THE DYNKIN DIAGRAMS PACKAGE
VERSION 3.11

BEN MCKAY

Contents

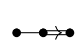
1. Quick introduction

Load the Dynkin diagram package (see options below)

```
\usepackage{dynkin-diagrams}
```

Invoke it

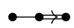
```
The Dynkin diagram of \(B_3\) is \dynkin{B}{3}.
```

The Dynkin diagram of B_3 is .

Date: 28 February 2018.

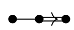
Inside a TikZ statement

```
\tikz \dynkin{B}{3};
```



Inside a TikZ environment

```
\begin{tikzpicture}
        \dynkin{B}{3}
\end{tikzpicture}
```



Indefinite rank Dynkin diagrams

```
\dynkin{B}{}
```

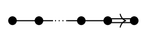
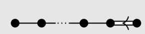
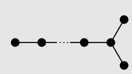
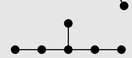
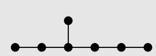
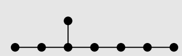
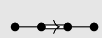


Table 1: The Dynkin diagrams of the reduced simple root systems [2] pp. 265–290, plates I–IX

A_n		`\dynkin{A}{}`
C_n		`\dynkin{C}{}`
D_n		`\dynkin{D}{}`
E_6		`\dynkin{E}{6}`
E_7		`\dynkin{E}{7}`
E_8		`\dynkin{E}{8}`
F_4		`\dynkin{F}{4}`
G_2		`\dynkin{G}{2}`

2. Set options globally

Most options set globally . . .

```
\pgfkeys{/Dynkin diagram,edgeLength=.5cm,foldradius=.5cm}
```

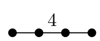
. . . or pass to the package

```
\usepackage[
     ordering=Kac,
     edge/.style=blue,
     mark=o,
     radius=.06cm]
     {dynkin-diagrams}
```

3. Coxeter diagrams

Coxeter diagram option

```
\dynkin[Coxeter]{F}{4}
```

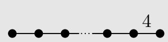
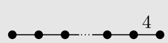
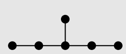
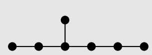
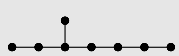
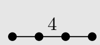


gonality option for G_2 and I_n Coxeter diagrams

```
\(G_2=\dynkin[Coxeter,gonality=n]{G}{2}\), \
\(I_n=\dynkin[Coxeter,gonality=n]{I}{}\)
```

$G_2 =$ , $I_n =$ 

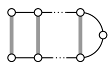
Table 2: The Coxeter diagrams of the simple reflection groups

A_n		`\dynkin[Coxeter]{A}{}`
B_n		`\dynkin[Coxeter]{B}{}`
C_n		`\dynkin[Coxeter]{C}{}`
E_6		`\dynkin[Coxeter]{E}{6}`
E_7		`\dynkin[Coxeter]{E}{7}`
E_8		`\dynkin[Coxeter]{E}{8}`
F_4		`\dynkin[Coxeter]{F}{4}`
G_2		`\dynkin[Coxeter,gonality=n]{G}{2}`
H_3		`\dynkin[Coxeter]{H}{3}`
H_4		`\dynkin[Coxeter]{H}{4}`
I_n		`\dynkin[Coxeter,gonality=n]{I}{}`

4. Satake diagrams

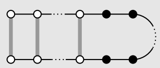
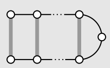
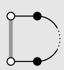
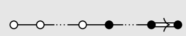
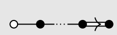
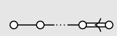
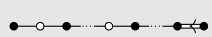
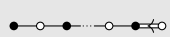
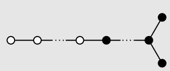
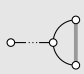
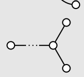
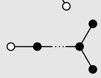
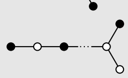
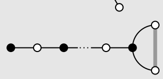
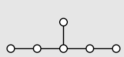
Satake diagrams use the standard name instead of a rank

```
\(A_{IIIb}=\dynkin{A}{IIIb}\)
```

$A_{IIIb} =$ 

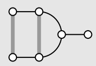
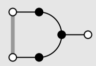
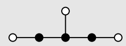
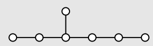
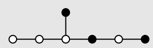
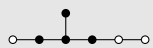
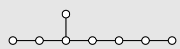
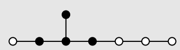
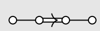
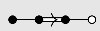
We use a solid gray bar to denote the folding of a Dynkin diagram, rather than the usual double arrow, since the diagrams turn out simpler and easier to read.

Table 3: The Satake diagrams of the real simple Lie algebras [9] p. 532–534

A_{I}		`\dynkin{A}{I}`
A_{II}		`\dynkin{A}{II}`
A_{IIIa}		`\dynkin{A}{IIIa}`
A_{IIIb}		`\dynkin{A}{IIIb}`
A_{IV}		`\dynkin{A}{IV}`
B_{I}		`\dynkin{B}{I}`
B_{II}		`\dynkin{B}{II}`
C_{I}		`\dynkin{C}{I}`
C_{IIa}		`\dynkin{C}{IIa}`
C_{IIb}		`\dynkin{C}{IIb}`
D_{Ia}		`\dynkin{D}{Ia}`
D_{Ib}		`\dynkin{D}{Ib}`
D_{Ic}		`\dynkin{D}{Ic}`
D_{II}		`\dynkin{D}{II}`
D_{IIIa}		`\dynkin{D}{IIIa}`
D_{IIIb}		`\dynkin{D}{IIIb}`
E_{I}		`\dynkin{E}{I}`

continued . . .

