagrams package has limited support for folding Dynkin diagrams.

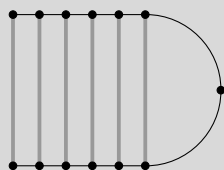
## Folding

```
\dynkin[fold]A{13}
```



## Big fold radius

```
\dynkin[fold,fold radius=1cm]A{13}
```



## Small fold radius

```
\dynkin[fold,fold radius=.2cm]A{13}
```



Some Dynkin diagrams have multiple foldings, which we attempt to distinguish (not entirely successfully) by their *ply*: the maximum number of roots folded together. Most diagrams can only allow a 2-ply folding, so `fold` is a synonym for `ply=2`.

## 3-ply

```
\dynkin[ply=3]D4
\dynkin[ply=3,fold right]D4
\dynkin[ply=3]D[1]4
```



4-ply

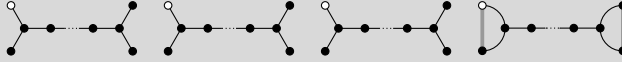
```
\dynkin[ply=4]D[1]4
```



The  $D_{\ell}^{(1)}$  diagrams can be folded on their left end and separately on their right end:

Left, right and both

```
\dynkin D[1]{ } \
\dynkin[fold left]D[1]{ } \
\dynkin[fold right]D[1]{ } \
\dynkin[fold]D[1]{ }
```



We have to be careful about the 4-ply foldings of  $D_{2\ell}^{(1)}$ , for which we can have two different patterns, so by default, the package only draws as much as it can without distinguishing the two:

Default  $D_{2\ell}^{(1)}$  and the two ways to finish it

```
\dynkin[ply=4]D[1]{****.*****.*****}%
\
\begin{dynkinDiagram}[ply=4]{D}{1}{****.*****.*****}%
  \dynkinFold[bend right=90]1{13}%
  \dynkinFold[bend right=90]0{14}%
\end{dynkinDiagram} \
\begin{dynkinDiagram}[ply=4]{D}{1}{****.*****.*****}%
  \dynkinFold01%
  \dynkinFold1{13}%
  \dynkinFold{13}{14}%
\end{dynkinDiagram}
```

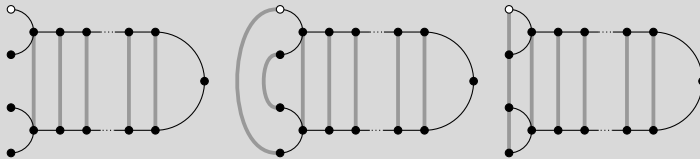


Table 21: Some foldings of Dynkin diagrams. For these diagrams, we want to compare a folding diagram with the diagram that results when we fold it, so it looks best to set `fold radius` and `edge length` to equal lengths.

$A_3$		<code>\dynkin[fold]A[0]3</code>
$C_2$		<code>\dynkin C[0]2</code>
$A_{2\ell-1}$		<code>\dynkin[fold]A{**.****.**}</code>
$C_\ell$		<code>\dynkin C{}</code>
$B_3$		<code>\dynkin[fold]B[0]3</code>
$G_2$		<code>\dynkin[reverse arrows]G[0]2</code>
$D_4$		<code>\dynkin[ply=3, fold right]D4</code>
$G_2$		<code>\dynkin G2</code>
$D_{\ell+1}$		<code>\dynkin[fold]D{}</code>
$B_\ell$		<code>\dynkin B{}</code>
$E_6$		<code>\dynkin[fold]E[0]6</code>
$F_4$		<code>\dynkin[reverse arrows]F[0]4</code>
$A_3^1$		<code>\dynkin[ply=4]A[1]3</code>
$A_1^1$		<code>\dynkin A[1]1</code>
$A_{2\ell-1}^1$		<code>\dynkin[fold]A[1]{**.****.**}</code>
$C_\ell^1$		<code>\dynkin C[1]{}</code>
$B_3^1$		<code>\dynkin[ply=3]B[1]3</code>
$A_2^2$		<code>\dynkin A[2]2</code>
$B_3^1$		<code>\dynkin[ply=2]B[1]3</code>
$G_2^1$		<code>\dynkin G[1]2</code>
$B_\ell^1$		<code>\dynkin[fold]B[1]{}</code>
$D_\ell^2$		<code>\dynkin D[2]{}</code>

continued ...

Table 21: ...continued

$D_4^1$		<code>\dynkin[ply=3]D[1]4</code>
$B_3^1$		<code>\dynkin B[1]3</code>
$D_4^1$		<code>\dynkin[ply=3]D[1]4</code>
$G_2^1$		<code>\dynkin G[1]2</code>
$D_{\ell+1}^1$		<code>\dynkin[fold]D[1]\{\}</code>
$D_\ell^2$		<code>\dynkin D[2]\{\}</code>
$D_{\ell+1}^1$		<code>\dynkin[fold right]D[1]\{\}</code>
$B_\ell^1$		<code>\dynkin B[1]\{\}</code>
$D_{2\ell}^1$		<code>\begin{dynkinDiagram}[ply=4]D[1]%</code> <code>{****,****,****}</code> <code>\dynkinFold01</code> <code>\dynkinFold1{13}</code> <code>\dynkinFold{13}{14}</code> <code>\end{dynkinDiagram}</code>
$A_{\text{odd}}^2$		<code>\dynkin A[2]{odd}</code>
$D_{2\ell}^1$		<code>\begin{dynkinDiagram}[ply=4]{D}[1]%</code> <code>{****,****,****}</code> <code>\dynkinFold[bend right=90]1{13}</code> <code>\dynkinFold[bend right=90]0{14}</code> <code>\end{dynkinDiagram}</code>
$A_{\text{even}}^2$		<code>\dynkin A[2]{even}</code>
$E_6^1$		<code>\dynkin[fold]E[1]6</code>
$F_4^1$		<code>\dynkin[reverse arrows]F[1]4</code>
$E_6^1$		<code>\dynkin[ply=3]E[1]6</code>
$D_4^3$		<code>\dynkin D[3]4</code>

continued ...



