

THE DYNKIN DIAGRAMS PACKAGE

BEN MCKAY

CONTENTS

1. Quick introduction	1
2. Syntax	4
3. Options	4
4. Finding the roots	6
5. Example: some parabolic subgroups	6
6. Example: the Hermitian symmetric spaces	7
7. Affine extended Dynkin diagrams	8
8. Coxeter diagrams	9
9. Folded Dynkin diagrams	10
10. Satake diagrams	11
11. Other stuff	13
References	13

1. QUICK INTRODUCTION

This is a test of the Dynkin diagram package. Load the package via `\usepackage{dynkin-diagrams}` (see below for options) and invoke it directly:

The flag variety of pointed lines in projective 3-space is associated to the Dynkin diagram $\times \bullet \bullet$.

1 The flag variety of pointed lines in
2 projective 3-space is associated to
3 the Dynkin diagram `\dynkin[parabolic=3]{A}{3}`.

or use the long form inside a `\tikz` statement:

$\times \bullet \bullet$

1 `\tikz \dynkin[parabolic=3]{A}{3};`

or a TikZ environment:

$\begin{array}{ccc} \times & \bullet & \bullet \\ 1 & 2 & 3 \end{array}$

1 `\begin{tikzpicture}`
2 `\dynkin[parabolic=3,label]{A}{3}`
3 `\end{tikzpicture}`

With labels for the roots:

$\begin{array}{ccc} \times & \bullet & \bullet \\ 1 & 2 & 3 \end{array}$

1 `\dynkin[parabolic=3,label]{A}{3}`

Make up your own labels for the roots:



```
1 \begin{tikzpicture}
2 \dynkin[parabolic=3]{A}{3}
3 \rootlabel{2}{\alpha_2}
4 \end{tikzpicture}
```

Use any text scale you like:



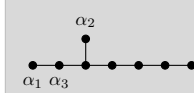
```
1 \begin{tikzpicture}
2 \dynkin[parabolic=3,textscale=1.2]{A}{3};
3 \rootlabel{2}{\alpha_2}
4 \end{tikzpicture}
```

and access root labels via TikZ:



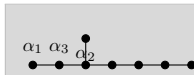
```
1 \begin{tikzpicture}
2 \dynkin[parabolic=3]{A}{3};
3 \node at (root label 2) {\alpha_2};
4 \end{tikzpicture}
```

The labels have default locations:



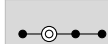
```
1 \begin{tikzpicture}
2 \dynkin{E}{8};
3 \rootlabel{1}{\alpha_1}
4 \rootlabel{2}{\alpha_2}
5 \rootlabel{3}{\alpha_3}
6 \end{tikzpicture}
```

You can use a starred form to flip labels to alternate locations:



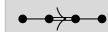
```
1 \begin{tikzpicture}
2 \dynkin{E}{8};
3 \rootlabel*{1}{\alpha_1}
4 \rootlabel*{2}{\alpha_2}
5 \rootlabel*{3}{\alpha_3}
6 \end{tikzpicture}
```

TikZ can access the roots themselves:



```
1 \begin{tikzpicture}
2 \dynkin{A}{4};
3 \fill [white,draw=black] (root 2) circle (.1cm);
4 \draw[black] (root 2) circle (.05cm);
5 \end{tikzpicture}
```

Some diagrams will have double edges:



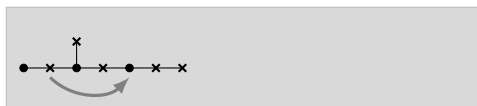
```
1 \dynkin{F}{4}
```

or triple edges:



```
1 \dynkin{G}{2}
```

Draw curves between the roots:



```
1 \begin{tikzpicture}
2 \dynkin[parabolic=429]{E}{8}
3 \draw[very thick, black!50,-latex]
  (root 3.south) to [out=-45, in=-135] (root 6.south);
4 \end{tikzpicture}
```

Draw dots on the roots:



```
1 \begin{tikzpicture}
2 \dynkin[label]{C}{8}
3 \dynkinopendot{3}
4 \dynkinopendot{7}
5 \end{tikzpicture}
```

Colours:



```
1 \dynkin[color=blue!50,backgroundcolor=red!20]{G}{2}
```

Edge lengths:



```
1 \dynkin[edgelength=1.2,parabolic=3]{A}{3}
```

Sizes of dots and crosses:



```
1 \dynkin[dotradius=.08cm,parabolic=3]{A}{3}
```