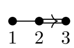
Table 3: ...continued

E_{II}		`\dynkin{E}{II}`
E_{III}		`\dynkin{E}{III}`
E_{IV}		`\dynkin{E}{IV}`
E_{V}		`\dynkin{E}{V}`
E_{VI}		`\dynkin{E}{VI}`
E_{VII}		`\dynkin{E}{VII}`
E_{VIII}		`\dynkin{E}{VIII}`
E_{IX}		`\dynkin{E}{IX}`
F_{I}		`\dynkin{F}{I}`
F_{II}		`\dynkin{F}{II}`
G_{I}		`\dynkin{G}{I}`

5. Labels for the roots

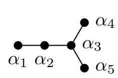
Label the roots by root number

```
\dynkin[label]{B}{3}
```



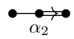
Make a macro to assign labels to roots

```
\dynkin[label,labelMacro/.code={\alpha_{#1}}]{D}{5}
```



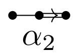
Label a single root

```
\begin{tikzpicture}
        \dynkin{B}{3}
        \dynkinLabelRoot{2}{\alpha_2}
\end{tikzpicture}
```



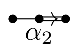
Use a text style

```
\begin{tikzpicture}
        \dynkin[text/.style={scale=1.2}]{B}{3};
        \dynkinLabelRoot{2}{\alpha_2}
\end{tikzpicture}
```



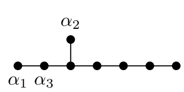
Access root labels via TikZ

```
\begin{tikzpicture}
        \dynkin{B}{3};
        \node[below] at (root 2) {\(\alpha_2\)};
\end{tikzpicture}
```



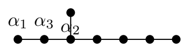
The labels have default locations

```
\begin{tikzpicture}
        \dynkin{E}{8};
        \dynkinLabelRoot{1}{\alpha_1}
        \dynkinLabelRoot{2}{\alpha_2}
        \dynkinLabelRoot{3}{\alpha_3}
\end{tikzpicture}
```



The starred form flips labels to alternate locations

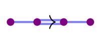
```
\begin{tikzpicture}
        \dynkin{E}{8};
        \dynkinLabelRoot*{1}{\alpha_1}
        \dynkinLabelRoot*{2}{\alpha_2}
        \dynkinLabelRoot*{3}{\alpha_3}
\end{tikzpicture}
```



6. Style

Colours

```
\dynkin[edge/.style={blue!50,thick},*/.style=blue!50!red]{F}{4}
```

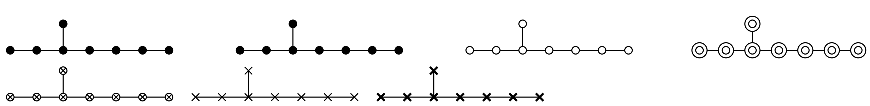


Edge lengths

```
\dynkin[edgeLength=1.2,parabolic=3]{A}{3}
```


Root marks

```
\dynkin{E}{8}
\dynkin[mark=*]{E}{8}
\dynkin[mark=o]{E}{8}
\dynkin[mark=O]{E}{8}
\dynkin[mark=t]{E}{8}
\dynkin[mark=x]{E}{8}
\dynkin[mark=X]{E}{8}
```

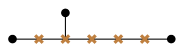


At the moment, you can only use:

- `*` solid dot
- `o` hollow circle
- `O` double hollow circle
- `t` tensor root
- `x` crossed root
- `X` thickly crossed root

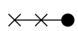
Mark styles

```
\dynkin[parabolic=124,x/.style={brown,very thick}]{E}{8}
```



Sizes of root marks

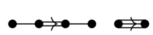
```
\dynkin[radius=.08cm,parabolic=3]{A}{3}
```



7. Suppress or reverse arrows

Some diagrams have double or triple edges

```
\dynkin{F}{4}
\dynkin{G}{2}
```

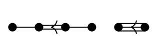


Suppress arrows

```
\dynkin[arrows=false]{F}{4}
\dynkin[arrows=false]{G}{2}
```


Reverse arrows

```
\dynkin[reverseArrows]{F}{4}
\dynkin[reverseArrows]{G}{2}
```

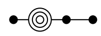


8. Drawing on top of a Dynkin diagram

TikZ can access the roots themselves

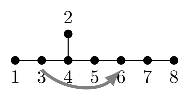
```
\begin{tikzpicture}
        \dynkin{A}{4};
        \fill[white,draw=black] (root 2) circle (.15cm);
        \fill[white,draw=black] (root 2) circle (.1cm);
        \draw[black] (root 2) circle (.05cm);
```

```
\end{tikzpicture}
```



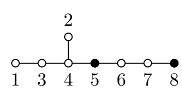
Draw curves between the roots

```
\begin{tikzpicture}
        \dynkin[label]{E}{8}
        \draw[very thick, black!50,-latex]
                (root 3.south) to [out=-45, in=-135] (root 6.south);
\end{tikzpicture}
```



Change marks

```
\begin{tikzpicture}
        \dynkin[mark=o,label]{E}{8};
        \dynkinRootMark{*}{5}
        \dynkinRootMark{*}{8}
\end{tikzpicture}
```

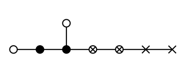


9. Mark lists

The package allows a list of root marks instead of a rank:

A mark list

```
\dynkin{E}{oo**ttxx}
```



The mark list `oo**ttxx` has one mark for each root: `o`, `o`, . . . , `x`. Roots are listed in the current default ordering. (Careful: in an affine root system, a mark list will *not* contain a mark for root zero.)

Table 4: Classical Lie superalgebras [7]. We need a slightly larger radius parameter to distinguish the tensor product symbols from the solid dots.

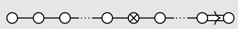
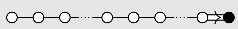
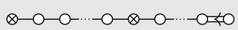
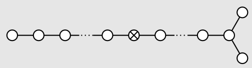
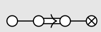
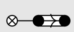
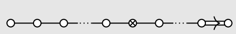
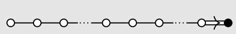
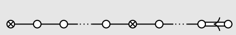
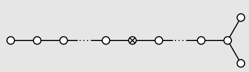
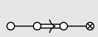
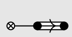
A_{mn}		`\dynkin{A}{ooo.oto.oo}`
B_{mn}		`\dynkin{B}{ooo.oto.oo}`
B_{0n}		`\dynkin{B}{ooo.ooo.o*}`
C_n		`\dynkin{C}{too.oto.oo}`
D_{mn}		`\dynkin{D}{ooo.oto.oooo}`
$D_{21\alpha}$		`\dynkin{A}{oto}`
F_4		`\dynkin{F}{ooot}`
G_3		`\dynkin[extended,affineMark=t]{G}{2}`

Table 5: Classical Lie superalgebras [7]. Here we see the problem with using the default radius parameter, which is too small for tensor product symbols.

