

THE DYNKIN DIAGRAMS PACKAGE

VERSION 3.13

BEN MCKAY

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1. QUICK INTRODUCTION

Load the Dynkin diagram package (see options below)

```
\documentclass{amsart}
\usepackage{dynkin-diagrams}
\begin{document}
The Dynkin diagram of  $(B_3)$  is  $\text{dynkin}\{B\}{3}$ .
\end{document}
```

Invoke it

The Dynkin diagram of $\backslash(B_3\backslash)$ is $\backslash\text{dynkin}\{B\}\{3\}$.

The Dynkin diagram of B_3 is $\bullet\text{---}\bullet\Rightarrow\bullet$.

Inside a *TikZ* statement

The Dynkin diagram of $\backslash(B_3\backslash)$ is
 $\backslash\text{tikz}[\text{baseline}=-0.5\text{ex}]\ \backslash\text{dynkin}\{B\}\{3\};$

The Dynkin diagram of B_3 is $\bullet\text{---}\bullet\Rightarrow\bullet$

Inside a *TikZ* environment

The Dynkin diagram of $\backslash(B_3\backslash)$ is
 $\backslash\text{begin}\{\text{tikzpicture}\}[\text{baseline}=-0.5\text{ex}]$
 $\quad\backslash\text{dynkin}\{B\}\{3\}$
 $\backslash\text{end}\{\text{tikzpicture}\}$

The Dynkin diagram of B_3 is $\bullet\text{---}\bullet\Rightarrow\bullet$

Indefinite rank Dynkin diagrams

$\backslash\text{dynkin}\{B\}\{\}$

$\bullet\text{---}\cdots\text{---}\bullet\Rightarrow\bullet$

Table 1: The Dynkin diagrams of the reduced simple root systems
 [2] pp. 265–290, plates I–IX

A_n	$\bullet\text{---}\cdots\text{---}\bullet$	$\backslash\text{dynkin}\{A\}\{\}$
C_n	$\bullet\text{---}\cdots\text{---}\bullet\Leftarrow\bullet$	$\backslash\text{dynkin}\{C\}\{\}$
D_n	$\bullet\text{---}\cdots\text{---}\bullet$ $\quad\quad\quad\diagup\bullet$ $\quad\quad\quad\diagdown\bullet$	$\backslash\text{dynkin}\{D\}\{\}$
E_6	$\bullet\text{---}\bullet\text{---}\bullet$ $\quad\quad\quad\uparrow\bullet$	$\backslash\text{dynkin}\{E\}\{6\}$
E_7	$\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet$ $\quad\quad\quad\uparrow\bullet$	$\backslash\text{dynkin}\{E\}\{7\}$
E_8	$\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet\text{---}\bullet$ $\quad\quad\quad\uparrow\bullet$	$\backslash\text{dynkin}\{E\}\{8\}$
F_4	$\bullet\Rightarrow\bullet\text{---}\bullet\text{---}\bullet$	$\backslash\text{dynkin}\{F\}\{4\}$
G_2	$\bullet\Rightarrow\bullet$	$\backslash\text{dynkin}\{G\}\{2\}$

2. SET OPTIONS GLOBALLY

Most options set globally ...

```
\pgfkeys{/Dynkin diagram,edgeLength=.5cm,foldradius=.5cm}
```

...or pass to the package

```
\usepackage[
  ordering=Kac,
  edge/.style=blue,
  mark=o,
  radius=.06cm]
{dynkin-diagrams}
```

3. COXETER DIAGRAMS

Coxeter diagram option

```
\dynkin[Coxeter]{F}{4}
```



gonality option for G_2 and I_n Coxeter diagrams

```
\(G_2=\dynkin[Coxeter,gonality=n]{G}{2}\), \
\ (I_n=\dynkin[Coxeter,gonality=n]{I}{})
```

$G_2 = \overset{n}{\bullet} \bullet$, $I_n = \bullet \overset{n}{\bullet}$

Table 2: The Coxeter diagrams of the simple reflection groups

A_n		<code>\dynkin[Coxeter]{A}{}</code>
B_n		<code>\dynkin[Coxeter]{B}{}</code>
C_n		<code>\dynkin[Coxeter]{C}{}</code>
E_6		<code>\dynkin[Coxeter]{E}{6}</code>
E_7		<code>\dynkin[Coxeter]{E}{7}</code>
E_8		<code>\dynkin[Coxeter]{E}{8}</code>
F_4		<code>\dynkin[Coxeter]{F}{4}</code>

continued ...

Table 2: ...continued

G_2		<code>\dynkin[Coxeter,gonality=n]{G}{2}</code>
H_3		<code>\dynkin[Coxeter]{H}{3}</code>
H_4		<code>\dynkin[Coxeter]{H}{4}</code>
I_n		<code>\dynkin[Coxeter,gonality=n]{I}{}</code>

4. SATAKE DIAGRAMS

Satake diagrams use the standard name instead of a rank

`\(A_{IIIb}=\dynkin{A}{IIIb}\)`

$$A_{IIIb} = \text{Diagram}$$

We use a solid gray bar to denote the folding of a Dynkin diagram, rather than the usual double arrow, since the diagrams turn out simpler and easier to read.

Table 3: The Satake diagrams of the real simple Lie algebras [11] p. 532–534

A_I		<code>\dynkin{A}{I}</code>
A_{II}		<code>\dynkin{A}{II}</code>
A_{IIIa}		<code>\dynkin{A}{IIIa}</code>
A_{IIIb}		<code>\dynkin{A}{IIIb}</code>
A_{IV}		<code>\dynkin{A}{IV}</code>
B_I		<code>\dynkin{B}{I}</code>
B_{II}		<code>\dynkin{B}{II}</code>
C_I		<code>\dynkin{C}{I}</code>
C_{IIa}		<code>\dynkin{C}{IIa}</code>
C_{IIb}		<code>\dynkin{C}{IIb}</code>
D_{Ia}		<code>\dynkin{D}{Ia}</code>
D_{Ib}		<code>\dynkin{D}{Ib}</code>

continued ...