Table 21: ...continued



$E_7^1$		<code>\dynkin[fold]E[1]7</code>
$E_6^2$		<code>\dynkin E[2]6</code>
$F_4^1$		<code>\dynkin[fold]F[1]4</code>
$G_2^1$		<code>\dynkin G[1]2</code>
$A_{\text{odd}}^2$		<code>\dynkin[odd,fold]A[2]{****.***}</code>
$A_{\text{even}}^2$		<code>\dynkin A[2]{even}</code>
$D_3^2$		<code>\dynkin[fold]D[2]3</code>
$A_2^2$		<code>\dynkin A[2]2</code>

Table 22: Frobenius fixed point subgroups of finite simple groups of Lie type [4] p. 15

$A_{\ell \geq 1}$		<code>\dynkin A{}</code>
${}^2A_{\ell \geq 2}$		<code>\dynkin[fold]A{}</code>
$B_{\ell \geq 2}$		<code>\dynkin B{}</code>
${}^2B_2$		<code>\dynkin[fold]B2</code>
$C_{\ell \geq 3}$		<code>\dynkin C{}</code>
$D_{\ell \geq 4}$		<code>\dynkin D{}</code>
${}^2D_{\ell \geq 4}$		<code>\dynkin[fold]D{}</code>
${}^3D_4$		<code>\dynkin[ply=3]D4</code>
$E_6$		<code>\dynkin E6</code>
${}^2E_6$		<code>\dynkin[fold]E6</code>
$E_7$		<code>\dynkin E7</code>
$E_8$		<code>\dynkin E8</code>
$F_4$		<code>\dynkin F4</code>
${}^2F_4$		<code>\dynkin[fold]F4</code>

continued ...

Table 22: ...continued

$G_2$		<code>\dynkin G2</code>
${}^2G_2$		<code>\dynkin[fold]G2</code>

## 26. TYPESETTING MATHEMATICAL NAMES OF DYNKIN DIAGRAMS

The `\dynkinName` command, with the same syntax as `\dynkin`, typesets a default name of your diagram in L<sup>A</sup>T<sub>E</sub>X. It is perhaps only useful when automatically generating a large collection of Dynkin diagrams in a computer program.

Name of a diagram

```
\dynkinName[label,extended]B7
\dynkinName A[2]{even}
\dynkinName[Coxeter]B7
\dynkinName[label,extended]B*
\dynkinName D[3]4
```

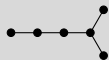
$B_7^1$   $A_{ev}^2$   $B_7$   $B_1^1$   $D_4^3$

## 27. CONNECTING DYNKIN DIAGRAMS

We can make some sophisticated folded diagrams by drawing multiple diagrams, each with a name:

Name a diagram

```
\dynkin[name=Bob]D6
```



We can then connect the two with folding edges:

Connect diagrams

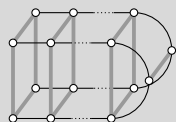
```
\begin{dynkinDiagram}[name=upper]A3
  \node (current) at ($(\text{upper root 1})+(0,-.3cm)$) {};
  \dynkin[at=(current),name=lower]A3
  \begin{pgfonlayer}{Dynkin behind}
    \foreach \i in {1,...,3}%
    {%
      \draw[/Dynkin diagram/fold style]
        ($(\text{upper root } \i)$)
        -- ($(\text{lower root } \i)$);%
    }%
  \end{pgfonlayer}
\end{dynkinDiagram}
```

```
\end{pgfonlayer}
\end{dynkinDiagram}
```

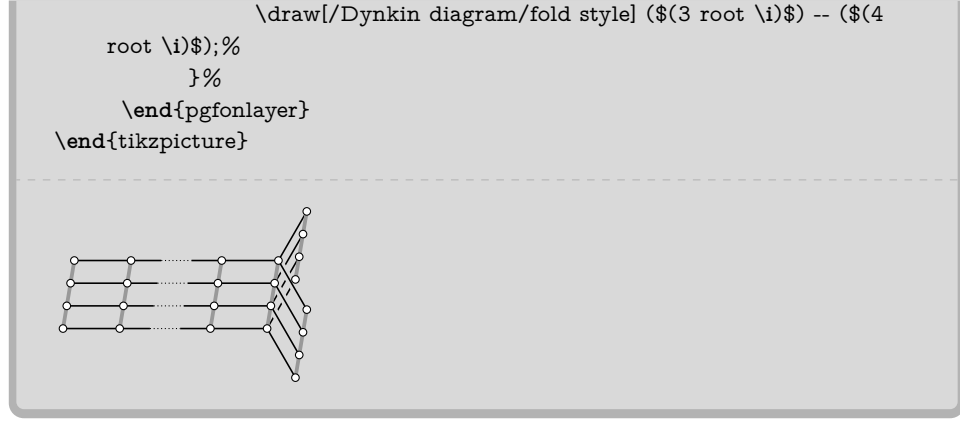


The following diagrams arise in the Satake diagrams of the pseudo-Riemannian symmetric spaces [2].

