THE DYNKIN DIAGRAMS PACKAGE
VERSION 3.1415926535

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Contents

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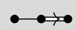
1. Quick introduction

Load the Dynkin diagram package (see options below)

```
\documentclass{amsart}
\usepackage{dynkin-diagrams}
\begin{document}
The Dynkin diagram of \(B_3\) is \dynkin B3.
\end{document}
```

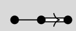
Invoke it

```
The Dynkin diagram of \(B_3\) is \dynkin B3.
```

The Dynkin diagram of B_3 is .

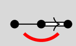
Inside a TikZ statement

```
The Dynkin diagram of \(B_3\) is
\tikz \dynkin B3;
```

The Dynkin diagram of B_3 is 

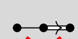
Inside a Dynkin diagram environment

```
The Dynkin diagram of \(B_3\) is
\begin{dynkinDiagram}B3
\draw[very thick,red] (root 1) to [out=-45, in=-135] (root 3);
\end{dynkinDiagram}
```

The Dynkin diagram of B_3 is 

Inside a TikZ environment

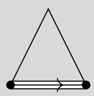
```
Baseline controls vertical alignment:
the Dynkin diagram of \(B_3\) is
\begin{tikzpicture}[baseline=(origin.base)]
\dynkin B3
\draw[very thick,red] (root 1) to [out=-45, in=-135] (root 3);
\end{tikzpicture}
```

Baseline controls vertical alignment: the Dynkin diagram of B_3 is 

In a TikZ picture, you might need to kill the default vertical shift (needed to allow inline Dynkin diagrams):

Inside TikZ pictures

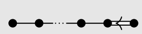
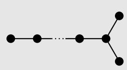
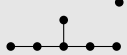
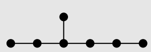
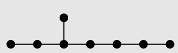
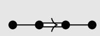
```
\begin{tikzpicture}
\draw (0,0) -- (.5,1) -- (1,0);
\dynkin[vertical shift=0,edge length=1cm]G2
\end{tikzpicture}
```



Indefinite rank Dynkin diagrams

```
\dynkin B{}
```

Table 1: The Dynkin diagrams of the reduced simple root systems [3] pp. 265–290, plates I–IX

A_n		`\dynkin A{}`
C_n		`\dynkin C{}`
D_n		`\dynkin D{}`
E_6		`\dynkin E6`
E_7		`\dynkin E7`
E_8		`\dynkin E8`
F_4		`\dynkin F4`
G_2		`\dynkin G2`

2. Set options globally

Most options set globally . . .

```
\pgfkeys{/Dynkin diagram,edge length=.5cm,fold radius=.5cm,
indefinite edge/.style={
   draw=black,fill=white,thin,densely dashed}}
```

You can also pass options to the package in `\usepackage`. *Danger:* spaces in option names are replaced with hyphens: `edge length=1cm` is `edge-length=1cm` as a global option; moreover you should drop the extension `/.style` on any option with spaces in its name (but not otherwise). For example,

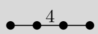
. . . or pass global options to the package

```
\usepackage[
    ordering=Kac,
    edge/.style=blue,
indefinite-edge={draw=green,fill=white,densely dashed},
indefinite-edge-ratio=5,
    mark=o,
    root-radius=.06cm]
    {dynkin-diagrams}
```

3. Coxeter diagrams

Coxeter diagram option

```
\dynkin[Coxeter]{F}{4}
```



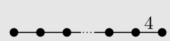
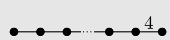
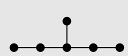
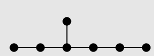
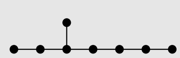
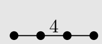
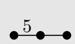
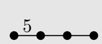
gonality option for G_2 and I_n Coxeter diagrams

```
\(G_2=\dynkin[Coxeter,gonality=n]G2\), \
\(I_n=\dynkin[Coxeter,gonality=n]I{}\)
```

$G_2 =$ 

, $I_n =$ 

Table 2: The Coxeter diagrams of the simple reflection groups

A_n		\dynkin[Coxeter]A{}
B_n		\dynkin[Coxeter]B{}
C_n		\dynkin[Coxeter]C{}
E_6		\dynkin[Coxeter]E6
E_7		\dynkin[Coxeter]E7
E_8		\dynkin[Coxeter]E8
F_4		\dynkin[Coxeter]F4
G_2		\dynkin[Coxeter,gonality=n]G2
H_3		\dynkin[Coxeter]H3
H_4		\dynkin[Coxeter]H4

continued . . .

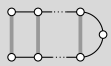
Table 2: ...continued

I_n		\dynkin[Coxeter,gonality=n]I{}

4. Satake diagrams

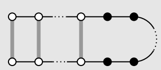
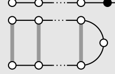
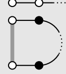
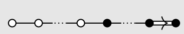
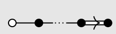
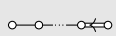
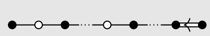
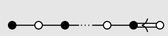
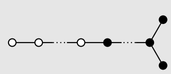
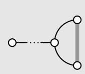
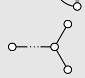
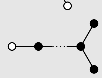
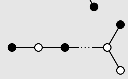
Satake diagrams use the standard name instead of a rank

```
\(A_{IIIb}=\dynkin A{IIIb}\)
```

$A_{IIIb} =$ 

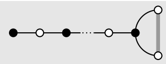
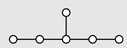
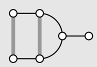
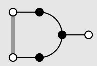
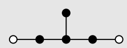
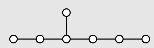
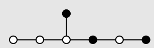
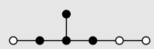
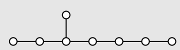
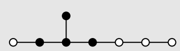
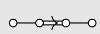
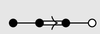
We use a solid gray bar to denote the folding of a Dynkin diagram, rather than the usual double arrow, since the diagrams turn out simpler and easier to read.

Table 3: The Satake diagrams of the real simple Lie algebras [13] p. 532–534

A_I		\dynkin AI
A_{II}		\dynkin A{II}
A_{IIIa}		\dynkin A{IIIa}
A_{IIIb}		\dynkin A{IIIb}
A_{IV}		\dynkin A{IV}
B_I		\dynkin BI
B_{II}		\dynkin B{II}
C_I		\dynkin CI
C_{IIa}		\dynkin C{IIa}
C_{IIb}		\dynkin C{IIb}
D_{Ia}		\dynkin D{Ia}
D_{Ib}		\dynkin D{Ib}
D_{Ic}		\dynkin D{Ic}
D_{II}		\dynkin D{II}
D_{IIIa}		\dynkin D{IIIa}

continued ...

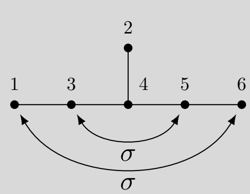
Table 3: . . . continued

D_{IIIb}		`\dynkin D{IIIb}`
E_I		`\dynkin EI`
E_{II}		`\dynkin E{II}`
E_{III}		`\dynkin E{III}`
E_{IV}		`\dynkin E{IV}`
E_V		`\dynkin EV`
E_{VI}		`\dynkin E{VI}`
E_{VII}		`\dynkin E{VII}`
E_{VIII}		`\dynkin E{VIII}`
E_{IX}		`\dynkin E{IX}`
F_I		`\dynkin FI`
F_{II}		`\dynkin F{II}`
G_I		`\dynkin GI`

5. How to fold

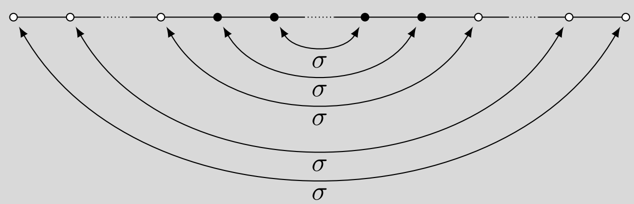
If you don't like the solid gray "folding bar", most people use arrows. Here is E_{II}

```
\newcommand{\invol}[2]{\draw[latex-latex] (root #1) to
[out=-60,in=-120] node[midway,below]{$\sigma$} (root #2);}
\begin{dynkinDiagram}[edge length=.75cm,labels*={1,...,6}]E6
\invol 16\invol 35
\end{dynkinDiagram}
```



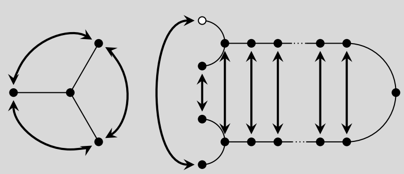
The double arrows for A_{IIIa} are big

```
\newcommand{\invol}[2]{\draw[latex-latex] (root #1) to
[out=-60,in=-120] node[midway,below]{$\sigma$} (root #2);}
\begin{dynkinDiagram}[edge length=.75cm]{A}{oo.o**.**o.oo}
\invol 1{10}\invol 29\invol 38\invol 47\invol 56
\end{dynkinDiagram}
```



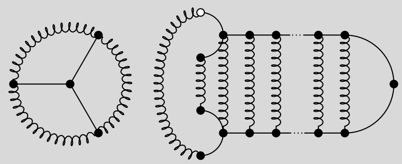
If you don’t like the solid gray “folding bar”, most people use arrows . . .

```
\tikzset{/Dynkin diagram/fold style/.style={stealth-stealth,thick,
shorten <=1mm,shorten >=1mm,}}
\dynkin[ply=3,edge length=.75cm]D4
\begin{dynkinDiagram}[ply=4]D[1]%
{****.*****.*****}
	\dynkinFold 1{13}
	\dynkinFold[bend right=90] 0{14}
\end{dynkinDiagram}
```



. . . but you could try springs pulling roots together

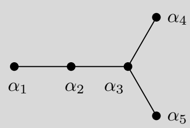
```
\tikzset{/Dynkin diagram/fold style/.style=
{decorate,decoration={name=coil,aspect=0.5,
segment length=1mm,amplitude=.6mm}}}
\dynkin[ply=3,edge length=.75cm]D4
\begin{dynkinDiagram}[ply=4]D[1]%
{****.*****.*****}
	\dynkinFold 1{13}
	\dynkinFold[bend right=90]0{14}
\end{dynkinDiagram}
```



6. Labels for the roots

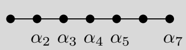
Make a macro to assign labels to roots

```
\dynkin[label,label macro/.code={\alpha_{\drlap{#1}}},edge
    length=.75cm]D5
```



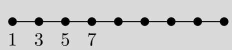
Labelling several roots

```
\dynkin[labels={,2,...,5,,7},label macro/.code={\alpha_{\drlap#1}}]A7
```



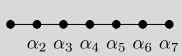
The foreach notation I

```
\dynkin[labels={1,3,...,7},]A9
```



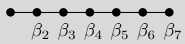
The foreach notation II

```
\dynkin[labels={,\alpha_2,\alpha_...,\alpha_7},]A7
```



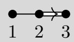
The foreach notation III

```
\dynkin[label macro/.code={\beta_{\drlap{#1}}},labels={,2,...,7},]A7
```



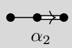
Label the roots individually by root number

```
\dynkin[label]B3
```



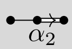
Label a single root

```
\begin{dynkinDiagram}B3
\dynkinLabelRoot 2{\alpha_{\drlap{2}}}
\end{dynkinDiagram}
```