Table 3: ...continued

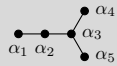
D_{Ic}		<code>\dynkin{D}{Ic}</code>
D_{II}		<code>\dynkin{D}{II}</code>
D_{IIIa}		<code>\dynkin{D}{IIIa}</code>
D_{IIIb}		<code>\dynkin{D}{IIIb}</code>
E_I		<code>\dynkin{E}{I}</code>
E_{II}		<code>\dynkin{E}{II}</code>
E_{III}		<code>\dynkin{E}{III}</code>
E_{IV}		<code>\dynkin{E}{IV}</code>
E_V		<code>\dynkin{E}{V}</code>
E_{VI}		<code>\dynkin{E}{VI}</code>
E_{VII}		<code>\dynkin{E}{VII}</code>
E_{VIII}		<code>\dynkin{E}{VIII}</code>
E_{IX}		<code>\dynkin{E}{IX}</code>
F_I		<code>\dynkin{F}{I}</code>
F_{II}		<code>\dynkin{F}{II}</code>
G_I		<code>\dynkin{G}{I}</code>

5. LABELS FOR THE ROOTS

Label the roots by root number

`\dynkin[label]{B}{3}`

Make a macro to assign labels to roots

`\dynkin[label,labelMacro/.code={\alpha_{#1}}]{D}{5}`

Label a single root

```
\begin{tikzpicture}
  \dynkin{B}{3}
  \dynkinLabelRoot{2}{\alpha_2}
\end{tikzpicture}
```



Use a text style

```
\begin{tikzpicture}
  \dynkin[text/.style={scale=1.2}]{B}{3};
  \dynkinLabelRoot{2}{\alpha_2}
\end{tikzpicture}
```



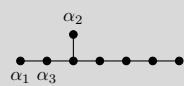
Access root labels via TikZ

```
\begin{tikzpicture}
  \dynkin{B}{3};
  \node[below] at (root 2) {\(\alpha_2\)};
\end{tikzpicture}
```



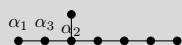
The labels have default locations

```
\begin{tikzpicture}
  \dynkin{E}{8};
  \dynkinLabelRoot{1}{\alpha_1}
  \dynkinLabelRoot{2}{\alpha_2}
  \dynkinLabelRoot{3}{\alpha_3}
\end{tikzpicture}
```



The starred form flips labels to alternate locations

```
\begin{tikzpicture}
  \dynkin{E}{8};
  \dynkinLabelRoot*{1}{\alpha_1}
  \dynkinLabelRoot*{2}{\alpha_2}
  \dynkinLabelRoot*{3}{\alpha_3}
\end{tikzpicture}
```



6. STYLE

Colours

```
\dynkin[
  edge/.style={blue!50,thick},
  */.style=blue!50!red,
  arrowColor=red]{F}{4}
```



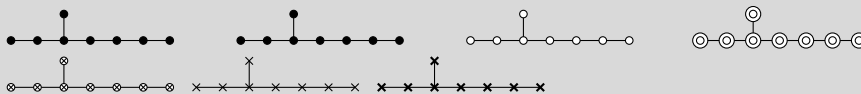
Edge lengths

```
\dynkin[edgeLength=1.2,parabolic=3]{A}{3}
```



Root marks

```
\dynkin{E}{8}
\dynkin[mark=*]{E}{8}
\dynkin[mark=o]{E}{8}
\dynkin[mark=0]{E}{8}
\dynkin[mark=t]{E}{8}
\dynkin[mark=x]{E}{8}
\dynkin[mark=X]{E}{8}
```



At the moment, you can only use:

- * solid dot
- o hollow circle
- ◻ double hollow circle
- t tensor root
- x crossed root
- X thickly crossed root

Mark styles

```
\dynkin[parabolic=124,x/.style={brown,very thick}]{E}{8}
```



Sizes of root marks

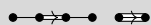
```
\dynkin[radius=.08cm,parabolic=3]{A}{3}
```



7. SUPPRESS OR REVERSE ARROWS

Some diagrams have double or triple edges

```
\dynkin{F}{4}
\dynkin{G}{2}
```



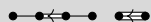
Suppress arrows

```
\dynkin[arrows=false]{F}{4}
\dynkin[arrows=false]{G}{2}
```



Reverse arrows

```
\dynkin[reverseArrows]{F}{4}
\dynkin[reverseArrows]{G}{2}
```



8. DRAWING ON TOP OF A DYNKIN DIAGRAM

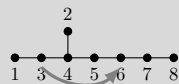
TikZ can access the roots themselves

```
\begin{tikzpicture}
  \dynkin{A}{4};
  \fill[white,draw=black] (root 2) circle (.15cm);
  \fill[white,draw=black] (root 2) circle (.1cm);
  \draw[black] (root 2) circle (.05cm);
\end{tikzpicture}
```



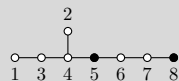
Draw curves between the roots

```
\begin{tikzpicture}
  \dynkin[label]{E}{8}
  \draw[very thick, black!50,-latex]
    (root 3.south) to [out=-45, in=-135] (root 6.south);
\end{tikzpicture}
```



Change marks

```
\begin{tikzpicture}
  \dynkin[mark=o,label]{E}{8};
  \dynkinRootMark{*}{5}
  \dynkinRootMark{*}{8}
\end{tikzpicture}
```

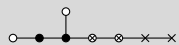


9. MARK LISTS

The package allows a list of root marks instead of a rank:

A mark list

```
\dynkin{E}{oo**ttxx}
```



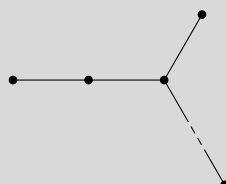
The mark list `oo**ttxx` has one mark for each root: `o`, `o`, \dots , `x`. Roots are listed in the current default ordering. (Careful: in an affine root system, a mark list will *not* contain a mark for root zero.)

Table 4: Classical Lie superalgebras [9]. We need a slightly larger radius parameter to distinguish the tensor product symbols from the solid dots.

		<code>\tikzset{/Dynkin</code> <code>diagram,radius=.07cm}</code>
A_{mn}		<code>\dynkin{A}{ooo.oto.oo}</code>
B_{mn}		<code>\dynkin{B}{ooo.oto.oo}</code>
B_{0n}		<code>\dynkin{B}{ooo.ooo.o*}</code>
C_n		<code>\dynkin{C}{too.oto.oo}</code>
D_{mn}		<code>\dynkin{D}{ooo.oto.oooo}</code>
$D_{21\alpha}$		<code>\dynkin{A}{oto}</code>
F_4		<code>\dynkin{F}{ooot}</code>
G_3		<code>\dynkin[extended,affineMark=t,</code> <code>reverseArrows]{G}{2}</code>

Table 5: Classical Lie superalgebras [9]. Here we see the problem with using the default radius parameter, which is too small for tensor product symbols.