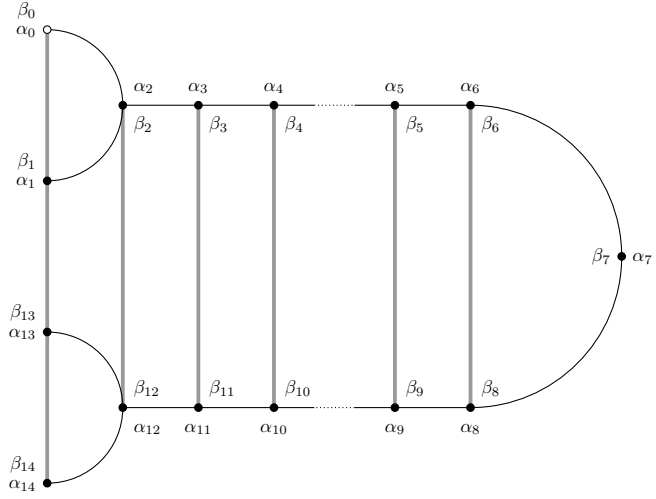
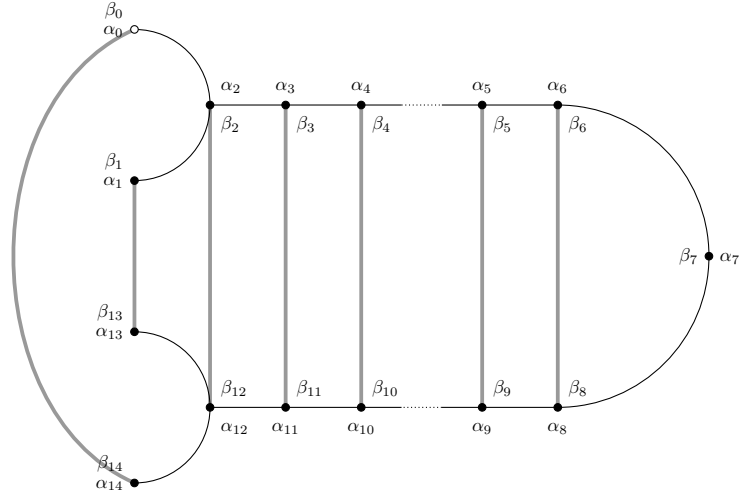
```
\pgfkeys{/Dynkin diagram,edge length=.5cm,fold radius=.5cm}
\begin{tikzpicture}
  \dynkin[name=1]A{IIIb}
  \node (a) at (-.3,-.4){};
  \dynkin[name=2,at=(a)]A{IIIb}
  \begin{pgfonlayer}{Dynkin behind}
    \foreach \i in {1,...,7}%
    {%
      \draw[/Dynkin diagram/fold style]
        ($ (1 root \i)$)
        --
        ($ (2 root \i)$);%
    }%
  \end{pgfonlayer}
\end{tikzpicture}
```



```
\pgfkeys{/Dynkin diagram,
edge length=.75cm,
edge/.style={draw=example-color,double=black,very thick}}
\begin{tikzpicture}
  \foreach \d in {1,...,4}
  {
    \node (current) at ($ (\d*.05,\d*.3)$){};
    \dynkin[name=\d,at=(current)]D{oo.oooo}
  }
  \begin{pgfonlayer}{Dynkin behind}
    \foreach \i in {1,...,6}%
    {%
      \draw[/Dynkin diagram/fold style] ($ (1 root \i)$) -- ($ (2
root \i)$);%
      \draw[/Dynkin diagram/fold style] ($ (2 root \i)$) -- ($ (3
root \i)$);%
    }%
  \end{pgfonlayer}
\end{tikzpicture}
```



## 28. OTHER EXAMPLES

 ${}^1D_4$  4-ply tied straight: ${}^1D_4$  4-ply tied bending:

```

\tikzset{/Dynkin diagram,
  edge length=1cm,
  fold radius=1cm,
  label,
  label*=true,
  label macro/.code={\alpha_{#1}},
  label macro*/.code={\beta_{#1}}}
\({}^1D_4\) 4-ply tied straight:
\begin{dynkinDiagram}[ply=4]D[1]%
{****,****,****}
\dynkinFold 01
\dynkinFold 1{13}
\dynkinFold{13}{14}
\end{dynkinDiagram}
\({}^1D_4\) 4-ply tied bending:
\begin{dynkinDiagram}[ply=4,label]D[1]%
{****,****,****}
\dynkinFold1{13}
\dynkinFold[bend right=65]0{14}
\end{dynkinDiagram}

```

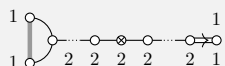
Below we draw the Vogan diagrams of some affine Lie superalgebras [21, 20].