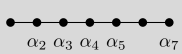
Access root labels via TikZ

```
\begin{dynkinDiagram}B3
\node[below] at (root 2) {\(\alpha_{\drlap{2}}\)};
\end{dynkinDiagram}
```



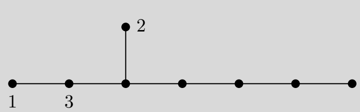
Commands to label several roots

```
\begin{dynkinDiagram}A7
\dynkinLabelRoots{,\alpha_2,\alpha_3,\alpha_4,\alpha_5,,\alpha_7}
\end{dynkinDiagram}
```



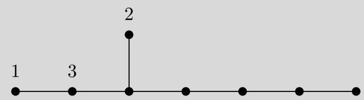
The labels have default locations, mostly below roots

```
\dynkin[edge length=.75cm,labels={1,2,3}]E8
```



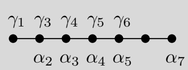
The starred form flips labels to alternate locations, mostly above roots

```
\dynkin[edge length=.75cm,labels*={1,2,3}]E8
```



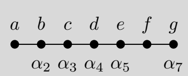
Labelling several roots and alternates

```
\dynkin[%
label macro/.code={\alpha_{\drlap{#1}}},
label macro*/.code={\gamma_{\drlap{#1}}},
labels={,2,...,5,,7},
labels*={1,3,4,5,6}]A7
```



Commands to label several roots

```
\begin{dynkinDiagram}A7
\dynkinLabelRoots{,\alpha_2,\alpha_3,\alpha_4,\alpha_5,,\alpha_7}
\dynkinLabelRoots*{a,b,c,d,e,f,g}
\end{dynkinDiagram}
```

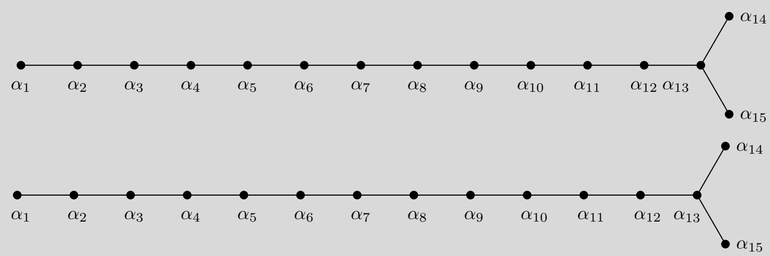


7. Label subscripts

Note the slight improvement that `\drlap` makes: the labels are centered on the middle of the letter α, ignoring the space taken up by the subscripts, using the `mathtools` command `\mathrlap`, but only for labels which are *not* placed to the left or right of a root.

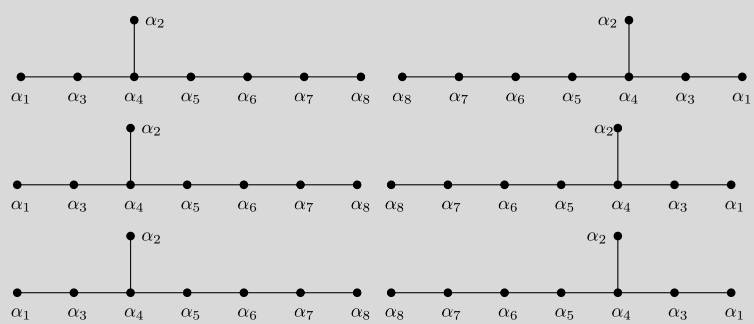
Label subscript spacing

```
\dynkin[label,label macro/.code={\alpha_{#1}},
      edge length=.75cm]D{15}
\par\noindent{}%
\dynkin[label,label macro/.code={\alpha_{\drlap{#1}}},
      edge length=.75cm]D{15}
```



Label subscript spacing

```
\dynkin[label,label macro/.code={\alpha_{#1}},
        edge length=.75cm]E8
\dynkin[label,label macro/.code={\alpha_{#1}},backwards,
        edge length=.75cm]E8
\par\noindent{}%
\dynkin[label,label macro/.code={\alpha_{\mathrlap{#1}}},
        edge length=.75cm]E8
\dynkin[label,label macro/.code={\alpha_{\mathrlap{#1}}},backwards,
        edge length=.75cm]E8
\par\noindent{}%
\dynkin[label,label macro/.code={\alpha_{\drlap{#1}}},
        edge length=.75cm]E8
\dynkin[label,label macro/.code={\alpha_{\drlap{#1}}},backwards,
        edge length=.75cm]E8
```

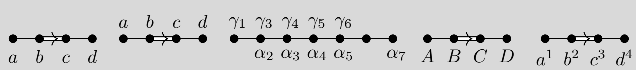


8. Height and depth of labels

Labels are set with default maximum height the height of the character b, and default maximum depth the depth of the character g. To change these, set `label height` and `label depth`:

Change height and depth of characters

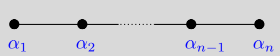
```
\dynkin[labels={a,b,c,d},label height=d,label depth=d]F4
\dynkin[labels*={a,b,c,d},label height=d,label depth=d]F4
\dynkin[%
label macro/.code={\alpha_{\drlap{#1}}},
label macro*/.code={\gamma_{\drlap{#1}}},
label height=$\alpha_1$,
label depth=$\alpha_1$,
labels={,2,...,5,,7},
labels*={1,3,4,5,6}]A7
\dynkin[labels={A,B,C,D},label height=$A$,label depth=$A$]F4
\dynkin[labels={a^1,b^2,c^3,d^4},label height=$X^X$]F4
```



9. Text style for the labels

Use a text style: big and blue

```
\begin{dynkinDiagram}[text style={scale=1.2,blue},
edge length=.75cm,
labels={1,2,n-1,n},
label macro/.code={\alpha_{\drlap{#1}}}
]A{}
\end{dynkinDiagram}
```



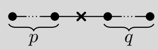
Use a text style; font selection is in the label macro

```
\begin{dynkinDiagram}[text style={scale=1.2,blue},
edge length=.75cm,
labels={1,2,n-1,n},
label macro/.code={\mathbb{A}_{\drlap{#1}}}]A{}
\end{dynkinDiagram}
```

10. Bracing roots

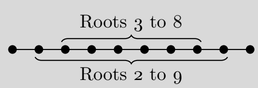
Bracing roots

```
\begin{dynkinDiagram}A{*.*x*.*}
\dynkinBrace[p]12
\dynkinBrace[q]45
\end{dynkinDiagram}
```



Bracing roots, and a starred form

```
\begin{dynkinDiagram}A{10}
\dynkinBrace[\text{Roots 2 to 9}]29
\dynkinBrace*[\text{Roots 3 to 8}]38
\end{dynkinDiagram}
```



Bracing roots

```
\newcommand\circleRoot[1]{\draw (root #1) circle (3pt);}
\begin{dynkinDiagram}A{**.***.***.***.***.**}
\circleRoot 4\circleRoot 7\circleRoot 10\circleRoot 13
\dynkinBrace[y-1]13
\dynkinBrace[z-1]56
\dynkinBrace[t-1]{11}{12}
\dynkinBrace[x-1]{14}{16}
\end{dynkinDiagram}
```

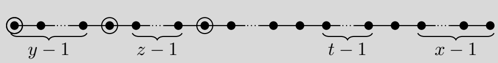
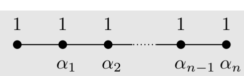
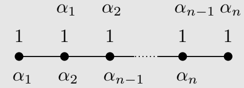


Table 4: Dynkin diagrams from Euler products [17]

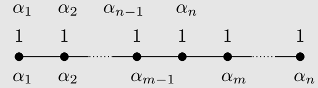
i.



ii.

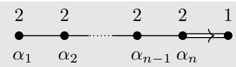
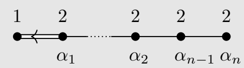
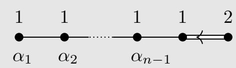
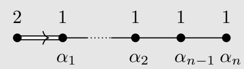
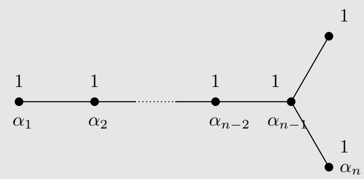
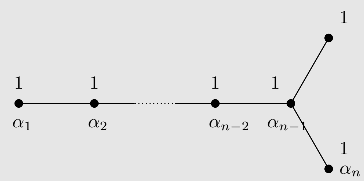
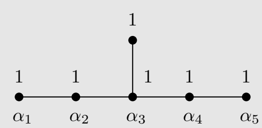
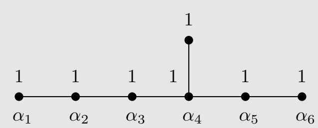
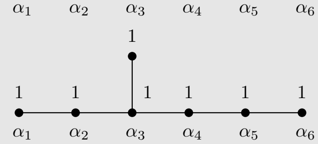
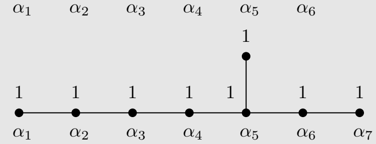
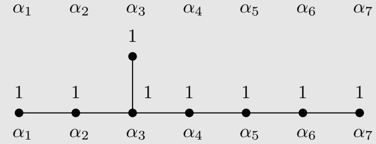
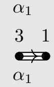
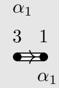
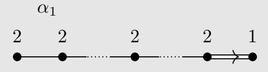


iii.



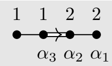
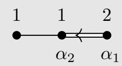
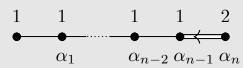
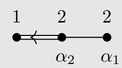
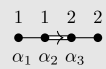
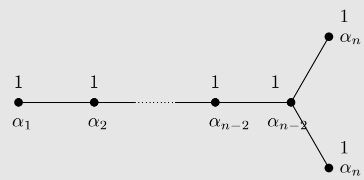
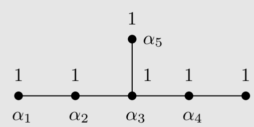
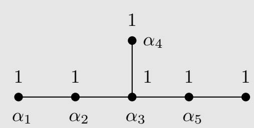
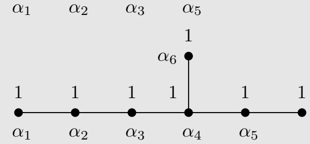
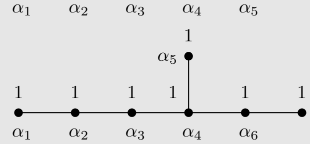
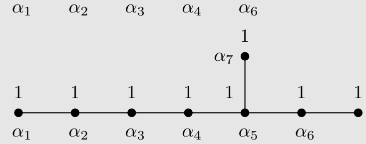
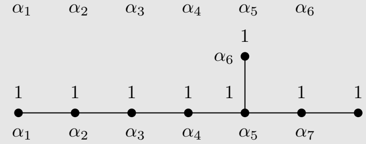
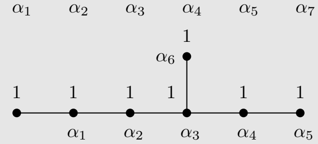
continued . . .

Table 4: . . . continued

iv.	
v.	
vi.	
vii.	
viii.	
ix.	
x.	
xi.	
xii.	
xiii.	
xiv.	
xv.	
xvi.	
xvii.	