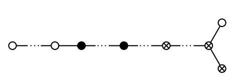
A_{mn}		`\dynkin{A}{ooo.oto.oo}`
B_{mn}		`\dynkin{B}{ooo.oto.oo}`
B_{0n}		`\dynkin{B}{ooo.ooo.o*}`
C_n		`\dynkin{C}{too.oto.oo}`
D_{mn}		`\dynkin{D}{ooo.oto.oooo}`
$D_{21\alpha}$		`\dynkin{A}{oto}`
F_4		`\dynkin{F}{ooot}`
G_3		`\dynkin[extended,affineMark=t]{G}{2}`

10. Indefinite edges

An *indefinite edge* is a dashed edge between two roots, indicating that an indefinite number of roots have been omitted from the Dynkin diagram. In between any two entries in a mark list, place a period to indicate an indefinite edge:

Indefinite edges

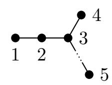
```
\dynkin{D}{o.o*.*.t.to.t}
```



In certain diagrams, roots may have an edge between them even though they are not subsequent in the ordering. For such rare situations, there is an option:

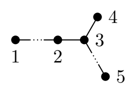
Indefinite edge option

```
\dynkin[makeIndefiniteEdge={3-5},label]{D}{5}
```



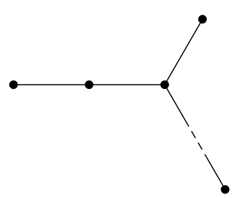
Give a list of edges to become indefinite

```
\dynkin[makeIndefiniteEdge/.list={1-2,3-5},label]{D}{5}
```



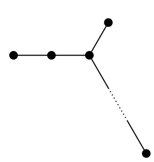
Indefinite edge style

```
\dynkin[indefiniteEdge/.style={draw=black,fill=white,thin,densely
    dashed},%
        edgeLength=1cm,%
        makeIndefiniteEdge={3-5}]
        {D}{5}
```



The ratio of the lengths of indefinite edges to those of other edges

```
\dynkin[edgeLength = .5cm,%
        indefiniteEdgeRatio=3,%
        makeIndefiniteEdge={3-5}]
        {D}{5}
```



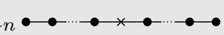
11. Parabolic subgroups

Each set of roots is assigned a number, with each binary digit zero or one to say whether the corresponding root is crossed or not:

```
The flag variety of pointed lines in
projective 3-space is associated to
the Dynkin diagram \dynkin[parabolic=3]{A}{3}.
```

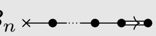
The flag variety of pointed lines in projective 3-space is associated to the Dynkin diagram .

Table 6: The Hermitian symmetric spaces

A_n 

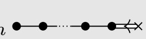
`\dynkin{A}{**.*x*.**}`

Grassmannian of k-planes in $\mathbb{C}^{n+1}$

B_n 

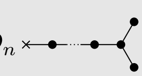
`\dynkin[parabolic=1]{B}{}`

$(2n-1)$-dimensional hyperquadric, i.e. the variety of null lines in $\mathbb{C}^{2n+1}$

C_n 

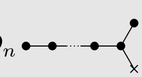
`\dynkin[parabolic=16]{C}{}`

space of Lagrangian n-planes in $\mathbb{C}^{2n}$

D_n 

`\dynkin[parabolic=1]{D}{}`

$(2n-2)$-dimensional hyperquadric, i.e. the variety of null lines in $\mathbb{C}^{2n}$

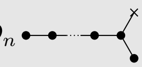
D_n 

`\dynkin[parabolic=32]{D}{}`

one component of the variety of maximal dimension null subspaces of $\mathbb{C}^{2n}$

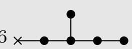
Table 6: continued ...

Table 6: . . . continued

D_n 

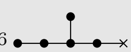
`\dynkin[parabolic=16]{D}{}`

the other component

E_6 

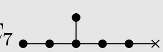
`\dynkin[parabolic=1]{E}{6}`

complexified octave projective plane

E_6 

`\dynkin[parabolic=32]{E}{6}`

its dual plane

E_7 

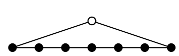
`\dynkin[parabolic=64]{E}{7}`

the space of null octave 3-planes in octave 6-space

12. Extended Dynkin diagrams

Extended Dynkin diagrams

```
\dynkin[extended]{A}{7}
```



The extended Dynkin diagrams are also described in the notation of Kac [11] p. 55 as affine untwisted Dynkin diagrams: we extend `\dynkin{A}{7}` to become `\dynkin{A}[1]{7}`:

Extended Dynkin diagrams

```
\dynkin{A}[1]{7}
```

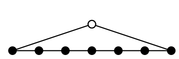
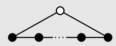
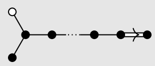
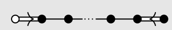
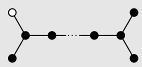
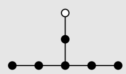
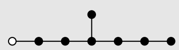
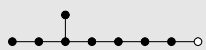
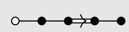
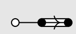


Table 7: The Dynkin diagrams of the extended simple root systems

A_1		`\dynkin[extended]{A}{1}`
A_n		`\dynkin[extended]{A}{}`
B_n		`\dynkin[extended]{B}{}`
C_n		`\dynkin[extended]{C}{}`
D_n		`\dynkin[extended]{D}{}`
E_6		`\dynkin[extended]{E}{6}`
E_7		`\dynkin[extended]{E}{7}`
E_8		`\dynkin[extended]{E}{8}`
F_4		`\dynkin[extended]{F}{4}`
G_2		`\dynkin[extended]{G}{2}`

13. Affine twisted and untwisted Dynkin diagrams

The affine Dynkin diagrams are described in the notation of Kac [11] p. 55:

Affine Dynkin diagrams

```
\(A^{(1)}_7=\dynkin{A}[1]{7}, \
E^{(2)}_6=\dynkin{E}[2]{6}, \
D^{(3)}_4=\dynkin{D}[3]{4}\)
```

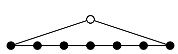
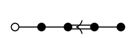
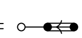
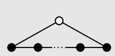
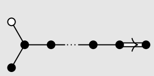
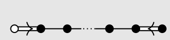
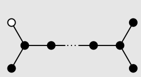
$A^{(1)}_7 =$ , $E^{(2)}_6 =$ , $D^{(3)}_4 =$ 

Table 8: The affine Dynkin diagrams

A^1_1		`\dynkin{A}[1]{1}`
A^1_n		`\dynkin{A}[1]{}`
B^1_n		`\dynkin{B}[1]{}`
C^1_n		`\dynkin{C}[1]{}`
D^1_n		`\dynkin{D}[1]{}`

continued . . .