A_{mn}		<code>\dynkin{A}{ooo.oto.oo}</code>
B_{mn}		<code>\dynkin{B}{ooo.oto.oo}</code>
B_{0n}		<code>\dynkin{B}{ooo.ooo.o*}</code>
C_n		<code>\dynkin{C}{too.oto.oo}</code>
D_{mn}		<code>\dynkin{D}{ooo.oto.oooo}</code>
$D_{21\alpha}$		<code>\dynkin{A}{oto}</code>
F_4		<code>\dynkin{F}{ooot}</code>
G_3		<code>\dynkin[extended,affineMark=t,</code> <code>reverseArrows]{G}{2}</code>



The ratio of the lengths of indefinite edges to those of other edges

```
\dynkin[edgeLength = .5cm,%
indefiniteEdgeRatio=3,%
makeIndefiniteEdge={3-5}]
{D}{5}
```

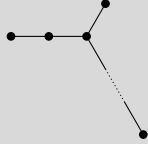


Table 6: Springer's table of indices [21], pp. 320-321, with one form of E_7 corrected

A_n		
A_n		
B_n		
C_n		
D_n		
E_6		<code>\dynkin{E}{*oooo*}</code>
E_6		<code>\dynkin{E}{o*o*oo}</code>
E_6		<code>\dynkin{E}{o*oooo}</code>
E_6		<code>\dynkin{E}{**oooo*}</code>
E_7		<code>\dynkin{E}{*oooooo}</code>
E_7		<code>\dynkin{E}{ooooo*o}</code>
E_7		<code>\dynkin{E}{oooooo*}</code>
E_7		<code>\dynkin{E}{*oooo*o}</code>
E_7		<code>\dynkin{E}{*oooo**}</code>
E_7		<code>\dynkin{E}{*o**o*o}</code>
E_8		<code>\dynkin{E}{*oooooooo}</code>

continued ...

Table 6: ...continued

E_8		<code>\dynkin{E}{oooooooo*}</code>
E_8		<code>\dynkin{E}{*oooooooo*}</code>
E_8		<code>\dynkin{E}{oooooooo**}</code>
E_8		<code>\dynkin{E}{*oooo***}</code>
F_4		<code>\dynkin{F}{ooo*}</code>
D_4		<code>\dynkin{D}{o*oo}</code>

11. PARABOLIC SUBGROUPS

Each set of roots is assigned a number, with each binary digit zero or one to say whether the corresponding root is crossed or not:

The flag variety of pointed lines in projective 3-space is associated to the Dynkin diagram `\dynkin[parabolic=3]{A}{3}`.

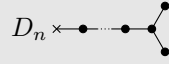
The flag variety of pointed lines in projective 3-space is associated to the Dynkin diagram $\times \rightarrow \bullet$.

Table 7: The Hermitian symmetric spaces

A_n		<code>\dynkin{A}{**.*x*.**}</code> Grassmannian of k -planes in \mathbb{C}^{n+1}
B_n		<code>\dynkin[parabolic=1]{B}{}</code> $(2n - 1)$ -dimensional hyperquadric, i.e. the variety of null lines in \mathbb{C}^{2n+1}
C_n		<code>\dynkin[parabolic=16]{C}{}</code> space of Lagrangian n -planes in \mathbb{C}^{2n}

Table 7: continued ...

Table 7: ...continued


 $\backslash\text{dynkin}[\text{parabolic}=1]\{D\}\{\}$

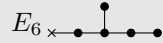
$(2n - 2)$ -dimensional hyperquadric, i.e.
the variety of null lines in \mathbb{C}^{2n}


 $\backslash\text{dynkin}[\text{parabolic}=32]\{D\}\{\}$

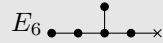
one component of the variety of maximal
dimension null subspaces of \mathbb{C}^{2n}


 $\backslash\text{dynkin}[\text{parabolic}=16]\{D\}\{\}$

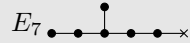
the other component


 $\backslash\text{dynkin}[\text{parabolic}=1]\{E\}\{6\}$

complexified octave projective plane


 $\backslash\text{dynkin}[\text{parabolic}=32]\{E\}\{6\}$

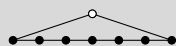
its dual plane


 $\backslash\text{dynkin}[\text{parabolic}=64]\{E\}\{7\}$

the space of null octave 3-planes in oc-
tave 6-space

12. EXTENDED DYNKIN DIAGRAMS

Extended Dynkin diagrams

`\dynkin[extended]{A}{7}`

The extended Dynkin diagrams are also described in the notation of Kac [13] p. 55 as affine untwisted Dynkin diagrams: we extend `\dynkin{A}{7}` to become `\dynkin{A}[1]{7}`:

Extended Dynkin diagrams

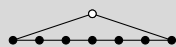
`\dynkin{A}[1]{7}`

Table 8: The Dynkin diagrams of the extended simple root systems

A_1^1		<code>\dynkin[extended]{A}{1}</code>
A_n^1		<code>\dynkin[extended]{A}{}</code>
B_n^1		<code>\dynkin[extended]{B}{}</code>
C_n^1		<code>\dynkin[extended]{C}{}</code>
D_n^1		<code>\dynkin[extended]{D}{}</code>
E_6^1		<code>\dynkin[extended]{E}{6}</code>
E_7^1		<code>\dynkin[extended]{E}{7}</code>
E_8^1		<code>\dynkin[extended]{E}{8}</code>
F_4^1		<code>\dynkin[extended]{F}{4}</code>
G_2^1		<code>\dynkin[extended]{G}{2}</code>

13. AFFINE TWISTED AND UNTWISTED DYNKIN DIAGRAMS

The affine Dynkin diagrams are described in the notation of Kac [13] p. 55:

Affine Dynkin diagrams

$\backslash(A^{\wedge\{1\}}_7=\text{\texttt{\textbackslashdynkin{A}[1]{7}}}, \backslash$
 $E^{\wedge\{2\}}_6=\text{\texttt{\textbackslashdynkin{E}[2]{6}}}, \backslash$
 $D^{\wedge\{3\}}_4=\text{\texttt{\textbackslashdynkin{D}[3]{4}}}\backslash$

$$A_7^{(1)} = \text{diagram}, E_6^{(2)} = \text{diagram}, D_4^{(3)} = \text{diagram}$$