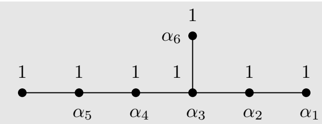
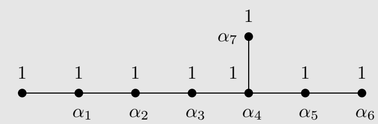
continued . . .

Table 4: ...continued

xviii.	
xix.	
xx.	
xxi.	
xxii.	
xxiii.	
xxiv.	
xxv.	
xxvi.	
xxvii.	
xxviii.	
xxix.	
xxx.	

continued ...

Table 4: …continued

xxxi.	
xxxii.	

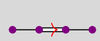
```
\tikzset{/Dynkin diagram,ordering=Dynkin,label macro/.code={\alpha_{\drlap{#1}}}}
\newcounter{EPNo}
\setcounter{EPNo}{0}
\NewDocumentCommand\EP{smmmm}%
{%
\stepcounter{EPNo}\roman{EPNo}. &%
\def\eL{.6cm}%
\IfStrEqCase{#2}%
{%
D{\gdef\eL{1cm}}%
E{\gdef\eL{.75cm}}%
F{\gdef\eL{.35cm}}%
G{\gdef\eL{.35cm}}%
}%
\IfBooleanTF{#1}%
{\dynkin[edge length=\eL,backwards,labels*={#4},labels={#5}]{#2}{#3}}
{\dynkin[edge length=\eL,labels*={#4},labels={#5}]{#2}{#3}}
\\
}%
\renewcommand*\do[1]{\EP#1}%
\begin{longtable}{MM}
\caption{Dynkin diagrams from Euler products \cite{Langlands:1967}}\\
\endfirsthead
\caption{\dots continued}\\
\endhead
\multicolumn{2}{c}{continued \dots}\\
\endfoot
\endlastfoot
\docsvlist{
A{***.**}{1,1,1,1,1}{,1,2,n-1,n},
A{***.**}{1,1,1,1,1}{1,2,n-1,n},
A{**.***.*}{1,1,1,1,1,1}{1,2,m-1,,m,n},
B{**.***}{2,2,2,2,1}{1,2,n-1,n},
*B{***.**}{2,2,2,2,1}{n,n-1,2,1,},
C{**.***}{1,1,1,1,2}{1,2,n-1,},
*C{***.**}{1,1,1,1,2}{n,n-1,2,1,},
D{**.****}{1,1,1,1,1,1}{1,2,n-2,n-1,n},
D{**.****}{1,1,1,1,1,1}{1,2,n-2,n-1,n},
E6{1,1,1,1,1,1}{1,...,5},
*E7{1,1,1,1,1,1,1}{6,...,1},
E7{1,1,1,1,1,1,1}{1,...,6},
```

```
*E8{1,1,1,1,1,1,1,1}{7,...,1},
E8{1,1,1,1,1,1,1,1}{1,...,7},
G2{1,3}{,1},
G2{1,3}{1},
B{**.*.**}{2,2,2,2,1}{,1,2,n-1,n},
F4{1,1,2,2}{,3,2,1},
C3{1,1,2}{,2,1},
C{**.***}{1,1,1,1,2}{,1,n-2,n-1,n},
*B3{2,2,1}{1,2},
F4{1,1,2,2}{1,2,3},
D{**.****}{1,1,1,1,1,1}{1,2,n-2,n-2,n,n},
E6{1,1,1,1,1,1}{1,2,3,4,,5},
E6{1,1,1,1,1,1}{1,2,3,5,,4},
*E7{1,1,1,1,1,1,1}{,5,...,1,6},
*E7{1,1,1,1,1,1,1}{,6,4,3,2,1,5},
*E8{1,1,1,1,1,1,1,1}{,6,...,1,7},
*E8{1,1,1,1,1,1,1,1}{,7,5,4,3,2,1,6},
*E7{1,1,1,1,1,1,1}{5,...,1,,6},
*E7{1,1,1,1,1,1,1}{1,...,5,,6},
*E8{1,1,1,1,1,1,1,1}{6,...,1,,7}}
\end{longtable}
```

11. Style

Colours

```
\dynkin[
    edge/.style={blue!50,thick},
    */.style=blue!50!red,
    arrow color=red]{F}{4}
```



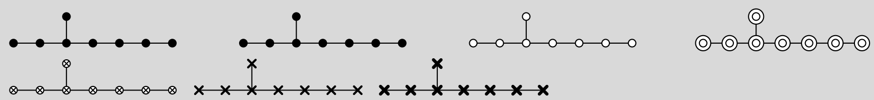
Edge lengths

```
The Dynkin diagram of \(A_3\) is \dynkin[edge length=1.2]A3
```

The Dynkin diagram of A_3 is

Root marks

```
\dynkin E8
\dynkin[mark=*]E8
\dynkin[mark=o]E8
\dynkin[mark=O]E8
\dynkin[mark=t]E8
\dynkin[mark=x]E8
\dynkin[mark=X]E8
```

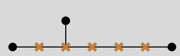


At the moment, you can only use:

- `*` • solid dot
- `o` ∘ hollow circle
- `O` ◎ double hollow circle
- `t` ⊗ tensor root
- `x` × crossed root
- `X` **×** thickly crossed root

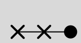
Mark styles

```
The parabolic subgroup \(E_{8,124}\) is
    \dynkin[parabolic=124,x/.style={brown,very thick}]E8
```

The parabolic subgroup $E_{8,124}$ is 

Sizes of root marks

```
\(A_{3,3}\) with big root marks is \dynkin[root
    radius=.08cm,parabolic=3]A3
```

$A_{3,3}$ with big root marks is 

12. Suppress or reverse arrows

Some diagrams have double or triple edges

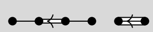
```
\dynkin F4
\dynkin G2
```

Suppress arrows

```
\dynkin[arrows=false]F4
\dynkin[arrows=false]G2
```


Reverse arrows

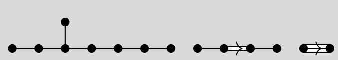
```
\dynkin[reverse arrows]F4
\dynkin[reverse arrows]G2
```



13. Backwards and upside down

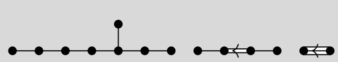
Default

```
\dynkin E8
\dynkin F4
\dynkin G2
```



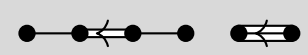
Backwards

```
\dynkin[backwards]E8
\dynkin[backwards]F4
\dynkin[backwards]G2
```



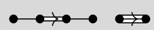
Reverse arrows

```
\dynkin[reverse arrows]F4
\dynkin[reverse arrows]G2
```



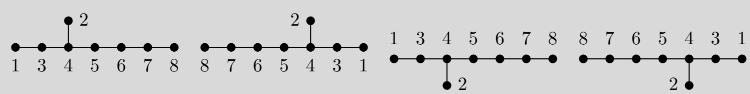
Backwards, reverse arrows

```
\dynkin[backwards,reverse arrows]F4
\dynkin[backwards,reverse arrows]G2
```



Backwards versus upside down

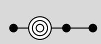
```
\dynkin[label]E8
\dynkin[label,backwards]E8
\dynkin[label,upside down]E8
\dynkin[label,backwards,upside down]E8
```



14. Drawing on top of a Dynkin diagram

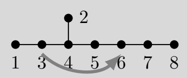
TikZ can access the roots themselves

```
\begin{dynkinDiagram}A4
    \fill[white,draw=black] (root 2) circle (.15cm);
    \fill[white,draw=black] (root 2) circle (.1cm);
    \draw[black] (root 2) circle (.05cm);
\end{dynkinDiagram}
```



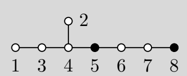
Draw curves between the roots

```
\begin{dynkinDiagram}[label]E8
	\draw[very thick, black!50,-latex]
		(root 3.south) to [out=-45, in=-135] (root 6.south);
\end{dynkinDiagram}
```



Change marks

```
\begin{dynkinDiagram}[mark=o,label]E8
	\dynkinRootMark{*}5
	\dynkinRootMark{*}8
\end{dynkinDiagram}
```

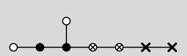


15. Mark lists

The package allows a list of root marks instead of a rank:

A mark list

```
\dynkin E{oo**ttxx}
```



The mark list `oo**ttxx` has one mark for each root: `o`, `o`, . . . , `x`. Roots are listed in the current default ordering. (Careful: in an affine root system, a mark list will *not* contain a mark for root zero.)

If you need to repeat a mark, you can give a *single digit* positive integer to indicate how many times to repeat it.

A mark list with repetitions

```
\dynkin A{x4o3t4}
```

Table 5: Classical Lie superalgebras [10]. We need a slightly larger root radius parameter to distinguish the tensor product symbols from the solid dots.

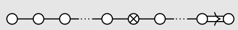
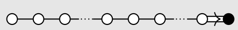
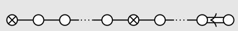
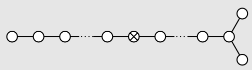
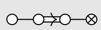
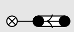
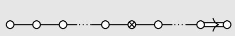
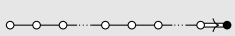
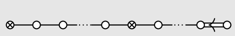
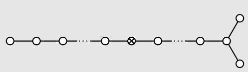
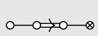
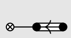
		\tikzset{/Dynkin diagram,root radius=.07cm}
A_{mn}		\dynkin A{o3.oto.oo}
B_{mn}		\dynkin B{o3.oto.oo}
B_{0n}		\dynkin B{o3.o3.o*}
C_n		\dynkin C{too.oto.oo}
D_{mn}		\dynkin D{o3.oto.o4}
$D_{21\alpha}$		\dynkin A{oto}
F_4		\dynkin F{ooot}
G_3		\dynkin[extended,affine mark=t, reverse arrows]G2

Table 6: Classical Lie superalgebras [10]. Here we see the problem with using the default root radius parameter, which is too small for tensor product symbols.