Edge styles:



```
1 \dynkin[edge=very thick,parabolic=3]{A}{3}
```

Open circles instead of closed dots:



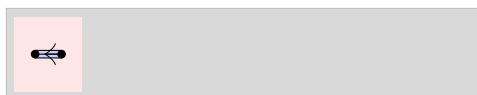
```
1 \dynkin[open]{E}{8}
```

Add closed dots to the open circles, at roots in the current ordering:



```
1 \begin{tikzpicture}
2 \dynkin[open]{E}{8};
3 \dynkincloseddot{5}
4 \dynkincloseddot{8}
5 \end{tikzpicture}
```

More colouring:



```
1 \begin{tikzpicture}[show background rectangle,
2   background rectangle/.style={ fill =red!10}]
3 \dynkin[parabolic=1,backgroundcolor=blue!20]{G}{2}
4 \end{tikzpicture}
```

Cross styles:



```
1 \dynkin[parabolic=124,cross=thin]{E}{8}
```

Suppress arrows:



```
1 \dynkin[arrows=false]{F}{4}
```



```
1 \dynkin[arrows=false]{G}{2}
```

2. SYNTAX

The syntax is `\dynkin[<options>]{<letter>}{<rank>}` where `<letter>` is `A, B, C, D, E, F` or `G`, the family of root system for the Dynkin diagram, and `<rank>` is an integer representing the rank, or is the symbol `*` to represent an indefinite rank:



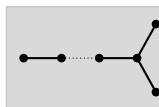
```
1 \dynkin[edge=thick,edgelength=.5cm]{A}{*}
```



```
1 \dynkin[edge=thick,edgelength=.5cm]{B}{*}
```



```
1 \dynkin[edge=thick,edgelength=.5cm]{C}{*}
```



```
1 \dynkin[edge=thick,edgelength=.5cm]{D}{*}
```

Outside a TikZ environment, the command builds its own TikZ environment.

3. OPTIONS

All `\dynkin` options (except `affine`, `folded`, `label` and `parabolic`) can also be passed to the package to force a global default option:

```
\usepackage[
  ordering=Kac,
  color=blue,
  open,
  dotradius=.06cm,
  backgroundcolor=red]
{dynkin-diagrams}
```

`parabolic = <integer>, default = 0`

A parabolic subgroup with specified integer, where the integer is computed as $n = \sum 2^i a_i$, $a_i = 0$ or 1 , to say that root i is crossed, i.e. a noncompact root.

`color = <color name>, default = black`

`backgroundcolor = <color name>, default = white`

This only says what color you have already set for the background rectangle. It is needed precisely for the G_2 root system, to draw the triple line correctly, and only when your background color is not white.

`dotradius = <number>cm, default = .05cm`

size of the dots and of the crosses in the Dynkin diagram

`edgelenlength = <number>cm, default = .35cm`

distance between nodes in the Dynkin diagram

`edge = <TikZ style data>, default = thin`

style of edges in the Dynkin diagram

`open = <true or false>, default = false`

use open circles rather than solid dots as default

`label = true or false, default = false`

whether to label the roots by their root numbers.

`arrows = <true or false>, default = true`

whether to draw the arrows that arise along the edges.

`folded = <true or false>, default = true`

whether, when drawing A , D or E_6 diagrams, to draw them folded.

`foldarrowstyle = <TikZ style>, default = stealth-stealth`

when drawing folded diagrams, style for the fold arrows.

`foldarrowcolor = <colour>, default = black!50`

when drawing folded diagrams, colour for the fold arrows.

`Coxeter = <true or false>, default = false`

whether to draw a Coxeter diagram, rather than a Dynkin diagram.

`ordering = <Adams, Bourbaki, Carter, Dynkin, Kac>, default = Bourbaki`

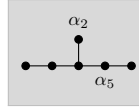
which ordering of the roots to use in exceptional root systems as follows:

	Adams	Bourbaki	Carter	Dynkin	Kac
E_6					
E_7					
E_8					
F_4					
G_2					

All other options are passed to TikZ.