Table 8: ... continued

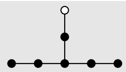
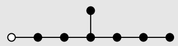
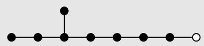
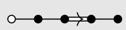
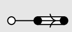
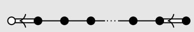
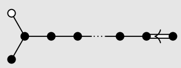
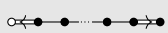
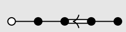
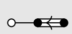
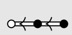
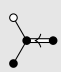
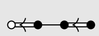
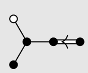
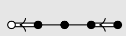
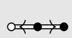
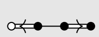
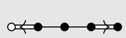
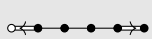
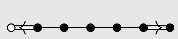
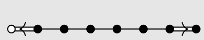
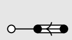
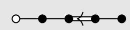
E_6^1		`\dynkin{E}[1]{6}`
E_7^1		`\dynkin{E}[1]{7}`
E_8^1		`\dynkin{E}[1]{8}`
F_4^1		`\dynkin{F}[1]{4}`
G_2^1		`\dynkin{G}[1]{2}`
A_2^2		`\dynkin{A}[2]{2}`
A_{even}^2		`\dynkin{A}[2]{even}`
A_{odd}^2		`\dynkin{A}[2]{odd}`
D_n^2		`\dynkin{D}[2]{}`
E_6^2		`\dynkin{E}[2]{6}`
D_4^3		`\dynkin{D}[3]{4}`

Table 9: Some more affine Dynkin diagrams

A_4^2		`\dynkin{A}[2]{4}`
A_5^2		`\dynkin{A}[2]{5}`
A_6^2		`\dynkin{A}[2]{6}`
A_7^2		`\dynkin{A}[2]{7}`
A_8^2		`\dynkin{A}[2]{8}`
D_3^2		`\dynkin{D}[2]{3}`
D_4^2		`\dynkin{D}[2]{4}`
D_5^2		`\dynkin{D}[2]{5}`
D_6^2		`\dynkin{D}[2]{6}`
D_7^2		`\dynkin{D}[2]{7}`
D_8^2		`\dynkin{D}[2]{8}`
D_4^3		`\dynkin{D}[3]{4}`

continued ...

Table 9: . . . continued

E_6^2		`\dynkin{E}[2]{6}`

14. Extended Coxeter diagrams

Extended and Coxeter options together

```
\dynkin[extended,Coxeter]{F}{4}
```

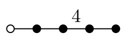
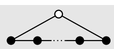
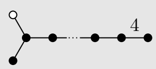
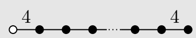
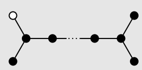
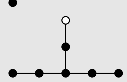
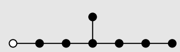
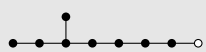


Table 10: The extended (affine) Coxeter diagrams

A_n		`\dynkin[extended,Coxeter]{A}{}`
B_n		`\dynkin[extended,Coxeter]{B}{}`
C_n		`\dynkin[extended,Coxeter]{C}{}`
D_n		`\dynkin[extended,Coxeter]{D}{}`
E_6		`\dynkin[extended,Coxeter]{E}{6}`
E_7		`\dynkin[extended,Coxeter]{E}{7}`
E_8		`\dynkin[extended,Coxeter]{E}{8}`
F_4		`\dynkin[extended,Coxeter]{F}{4}`
G_2		`\dynkin[extended,Coxeter]{G}{2}`
H_3		`\dynkin[extended,Coxeter]{H}{3}`
H_4		`\dynkin[extended,Coxeter]{H}{4}`
I_1		`\dynkin[extended,Coxeter]{I}{1}`

15. Kac style

We include a style called `Kac` which tries to imitate the style of [11].

Kac style

```
\dynkin[Kac]{F}{4}
```

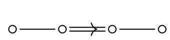
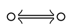
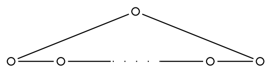
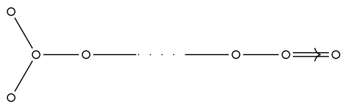
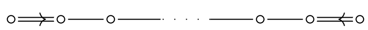
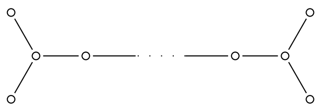
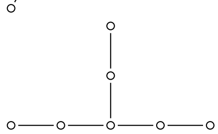
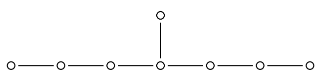
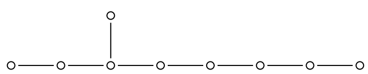
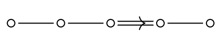
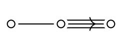


Table 11: The Dynkin diagrams of the extended simple root systems in Kac style. At the moment, it only works on a white background.

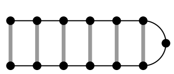
A_1		`\dynkin[extended]{A}{1}`
A_n		`\dynkin[extended]{A}{}`
B_n		`\dynkin[extended]{B}{}`
C_n		`\dynkin[extended]{C}{}`
D_n		`\dynkin[extended]{D}{}`
E_6		`\dynkin[extended]{E}{6}`
E_7		`\dynkin[extended]{E}{7}`
E_8		`\dynkin[extended]{E}{8}`
F_4		`\dynkin[extended]{F}{4}`
G_2		`\dynkin[extended]{G}{2}`

16. Folded Dynkin diagrams

The Dynkin diagrams package has limited support for folding Dynkin diagrams.

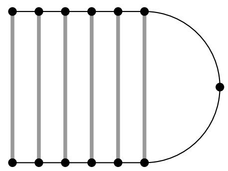
Folding

```
\dynkin[fold]{A}{13}
```



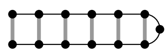
Big fold radius

```
\dynkin[fold,foldradius=1cm]{A}{13}
```



Small fold radius

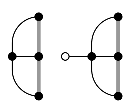
```
\dynkin[fold,foldradius=.2cm]{A}{13}
```



Some Dynkin diagrams have multiple foldings, which we attempt to distinguish (not entirely successfully) by their *ply*: the maximum number of roots folded together. Most diagrams can only allow a 2-ply folding, so `fold` is a synonym form `ply=2`.