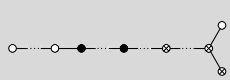
A_{mn}		\dynkin A{o3.oto.oo}
B_{mn}		\dynkin B{o3.oto.oo}
B_{0n}		\dynkin B{o3.o3.o*}
C_n		\dynkin C{too.oto.oo}
D_{mn}		\dynkin D{o3.oto.o4}
$D_{21\alpha}$		\dynkin A{oto}
F_4		\dynkin F{ooot}
G_3		\dynkin[extended,affine mark=t, reverse arrows]G2

16. Indefinite edges

An *indefinite edge* is a dashed edge between two roots, indicating that an indefinite number of roots have been omitted from the Dynkin diagram. In between any two entries in a mark list, place a period to indicate an indefinite edge:

Indefinite edges

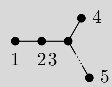
```
\dynkin D{o.o*.*.t.to.t}
```



In certain diagrams, roots may have an edge between them even though they are not subsequent in the ordering. For such rare situations, there is an option:

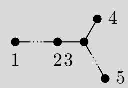
Indefinite edge option

```
\dynkin[make indefinite edge={3-5},label]D5
```



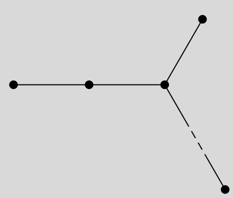
Give a list of edges to become indefinite

```
\dynkin[make indefinite edge/.list={1-2,3-5},label]D5
```



Indefinite edge style

```
\dynkin[indefinite edge/.style={
        draw=black,fill=white,thin,densely dashed},
        edge length=1cm,
        make indefinite edge={3-5}]D5
```



The ratio of the lengths of indefinite edges to those of other edges

```
\dynkin[edge length = .5cm,
        indefinite edge ratio=3,
        make indefinite edge={3-5}]D5
```

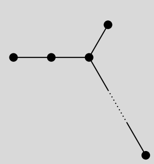
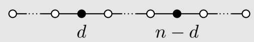
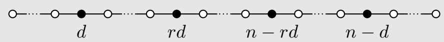
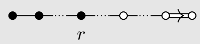
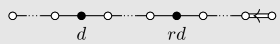
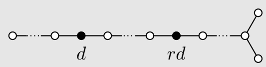
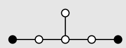
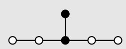
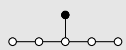
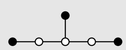
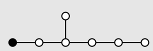
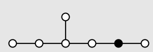
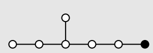
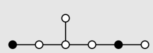
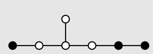
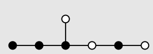
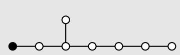
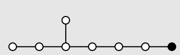
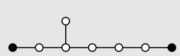
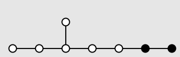
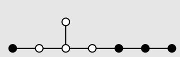
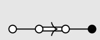
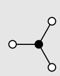


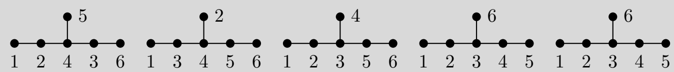
Table 7: Springer's table of indices [24], pp. 320-321, with one form of E_7 corrected

A_n		
A_n		
B_n		
C_n		
D_n		
E_6		`\dynkin E{*oooo*}`
E_6		`\dynkin E{o*o*oo}`
E_6		`\dynkin E{o*oooo}`
E_6		`\dynkin E{**ooo*}`
E_7		`\dynkin E{*oooooo}`
E_7		`\dynkin E{ooooo*o}`
E_7		`\dynkin E{oooooo*}`
E_7		`\dynkin E{*oooo*o}`
E_7		`\dynkin E{*oooo**}`
E_7		`\dynkin E{*o**o*o}`
E_8		`\dynkin E{*ooooooo}`
E_8		`\dynkin E{ooooooo*}`
E_8		`\dynkin E{*oooooo*}`
E_8		`\dynkin E{oooooo**}`
E_8		`\dynkin E{*oooo***}`
F_4		`\dynkin F{ooo*}`
D_4		`\dynkin D{o*oo}`

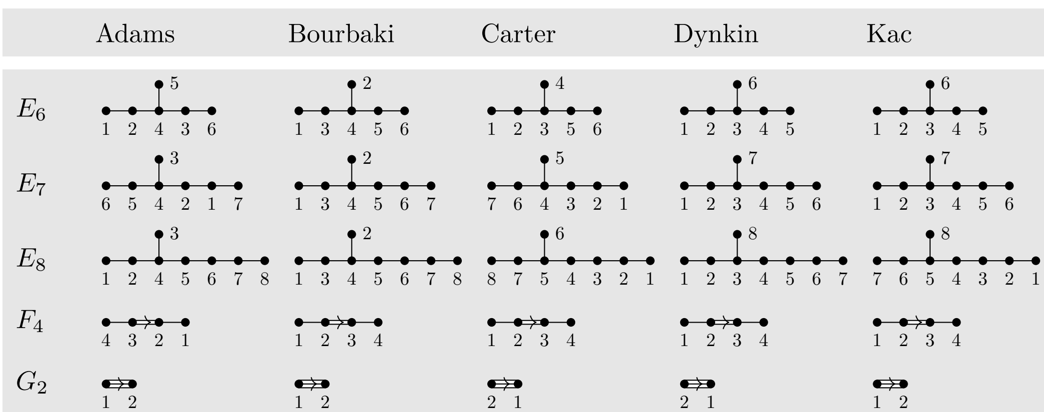
17. Root ordering

Root ordering

```
\dynkin[label,ordering=Adams]E6
\dynkin[label,ordering=Bourbaki]E6
\dynkin[label,ordering=Carter]E6
\dynkin[label,ordering=Dynkin]E6
\dynkin[label,ordering=Kac]E6
```

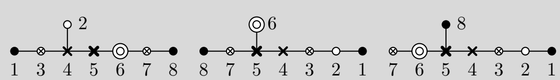


Default is Bourbaki. Sources are Adams [1] p. 56–57, Bourbaki [3] p. pp. 265–290 plates I-IX, Carter [5] p. 540–609, Dynkin [8], Kac [15] p. 43.



The marks are set down in order according to the current root ordering:

```
\dynkin[label]E{*otxXOt*}
\dynkin[label,ordering=Carter]E{*otxXOt*}
\dynkin[label,ordering=Kac]E{*otxXOt*}
```



Convert between orderings

```
\newcount\r
\dynkinOrder E8.Carter::6->Bourbaki.{\r}
```

```
In \(E_8\), root 6 in Carter's ordering is root \the\r{} in Bourbaki's
    ordering.
```

In E_8, root 6 in Carter's ordering is root 2 in Bourbaki's ordering.

18. Parabolic subgroups

Each set of roots is assigned a number, with each binary digit zero or one to say whether the corresponding root is crossed or not:

```
The flag variety of pointed lines in
projective 3-space is associated to
the Dynkin diagram \dynkin[parabolic=3]A3.
```

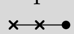
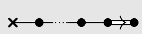
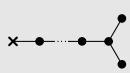
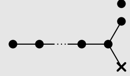
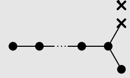
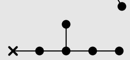
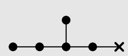
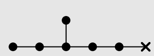
The flag variety of pointed lines in projective 3-space is associated to the Dynkin diagram .

Table 9: The Hermitian symmetric spaces

A_n		Grassmannian of k-planes in $\mathbb{C}^{n+1}$
B_n		$(2n-1)$-dimensional hyperquadric, i.e. the variety of null lines in $\mathbb{C}^{2n+1}$
C_n		space of Lagrangian n-planes in $\mathbb{C}^{2n}$
D_n		$(2n-2)$-dimensional hyperquadric, i.e. the variety of null lines in $\mathbb{C}^{2n}$
D_n		one component of the variety of maximal dimension null subspaces of $\mathbb{C}^{2n}$
D_n		the other component
E_6		complexified octave projective plane
E_6		its dual plane
E_7		the space of null octave 3-planes in octave 6-space

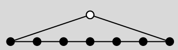
```
\NewDocumentCommand\HSS{mommm}
{#1&\IfNoValueTF{#2}{\dynkin{#3}{#4}}{\dynkin[parabolic=#2]{#3}{#4}}&#5\\}
\renewcommand*{\arraystretch}{1.5}
\begin{longtable}
{>{\columncolor[gray]{.9}}>$l<$>{\columncolor[gray]{.9}}>$l<$>{\columncolor[gray]{.9}}l}
\caption{The Hermitian symmetric spaces}\endfirsthead
\caption{\dots continued}\\ \endhead
\caption{continued \dots}\\ \endfoot
\endlastfoot
\HSS{A_n}A{**.*x*.**}{Grassmannian of $k$-planes in $\C{n+1}$}
\HSS{B_n}[1]B{}{$(2n-1)$-dimensional hyperquadric, i.e. the variety of null lines in $\C{2n+1}$}
\HSS{C_n}[16]C{}{space of Lagrangian $n$-planes in $\C{2n}$}
```

```
\HSS{D_n}[1]D{}{$(2n-2)$-dimensional hyperquadric, i.e. the variety of null lines in $\C{2n}$}
\HSS{D_n}[32]D{}{one component of the variety of maximal dimension null subspaces of $\C{2n}$}
\HSS{D_n}[16]D{}{the other component}
\HSS{E_6}[1]E6{complexified octave projective plane}
\HSS{E_6}[32]E6{its dual plane}
\HSS{E_7}[64]E7{the space of null octave 3-planes in octave 6-space}
\end{longtable}
```

19. Extended Dynkin diagrams

Extended Dynkin diagrams

```
\dynkin[extended]A7
```



The extended Dynkin diagrams are also described in the notation of Kac [15] p. 55 as affine untwisted Dynkin diagrams: we extend `\dynkin{A}{7}` to become `\dynkin A[1]7`:

Extended Dynkin diagrams

```
\dynkin A[1]7
```

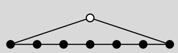
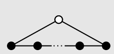
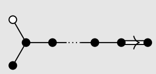
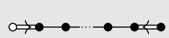
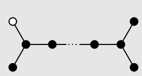
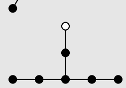
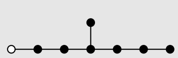
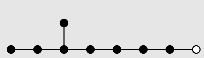
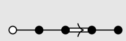
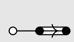


Table 10: The Dynkin diagrams of the extended simple root systems

A_1^1		`\dynkin[extended]A1`
A_n^1		`\dynkin[extended]A{}`
B_n^1		`\dynkin[extended]B{}`
C_n^1		`\dynkin[extended]C{}`
D_n^1		`\dynkin[extended]D{}`
E_6^1		`\dynkin[extended]E6`
E_7^1		`\dynkin[extended]E7`
E_8^1		`\dynkin[extended]E8`
F_4^1		`\dynkin[extended]F4`
G_2^1		`\dynkin[extended]G2`

20. Affine twisted and untwisted Dynkin diagrams

The affine Dynkin diagrams are described in the notation of Kac [15] p. 55:

Affine Dynkin diagrams

```
\(A^{(1)}_7=\dynkin A[1]7, \
E^{(2)}_6=\dynkin E[2]6, \
D^{(3)}_4=\dynkin D[3]4\)
```

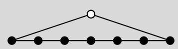
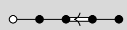
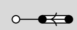
$A^{(1)}_7 =$ , $E^{(2)}_6 =$ , $D^{(3)}_4 =$ 

Table 11: The affine Dynkin diagrams

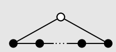
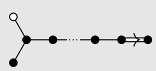
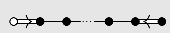
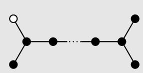
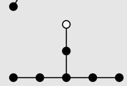
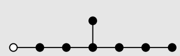
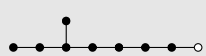
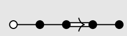
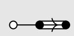
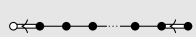
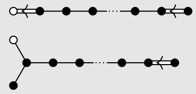
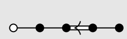
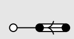
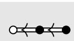
A^1_1		`\dynkin A[1]1`
A^1_n		`\dynkin A[1]{}`
B^1_n		`\dynkin B[1]{}`
C^1_n		`\dynkin C[1]{}`
D^1_n		`\dynkin D[1]{}`
E^1_6		`\dynkin E[1]6`
E^1_7		`\dynkin E[1]7`
E^1_8		`\dynkin E[1]8`
F^1_4		`\dynkin F[1]4`
G^1_2		`\dynkin G[1]2`
A^2_2		`\dynkin A[2]2`
A^2_{ev}		`\dynkin A[2]{even}`
A^2_{od}		`\dynkin A[2]{odd}`
D^2_n		`\dynkin D[2]{}`
E^2_6		`\dynkin E[2]6`
D^3_4		`\dynkin D[3]4`

Table 12: Some more affine Dynkin diagrams

A^2_4		`\dynkin A[2]4`

continued ...