$\mathfrak{sl}(2m|2n)^{(2)}$

```

\begin{dynkinDiagram}[ply=2,label]{B}[1]{oo.oto.oo}
\dynkinLabelRoot*71
\end{dynkinDiagram}

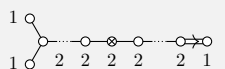
```



```

\dynkin[label]B[1]{oo.oto.oo}

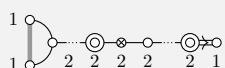
```

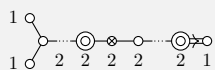
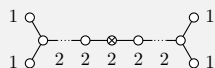
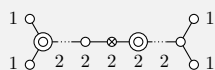
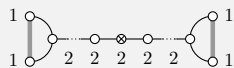
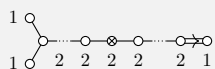
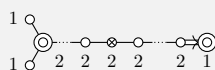


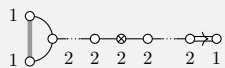
```

\dynkin[ply=2,label]B[1]{oo.Oto.Oo}

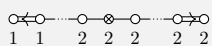
```

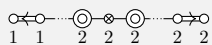


$\backslash\text{dynkin}[\text{label}]\text{B}[1]\{\text{oo.Oto.Oo}\}$ 

 $\backslash\text{dynkin}[\text{label}]\text{D}[1]\{\text{oo.oto.ooo}\}$ 

 $\backslash\text{dynkin}[\text{label}]\text{D}[1]\{\text{oO.otO.ooo}\}$ 

 $\backslash\text{dynkin}[\text{label},\text{fold}]\text{D}[1]\{\text{oo.oto.ooo}\}$ 

 $\mathfrak{sl}(2m+1|2n)^2$ 
 $\backslash\text{dynkin}[\text{label}]\text{B}[1]\{\text{oo.oto.oo}\}$ 

 $\backslash\text{dynkin}[\text{label}]\text{B}[1]\{\text{oO.oto.oO}\}$ 


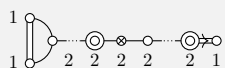
$$\backslash\mathrm{dynkin}[\mathrm{label},\mathrm{fold}]\mathrm{B}[1]\{\mathrm{oo.oto.oo}\}$$


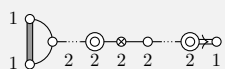
$$\mathfrak{sl}(2m+1|2n+1)^2$$

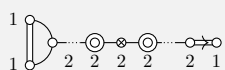
$$\backslash\mathrm{dynkin}[\mathrm{label}]\mathrm{D}[2]\{\mathrm{o.oto.oo}\}$$


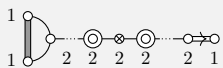
$$\backslash\mathrm{dynkin}[\mathrm{label}]\mathrm{D}[2]\{\mathrm{o.OtO.oo}\}$$


$$\mathfrak{sl}(2|2n+1)^{(2)}$$

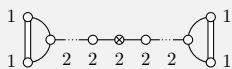
$$\backslash\mathrm{dynkin}[\mathrm{ply}=2,\mathrm{label},\mathrm{double\ edges}]\mathrm{B}[1]\{\mathrm{oo.Oto.Oo}\}$$


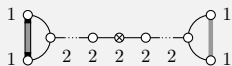
$$\backslash\mathrm{dynkin}[\mathrm{ply}=2,\mathrm{label},\mathrm{double\ fold}]\mathrm{B}[1]\{\mathrm{oo.Oto.Oo}\}$$


$$\backslash\mathrm{dynkin}[\mathrm{ply}=2,\mathrm{label},\mathrm{double\ edges}]\mathrm{B}[1]\{\mathrm{oo.OtO.oo}\}$$


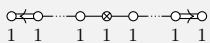
$$\backslash\text{dynkin}[ply=2,label,double\ fold]B[1]\{oo.OtO.oo\}$$


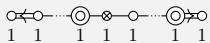
$$\mathfrak{sl}(2|2n)^{(2)}$$

$$\backslash\text{dynkin}[ply=2,label,double\ edges]D[1]\{oo.oto.ooo\}$$


$$\backslash\text{dynkin}[ply=2,label,double\ fold\ left]D[1]\{oo.oto.ooo\}$$


$$\mathfrak{osp}(2m|2n)^{(2)}$$

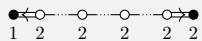
$$\backslash\text{dynkin}[label,label\ macro/.code=\{1\}]D[2]\{o.oto.oo\}$$


$$\backslash\text{dynkin}[label,label\ macro/.code=\{1\}]D[2]\{o.Oto.Oo\}$$


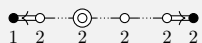


$\mathfrak{osp}(2|2n)^{(2)}$ 

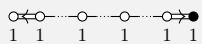
```
\dynkin[label,label macro/.code=\lablIt{#1},
  affine mark=*]
D[2]{o.o.o.o*}
```



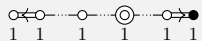
```
\dynkin[label,label macro/.code=\lablIt{#1},
  affine mark=*]
D[2]{o.O.o.o*}
```


 $\mathfrak{sl}(1|2n+1)^4$ 

```
\dynkin[label,label macro/.code={1}]D[2]{o.o.o.o*}
```



```
\dynkin[label,label macro/.code={1}]D[2]{o.o.O.o*}
```