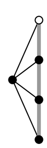
3-ply

```
\dynkin[ply=3]{D}{4}
\dynkin[ply=3]{D}[1]{4}
```



4-ply

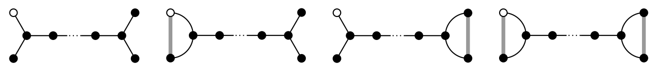
```
\dynkin[ply=4]{D}[1]{4}
```



The $D_\ell^{(1)}$ diagrams can be folded on their left end and separately on their right end:

Left, right and both

```
\dynkin{D}[1]{} \
\dynkin[foldleft]{D}[1]{} \
\dynkin[foldright]{D}[1]{} \
\dynkin[fold]{D}[1]{}
```



We have to be careful about the 4-ply foldings of $D^{(1)}_{2\ell}$, for which we can have two different patterns, so by default, the package only draws as much as it can without distinguishing the two:

Default $D^{(1)}_{2\ell}$ and the two ways to finish it

```
\begin{tikzpicture}
        \dynkin[ply=4]{D}[1]{****.*****.*****}%
\end{tikzpicture} \
\begin{tikzpicture}
        \dynkin[ply=4]{D}[1]{****.*****.*****}%
        \dynkinFold[bend right=65]{1}{13}%
        \dynkinFold[bend right=65]{0}{14}%
\end{tikzpicture} \
\begin{tikzpicture}
        \dynkin[ply=4]{D}[1]{****.*****.*****}%
        \dynkinFold{0}{1}%
        \dynkinFold{1}{13}%
        \dynkinFold{13}{14}%
\end{tikzpicture}
```

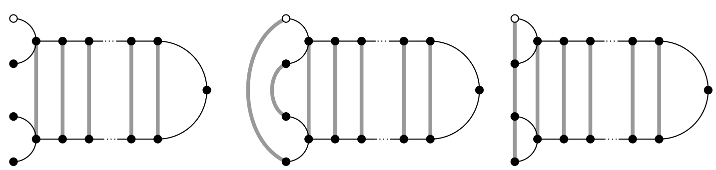
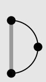
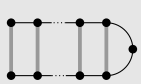
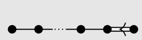
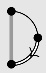
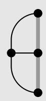
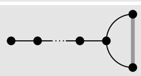
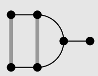
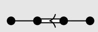
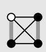
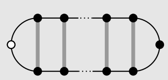
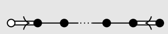
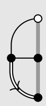
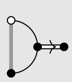
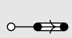
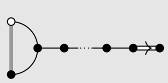
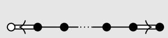


Table 12: Some foldings of Dynkin diagrams

A_3		`\dynkin[fold]{A}[0]{3}`
C_2		`\dynkin{C}[0]{2}`
$A_{2\ell-1}$		`\dynkin[fold]{A}{**.*****.**}`
C_ℓ		`\dynkin{C}{}`

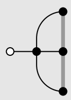
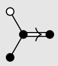
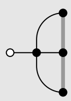
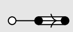
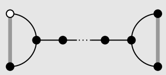
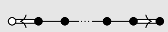
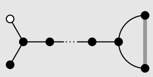
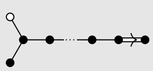
continued . . .

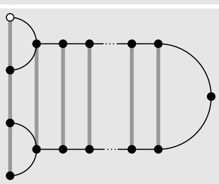
Table 12: ...continued

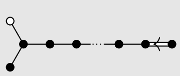
B_3		`\dynkin[fold]{B}[0]{3}`
G_2		`\dynkin[reverseArrows]{G}[0]{2}`
D_4		`\dynkin[ply=3]{D}{4}`
G_2		`\dynkin{G}{2}`
$D_{\ell+1}$		`\dynkin[fold]{D}{}`
B_ℓ		`\dynkin{B}{}`
E_6		`\dynkin[fold]{E}[0]{6}`
F_4		`\dynkin[reverseArrows]{F}[0]{4}`
A_3^1		`\dynkin[ply=4]{A}[1]{3}`
A_1^1		`\dynkin{A}[1]{1}`
$A_{2\ell-1}^1$		`\dynkin[fold]{A}[1]{**.*****.**}`
C_ℓ^1		`\dynkin{C}[1]{}`
B_3^1		`\dynkin[ply=3]{B}[1]{3}`
A_2^2		`\dynkin{A}[2]{2}`
B_3^1		`\dynkin[ply=2]{B}[1]{3}`
G_2^1		`\dynkin{G}[1]{2}`
B_ℓ^1		`\dynkin[fold]{B}[1]{}`
D_ℓ^2		`\dynkin{D}[2]{}`

continued ...

Table 12: ...continued

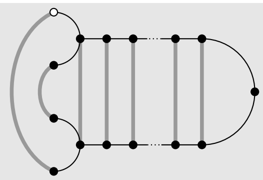
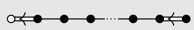
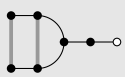
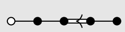
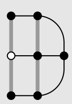
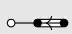
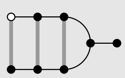
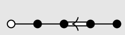
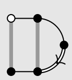
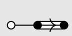
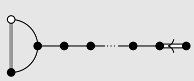
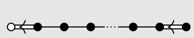
D_4^1		`\dynkin[ply=3]{D}[1]{4}`
B_3^1		`\dynkin{B}[1]{3}`
D_4^1		`\dynkin[ply=3]{D}[1]{4}`
G_2^1		`\dynkin{G}[1]{2}`
$D_{\ell+1}^1$		`\dynkin[fold]{D}[1]{}`
D_ℓ^2		`\dynkin{D}[2]{}`
$D_{\ell+1}^1$		`\dynkin[foldright]{D}[1]{}`
B_ℓ^1		`\dynkin{B}[1]{}`

$D_{2\ell}^1$		`\begin{tikzpicture}`
		` \dynkin[ply=4]{D}[1]{****.*****.*****}`
		` \dynkinFold{0}{1}`
		` \dynkinFold{1}{13}`
		` \dynkinFold{13}{14}`
		`\end{tikzpicture}`

A_{odd}^2		`\dynkin{A}[2]{odd}`

continued ...