Table 12: . . . continued

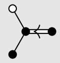
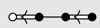
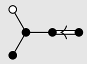
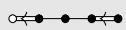
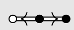
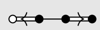
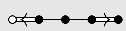
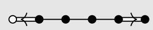
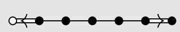
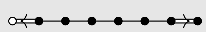
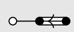
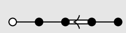
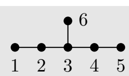
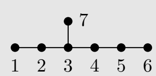
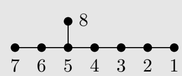
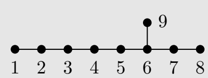
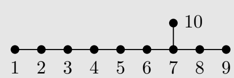
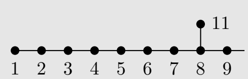
A_5^2		\dynkin A[2]5
A_6^2		\dynkin A[2]6
A_7^2		\dynkin A[2]7
A_8^2		\dynkin A[2]8
D_3^2		\dynkin D[2]3
D_4^2		\dynkin D[2]4
D_5^2		\dynkin D[2]5
D_6^2		\dynkin D[2]6
D_7^2		\dynkin D[2]7
D_8^2		\dynkin D[2]8
D_4^3		\dynkin D[3]4
E_6^2		\dynkin E[2]6

Table 13: Some more Kac–Moody Dynkin diagrams, only allowed in Kac ordering

E_6		\dynkin[ordering=Kac,label]E6
E_7		\dynkin[ordering=Kac,label]E7
E_8		\dynkin[ordering=Kac,label]E8
E_9		\dynkin[ordering=Kac,label]E9
E_{10}		\dynkin[ordering=Kac,label]E{10}
E_{11}		\dynkin[ordering=Kac,label]E{11}

21. Extended Coxeter diagrams

Extended and Coxeter options together

```
\dynkin[extended,Coxeter]F4
```

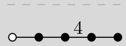
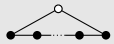
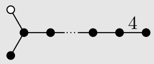
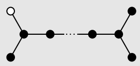
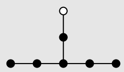
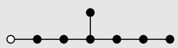
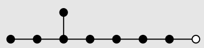
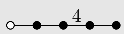
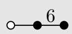
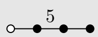
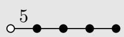


Table 14: The extended (affine) Coxeter diagrams

A_n		`\dynkin[extended,Coxeter]A{}`
B_n		`\dynkin[extended,Coxeter]B{}`
C_n		`\dynkin[extended,Coxeter]C{}`
D_n		`\dynkin[extended,Coxeter]D{}`
E_6		`\dynkin[extended,Coxeter]E6`
E_7		`\dynkin[extended,Coxeter]E7`
E_8		`\dynkin[extended,Coxeter]E8`
F_4		`\dynkin[extended,Coxeter]F4`
G_2		`\dynkin[extended,Coxeter]G2`
H_3		`\dynkin[extended,Coxeter]H3`
H_4		`\dynkin[extended,Coxeter]H4`
I_1		`\dynkin[extended,Coxeter]I1`

22. Kac style

We include a style called `Kac` which tries to imitate the style of [15].

Kac style

```
\dynkin[Kac]F4
```

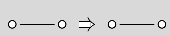
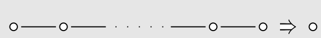
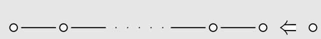
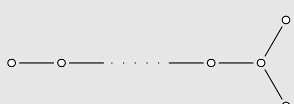


Table 15: The Dynkin diagrams of the simple root systems in Kac style

A_n		`\dynkin A{}`
B_n		`\dynkin B{}`
C_n		`\dynkin C{}`
D_n		`\dynkin D{}`
E_6		`\dynkin E6`

continued . . .

Table 15: ...continued

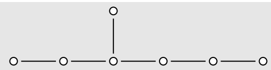
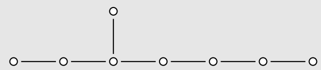
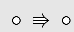
E_7		\dynkin E7
E_8		\dynkin E8
F_4		\dynkin F4
G_2		\dynkin G2

Table 16: The Dynkin diagrams of the extended simple root systems in Kac style

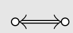
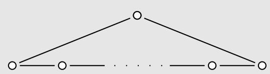
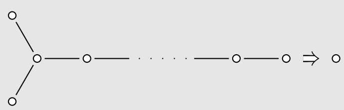
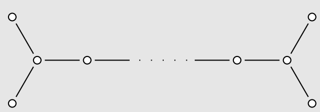
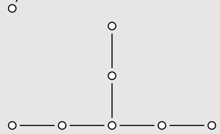
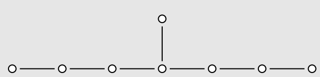
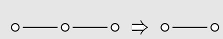
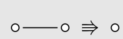
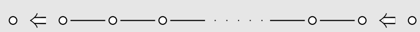
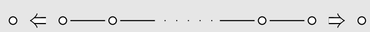
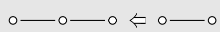
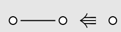
A_1^1		\dynkin[extended]A1
A_n^1		\dynkin[extended]A{}
B_n^1		\dynkin[extended]B{}
C_n^1		\dynkin[extended]C{}
D_n^1		\dynkin[extended]D{}
E_6^1		\dynkin[extended]E6
E_7^1		\dynkin[extended]E7
E_8^1		\dynkin[extended]E8
F_4^1		\dynkin[extended]F4
G_2^1		\dynkin[extended]G2

Table 17: The Dynkin diagrams of the twisted simple root systems in Kac style

A_2^2		`\dynkin A[2]2`
A_{ev}^2		`\dynkin A[2]{even}`
A_{od}^2		`\dynkin A[2]{odd}`
D_n^2		`\dynkin D[2]{}`
E_6^2		`\dynkin E[2]6`
D_4^3		`\dynkin D[3]4`

23. Ceref style

We include a style called `ceref` which paints oblong root markers with shadows. The word “ceref” is an old form of the word “serif”.

Ceref style

```
\dynkin[ceref]F4
```

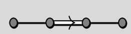


Table 18: The Dynkin diagrams of the simple root systems in ceref style

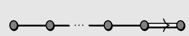
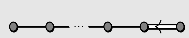
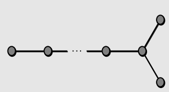
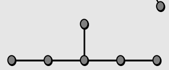
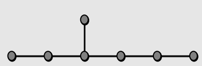
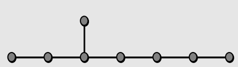
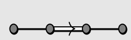
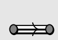
A_n		`\dynkin A{}`
B_n		`\dynkin B{}`
C_n		`\dynkin C{}`
D_n		`\dynkin D{}`
E_6		`\dynkin E6`
E_7		`\dynkin E7`
E_8		`\dynkin E8`
F_4		`\dynkin F4`
G_2		`\dynkin G2`

Table 19: The Dynkin diagrams of the extended simple root systems in ceref style

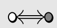
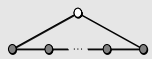
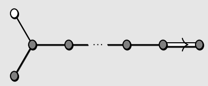
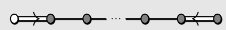
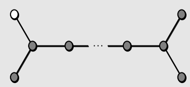
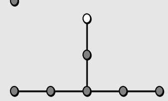
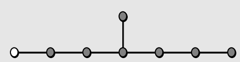
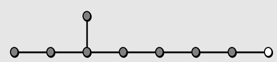
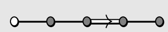
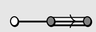
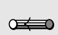
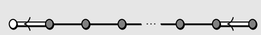
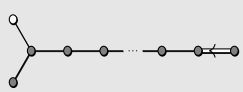
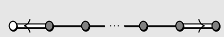
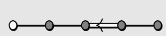
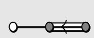
A_1^1		\dynkin[extended]A1
A_n^1		\dynkin[extended]A{}
B_n^1		\dynkin[extended]B{}
C_n^1		\dynkin[extended]C{}
D_n^1		\dynkin[extended]D{}
E_6^1		\dynkin[extended]E6
E_7^1		\dynkin[extended]E7
E_8^1		\dynkin[extended]E8
F_4^1		\dynkin[extended]F4
G_2^1		\dynkin[extended]G2

Table 20: The Dynkin diagrams of the twisted simple root systems in ceref style

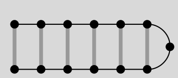
A_2^2		\dynkin A[2]2
A_{ev}^2		\dynkin A[2]{even}
A_{od}^2		\dynkin A[2]{odd}
D_n^2		\dynkin D[2]{}
E_6^2		\dynkin E[2]6
D_4^3		\dynkin D[3]4

24. More on folded Dynkin diagrams

The Dynkin diagrams package has limited support for folding Dynkin diagrams.

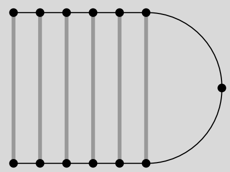
Folding

```
\dynkin[fold]A{13}
```



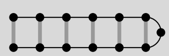
Big fold radius

```
\dynkin[fold,fold radius=1cm]A{13}
```



Small fold radius

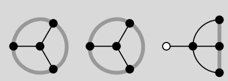
```
\dynkin[fold,fold radius=.2cm]A{13}
```



Some Dynkin diagrams have multiple foldings, which we attempt to distinguish (not entirely successfully) by their *ply*: the maximum number of roots folded together. Most diagrams can only allow a 2-ply folding, so `fold` is a synonym for `ply=2`.

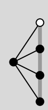
3-ply

```
\dynkin[ply=3]D4
\dynkin[ply=3,fold right]D4
\dynkin[ply=3]D[1]4
```



4-ply

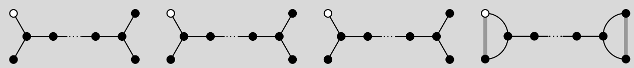
```
\dynkin[ply=4]D[1]4
```



The $D_\ell^{(1)}$ diagrams can be folded on their left end and separately on their right end:

Left, right and both

```
\dynkin D[1]{} \
\dynkin[fold left]D[1]{} \
\dynkin[fold right]D[1]{} \
\dynkin[fold]D[1]{}
```



We have to be careful about the 4-ply foldings of $D_{2\ell}^{(1)}$, for which we can have two different patterns, so by default, the package only draws as much as it can without distinguishing the two:

Default $D_{2\ell}^{(1)}$ and the two ways to finish it

```
    \dynkin[ply=4]D[1]{****.*****.*****}%
 \
\begin{dynkinDiagram}[ply=4]{D}[1]{****.*****.*****}%
    \dynkinFold[bend right=90]1{13}%
    \dynkinFold[bend right=90]0{14}%
\end{dynkinDiagram} \
\begin{dynkinDiagram}[ply=4]{D}[1]{****.*****.*****}%
    \dynkinFold01%
    \dynkinFold1{13}%
    \dynkinFold{13}{14}%
\end{dynkinDiagram}
```

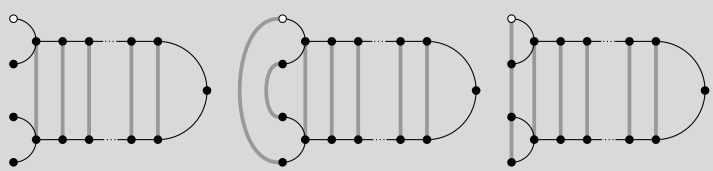
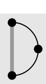
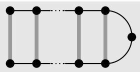
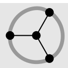
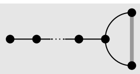
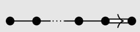
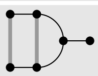
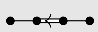
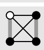
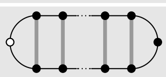
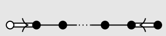
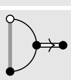
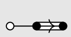
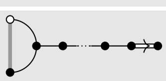
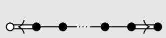
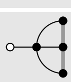
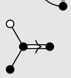


Table 21: Some foldings of Dynkin diagrams. For these diagrams, we want to compare a folding diagram with the diagram that results when we fold it, so it looks best to set `fold radius` and `edge length` to equal lengths.