4. FINDING THE ROOTS

The roots are labelled from 1 to r , where r is the rank. The command sets up TikZ nodes (`root 1`), (`root 2`), and so on. Affine extended Dynkin diagrams have affine root are at (`root 0`). Use these tikz nodes to draw on the Dynkin diagram, as above. It also sets up TikZ nodes (`root label 0`), (`root label 1`), and so on for the labels, and TikZ nodes (`root label swap 0`), (`root label swap 1`), and so on as alternative label locations, in case you want two labels on the same root, or the default choice doesn't look the way you like.



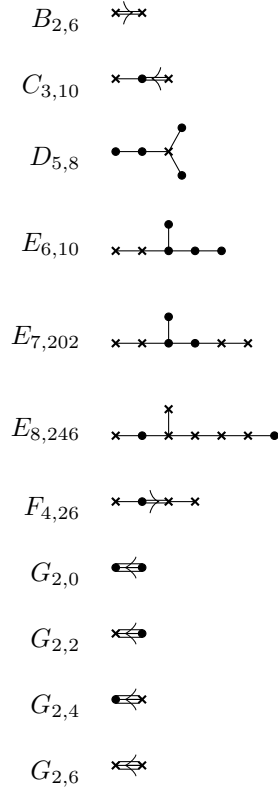
```

1 \begin{tikzpicture}
2 \dynkin{E}{6};
3 \rootlabel{2}{\alpha_2}
4 \rootlabel{5}{\alpha_5}
5 \end{tikzpicture}

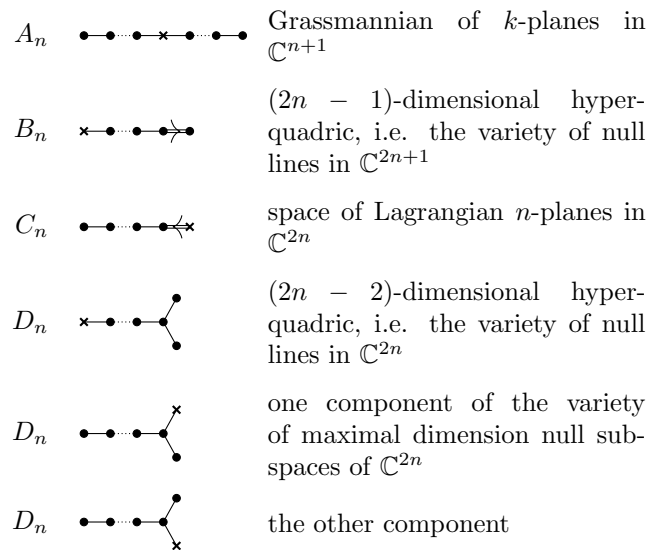
```

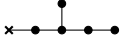
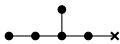
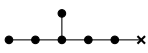
5. EXAMPLE: SOME PARABOLIC SUBGROUPS

$$\begin{array}{ll}
 A_{1,0} & \bullet \\
 A_{1,2} & \times \\
 A_{2,0} & \bullet \text{---} \bullet \\
 A_{2,2} & \times \text{---} \bullet \\
 A_{2,4} & \bullet \text{---} \times \\
 A_{2,6} & \times \text{---} \times
 \end{array}$$

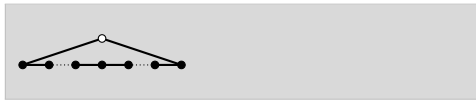


6. EXAMPLE: THE HERMITIAN SYMMETRIC SPACES

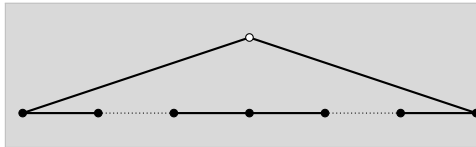


E_6		complexified octave projective plane
E_6		its dual plane
E_7		the space of null octave 3-planes in octave 6-space

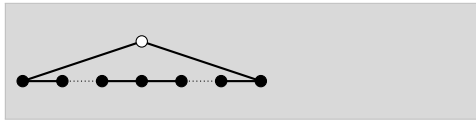
7. AFFINE EXTENDED DYNKIN DIAGRAMS



```
1 \dynkin[affine,edge=thick]{A}{*}
```



```
1 \dynkin[edglength=1cm,edge=thick,affine]{A}{*}
```



```
1 \dynkin[scale=1.5,edge=thick,affine]{A}{*}
```



```
1 \begin{tikzpicture}
2 \dynkin[affine,label]{A}{8};
3 \end{tikzpicture}
```



```
1 \begin{tikzpicture}
2 \dynkin[affine]{A}{*};
3 \node at (root label 0) {\(\alpha_0\)};
4 \end{tikzpicture}
```