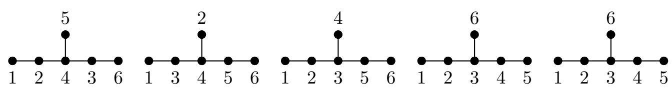
Table 12: …continued

$D^1_{2\ell}$		\begin{tikzpicture} \dynkin[ply=4]{D}[1]{****.*****.*****} \dynkinFold[bend right=65]{1}{13} \dynkinFold[bend right=65]{0}{14} \end{tikzpicture}
A^2_{even}		\dynkin{A}[2]{even}
E^1_6		\dynkin[fold]{E}[1]{6}
F^1_4		\dynkin[reverseArrows]{F}[1]{4}
E^1_6		\dynkin[ply=3]{E}[1]{6}
D^3_4		\dynkin{D}[3]{4}
E^1_7		\dynkin[fold]{E}[1]{7}
E^2_6		\dynkin{E}[2]{6}
F^1_4		\dynkin[fold]{F}[1]{4}
G^1_2		\dynkin{G}[1]{2}
A^2_{odd}		\dynkin[odd,fold]{A}[2]{****.***}
A^2_{even}		\dynkin{A}[2]{even}
D^2_3		\dynkin[fold]{D}[2]{3}
A^2_2		\dynkin{A}[2]{2}

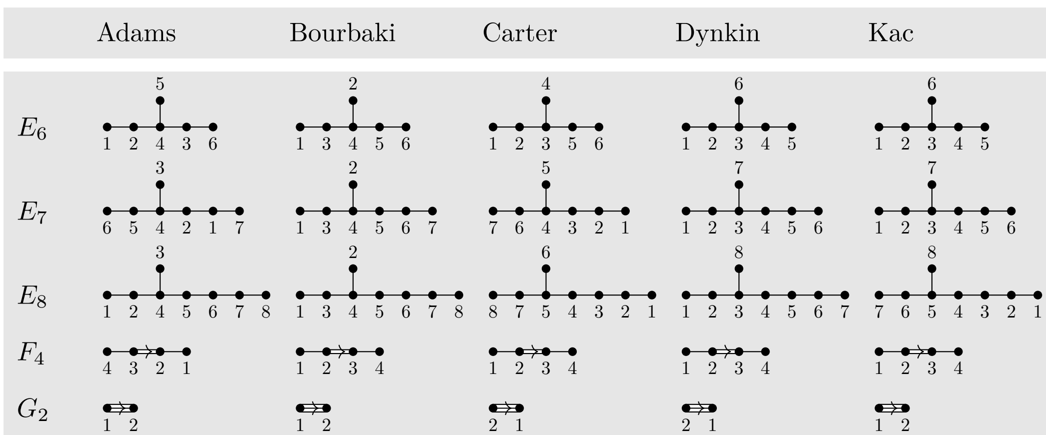
17. Root ordering

Root ordering

```
\dynkin[label,ordering=Adams]{E}{6}
\dynkin[label,ordering=Bourbaki]{E}{6}
\dynkin[label,ordering=Carter]{E}{6}
\dynkin[label,ordering=Dynkin]{E}{6}
\dynkin[label,ordering=Kac]{E}{6}
```



Default is Bourbaki.

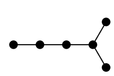


18. Connecting Dynkin diagrams

We can make some sophisticated folded diagrams by drawing multiple diagrams, each with a name:

Name a diagram

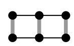
```
\dynkin[name=Bob]{D}{6}
```



We can then connect the two with folding edges:

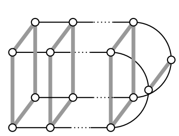
Connect diagrams

```
\begin{tikzpicture}
	\dynkin[name=upper]{A}{3}
	\node (current) at ($(upper root 1)+(0,-.3cm)$) {};
	\dynkin[at=(current),name=lower]{A}{3}
	\begin{scope}[on background layer]
		\foreach \i in {1,...,3}%
		{%
			\draw[/Dynkin diagram/foldStyle]
				($(upper root \i)$) -- ($(lower
    root \i)$);%
		}%
	\end{scope}
\end{tikzpicture}
```



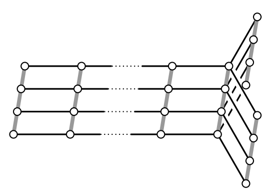
The following diagrams arise in the Satake diagrams of the pseudo-Riemannian symmetric spaces [1].

```
\pgfkeys{/Dynkin diagram,edgeLength=.5cm,foldradius=.5cm}
\begin{tikzpicture}
	\dynkin[name=1]{A}{IIIb}
	\node (a) at (.3,.4){};
	\dynkin[name=2,at=(a)]{A}{IIIb}
	\begin{scope}[on background layer]
		\foreach \i in {1,...,7}%
		{%
			\draw[/Dynkin diagram/foldStyle]
				($(1 root \i)$)
				--
				($(2 root \i)$);%
		}%
	\end{scope}
\end{tikzpicture}
```



```
\pgfkeys{/Dynkin diagram/edgeLength=.75cm,/Dynkin
    diagram/edge/.style={draw=white,double=black,very thick},
}
```

```
\begin{tikzpicture}
        \foreach \d in {1,...,4}
        {
                \node (current) at ($(\d*.05,\d*.3)$){};
                \dynkin[name=\d,at=(current)]{D}{oo.oooo}
        }
        \begin{scope}[on background layer]
                \foreach \i in {1,...,6}%
                {%
                        \draw[/Dynkin diagram/foldStyle] ($(1 root
    \i)$) -- ($(2 root \i)$);%
                        \draw[/Dynkin diagram/foldStyle] ($(2 root
    \i)$) -- ($(3 root \i)$);%
                        \draw[/Dynkin diagram/foldStyle] ($(3 root
    \i)$) -- ($(4 root \i)$);%
                }%
        \end{scope}
\end{tikzpicture}
```

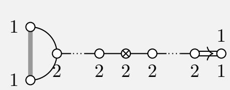


19. Other examples

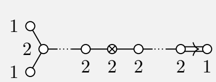
Below we draw the Vogan diagrams of some affine Lie superalgebras [16, 15].

$\mathfrak{sl}(2m|2n)^{(2)}$

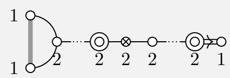
```
\begin{tikzpicture}
        \dynkin[ply=2,label]{B}[1]{oo.oto.oo}
        \dynkinLabelRoot*{7}{1}
\end{tikzpicture}
```



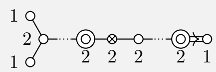

```
\dynkin[label]{B}[1]{oo.oto.oo}
```



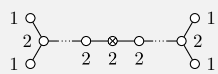
```
\dynkin[ply=2,label]{B}[1]{oo.Oto.Oo}
```



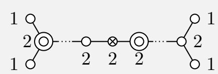
```
\dynkin[label]{B}[1]{oo.Oto.Oo}
```



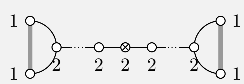
```
\dynkin[label]{D}[1]{oo.oto.ooo}
```



```
\dynkin[label]{D}[1]{oO.otO.ooo}
```

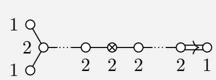


```
\dynkin[label,fold]{D}[1]{oo.oto.ooo}
```