A_3		`\dynkin[fold]A[0]3`
C_2		`\dynkin C[0]2`

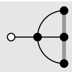
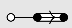
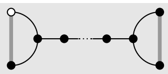
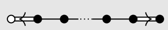
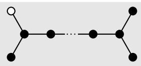
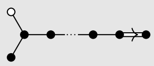
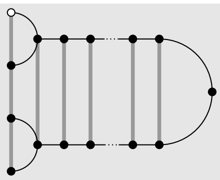
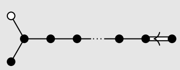
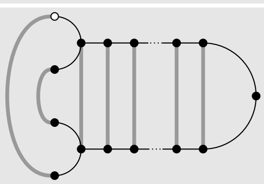
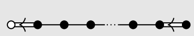
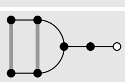
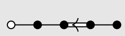
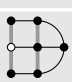
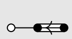
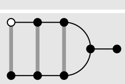
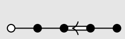
continued ...

Table 21: ... continued

$A_{2\ell-1}$		\dynkin[fold]A{**.*****.**}
C_ℓ		\dynkin C{}
B_3		\dynkin[fold]B[0]3
G_2		\dynkin[reverse arrows]G[0]2
D_4		\dynkin[ply=3,fold right]D4
G_2		\dynkin G2
$D_{\ell+1}$		\dynkin[fold]D{}
B_ℓ		\dynkin B{}
E_6		\dynkin[fold]E[0]6
F_4		\dynkin[reverse arrows]F[0]4
A_3^1		\dynkin[ply=4]A[1]3
A_1^1		\dynkin A[1]1
$A_{2\ell-1}^1$		\dynkin[fold]A[1]{**.*****.**}
C_ℓ^1		\dynkin C[1]{}
B_3^1		\dynkin[ply=3]B[1]3
A_2^2		\dynkin A[2]2
B_3^1		\dynkin[ply=2]B[1]3
G_2^1		\dynkin G[1]2
B_ℓ^1		\dynkin[fold]B[1]{}
D_ℓ^2		\dynkin D[2]{}
D_4^1		\dynkin[ply=3]D[1]4
B_3^1		\dynkin B[1]3

continued ...

Table 21: . . . continued

D_4^1		\dynkin[ply=3]D[1]4
G_2^1		\dynkin G[1]2
$D_{\ell+1}^1$		\dynkin[fold]D[1]{}
D_ℓ^2		\dynkin D[2]{}
$D_{\ell+1}^1$		\dynkin[fold right]D[1]{}
B_ℓ^1		\dynkin B[1]{}
$D_{2\ell}^1$		\begin{dynkinDiagram}[ply=4]D[1]% {****.*****.*****} \dynkinFold01 \dynkinFold1{13} \dynkinFold{13}{14} \end{dynkinDiagram}
A_{odd}^2		\dynkin A[2]{odd}
$D_{2\ell}^1$		\begin{dynkinDiagram}[ply=4]{D}[1]% {****.*****.*****} \dynkinFold[bend right=90]1{13} \dynkinFold[bend right=90]0{14} \end{dynkinDiagram}
A_{even}^2		\dynkin A[2]{even}
E_6^1		\dynkin[fold]E[1]6
F_4^1		\dynkin[reverse arrows]F[1]4
E_6^1		\dynkin[ply=3]E[1]6
D_4^3		\dynkin D[3]4
E_7^1		\dynkin[fold]E[1]7
E_6^2		\dynkin E[2]6

continued . . .

Table 21: . . . continued

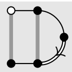
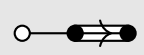
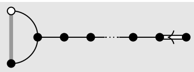
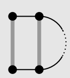
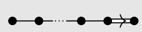
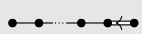
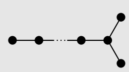
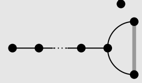
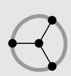
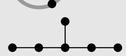
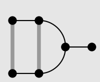
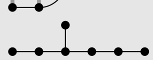
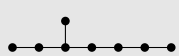
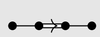
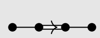
F_4^1		`\dynkin[fold]F[1]4`
G_2^1		`\dynkin G[1]2`
A^2_{odd}		`\dynkin[odd,fold]A[2]{****.***}`
A^2_{even}		`\dynkin A[2]{even}`
D_3^2		`\dynkin[fold]D[2]3`
A_2^2		`\dynkin A[2]2`

Table 22: Frobenius fixed point subgroups of finite simple groups of Lie type [4] p. 15

$A_{\ell \geq 1}$		`\dynkin A{}`
${}^2A_{\ell \geq 2}$		`\dynkin[fold]A{}`
$B_{\ell \geq 2}$		`\dynkin B{}`
${}^2B_2$		`\dynkin[fold]B2`
$C_{\ell \geq 3}$		`\dynkin C{}`
$D_{\ell \geq 4}$		`\dynkin D{}`
${}^2D_{\ell \geq 4}$		`\dynkin[fold]D{}`
${}^3D_4$		`\dynkin[ply=3]D4`
E_6		`\dynkin E6`
${}^2E_6$		`\dynkin[fold]E6`
E_7		`\dynkin E7`
E_8		`\dynkin E8`
F_4		`\dynkin F4`
${}^2F_4$		`\dynkin[fold]F4`
G_2		`\dynkin G2`
${}^2G_2$		`\dynkin[fold]G2`

25. Typesetting mathematical names of Dynkin diagrams

The `\dynkinName` command, with the same syntax as `\dynkin`, typesets a default name of your diagram in LaTeX. It is perhaps only useful when automatically generating a large collection of Dynkin diagrams in a computer program.

Name of a diagram

```
\dynkinName[label,extended]B7
\dynkinName A[2]{even}
\dynkinName[Coxeter]B7
\dynkinName[label,extended]B*
\dynkinName D[3]4
```

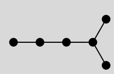
B^1_7 A^2_{ev} B_7 B^1_1 D^3_4

26. Connecting Dynkin diagrams

We can make some sophisticated folded diagrams by drawing multiple diagrams, each with a name:

Name a diagram

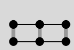
```
\dynkin[name=Bob]D6
```



We can then connect the two with folding edges:

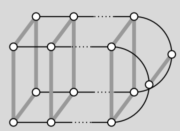
Connect diagrams

```
\begin{dynkinDiagram}[name=upper]A3
	\node (current) at ($(upper root 1)+(0,-.3cm)$) {};
	\dynkin[at=(current),name=lower]A3
	\begin{pgfonlayer}{Dynkin behind}
		\foreach \i in {1,...,3}%
		{%
			\draw[/Dynkin diagram/fold style]
				($(upper root \i)$)
				-- ($(lower root \i)$);%
		}%
	\end{pgfonlayer}
\end{dynkinDiagram}
```

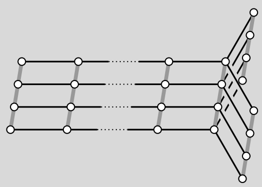


The following diagrams arise in the Satake diagrams of the pseudo-Riemannian symmetric spaces [2].

```
\pgfkeys{/Dynkin diagram,edge length=.5cm,fold radius=.5cm}
\begin{tikzpicture}
	\dynkin[name=1]A{IIIb}
	\node (a) at (-.3,-.4){};
	\dynkin[name=2,at=(a)]A{IIIb}
	\begin{pgfonlayer}{Dynkin behind}
		\foreach \i in {1,...,7}%
		{%
			\draw[/Dynkin diagram/fold style]
				($(1 root \i)$)
				--
				($(2 root \i)$);%
		}%
	\end{pgfonlayer}
\end{tikzpicture}
```

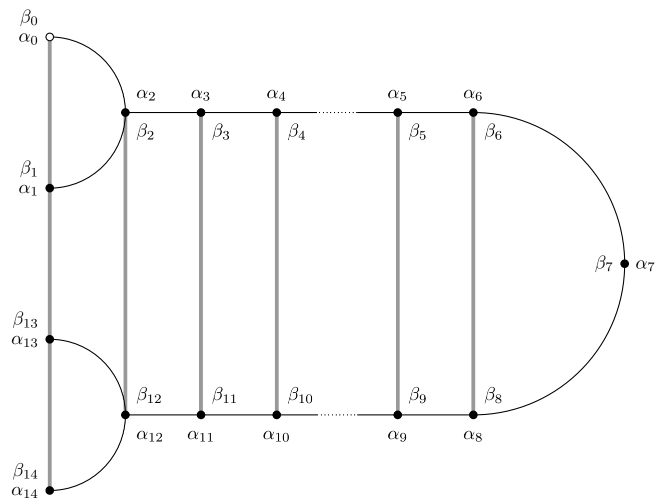


```
\pgfkeys{/Dynkin diagram,
edge length=.75cm,
edge/.style={draw=example-color,double=black,very thick}}
\begin{tikzpicture}
	\foreach \d in {1,...,4}
	{
		\node (current) at ($(\d*.05,\d*.3)$){};
		\dynkin[name=\d,at=(current)]D{oo.oooo}
	}
	\begin{pgfonlayer}{Dynkin behind}
		\foreach \i in {1,...,6}%
		{%
			\draw[/Dynkin diagram/fold style] ($(1 root \i)$) -- ($(2
	root \i)$);%
			\draw[/Dynkin diagram/fold style] ($(2 root \i)$) -- ($(3
	root \i)$);%
			\draw[/Dynkin diagram/fold style] ($(3 root \i)$) -- ($(4
	root \i)$);%
		}%
	\end{pgfonlayer}
\end{tikzpicture}
```

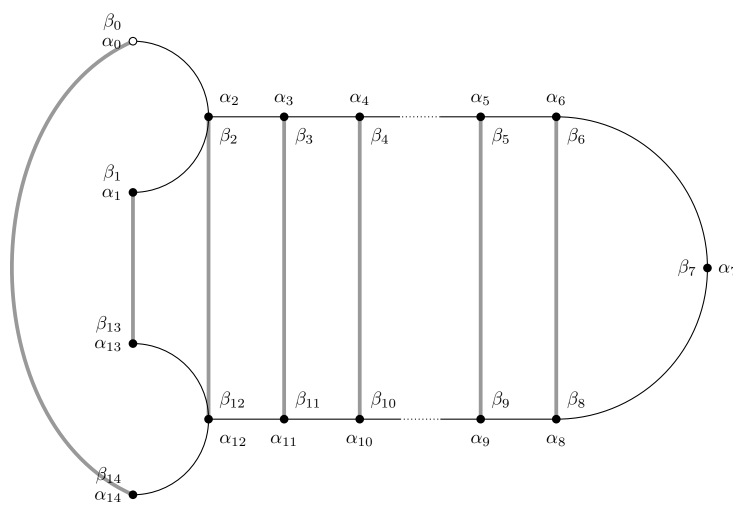


27. Other examples

$^1D_4$ 4-ply tied straight:



$^1D_4$ 4-ply tied bending:



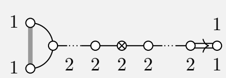

```
\tikzset{/Dynkin diagram,edge length=1cm,fold radius=1cm}
\tikzset{/Dynkin diagram,label macro/.code={\alpha_{#1}},label macro*/.code={\beta_{#1}}}
\({}^1 D_4\) 4-ply tied straight:
\begin{dynkinDiagram}[ply=4]D[1]%
{****.*****.*****}
 \dynkinFold 01
 \dynkinFold 1{13}
```

```
 \dynkinFold{13}{14}
\dynkinLabelRoots{0,...,14}
\dynkinLabelRoots*{0,...,14}
\end{dynkinDiagram}
\({}^1 D_4\) 4-ply tied bending:
\begin{dynkinDiagram}[ply=4]D[1]%
{****.*****.*****}
\dynkinFold1{13}
\dynkinFold[bend right=65]0{14}
\dynkinLabelRoots{0,...,14}
\dynkinLabelRoots*{0,...,14}
\end{dynkinDiagram}
```

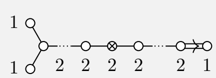
Below we draw the Vogan diagrams of some affine Lie superalgebras [21, 20].