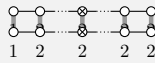


$A^1$ 

```

\begin{tikzpicture}
  \dynkin[name=upper]A{oo.t.oo}
  \node (Dynkin current) at (upper root 1){};
  \dynkinSouth
  \dynkin[at=(Dynkin current),name=lower]A{oo.t.oo}
  \begin{pgfonlayer}{Dynkin behind}
    \foreach \i in {1,...,5}{
      \draw[/Dynkin diagram/fold style]
        ($(\text{upper root } \i)$) -- ($(\text{lower root } \i)$);
    }
  \end{pgfonlayer}
\end{tikzpicture}

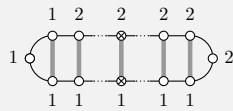
```



```

\dynkin[fold]A[1]{oo.t.ooooo.t.oo}

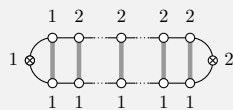
```



```

\dynkin[fold,affine mark=t]A[1]{oo.o.ootoo.o.oo}

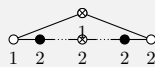
```

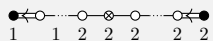
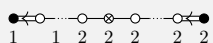
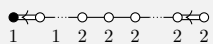
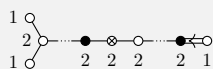
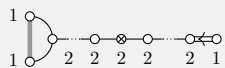
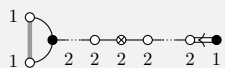


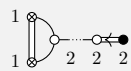
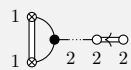
```

\dynkin[affine mark=t]A[1]{o*.t.*o}

```

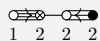


$B^1$ `\dynkin[affine mark=*]A[2]{o.oto.o*}``\dynkin[affine mark=*]A[2]{o.oto.o*}``\dynkin[affine mark=*]A[2]{o.ooo.o}``\dynkin[odd]A[2]{oo.*to.*o}``\dynkin[odd,fold]A[2]{oo.oto.oo}``\dynkin[odd,fold]A[2]{o*.oto.o*}`

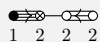
$D^1$ `\dynkin D{otoo}``\dynkin D{ot*o}``\dynkin[fold]D{otoo}` $C^1$ `\dynkin[double edges,fold,affine mark=t,odd]A[2]{to.o*}``\dynkin[double edges,fold,affine mark=t,odd]A[2]{t*.oo}`

$F^1$ 

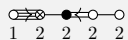
```
\begin{dynkinDiagram}A{oto*}%
  \dynkinQuadrupleEdge 12%
  \dynkinTripleEdge 43%
\end{dynkinDiagram}%
```



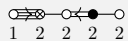
```
\begin{dynkinDiagram}A{*too}%
  \dynkinQuadrupleEdge 12%
  \dynkinTripleEdge 43%
\end{dynkinDiagram}%
```

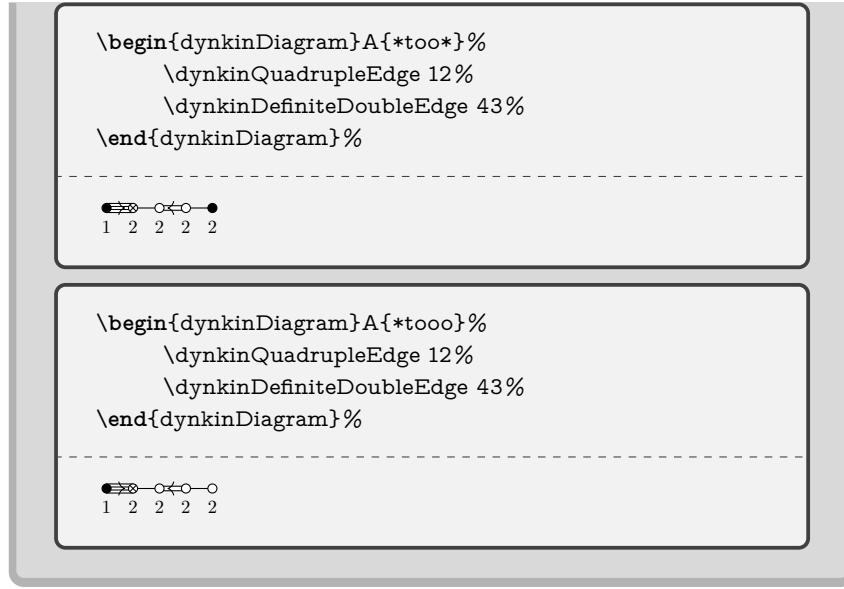
 $G^1$ 

```
\begin{dynkinDiagram}A{ot*oo}%
  \dynkinQuadrupleEdge 12%
  \dynkinDefiniteDoubleEdge 43%
\end{dynkinDiagram}%
```




```
\begin{dynkinDiagram}A{oto*o}%
  \dynkinQuadrupleEdge 12%
  \dynkinDefiniteDoubleEdge 43%
\end{dynkinDiagram}%
```