Table 9: The affine Dynkin diagrams

A_1^1		$\text{\texttt{\textbackslashdynkin{A}[1]{1}}}$
A_n^1		$\text{\texttt{\textbackslashdynkin{A}[1]{n}}}$
B_n^1		$\text{\texttt{\textbackslashdynkin{B}[1]{n}}}$
C_n^1		$\text{\texttt{\textbackslashdynkin{C}[1]{n}}}$
D_n^1		$\text{\texttt{\textbackslashdynkin{D}[1]{n}}}$
E_6^1		$\text{\texttt{\textbackslashdynkin{E}[1]{6}}}$
E_7^1		$\text{\texttt{\textbackslashdynkin{E}[1]{7}}}$
E_8^1		$\text{\texttt{\textbackslashdynkin{E}[1]{8}}}$
F_4^1		$\text{\texttt{\textbackslashdynkin{F}[1]{4}}}$
G_2^1		$\text{\texttt{\textbackslashdynkin{G}[1]{2}}}$
A_2^2		$\text{\texttt{\textbackslashdynkin{A}[2]{2}}}$
A_{ev}^2		$\text{\texttt{\textbackslashdynkin{A}[2]{even}}}$
A_{od}^2		$\text{\texttt{\textbackslashdynkin{A}[2]{odd}}}$
D_n^2		$\text{\texttt{\textbackslashdynkin{D}[2]{n}}}$
E_6^2		$\text{\texttt{\textbackslashdynkin{E}[2]{6}}}$
D_4^3		$\text{\texttt{\textbackslashdynkin{D}[3]{4}}}$

Table 10: Some more affine Dynkin diagrams

A_4^2		$\text{\texttt{\textbackslashdynkin{A}[2]{4}}}$
---------	--	---

continued ...

Table 10: ...continued

A_5^2		<code>\dynkin{A}[2]{5}</code>
A_6^2		<code>\dynkin{A}[2]{6}</code>
A_7^2		<code>\dynkin{A}[2]{7}</code>
A_8^2		<code>\dynkin{A}[2]{8}</code>
D_3^2		<code>\dynkin{D}[2]{3}</code>
D_4^2		<code>\dynkin{D}[2]{4}</code>
D_5^2		<code>\dynkin{D}[2]{5}</code>
D_6^2		<code>\dynkin{D}[2]{6}</code>
D_7^2		<code>\dynkin{D}[2]{7}</code>
D_8^2		<code>\dynkin{D}[2]{8}</code>
D_4^3		<code>\dynkin{D}[3]{4}</code>
E_6^2		<code>\dynkin{E}[2]{6}</code>

14. EXTENDED COXETER DIAGRAMS

Extended and Coxeter options together

`\dynkin[extended,Coxeter]{F}{4}`

Table 11: The extended (affine) Coxeter diagrams

A_n		<code>\dynkin[extended,Coxeter]{A}{}</code>
B_n		<code>\dynkin[extended,Coxeter]{B}{}</code>
C_n		<code>\dynkin[extended,Coxeter]{C}{}</code>
D_n		<code>\dynkin[extended,Coxeter]{D}{}</code>
E_6		<code>\dynkin[extended,Coxeter]{E}{6}</code>
E_7		<code>\dynkin[extended,Coxeter]{E}{7}</code>

continued ...

Table 11: ...continued

E_8		<code>\dynkin[extended,Coxeter]{E}{8}</code>
F_4		<code>\dynkin[extended,Coxeter]{F}{4}</code>
G_2		<code>\dynkin[extended,Coxeter]{G}{2}</code>
H_3		<code>\dynkin[extended,Coxeter]{H}{3}</code>
H_4		<code>\dynkin[extended,Coxeter]{H}{4}</code>
I_1		<code>\dynkin[extended,Coxeter]{I}{1}</code>

15. KAC STYLE

We include a style called `Kac` which tries to imitate the style of [13].

Kac style

`\dynkin[Kac]{F}{4}`

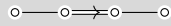


Table 12: The Dynkin diagrams of the extended simple root systems in Kac style. At the moment, it only works on a white background.

A_1^1		<code>\dynkin[extended]{A}{1}</code>
A_n^1		<code>\dynkin[extended]{A}{}</code>
B_n^1		<code>\dynkin[extended]{B}{}</code>
C_n^1		<code>\dynkin[extended]{C}{}</code>
D_n^1		<code>\dynkin[extended]{D}{}</code>
E_6^1		<code>\dynkin[extended]{E}{6}</code>
E_7^1		<code>\dynkin[extended]{E}{7}</code>
E_8^1		<code>\dynkin[extended]{E}{8}</code>

continued ...

Table 12: ...continued

F_4^1		<code>\dynkin[extended]{F}{4}</code>
G_2^1		<code>\dynkin[extended]{G}{2}</code>

16. FOLDED DYNKIN DIAGRAMS

The Dynkin diagrams package has limited support for folding Dynkin diagrams.

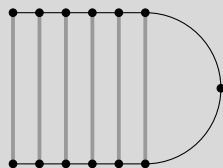
Folding

```
\dynkin[fold]{A}{13}
```



Big fold radius

```
\dynkin[fold,foldradius=1cm]{A}{13}
```



Small fold radius

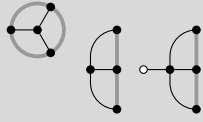
```
\dynkin[fold,foldradius=.2cm]{A}{13}
```



Some Dynkin diagrams have multiple foldings, which we attempt to distinguish (not entirely successfully) by their *ply*: the maximum number of roots folded together. Most diagrams can only allow a 2-ply folding, so `fold` is a synonym for `ply=2`.

3-ply

```
\dynkin[ply=3]{D}{4}
\dynkin[ply=3,foldright]{D}{4}
\dynkin[ply=3]{D}[1]{4}
```

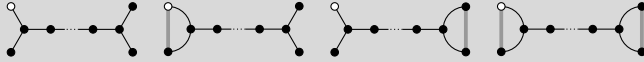


4-ply

 $\backslash\text{dynkin}[\text{ply}=4]\{D\}[1]\{4\}$


The $D_{\ell}^{(1)}$ diagrams can be folded on their left end and separately on their right end:

Left, right and both

 $\backslash\text{dynkin}\{D\}[1]\{\}\ \backslash$
 $\backslash\text{dynkin}[\text{foldleft}]\{D\}[1]\{\}\ \backslash$
 $\backslash\text{dynkin}[\text{foldright}]\{D\}[1]\{\}\ \backslash$
 $\backslash\text{dynkin}[\text{fold}]\{D\}[1]\{\}$


We have to be careful about the 4-ply foldings of $D_{2\ell}^{(1)}$, for which we can have two different patterns, so by default, the package only draws as much as it can without distinguishing the two:

Default $D_{2\ell}^{(1)}$ and the two ways to finish it
 $\backslash\text{begin}\{\text{tikzpicture}\}$
 $\quad\backslash\text{dynkin}[\text{ply}=4]\{D\}[1]\{****.*****.*****\}\%$
 $\backslash\text{end}\{\text{tikzpicture}\}\ \backslash$
 $\backslash\text{begin}\{\text{tikzpicture}\}$
 $\quad\backslash\text{dynkin}[\text{ply}=4]\{D\}[1]\{****.*****.*****\}\%$
 $\quad\backslash\text{dynkinFold}[\text{bend right}=65]\{1\}\{13\}\%$
 $\quad\backslash\text{dynkinFold}[\text{bend right}=65]\{0\}\{14\}\%$
 $\backslash\text{end}\{\text{tikzpicture}\}\ \backslash$
 $\backslash\text{begin}\{\text{tikzpicture}\}$
 $\quad\backslash\text{dynkin}[\text{ply}=4]\{D\}[1]\{****.*****.*****\}\%$
 $\quad\backslash\text{dynkinFold}\{0\}\{1\}\%$
 $\quad\backslash\text{dynkinFold}\{1\}\{13\}\%$
 $\quad\backslash\text{dynkinFold}\{13\}\{14\}\%$

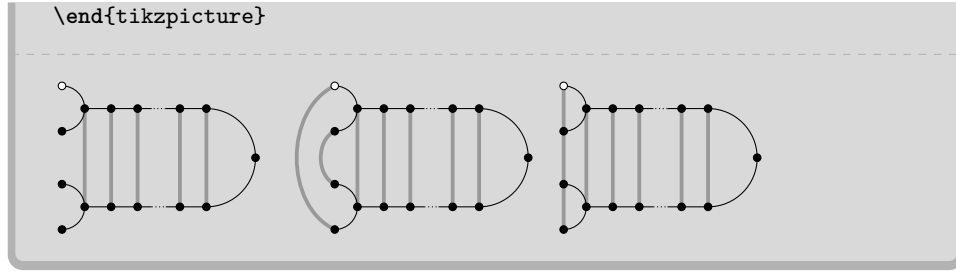


Table 13: Some foldings of Dynkin diagrams. For these diagrams, we want to compare a folding diagram with the diagram that results when we fold it, so it looks best to set `foldradius` and `edgeLength` to equal lengths.

A_3		<code>\dynkin[fold]{A}[0]{3}</code>
C_2		<code>\dynkin{C}[0]{2}</code>
$A_{2\ell}$		<code>\dynkin[fold]{A}{**.****.**}</code>
C_ℓ		<code>\dynkin{C}{}</code>
B_3		<code>\dynkin[fold]{B}[0]{3}</code>
G_2		<code>\dynkin[reverseArrows]{G}[0]{2}</code>
D_4		<code>\dynkin[ply=3,foldright]{D}{4}</code>
G_2		<code>\dynkin{G}{2}</code>
$D_{\ell+1}$		<code>\dynkin[fold]{D}{}</code>
B_ℓ		<code>\dynkin{B}{}</code>
E_6		<code>\dynkin[fold]{E}[0]{6}</code>
F_4		<code>\dynkin[reverseArrows]{F}[0]{4}</code>
A_3^1		<code>\dynkin[ply=4]{A}[1]{3}</code>
A_1^1		<code>\dynkin{A}[1]{1}</code>

continued ...

Table 13: ...continued

$A_{2\ell}^1$		<code>\dynkin[fold]{A}[1]{**.*.....**}</code>
C_ℓ^1		<code>\dynkin{C}[1]{}</code>
B_3^1		<code>\dynkin[ply=3]{B}[1]{3}</code>
A_2^2		<code>\dynkin{A}[2]{2}</code>
B_3^1		<code>\dynkin[ply=2]{B}[1]{3}</code>
G_2^1		<code>\dynkin{G}[1]{2}</code>
B_ℓ^1		<code>\dynkin[fold]{B}[1]{}</code>
D_ℓ^2		<code>\dynkin{D}[2]{}</code>
D_4^1		<code>\dynkin[ply=3]{D}[1]{4}</code>
B_3^1		<code>\dynkin{B}[1]{3}</code>
D_4^1		<code>\dynkin[ply=3]{D}[1]{4}</code>
G_2^1		<code>\dynkin{G}[1]{2}</code>
$D_{\ell+1}^1$		<code>\dynkin[fold]{D}[1]{}</code>
D_ℓ^2		<code>\dynkin{D}[2]{}</code>
$D_{\ell+1}^1$		<code>\dynkin[foldright]{D}[1]{}</code>
B_ℓ^1		<code>\dynkin{B}[1]{}</code>

continued ...

Table 13: ...continued

$D_{2\ell}^1$		<pre>\begin{tikzpicture}[baseline=0pt] \dynkin[ply=4]{D}[1]{****.*****.*****} \dynkinFold{0}{1} \dynkinFold{1}{13} \dynkinFold{13}{14} \end{tikzpicture}</pre>
A_{od}^2		<pre>\dynkin{A}[2]{odd}</pre>
$D_{2\ell}^1$		<pre>\begin{tikzpicture}[baseline=0pt] \dynkin[ply=4]{D}[1]{****.*****.*****} \dynkinFold[bend right=65]{1}{13} \dynkinFold[bend right=65]{0}{14} \end{tikzpicture}</pre>
A_{ev}^2		<pre>\dynkin{A}[2]{even}</pre>
E_6^1		<pre>\dynkin[fold]{E}[1]{6}</pre>
F_4^1		<pre>\dynkin[reverseArrows]{F}[1]{4}</pre>
E_6^1		<pre>\dynkin[ply=3]{E}[1]{6}</pre>
D_4^3		<pre>\dynkin{D}[3]{4}</pre>
E_7^1		<pre>\dynkin[fold]{E}[1]{7}</pre>
E_6^2		<pre>\dynkin{E}[2]{6}</pre>
F_4^1		<pre>\dynkin[fold]{F}[1]{4}</pre>
G_2^1		<pre>\dynkin{G}[1]{2}</pre>

continued ...