$\mathfrak{sl}(2m|2n)^{(2)}$

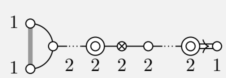
```
\begin{dynkinDiagram}[ply=2,label]{B}[1]{oo.oto.oo}
      \dynkinLabelRoot*71
\end{dynkinDiagram}
```



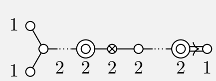
```
\dynkin[label]B[1]{oo.oto.oo}
```



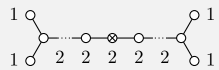
```
\dynkin[ply=2,label]B[1]{oo.Oto.Oo}
```



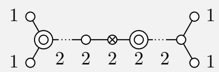
```
\dynkin[label]B[1]{oo.Oto.Oo}
```



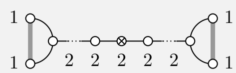
```
\dynkin[label]D[1]{oo.oto.ooo}
```



```
\dynkin[label]D[1]{oO.otO.ooo}
```

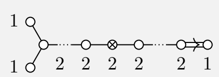


```
\dynkin[label,fold]D[1]{oo.oto.ooo}
```

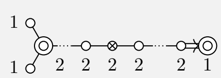


$\mathfrak{sl}(2m+1|2n)^2$

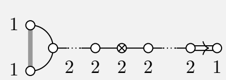
```
\dynkin[label]B[1]{oo.oto.oo}
```



```
\dynkin[label]B[1]{oO.oto.oO}
```

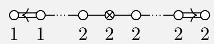


```
\dynkin[label,fold]B[1]{oo.oto.oo}
```

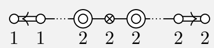


$\mathfrak{sl}\left(2m+1|2n+1\right)^2$

```
\dynkin[label]D[2]{o.oto.oo}
```

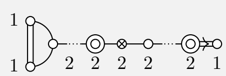



```
\dynkin[label]D[2]{o.OtO.oo}
```

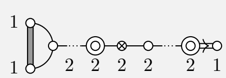


$\mathfrak{sl}\left(2|2n+1\right)^{(2)}$

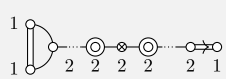
```
\dynkin[ply=2,label,double edges]B[1]{oo.Oto.Oo}
```



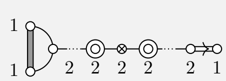

```
\dynkin[ply=2,label,double fold]B[1]{oo.Oto.Oo}
```




```
\dynkin[ply=2,label,double edges]B[1]{oo.OtO.oo}
```

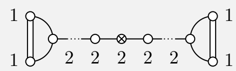



```
\dynkin[ply=2,label,double fold]B[1]{oo.OtO.oo}
```

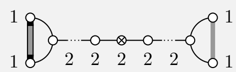


$\mathfrak{sl}\left(2|2n\right)^{(2)}$

```
\dynkin[ply=2,label,double edges]D[1]{oo.oto.ooo}
```

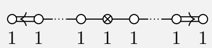



```
\dynkin[ply=2,label,double fold left]D[1]{oo.oto.ooo}
```

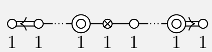


$\mathfrak{osp}\left(2m|2n\right)^{(2)}$

```
\dynkin[label,label macro/.code={1}]D[2]{o.oto.oo}
```

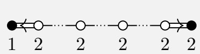



```
\dynkin[label,label macro/.code={1}]D[2]{o.Oto.Oo}
```

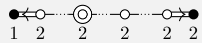


$\mathfrak{osp}\left(2|2n\right)^{(2)}$

```
\dynkin[label,label macro/.code=\lablIt{#1},
        affine mark=*]
        D[2]{o.o.o.o*}
```

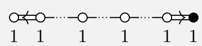


```
\dynkin[label,label macro/.code=\lablIt{#1},
        affine mark=*]
        D[2]{o.O.o.o*}
```

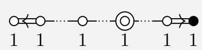


$\mathfrak{sl}(1|2n+1)^4$

```
\dynkin[label,label macro/.code={1}]D[2]{o.o.o.o*}
```

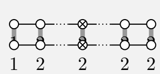



```
\dynkin[label,label macro/.code={1}]D[2]{o.o.O.o*}
```

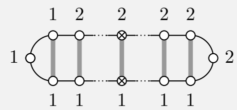


A^1

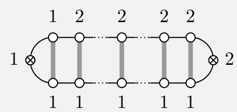
```
\begin{tikzpicture}
        \dynkin[name=upper]A{oo.t.oo}
        \node (Dynkin current) at (upper root 1){};
        \dynkinSouth
        \dynkin[at=(Dynkin current),name=lower]A{oo.t.oo}
        \begin{pgfonlayer}{Dynkin behind}
        \foreach \i in {1,...,5}{
                \draw[/Dynkin diagram/fold style]
                        ($(upper root \i)$) -- ($(lower root \i)$);
        }
        \end{pgfonlayer}
\end{tikzpicture}
```



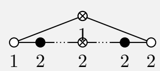
```
\dynkin[fold]A[1]{oo.t.ooooo.t.oo}
```




```
\dynkin[fold,affine mark=t]A[1]{oo.o.ootoo.o.oo}
```

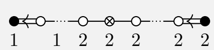



```
\dynkin[affine mark=t]A[1]{o*.t.*o}
```

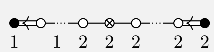


B^1

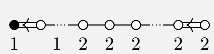
```
\dynkin[affine mark=*]A[2]{o.oto.o*}
```



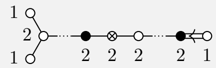

```
\dynkin[affine mark=*]A[2]{o.oto.o*}
```



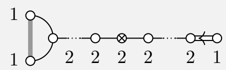

```
\dynkin[affine mark=*]A[2]{o.ooo.oo}
```



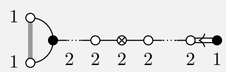
```
\dynkin[odd]A[2]{oo.*to.*o}
```




```
\dynkin[odd,fold]A[2]{oo.oto.oo}
```

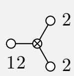



```
\dynkin[odd,fold]A[2]{o*.oto.o*}
```

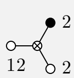


D^1

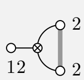
```
\dynkin D{otoo}
```




```
\dynkin D{ot*o}
```

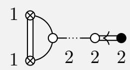



```
\dynkin[fold]D{otoo}
```

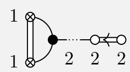


C^1

```
\dynkin[double edges,fold,affine mark=t,odd]A[2]{to.o*}
```

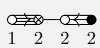



```
\dynkin[double edges,fold,affine mark=t,odd]A[2]{t*.oo}
```

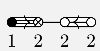


F^1

```
\begin{dynkinDiagram}A{oto*}%
    \dynkinQuadrupleEdge 12%
    \dynkinTripleEdge 43%
\end{dynkinDiagram}%
```

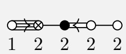



```
\begin{dynkinDiagram}A{*too}%
    \dynkinQuadrupleEdge 12%
    \dynkinTripleEdge 43%
\end{dynkinDiagram}%
```

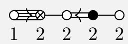


G^1

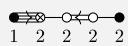
```
\begin{dynkinDiagram}A{ot*oo}%
    \dynkinQuadrupleEdge 12%
    \dynkinDefiniteDoubleEdge 43%
\end{dynkinDiagram}%
```



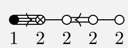
```
\begin{dynkinDiagram}A{oto*o}%
    \dynkinQuadrupleEdge 12%
    \dynkinDefiniteDoubleEdge 43%
\end{dynkinDiagram}%
```



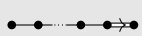
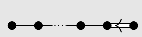
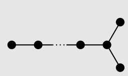
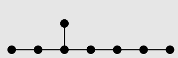

```
\begin{dynkinDiagram}A{*too*}%
    \dynkinQuadrupleEdge 12%
    \dynkinDefiniteDoubleEdge 43%
\end{dynkinDiagram}%
```

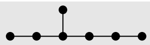
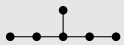
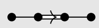



```
\begin{dynkinDiagram}A{*tooo}%
    \dynkinQuadrupleEdge 12%
    \dynkinDefiniteDoubleEdge 43%
\end{dynkinDiagram}%
```



28. Example: the complex simple Lie algebras

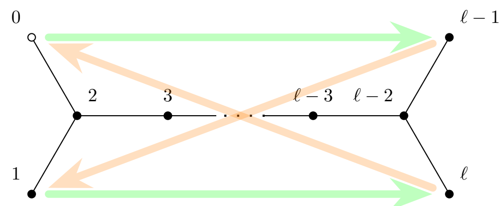
$\mathfrak{g}$	Diagram	Weights	Roots	Simple roots
A_n		$\frac{1}{n+1}\mathbb{Z}^{n+1}/\left\langle\sum e_j\right\rangle$	$e_i - e_j$	$e_i - e_{i+1}$
B_n		$\frac{1}{2}\mathbb{Z}^n$	$\pm e_i, \pm e_i \pm e_j, i \ne j$	$e_i - e_{i+1}, e_n$
C_n		$\mathbb{Z}^n$	$\pm 2e_i, \pm e_i \pm e_j, i \ne j$	$e_i - e_{i+1}, 2e_n$
D_n		$\frac{1}{2}\mathbb{Z}^n$	$\pm e_i \pm e_j, i \ne j$	$e_i - e_{i+1}, \quad i \le n-1$ $e_{n-1} + e_n$
E_8		$\frac{1}{2}\mathbb{Z}^8$	$\pm 2e_i \pm 2e_j, \quad i \ne j,$ $\sum_i (-1)^{m_i} e_i, \quad \sum m_i$ even	$2e_1 - 2e_2,$ $2e_2 - 2e_3,$ $2e_3 - 2e_4,$ $2e_4 - 2e_5,$ $2e_5 - 2e_6,$ $2e_6 + 2e_7,$ $-\sum e_j,$ $2e_6 - 2e_7$