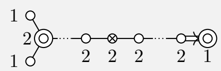


$\mathfrak{sl}(2m+1|2n)^2$

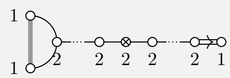
```
\dynkin[label]{B}[1]{oo.oto.oo}
```



```
\dynkin[label]{B}[1]{oO.oto.oO}
```

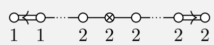



```
\dynkin[label,fold]{B}[1]{oo.oto.oo}
```

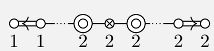


$\mathfrak{sl}\left(2m+1|2n+1\right)^2$

```
\dynkin[label]{D}[2]{o.oto.oo}
```

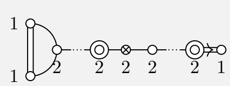



```
\dynkin[label]{D}[2]{o.OtO.oo}
```

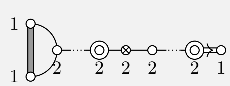


$\mathfrak{sl}\left(2|2n+1\right)^{(2)}$

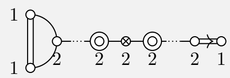
```
\dynkin[ply=2,label,doubleEdges]{B}[1]{oo.Oto.Oo}
```



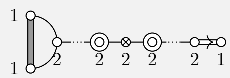

```
\dynkin[ply=2,label,doubleFold]{B}[1]{oo.Oto.Oo}
```



```
\dynkin[ply=2,label,doubleEdges]{B}[1]{oo.OtO.oo}
```

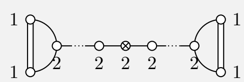



```
\dynkin[ply=2,label,doubleFold]{B}[1]{oo.OtO.oo}
```

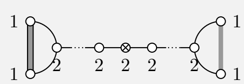


$\mathfrak{sl}\,(2|2n)^{(2)}$

```
\dynkin[ply=2,label,doubleEdges]{D}[1]{oo.oto.ooo}
```

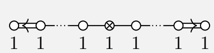



```
\dynkin[ply=2,label,doubleFoldLeft]{D}[1]{oo.oto.ooo}
```

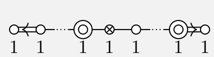


$\mathfrak{osp}\,(2m|2n)^{(2)}$

```
\dynkin[label,labelMacro/.code={1}]{D}[2]{o.oto.oo}
```

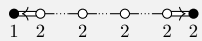



```
\dynkin[label,labelMacro/.code={1}]{D}[2]{o.Oto.Oo}
```

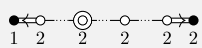


$\mathfrak{osp}\left(2|2n\right)^{(2)}$

```
\dynkin[label,labelMacro/.code=\lablIt{#1},
        affineMark=*]
        {D}[2]{o.o.o.o*}
```

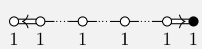



```
\dynkin[label,labelMacro/.code=\lablIt{#1},
        affineMark=*]
        {D}[2]{o.O.o.o*}
```

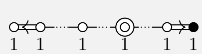


$\mathfrak{sl}\left(1|2n+1\right)^{4}$

```
\dynkin[label,labelMacro/.code={1}]{D}[2]{o.o.o.o*}
```

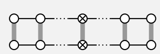



```
\dynkin[label,labelMacro/.code={1}]{D}[2]{o.o.O.o*}
```

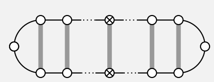


A^1

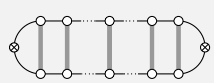
```
\begin{tikzpicture}
        \dynkin[name=upper]{A}{oo.t.oo}
        \node (Dynkin current) at (upper root 1){};
        \dynkinSouth
        \dynkin[at=(Dynkin
    current),name=lower]{A}{oo.t.oo}
        \begin{scope}[on background layer]
        \foreach \i in {1,...,5}{
                \draw[/Dynkin diagram/foldStyle]
                        ($(upper root \i)$) --
    ($(lower root \i)$);
        }
        \end{scope}
\end{tikzpicture}
```



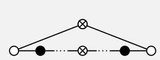
```
\dynkin[fold]{A}[1]{oo.t.ooooo.t.oo}
```



```
\dynkin[fold,affineMark=t]{A}[1]{oo.o.ootoo.o.oo}
```

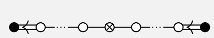


```
\dynkin[affineMark=t]{A}[1]{o*.t.*o}
```

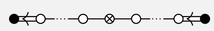


B^1

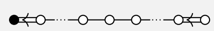
```
\dynkin[affineMark=*]{A}[2]{o.oto.o*}
```



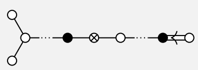
```
\dynkin[affineMark=*]{A}[2]{o.oto.o*}
```



```
\dynkin[affineMark=*]{A}[2]{o.ooo.oo}
```

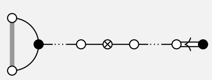


```
\dynkin[odd]{A}[2]{oo.*to.*o}
```



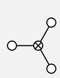
```
\dynkin[odd,fold]{A}[2]{oo.oto.oo}
```

```
\dynkin[odd,fold]{A}[2]{o*.oto.o*}
```

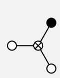


D^1

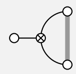
```
\dynkin{D}{otoo}
```



```
\dynkin{D}{ot*o}
```

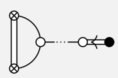


```
\dynkin[fold]{D}{otoo}
```

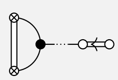


C^1

```
\dynkin[doubleEdges,fold,affineMark=t,odd]{A}[2]{to.o*}
```

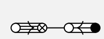


```
\dynkin[doubleEdges,fold,affineMark=t,odd]{A}[2]{t*.oo}
```

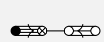


F^1

```
\begin{tikzpicture}%
        \dynkin{A}{oto*}%
        \dynkinQuadrupleEdge{1}{2}%
        \dynkinTripleEdge{4}{3}%
\end{tikzpicture}%
```

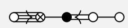


```
\begin{tikzpicture}%
        \dynkin{A}{*too}%
        \dynkinQuadrupleEdge{1}{2}%
        \dynkinTripleEdge{4}{3}%
\end{tikzpicture}%
```

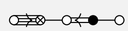


G^1

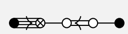
```
\begin{tikzpicture}%
        \dynkin{A}{ot*oo}%
        \dynkinQuadrupleEdge{1}{2}%
        \dynkinDefiniteDoubleEdge{4}{3}%
\end{tikzpicture}%
```