Table 13: ...continued

A_{od}^2		<code>\dynkin[odd,fold]{A}[2]{****.***}</code>
A_{ev}^2		<code>\dynkin{A}[2]{even}</code>
D_3^2		<code>\dynkin[fold]{D}[2]{3}</code>
A_2^2		<code>\dynkin{A}[2]{2}</code>

Table 14: Frobenius fixed point subgroups of finite simple groups of Lie type [3] p. 15

$A_{\ell \geq 1}$		<code>\dynkin{A}{}</code>
${}^2A_{\ell \geq 2}$		<code>\dynkin[fold]{A}{}</code>
$B_{\ell \geq 2}$		<code>\dynkin{B}{}</code>
2B_2		<code>\dynkin[fold]{B}{2}</code>
$C_{\ell \geq 3}$		<code>\dynkin{C}{}</code>
$D_{\ell \geq 4}$		<code>\dynkin{D}{}</code>
${}^2D_{\ell \geq 4}$		<code>\dynkin[fold]{D}{}</code>
3D_4		<code>\dynkin[ply=3]{D}{4}</code>
E_6		<code>\dynkin{E}{6}</code>
2E_6		<code>\dynkin[fold]{E}{6}</code>
E_7		<code>\dynkin{E}{7}</code>
E_8		<code>\dynkin{E}{8}</code>
F_4		<code>\dynkin{F}{4}</code>
2F_4		<code>\dynkin[fold]{F}{4}</code>
G_2		<code>\dynkin{G}{2}</code>

continued ...

Table 14: ...continued

 2G_2 `\dynkin[fold]{G}{2}`

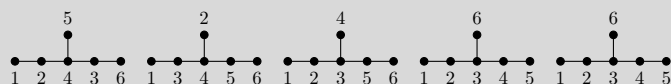
17. ROOT ORDERING

Root ordering

```

\dynkin[label,ordering=Adams]{E}{6}
\dynkin[label,ordering=Bourbaki]{E}{6}
\dynkin[label,ordering=Carter]{E}{6}
\dynkin[label,ordering=Dynkin]{E}{6}
\dynkin[label,ordering=Kac]{E}{6}

```



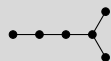
Default is Bourbaki.

	Adams	Bourbaki	Carter	Dynkin	Kac
E_6					
E_7					
E_8					
F_4					
G_2					

18. CONNECTING DYNKIN DIAGRAMS

We can make some sophisticated folded diagrams by drawing multiple diagrams, each with a name:

Name a diagram

`\dynkin[name=Bob]{D}{6}`

We can then connect the two with folding edges:

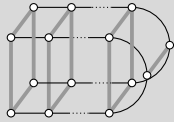
Connect diagrams

```
\begin{tikzpicture}
  \dynkin[name=upper]{A}{3}
  \node (current) at ($(upper root 1)+(0,-.3cm)$) {};
  \dynkin[at=(current),name=lower]{A}{3}
  \begin{scope}[on background layer]
    \foreach \i in {1,...,3}%
    {%
      \draw[/Dynkin diagram/foldStyle]
        ($(upper root \i)$) -- ($(lower
        root \i)$);%
    }%
  \end{scope}
\end{tikzpicture}
```



The following diagrams arise in the Satake diagrams of the pseudo-Riemannian symmetric spaces [1].

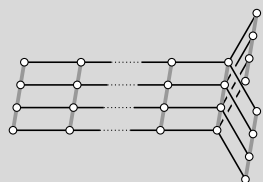
```
\pgfkeys{/Dynkin diagram,edgeLength=.5cm,foldradius=.5cm}
\begin{tikzpicture}
  \dynkin[name=1]{A}{IIIb}
  \node (a) at (.3,.4){};
  \dynkin[name=2,at=(a)]{A}{IIIb}
  \begin{scope}[on background layer]
    \foreach \i in {1,...,7}%
    {%
      \draw[/Dynkin diagram/foldStyle]
        ($(1 root \i)$)
        --
        ($(2 root \i)$);%
    }%
  \end{scope}
\end{tikzpicture}
```



```

\pgfkeys{/Dynkin diagram/edgeLength=.75cm,/Dynkin
diagram/edge/.style={draw=example-color,double=black,very
thick},
}
\begin{tikzpicture}
  \foreach \d in {1,...,4}
  {
    \node (current) at ($(\d*.05,\d*.3)$){};
    \dynkin[name=\d,at=(current)]{D}{oo.oooo}
  }
  \begin{scope}[on background layer]
    \foreach \i in {1,...,6}%
    {%
      \draw[/Dynkin diagram/foldStyle] ($(1 root
\i)$) -- ($(2 root \i)$);%
      \draw[/Dynkin diagram/foldStyle] ($(2 root
\i)$) -- ($(3 root \i)$);%
      \draw[/Dynkin diagram/foldStyle] ($(3 root
\i)$) -- ($(4 root \i)$);%
    }%
  \end{scope}
\end{tikzpicture}

```



19. OTHER EXAMPLES

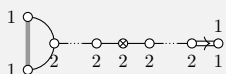
Below we draw the Vogan diagrams of some affine Lie superalgebras [18, 17].

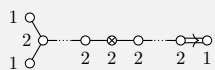
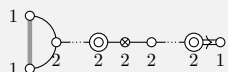
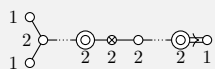
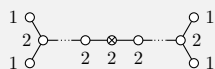
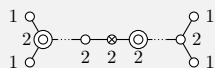
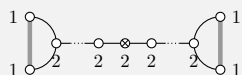
$\mathfrak{sl}(2m|2n)^{(2)}$

```

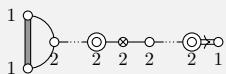
\begin{tikzpicture}
  \dynkin[ply=2,label]{B}{1}{oo.oto.oo}
  \dynkinLabelRoot*{7}{1}
\end{tikzpicture}

```

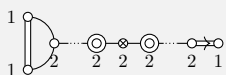


$\backslash\text{dynkin}[\text{label}]\{\text{B}\}[1]\{\text{oo.oto.oo}\}$

 $\backslash\text{dynkin}[\text{ply}=2,\text{label}]\{\text{B}\}[1]\{\text{oo.Oto.Oo}\}$

 $\backslash\text{dynkin}[\text{label}]\{\text{B}\}[1]\{\text{oo.Oto.Oo}\}$

 $\backslash\text{dynkin}[\text{label}]\{\text{D}\}[1]\{\text{oo.oto.ooo}\}$

 $\backslash\text{dynkin}[\text{label}]\{\text{D}\}[1]\{\text{oO.otO.ooo}\}$

 $\backslash\text{dynkin}[\text{label},\text{fold}]\{\text{D}\}[1]\{\text{oo.oto.ooo}\}$