$\mathfrak{g}$	Diagram	Weights	Roots	Simple roots
E_7		$\frac{1}{2}\mathbb{Z}^8/\left<e_1-e_2\right>$	quotient of E_8	quotient of E_8
E_6		$\frac{1}{3}\mathbb{Z}^8/\left<e_1-e_2,e_2-e_3\right>$	quotient of E_8	quotient of E_8
F_4		$\mathbb{Z}^4$	$\pm 2e_i$, $\pm 2e_i \pm 2e_j, \quad i \ne j$, $\pm e_1 \pm e_2 \pm e_3 \pm e_4$	$2e_2-2e_3$, $2e_3-2e_4$, $2e_4$, $e_1-e_2-e_3-e_4$
G_2		$\mathbb{Z}^3/\left<\sum e_j\right>$	$\pm(1,-1,0)$, $\pm(-1,0,1)$, $\pm(0,-1,1)$, $\pm(2,-1,-1)$, $\pm(1,-2,1)$, $\pm(-1,-1,2)$	$(-1,0,1)$, $(2,-1,-1)$

```
\NewDocumentEnvironment{bunch}{}%
{\renewcommand*{\arraystretch}{1}\begin{array}{@{}ll@{}}\\ \midrule}{\\ \midrule\end{array}}
\small
\NewDocumentCommand\nct{mm}{\newcolumntype{#1}{>{\columncolor[gray]{.9}}>{$}m{#2cm}<{$}}}
\nct{G}{.3}\nct{D}{2.1}\nct{W}{3}\nct{R}{3.7}\nct{S}{3}
\NewDocumentCommand\LieG{}{\mathfrak{g}}
\NewDocumentCommand\W{om}{\ensuremath{\mathbb{Z}^{#2}\IfValueT{#1}{/\left<#1\right>}}}
\renewcommand*{\arraystretch}{1.5}
\NewDocumentCommand\quo{}{\text{quotient of } E_8}
\begin{longtable}{@{}GDWRS@{}}
\LieG&\text{Diagram}&\text{Weights}&\text{Roots}&\text{Simple roots}\\ \midrule\endfirsthead
\LieG&\text{Diagram}&\text{Weights}&\text{Roots}&\text{Simple roots}\\ \midrule\endhead
A_n&\dynkin A{}&\frac1{n+1}\W[\sum e_j]{n+1}&e_i-e_j&e_i-e_{i+1}\\
B_n&\dynkin B{}&\frac12\W n& \pm e_i, \pm e_i \pm e_j, i\ne j&e_i-e_{i+1}, e_n\\
C_n&\dynkin C{}&\W n& \pm 2 e_i, \pm e_i \pm e_j, i\ne j&e_i-e_{i+1}, 2e_n\\
D_n&\dynkin D{}&\frac12\W n& \pm e_i \pm e_j, i\ne j &
\begin{bunch}e_i-e_{i+1},&i\le n-1\\e_{n-1}+e_n\end{bunch}\\
E_8&\dynkin E8&\frac12\W 8&
\begin{bunch}\pm2e_i\pm2e_j,&i\ne j,\\ \sum_i(-1)^{m_i}e_i,&\sum m_i \text{ even}\end{bunch}&
\begin{bunch}
2e_1-2e_2,\\2e_2-2e_3,\\2e_3-2e_4,\\2e_4-2e_5,\\2e_5-2e_6,\\2e_6+2e_7,\\
-\sum e_j,\\2e_6-2e_7
\end{bunch}\\
E_7&\dynkin E7&\frac12\W[e_1-e_2]8&\quo&\quo\\
E_6&\dynkin E6&\frac13\W[e_1-e_2,e_2-e_3]8&\quo&\quo\\
F_4& \dynkin F4&\W4&
\begin{bunch}\pm 2e_i,\\ \pm 2e_i \pm 2e_j, \quad i \ne j,\\ \pm e_1 \pm e_2 \pm e_3 \pm e_4
\end{bunch}&
\begin{bunch}2e_2-2e_3,\\2e_3-2e_4,\\2e_4,\\e_1-e_2-e_3-e_4\end{bunch}\\
G_2&\dynkin G2&\W[\sum e_j]3&
\begin{bunch}
\pm(1,-1,0),\\ \pm(-1,0,1),\\ \pm(0,-1,1),\\ \pm(2,-1,-1),\\ \pm(1,-2,1),\\ \pm(-1,-1,2)
```

```
\end{bunch}&
\begin{bunch}(-1,0,1),\\(2,-1,-1)\end{bunch}
\end{longtable}
```

29. An example of Mikhail Borovoi




```
\tikzset{big arrow/.style={
-Stealth,line cap=round,line width=1mm,
shorten <=1mm,shorten >=1mm}}
\newcommand\catholic[2]{\draw[big arrow,green!25!white]
(root #1) to (root #2);}
\newcommand\protestant[2]{
\begin{scope}[transparency group, opacity=.25]
\draw[big arrow,orange] (root #1) to (root #2);
\end{scope}}
\begin{dynkinDiagram}[edge length=1.2cm,
indefinite edge/.style={thick,loosely dotted},
labels*={0,1,2,3,\ell-3,\ell-2,\ell-1,\ell}]D[1]{}
\catholic 06\catholic 17
\protestant 70\protestant 61
\end{dynkinDiagram}
```

30. Syntax

The syntax is \dynkin[<options>]{<letter>}[<twisted rank>]{<rank>} where <letter> is A, B, C, D, E, F or G, the family of root system for the Dynkin diagram, <twisted rank> is 0, 1, 2, 3 (default is 0) representing:

0	finite root system
1	affine extended root system, i.e. of type $^{(1)}$
2	affine twisted root system of type $^{(2)}$
3	affine twisted root system of type $^{(3)}$

and <rank> is

(1) an integer representing the rank or
(2) blank to represent an indefinite rank or
(3) the name of a Satake diagram as in section 4.

The environment syntax is \begin{dynkinDiagram} followed by the same parameters as \dynkin, then various Dynkin diagram and Ti*k*Z commands, and then \end{dynkinDiagram}.

31. Options

ceref = ⟨true or false⟩,
default : false

continued ...

Table 24: ...continued

whether to draw roots in a "ceref" style.
edge length = ⟨number⟩cm,
default : .35cm
distance between nodes in the Dynkin diagram
edge/.style = TikZ style data,
default : solid,draw=black,fill=white,thin
style of edges in the Dynkin diagram
edge label/.style = TikZ style data,
default : text height=0,text depth=0,label distance=-2pt
style of edge labels in the Dynkin diagram, as found, for example, on some Coxeter diagrams
Kac = ⟨true or false⟩,
default : false
whether to draw in the style of [15]
name = ⟨string⟩,
default : anonymous
A name for the Dynkin diagram, with anonymous treated as a blank; see section 26.
parabolic = ⟨integer⟩,
default : 0
A parabolic subgroup with specified integer, where the integer is computed as $n = \sum 2^{i-1} a_i$, $a_i = 0$ or 1, to say that root i is crossed, i.e. a noncompact root.
root radius = ⟨number⟩cm,
default : .05cm
size of the dots and of the crosses in the Dynkin diagram
text style = ⟨TikZ style data⟩,
default : scale=.7
Style for any labels on the roots.
mark = ⟨o,O,t,x,X,*⟩,
default : *
default root mark
affine mark = o,O,t,x,X,*,
default : *
default root mark for root zero in an affine Dynkin diagram
label = true or false,
default : false
whether to label the roots according to the current labelling scheme.
label macro = ⟨1-parameter TEX macro⟩,
default : #1
the current labelling scheme for roots.
label macro* = ⟨1-parameter TEX macro⟩,
default : #1
the current labelling scheme for alternate roots.
label height = ⟨1-parameter TEX macro⟩,
default : b

continued ...

Table 24: . . . continued

the current maximal height of text labels for the roots, set by giving mathematics text of that height.

label depth = ⟨1-parameter TEX macro⟩,
default : g

the current maximal depth of text labels for the roots, set by giving mathematics text of that depth.

make indefinite edge = ⟨edge pair i-j or list of such⟩,
default : {}

edge pair or list of edge pairs to treat as having indefinitely many roots on them.

indefinite edge ratio = ⟨float⟩,
default : 1.6

ratio of indefinite edge lengths to other edge lengths.

indefinite edge/.style = ⟨TikZ style data⟩,
default : solid,draw=black,fill=white,thin,densely dotted

style of the dotted or dashed middle third of each indefinite edge.

backwards = ⟨true or false⟩,
default : false

whether to reverse right to left.

upside down = ⟨true or false⟩,
default : false

whether to reverse up to down.

arrows = ⟨true or false⟩,
default : true

whether to draw the arrows that arise along the edges.

reverse arrows = ⟨true or false⟩,
default : true

whether to reverse the direction of the arrows that arise along the edges.

fold = ⟨true or false⟩,
default : true

whether, when drawing Dynkin diagrams, to draw them 2-ply.

ply = ⟨0,1,2,3,4⟩,
default : 0

how many roots get folded together, at most.

fold left = ⟨true or false⟩,
default : true

whether to fold the roots on the left side of a Dynkin diagram.

fold right = ⟨true or false⟩,
default : true

whether to fold the roots on the right side of a Dynkin diagram.

fold radius = ⟨length⟩,
default : .3cm

the radius of circular arcs used in curved edges of folded Dynkin diagrams.

fold style/.style = ⟨TikZ style data⟩,

continued . . .

Table 24: ... continued

default : solid,draw=black!40,fill=none,line width=radius
when drawing folded diagrams, style for the fold indicators.

*/.style = ⟨TikZ style data⟩,
default : solid,draw=black,fill=black
style for roots like •

o/.style = ⟨TikZ style data⟩,
default : solid,draw=black,fill=white
style for roots like ∘

O/.style = ⟨TikZ style data⟩,
default : solid,draw=black,fill=white
style for roots like ◎

t/.style = ⟨TikZ style data⟩,
default : solid,draw=black,fill=black
style for roots like ⊗

x/.style = ⟨TikZ style data⟩,
default : solid,draw=black,line cap=round
style for roots like ×

X/.style = ⟨TikZ style data⟩,
default : solid,draw=black,thick,line cap=round
style for roots like ✖

fold left style/.style = ⟨TikZ style data⟩,
default :
style to override the fold style when folding roots together on the left half of a Dynkin diagram

fold right style/.style = ⟨TikZ style data⟩,
default :
style to override the fold style when folding roots together on the right half of a Dynkin diagram

double edges = ⟨⟩,
default : not set
set to override the fold style when folding roots together in a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows).

double fold = ⟨⟩,
default : not set
set to override the fold style when folding roots together in a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows), but filled in solidly.

double left = ⟨⟩,
default : not set
set to override the fold style when folding roots together at the left side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows).

double fold left = ⟨⟩,

continued ...

Table 24: ... continued

default : not set

set to override the fold style when folding roots together at the left side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows), but filled in solidly.

double right = ⟨⟩,

default : not set

set to override the fold style when folding roots together at the right side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows).

double fold right = ⟨⟩,

default : not set

set to override the fold style when folding roots together at the right side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows), but filled in solidly.

arrow color = ⟨⟩,

default : black

set to override the default color for the arrows in nonsimply laced Dynkin diagrams.

Coxeter = ⟨true or false⟩,

default : false

whether to draw a Coxeter diagram, rather than a Dynkin diagram.

ordering = ⟨Adams, Bourbaki, Carter, Dynkin, Kac⟩,

default : Bourbaki

which ordering of the roots to use in exceptional root systems as in section 17.

All other options are passed to TikZ.

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