## 29. EXAMPLE: THE COMPLEX SIMPLE LIE ALGEBRAS

$\mathfrak{g}$	Diagram	Weights	Roots	Simple roots
$A_n$		$\frac{1}{n+1}\mathbb{Z}^{n+1} / \langle \sum e_j \rangle$	$e_i - e_j$	$e_i - e_{i+1}$
$B_n$		$\frac{1}{2}\mathbb{Z}^n$	$\pm e_i, \pm e_i \pm e_j, i \neq j$	$e_i - e_{i+1}, e_n$
$C_n$		$\mathbb{Z}^n$	$\pm 2e_i, \pm e_i \pm e_j, i \neq j$	$e_i - e_{i+1}, 2e_n$
$D_n$		$\frac{1}{2}\mathbb{Z}^n$	$\pm e_i \pm e_j, i \neq j$	$e_i - e_{i+1}, \quad i \leq n-1$ $e_{n-1} + e_n$
$E_8$		$\frac{1}{2}\mathbb{Z}^8$	$\pm 2e_i \pm 2e_j, \quad i \neq j,$ $\sum_i (-1)^{m_i} e_i, \quad \sum m_i \text{ even}$	$2e_1 - 2e_2,$ $2e_2 - 2e_3,$ $2e_3 - 2e_4,$ $2e_4 - 2e_5,$ $2e_5 - 2e_6,$ $2e_6 + 2e_7,$ $-\sum e_j,$ $2e_6 - 2e_7$
$E_7$		$\frac{1}{2}\mathbb{Z}^8 / \langle e_1 - e_2 \rangle$	quotient of $E_8$	quotient of $E_8$
$E_6$		$\frac{1}{3}\mathbb{Z}^8 / \langle e_1 - e_2, e_2 - e_3 \rangle$	quotient of $E_8$	quotient of $E_8$
$F_4$		$\mathbb{Z}^4$	$\pm 2e_i,$ $\pm 2e_i \pm 2e_j, \quad i \neq j,$ $\pm e_1 \pm e_2 \pm e_3 \pm e_4$	$2e_2 - 2e_3,$ $2e_3 - 2e_4,$ $2e_4,$ $e_1 - e_2 - e_3 - e_4$

g	Diagram	Weights	Roots	Simple roots
$G_2$		$\mathbb{Z}^3 / \langle \sum e_j \rangle$	$\pm(1, -1, 0),$ $\pm(-1, 0, 1),$ $\pm(0, -1, 1),$ $\pm(2, -1, -1),$ $\pm(1, -2, 1),$ $\pm(-1, -1, 2)$	$(-1, 0, 1),$ $(2, -1, -1)$

```

\NewDocumentEnvironment{bunch}{}%
{
  \renewcommand*{\arraystretch}{1}
  \begin{array}{@{}l@{}}
    \midrule
  \{
    \midrule\end{array}
  }
  \small
  \NewDocumentCommand\nct{mm}
  {
    \newcolumntype{#1}{>\columncolor[gray]{.9}>{\$}m{#2cm}<{\$}}
  }
  \nct{G}{.3}
  \nct{D}{2.1}
  \nct{W}{3}
  \nct{R}{3.7}
  \nct{S}{3}
  \NewDocumentCommand\LieG{}{\mathfrak{g}}
  \NewDocumentCommand\W{om}
  {
    \ensuremath{
      \mathbb{Z}^{\sim\#2}
    }
    \IfValueT{#1}{/\left<#1\right>}
  }
}
\renewcommand*{\arraystretch}{1.5}
\NewDocumentCommand\quo{}{\text{quotient of } E_8}
\begin{longtable}{@{}GDWRS@{}}
\LieG&
  \text{Diagram}&
  \text{Weights}&
  \text{Roots}&
  \text{Simple roots}\\
\midrule\endfirsthead
\LieG&
  \text{Diagram}&
  \text{Weights}&
  \text{Roots}&
  \text{Simple roots}\\
\midrule\endhead
A_n&

```

```

\dynkin A{}&
\frac{1}{n+1}W[\sum e_j]{n+1}&
e_i-e_j&
e_i-e_{i+1}\\
B_n&
\dynkin B{}&
\frac{1}{2}W n&
\pm e_i, \pm e_i \pm e_j, i \ne j&
e_i-e_{i+1}, e_n\\
C_n&
\dynkin C{}&
W n&
\pm 2 e_i, \pm e_i \pm e_j, i \ne j&
e_i-e_{i+1}, 2e_n\\
D_n&
\dynkin D{}&
\frac{1}{2}W n&
\pm e_i \pm e_j, i \ne j &
\begin{bunch}
e_i-e_{i+1}, & i \leq n-1 \\
e_{n-1}+e_n
\end{bunch}\\
E_8&
\dynkin E8&
\frac{1}{2}W 8&
\begin{bunch}
\pm 2e_i \pm 2e_j, & i \ne j, \\
\sum_i (-1)^{m_i} e_i, & \sum m_i \text{ even}
\end{bunch}&
\begin{bunch}
2e_1-2e_2, \\
2e_2-2e_3, \\
2e_3-2e_4, \\
2e_4-2e_5, \\
2e_5-2e_6, \\
2e_6+2e_7, \\
-\sum e_j, & 2e_6-2e_7
\end{bunch}\\
E_7&
\dynkin E7&
\frac{1}{2}W[e_1-e_2]8&
\text{quo}&
\text{quo}\\
E_6&
\dynkin E6&
\frac{1}{3}W[e_1-e_2, e_2-e_3]8&
\text{quo}&
\text{quo}\\
F_4&
\dynkin F4&
W4&
\begin{bunch}
\pm 2e_i,