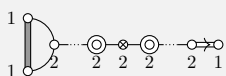

```
\dynkin[ply=2,label,doubleFold]{B}[1]{oo.Oto.Oo}
```



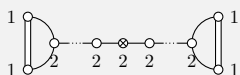
```
\dynkin[ply=2,label,doubleEdges]{B}[1]{oo.OtO.Oo}
```



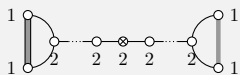
```
\dynkin[ply=2,label,doubleFold]{B}[1]{oo.OtO.Oo}
```


 $\mathfrak{sl}(2|2n)^{(2)}$

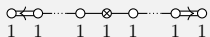
```
\dynkin[ply=2,label,doubleEdges]{D}[1]{oo.Oto.Ooo}
```



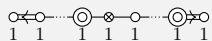
```
\dynkin[ply=2,label,doubleFoldLeft]{D}[1]{oo.Oto.Ooo}
```


 $\mathfrak{osp}(2m|2n)^{(2)}$

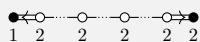
```
\dynkin[label,labelMacro/.code={1}]{D}[2]{o.Oto.Oo}
```



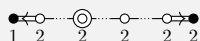
```
\dynkin[label,labelMacro/.code={1}]{D}[2]{o.Oto.Oo}
```


 $\mathfrak{osp}(2|2n)^{(2)}$

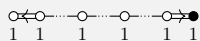
```
\dynkin[label,labelMacro/.code=\lablIt{#1},
  affineMark=*]
{D}[2]{o.o.o.o*}
```



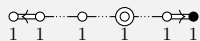
```
\dynkin[label,labelMacro/.code=\lablIt{#1},
  affineMark=*]
{D}[2]{o.O.o.o*}
```


 $\mathfrak{sl}(1|2n+1)^4$

```
\dynkin[label,labelMacro/.code={1}]{D}[2]{o.o.o.o*}
```



```
\dynkin[label,labelMacro/.code={1}]{D}[2]{o.o.O.o*}
```



A^1

```

\begin{tikzpicture}
  \dynkin[name=upper]{A}{oo.t.oo}
  \node (Dynkin current) at (upper root 1){};
  \dynkinSouth
  \dynkin[at=(Dynkin
current),name=lower]{A}{oo.t.oo}
  \begin{scope}[on background layer]
    \foreach \i in {1,...,5}{
      \draw[/Dynkin diagram/foldStyle]
        ($(\text{upper root \i})$) --
        ($(\text{lower root \i})$);
    }
  \end{scope}
\end{tikzpicture}

```



```

\dynkin[fold]{A}[1]{oo.t.ooooo.t.oo}

```



```

\dynkin[fold,affineMark=t]{A}[1]{oo.o.ootoo.o.oo}

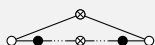
```



```

\dynkin[affineMark=t]{A}[1]{o*.t.*o}

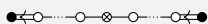
```

 B^1

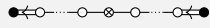
```

\dynkin[affineMark=*]{A}[2]{o.oto.o*}

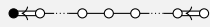
```



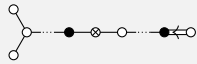
```
\dynkin[affineMark=*]{A}[2]{o.oto.o*}
```



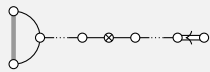
```
\dynkin[affineMark=*]{A}[2]{o.ooo.oo}
```



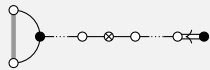
```
\dynkin[odd]{A}[2]{oo.*to.*o}
```



```
\dynkin[odd,fold]{A}[2]{oo.oto.oo}
```



```
\dynkin[odd,fold]{A}[2]{o*.oto.o*}
```


 D^1

```
\dynkin{D}{otoo}
```



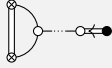
```
\dynkin{D}{ot*o}
```



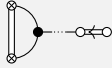
```
\dynkin[fold]{D}{otoo}
```


 C^1

```
\dynkin[doubleEdges,fold,affineMark=t,odd]{A}[2]{to.o*}
```



```
\dynkin[doubleEdges,fold,affineMark=t,odd]{A}[2]{t*.oo}
```


 F^1

```
\begin{tikzpicture}%
  \dynkin{A}{oto*}%
  \dynkinQuadrupleEdge{1}{2}%
  \dynkinTripleEdge{4}{3}%
\end{tikzpicture}%
```



```
\begin{tikzpicture}%
  \dynkin{A}{*too}%
  \dynkinQuadrupleEdge{1}{2}%
  \dynkinTripleEdge{4}{3}%
\end{tikzpicture}%
```





\mathfrak{g}	diagram	V	Δ	α_i
A_n		$\mathbb{R}^{n+1} / \langle \sum e_j \rangle$	$e_i - e_j$	$e_i - e_{i+1}$
B_n		\mathbb{R}^n	$\pm e_i, \pm e_i \pm e_j, i \neq j$	$e_i - e_{i+1}, e_n$
C_n		\mathbb{R}^n	$\pm 2e_i, \pm e_i \pm e_j, i \neq j$	$e_i - e_{i+1}, 2e_n$
D_r		\mathbb{R}^n	$\pm e_i \pm e_j, i \neq j$	$\begin{cases} e_i - e_{i+1}, & i \leq n-2 \\ e_{n-1} + e_n \end{cases}$

\mathfrak{g}	diagram	V	Δ	α_i
E_8		\mathbb{R}^8	$\begin{cases} \pm 2e_i \pm 2e_j, & i \neq j, \\ \sum_i (-1)^{m_i} e_i, & \sum m_i \text{ even} \end{cases}$	$\begin{cases} 2e_1 - 2e_2, \\ 2e_2 - 2e_3, \\ 2e_3 - 2e_4, \\ 2e_4 - 2e_5, \\ 2e_5 - 2e_6, \\ 2e_6 + 2e_7, \\ -\sum e_j, \\ 2e_6 - 2e_7 \end{cases}$
E_7		$\mathbb{R}^8 / \langle e_1 - e_2 \rangle$	quotient of E_8	quotient of E_8
E_6		$\mathbb{R}^8 / \langle e_1 - e_2, e_2 - e_3 \rangle$	quotient of E_8	quotient of E_8
F_4		\mathbb{R}^4	$\begin{cases} \pm 2e_i, \\ \pm 2e_i \pm 2e_j, i \neq j, \\ \pm e_1 \pm e_2 \pm e_3 \pm e_4 \end{cases}$	$\begin{cases} 2e_2 - 2e_3, \\ 2e_3 - 2e_4, \\ 2e_4, \\ e_1 - e_2 - e_3 - e_4 \end{cases}$
G_2		$\mathbb{R}^3 / \langle \sum e_j \rangle$	$\begin{cases} \pm(1, -1, 0), \\ \pm(-1, 0, 1), \\ \pm(0, -1, 1), \\ \pm(2, -1, -1), \\ \pm(1, -2, 1), \\ \pm(-1, -1, 2) \end{cases}$	$\begin{cases} (-1, 0, 1), \\ (2, -1, -1) \end{cases}$