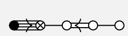
```
\begin{tikzpicture}%
        \dynkin{A}{oto*o}%
        \dynkinQuadrupleEdge{1}{2}%
        \dynkinDefiniteDoubleEdge{4}{3}%
\end{tikzpicture}%
```



```
\begin{tikzpicture}%
        \dynkin{A}{*too*}%
        \dynkinQuadrupleEdge{1}{2}%
        \dynkinDefiniteDoubleEdge{4}{3}%
\end{tikzpicture}%
```



```
\begin{tikzpicture}%
        \dynkin{A}{*tooo}%
        \dynkinQuadrupleEdge{1}{2}%
        \dynkinDefiniteDoubleEdge{4}{3}%
\end{tikzpicture}%
```



20. Syntax

The syntax is `\dynkin[<options>]{<letter>}[<twisted rank>]{<rank>}` where `<letter>` is `A`, `B`, `C`, `D`, `E`, `F` or `G`, the family of root system for the Dynkin diagram,

`<twisted rank>` is `0`, `1`, `2`, `3` (default is `0`) representing:

0 finite root system
1 affine extended root system, i.e. of type $^{(1)}$
2 affine twisted root system of type $^{(2)}$
3 affine twisted root system of type $^{(3)}$

and `<rank>` is

(1) an integer representing the rank or
(2) blank to represent an indefinite rank or
(3) the name of a Satake diagram as in section 4.

21. Options

`text/.style` = ⟨`TikZ style data`⟩,
default : `scale=.7`
Style for any labels on the roots.

`name` = ⟨`string`⟩,
default : `anonymous`
A name for the Dynkin diagram, with `anonymous` treated as a blank; see section 18.

`parabolic` = ⟨`integer`⟩,
default : `0`
A parabolic subgroup with specified integer, where the integer is computed as $n = \sum 2^{i-1} a_i$, $a_i = 0$ or 1, to say that root i is crossed, i.e. a noncompact root.

`radius` = ⟨`number`⟩`cm`,
default : `.05cm`
size of the dots and of the crosses in the Dynkin diagram

`edgeLength` = ⟨`number`⟩`cm`,
default : `.35cm`
distance between nodes in the Dynkin diagram

`edge/.style` = `TikZ style data`,
default : `thin`
style of edges in the Dynkin diagram

`mark` = ⟨`o,O,t,x,X,*`⟩,
default : `*`
default root mark

`affineMark` = `o,O,t,x,X,*`,
default : `*`
default root mark for root zero in an affine Dynkin diagram

`label` = `true or false`,
default : `false`
whether to label the roots according to the current labelling scheme.

`labelMacro` = ⟨`1-parameter TeX macro`⟩,
default : `#1`
the current labelling scheme.

`makeIndefiniteEdge` = ⟨`edge pair` i–j `or list of such`⟩,
default : `{}`

continued . . .

Table 14: . . . continued

edge pair or list of edge pairs to treat as having indefinitely many roots on them.
`indefiniteEdgeRatio` = ⟨`float`⟩,
default : `1.6`
ratio of indefinite edge lengths to other edge lengths.
`indefiniteEdge/.style` = ⟨`TikZ style data`⟩,
default : `draw=black,fill=white,thin,densely dotted`
style of the dotted or dashed middle third of each indefinite edge.
`arrows` = ⟨`true or false`⟩,
default : `true`
whether to draw the arrows that arise along the edges.
`reverseArrows` = ⟨`true or false`⟩,
default : `true`
whether to reverse the direction of the arrows that arise along the edges.
`fold` = ⟨`true or false`⟩,
default : `true`
whether, when drawing Dynkin diagrams, to draw them 2-ply.
`ply` = ⟨`0,1,2,3,4`⟩,
default : `0`
how many roots get folded together, at most.
`foldleft` = ⟨`true or false`⟩,
default : `true`
whether to fold the roots on the left side of a Dynkin diagram.
`foldright` = ⟨`true or false`⟩,
default : `true`
whether to fold the roots on the right side of a Dynkin diagram.
`foldradius` = ⟨`length`⟩,
default : `.3cm`
the radius of circular arcs used in curved edges of folded Dynkin diagrams.
`foldStyle` = ⟨`TikZ style data`⟩,
default : `draw=black!40,fill=none,line width=radius`
when drawing folded diagrams, style for the fold indicators.
`*/.style` = ⟨`TikZ style data`⟩,
default : `draw=black,fill=black`
style for roots like •
`o/.style` = ⟨`TikZ style data`⟩,
default : `draw=black,fill=black`
style for roots like ∘
`O/.style` = ⟨`TikZ style data`⟩,
default : `draw=black,fill=black`
style for roots like ◎
`t/.style` = ⟨`TikZ style data`⟩,
default : `draw=black,fill=black`
style for roots like ⊗

continued . . .

Table 14: ... continued

`x/.style` = ⟨`TikZ style data`⟩,
default : `draw=black`
style for roots like ×

`X/.style` = ⟨`TikZ style data`⟩,
default : `draw=black,thick`
style for roots like **×**

`leftFold/.style` = ⟨`TikZ style data`⟩,
default :
style to override the `fold` style when folding roots together on the left half of a Dynkin diagram

`rightFold/.style` = ⟨`TikZ style data`⟩,
default :
style to override the `fold` style when folding roots together on the right half of a Dynkin diagram

`doubleEdges` = ⟨⟩,
default : `not set`
set to override the `fold` style when folding roots together in a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows).

`doubleFold` = ⟨⟩,
default : `not set`
set to override the `fold` style when folding roots together in a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows), but filled in solidly.

`doubleLeft` = ⟨⟩,
default : `not set`
set to override the `fold` style when folding roots together at the left side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows).

`doubleFoldLeft` = ⟨⟩,
default : `not set`
set to override the `fold` style when folding roots together at the left side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows), but filled in solidly.

`doubleRight` = ⟨⟩,
default : `not set`
set to override the `fold` style when folding roots together at the right side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows).

`doubleFoldRight` = ⟨⟩,
default : `not set`

continued ...

Table 14: ...continued

set to override the `fold` style when folding roots together at the right side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows), but filled in solidly.

`Coxeter` = ⟨`true or false`⟩,
default : `false`
whether to draw a Coxeter diagram, rather than a Dynkin diagram.

`ordering` = ⟨`Adams, Bourbaki, Carter, Dynkin, Kac`⟩,
default : `Bourbaki`
which ordering of the roots to use in exceptional root systems as in section 17.

All other options are passed to TikZ.

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