```

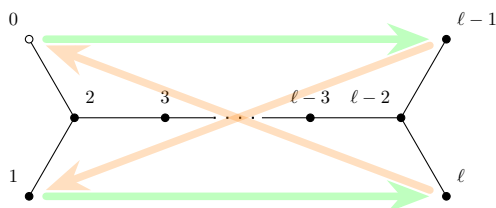


```

\pm 2e_i \pm 2e_j, \quad i \ne j, \\
\pm e_1 \pm e_2 \pm e_3 \pm e_4
\end{bunch}&
\begin{bunch}
2e_2-2e_3, \\
2e_3-2e_4, \\
2e_4, \\
e_1-e_2-e_3-e_4
\end{bunch} \\
G_2&
\dynkin G2&
\W[\sum e_j]3&
\begin{bunch}
\pm(1,-1,0), \\
\pm(-1,0,1), \\
\pm(0,-1,1), \\
\pm(2,-1,-1), \\
\pm(1,-2,1), \\
\pm(-1,-1,2)
\end{bunch}
&
\begin{bunch}
(-1,0,1), \\
(2,-1,-1)
\end{bunch}
\end{longtable}

```

## 30. AN EXAMPLE OF MIKHAIL BOROVoi



```

\tikzset{
  big arrow/.style={
    -Stealth,
    line cap=round,
    line width=1mm,
    shorten <=1mm,
    shorten >=1mm}
}
\newcommand\catholic[2]{
  \draw[big arrow,green!25!white] (root #1) to (root #2);
}
\newcommand\protestant[2]{
  \begin{scope}[transparency group, opacity=.25]
    \draw[big arrow,orange] (root #1) to (root #2);
  \end{scope}
}
\begin{dynkinDiagram}{%

```

```

edge length=1.2cm,
indefinite edge/.style={
    thick,
    loosely dotted
},
labels*={0,1,2,3,\ell-3,\ell-2,\ell-1,\ell]}
D[1]{ }
\catholic 06\catholic 17
\protestant 70\protestant 61
\end{dynkinDiagram}

```

### 31. SYNTAX

The syntax is `\dynkin[<options>]{<letter>}[<twisted rank>]{<rank>}` where `<letter>` is A, B, C, D, E, F or G, the family of root system for the Dynkin diagram, `<twisted rank>` is 0, 1, 2, 3 (default is 0) representing:

- 0 finite root system
- 1 affine extended root system, i.e. of type <sup>(1)</sup>
- 2 affine twisted root system of type <sup>(2)</sup>
- 3 affine twisted root system of type <sup>(3)</sup>

and `<rank>` is

- (1) an integer representing the rank or
- (2) blank to represent an indefinite rank or
- (3) the name of a Satake diagram as in section 4.

The environment syntax is `\begin{dynkinDiagram}` followed by the same parameters as `\dynkin`, then various Dynkin diagram and TikZ commands, and then `\end{dynkinDiagram}`.

### 32. OPTIONS

```

*/.style = <TikZ style data>,
default : solid,draw=black,fill=black
           style for roots like •
o/.style = <TikZ style data>,
default : solid,draw=black,fill=white
           style for roots like ◦
O/.style = <TikZ style data>,
default : solid,draw=black,fill=white
           style for roots like ⊙
t/.style = <TikZ style data>,
default : solid,draw=black,fill=black
           style for roots like ⊗
x/.style = <TikZ style data>,
default : solid,draw=black,line cap=round
           style for roots like ✕
X/.style = <TikZ style data>,
default : solid,draw=black,thick,line cap=round
           style for roots like ✕

```

continued ...

Table 24: ...continued

**affine mark** = o,O,t,x,X,\*,  
**default** : \*  
 default root mark for root zero in an affine Dynkin diagram  
**arrow shape/.style** = TikZ style data,  
**default** : -{Computer Modern Rightarrow[black]}  
 shape of arrow heads for most Dynkin diagrams that have arrows  
**arrow style** = TikZ style data,  
**default** : black  
 set to override the default style for the arrows in nonsimply laced  
 Dynkin diagrams, including length, width, line width and color  
**arrows** =  $\langle \text{true or false} \rangle$ ,  
**default** : true  
 whether to draw the arrows that arise along the edges  
**backwards** =  $\langle \text{true or false} \rangle$ ,  
**default** : false  
 whether to reverse right to left  
**ceref** =  $\langle \text{true or false} \rangle$ ,  
**default** : false  
 whether to draw roots in a “ceref” style  
**Coxeter** =  $\langle \text{true or false} \rangle$ ,  
**default** : false  
 whether to draw a Coxeter diagram, rather than a Dynkin diagram  
**double edges** =  $\langle \rangle$ ,  
**default** : not set  
 set to override the **fold** style when folding roots together in a  
 Dynkin diagram, so that the foldings are indicated with double  
 edges (like those of an  $F_4$  Dynkin diagram without arrows)  
**double fold** =  $\langle \rangle$ ,  
**default** : not set  
 set to override the **fold** style when folding roots together in a  
 Dynkin diagram, so that the foldings are indicated with double  
 edges (like those of an  $F_4$  Dynkin diagram without arrows), but  
 filled in solidly  
**double left** =  $\langle \rangle$ ,  
**default** : not set  
 set to override the **fold** style when folding roots together at the  
 left side of a Dynkin diagram, so that the foldings are indicated  
 with double edges (like those of an  $F_4$  Dynkin diagram without  
 arrows)  
**double fold left** =  $\langle \rangle$ ,  
**default** : not set  
 set to override the **fold** style when folding roots together at the  
 left side of a Dynkin diagram, so that the foldings are indicated  
 with double edges (like those of an  $F_4$  Dynkin diagram without  
 arrows), but filled in solidly  
**double right** =  $\langle \rangle$ ,

continued ...