20. SYNTAX

The syntax is `\dynkin[<options>]{<letter>}[<twisted rank>]{<rank>}` where `<letter>` is A, B, C, D, E, F or G, the family of root system for the Dynkin diagram, `<twisted rank>` is 0, 1, 2, 3 (default is 0) representing:

- 0 finite root system
- 1 affine extended root system, i.e. of type ⁽¹⁾
- 2 affine twisted root system of type ⁽²⁾
- 3 affine twisted root system of type ⁽³⁾

and `<rank>` is

- (1) an integer representing the rank or
- (2) blank to represent an indefinite rank or
- (3) the name of a Satake diagram as in section 4.

21. OPTIONS

```
text/.style = <TikZ style data>,
default : scale=.7
          Style for any labels on the roots.
name = <string>,
default : anonymous
```

continued ...

Table 17: ...continued

	A name for the Dynkin diagram, with <code>anonymous</code> treated as a blank; see section 18.
<code>parabolic = <integer></code> , default : 0	A parabolic subgroup with specified integer, where the integer is computed as $n = \sum 2^{i-1} a_i$, $a_i = 0$ or 1, to say that root i is crossed, i.e. a noncompact root.
<code>radius = <number>cm</code> , default : .05cm	size of the dots and of the crosses in the Dynkin diagram
<code>edgeLength = <number>cm</code> , default : .35cm	distance between nodes in the Dynkin diagram
<code>edge/.style = TikZ style data</code> , default : <code>thin</code>	style of edges in the Dynkin diagram
<code>mark = <o,0,t,x,X,*></code> , default : <code>*</code>	default root mark
<code>affineMark = o,0,t,x,X,*</code> , default : <code>*</code>	default root mark for root zero in an affine Dynkin diagram
<code>label = true or false</code> , default : <code>false</code>	whether to label the roots according to the current labelling scheme.
<code>labelMacro = <1-parameter TeX macro></code> , default : <code>#1</code>	the current labelling scheme.
<code>makeIndefiniteEdge = <edge pair i-j or list of such></code> , default : <code>{}</code>	edge pair or list of edge pairs to treat as having indefinitely many roots on them.
<code>indefiniteEdgeRatio = <float></code> , default : 1.6	ratio of indefinite edge lengths to other edge lengths.
<code>indefiniteEdge/.style = <TikZ style data></code> , default : <code>draw=black,fill=white,thin,densely dotted</code>	style of the dotted or dashed middle third of each indefinite edge.
<code>arrows = <true or false></code> , default : <code>true</code>	whether to draw the arrows that arise along the edges.
<code>reverseArrows = <true or false></code> , default : <code>true</code>	whether to reverse the direction of the arrows that arise along the edges.
<code>fold = <true or false></code> ,	

continued ...

Table 17: ...continued

```

default : true
    whether, when drawing Dynkin diagrams, to draw them 2-ply.
ply = ⟨0,1,2,3,4⟩,
default : 0
    how many roots get folded together, at most.
foldleft = ⟨true or false⟩,
default : true
    whether to fold the roots on the left side of a Dynkin diagram.
foldright = ⟨true or false⟩,
default : true
    whether to fold the roots on the right side of a Dynkin diagram.
foldradius = ⟨length⟩,
default : .3cm
    the radius of circular arcs used in curved edges of folded Dynkin
    diagrams.
foldStyle = ⟨TikZ style data⟩,
default : draw=black!40,fill=none,line width=radius
    when drawing folded diagrams, style for the fold indicators.
*/.style = ⟨TikZ style data⟩,
default : draw=black,fill=black
    style for roots like •
o/.style = ⟨TikZ style data⟩,
default : draw=black,fill=black
    style for roots like ◦
O/.style = ⟨TikZ style data⟩,
default : draw=black,fill=black
    style for roots like ⊙
t/.style = ⟨TikZ style data⟩,
default : draw=black,fill=black
    style for roots like ⊗
x/.style = ⟨TikZ style data⟩,
default : draw=black
    style for roots like ×
X/.style = ⟨TikZ style data⟩,
default : draw=black,thick
    style for roots like ✕
leftFold/.style = ⟨TikZ style data⟩,
default :
    style to override the fold style when folding roots together on the
    left half of a Dynkin diagram
rightFold/.style = ⟨TikZ style data⟩,
default :
    style to override the fold style when folding roots together on the
    right half of a Dynkin diagram
doubleEdges = ⟨⟩,
default : not set

```

continued ...

Table 17: ...continued

	set to override the <code>fold</code> style when folding roots together in a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows).
<code>doubleFold = $\langle \rangle$,</code>	
<code>default : not set</code>	set to override the <code>fold</code> style when folding roots together in a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows), but filled in solidly.
<code>doubleLeft = $\langle \rangle$,</code>	
<code>default : not set</code>	set to override the <code>fold</code> style when folding roots together at the left side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows).
<code>doubleFoldLeft = $\langle \rangle$,</code>	
<code>default : not set</code>	set to override the <code>fold</code> style when folding roots together at the left side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows), but filled in solidly.
<code>doubleRight = $\langle \rangle$,</code>	
<code>default : not set</code>	set to override the <code>fold</code> style when folding roots together at the right side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows).
<code>doubleFoldRight = $\langle \rangle$,</code>	
<code>default : not set</code>	set to override the <code>fold</code> style when folding roots together at the right side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows), but filled in solidly.
<code>arrowColor = $\langle \rangle$,</code>	
<code>default : black</code>	set to override the default color for the arrows in nonsimply laced Dynkin diagrams.
<code>Coxeter = \langletrue or false\rangle,</code>	
<code>default : false</code>	whether to draw a Coxeter diagram, rather than a Dynkin diagram.
<code>ordering = \langleAdams, Bourbaki, Carter, Dynkin, Kac\rangle,</code>	
<code>default : Bourbaki</code>	which ordering of the roots to use in exceptional root systems as in section 17.

All other options are passed to TikZ.

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