Table 24: ...continued

default : **not set**  
 set to override the **fold** style when folding roots together at the right side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an  $F_4$  Dynkin diagram without arrows)  
 double fold right =  $\langle \rangle$ ,  
 default : **not set**  
 set to override the **fold** style when folding roots together at the right side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an  $F_4$  Dynkin diagram without arrows), but filled in solidly  
 edge label/.style = **TikZ style data**,  
 default : **text height=0,text depth=0,label distance=-2pt**  
 style of edge labels in the Dynkin diagram, as found, for example, on some Coxeter diagrams  
 edge length =  $\langle \text{number} \rangle \text{cm}$ ,  
 default : **.35cm**  
 distance between nodes in the Dynkin diagram  
 edge/.style = **TikZ style data**,  
 default : **solid,draw=black,fill=white,thin**  
 style of edges in the Dynkin diagram  
 fold =  $\langle \text{true or false} \rangle$ ,  
 default : **true**  
 whether, when drawing Dynkin diagrams, to draw them 2-ply  
 fold left =  $\langle \text{true or false} \rangle$ ,  
 default : **true**  
 whether to fold the roots on the left side of a Dynkin diagram  
 fold radius =  $\langle \text{length} \rangle$ ,  
 default : **.3cm**  
 the radius of circular arcs used in curved edges of folded Dynkin diagrams  
 fold right =  $\langle \text{true or false} \rangle$ ,  
 default : **true**  
 whether to fold the roots on the right side of a Dynkin diagram  
 fold left style/.style =  $\langle \text{TikZ style data} \rangle$ ,  
 default :  
 style to override the **fold** style when folding roots together on the left half of a Dynkin diagram  
 fold right style/.style =  $\langle \text{TikZ style data} \rangle$ ,  
 default :  
 style to override the **fold** style when folding roots together on the right half of a Dynkin diagram  
 fold style/.style =  $\langle \text{TikZ style data} \rangle$ ,  
 default : **solid,draw=black!40,fill=none,line width=radius**  
 when drawing folded diagrams, style for the fold indicators  
 indefinite edge ratio =  $\langle \text{float} \rangle$ ,  
 continued ...

Table 24: ...continued

default : 1.6  
     ratio of indefinite edge lengths to other edge lengths  
 indefinite edge/.style =  $\langle$ TikZ style data $\rangle$ ,  
 default : solid,draw=black,fill=white,thin,densely dotted  
     style of the dotted or dashed middle third of each indefinite edge  
 involutions =  $\langle$ semicolon separated list of pairs $\rangle$ ,  
 default :  
     involution double arrows to draw  
 Kac =  $\langle$ true or false $\rangle$ ,  
 default : false  
     whether to draw in the style of [15]  
 label = true or false,  
 default : false  
     whether to label the roots according to the current labelling scheme  
 label\* =  $\langle$ true or false $\rangle$ ,  
 default : false  
     whether to label the roots at alternative label locations according  
     to the current labelling scheme  
 label macro =  $\langle$ 1-parameter T<sub>E</sub>X macro $\rangle$ ,  
 default : #1  
     the current labelling scheme for roots  
 label macro\* =  $\langle$ 1-parameter T<sub>E</sub>X macro $\rangle$ ,  
 default : #1  
     the current labelling scheme for alternate roots  
 label height =  $\langle$ 1-parameter T<sub>E</sub>X macro $\rangle$ ,  
 default : b  
     the current maximal height of text labels for the roots, set by  
     giving mathematics text of that height  
 label depth =  $\langle$ 1-parameter T<sub>E</sub>X macro $\rangle$ ,  
 default : g  
     the current maximal depth of text labels for the roots, set by  
     giving mathematics text of that depth  
 make indefinite edge =  $\langle$ edge pair  $i$ - $j$  or list of such $\rangle$ ,  
 default : {}  
     edge pair or list of edge pairs to treat as having indefinitely many  
     roots on them  
 mark =  $\langle$ o,O,t,x,X,\* $\rangle$ ,  
 default : \*  
     default root mark  
 name =  $\langle$ string $\rangle$ ,  
 default : anonymous  
     A name for the Dynkin diagram, with `anonymous` treated as a  
     blank; see section 27  
 ordering =  $\langle$ Adams, Bourbaki, Carter, Dynkin, Kac $\rangle$ ,  
 default : Bourbaki

continued ...

Table 24: ...continued

which ordering of the roots to use in exceptional root systems as in section 18

`parabolic = <integer>`,  
 default : 0  
 A parabolic subgroup with specified integer, where the integer is computed as  $n = \sum 2^{i-1}a_i$ ,  $a_i = 0$  or 1, to say that root  $i$  is crossed, i.e. a noncompact root

`ply = <0,1,2,3,4>`,  
 default : 0  
 how many roots get folded together, at most

`reverse arrows = <true or false>`,  
 default : `true`  
 whether to reverse the direction of the arrows that arise along the edges

`root radius = <number>cm`,  
 default : `.05cm`  
 size of the dots and of the crosses in the Dynkin diagram

`text style = <TikZ style data>`,  
 default : `scale=.7`  
 Style for any labels on the roots

`upside down = <true or false>`,  
 default : `false`  
 whether to reverse up to down

All other options are passed to TikZ.

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