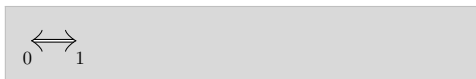


```
1 \begin{tikzpicture}
2 \dynkin[affine]{A}{9}
3 \node at (root label 0) {\(\alpha_0\)};
4 \end{tikzpicture}
```

You can use TikZ to put in labels:



```
1 \begin{tikzpicture}
2 \dynkin[affine]{A}{9};
3 \node at (root label 0) {\(\alpha_0\)};
4 \node at (root label 1) {\(\alpha_1\)};
5 \node at (root label 2) {\(\alpha_2\)};
6 \node at (root label 3) {\(\alpha_3\)};
7 \end{tikzpicture}
```






```
1 \dynkin[affine,label]{A}{1}
```


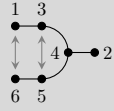





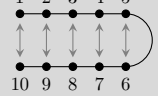
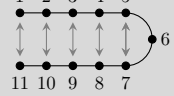

	<code>1 \dynkin[affine,label]{B}{8}</code>
	<code>1 \dynkin[affine,label]{B}{*}</code>
	<code>1 \dynkin[affine,label]{C}{8}</code>
	<code>1 \dynkin[affine,label]{C}{*}</code>
	<code>1 \dynkin[affine,label]{D}{8}</code>
	<code>1 \dynkin[affine,label]{D}{*}</code>
	<code>1 \dynkin[affine,label]{E}{6}</code>
	<code>1 \dynkin[affine,label]{E}{7}</code>
	<code>1 \dynkin[affine,label]{E}{8}</code>
Open circles instead of closed dots:	
	<code>1 \dynkin[affine,open,label]{E}{8}</code>
	<code>1 \dynkin[affine,label]{F}{4}</code>
	<code>1 \dynkin[affine,label]{G}{2}</code>

8. COXETER DIAGRAMS

	<code>1 \dynkin[Coxeter]{B}{7}</code>
	<code>1 \dynkin[Coxeter]{F}{4}</code>

	$1 \backslash \text{dynkin}[\text{Coxeter}]\{\text{G}\}\{2\}$
	$1 \backslash \text{dynkin}[\text{Coxeter}]\{\text{H}\}\{7\}$
	$1 \backslash \text{dynkin}[\text{Coxeter}]\{\text{I}\}\{7\}$

9. FOLDED DYNKIN DIAGRAMS

	$1 \backslash \text{dynkin}[\text{folded}]\{\text{E}\}\{6\}$
	$1 \backslash \text{dynkin}[\text{folded}, \text{label}]\{\text{E}\}\{6\}$
	$1 \backslash \text{dynkin}[\text{folded}]\{\text{A}\}\{*\}$
	$1 \backslash \text{dynkin}[\text{folded}, \text{label}]\{\text{A}\}\{1\}$
	$1 \backslash \text{dynkin}[\text{folded}, \text{label}]\{\text{A}\}\{2\}$
	$1 \backslash \text{dynkin}[\text{folded}, \text{label}]\{\text{A}\}\{3\}$
	$1 \backslash \text{dynkin}[\text{folded}, \text{label}]\{\text{A}\}\{4\}$
	$1 \backslash \text{dynkin}[\text{folded}, \text{label}]\{\text{A}\}\{10\}$
	$1 \backslash \text{dynkin}[\text{folded}, \text{label}]\{\text{A}\}\{11\}$

	<code>1 \dynkin[folded,label,arrows=false]{A}{11}</code>
	<code>1 \dynkin[folded]{D}{*}</code>
	<code>1 \dynkin[folded,label]{D}{1}</code>
	<code>1 \dynkin[folded,label]{D}{2}</code>
	<code>1 \dynkin[folded,label]{D}{3}</code>
	<code>1 \dynkin[folded,label]{D}{4}</code>
	<code>1 \dynkin[folded,label]{D}{10}</code>
	<code>1 \dynkin[folded,label]{D}{11}</code>

10. SATAKE DIAGRAMS

We have incomplete support for Satake diagrams as yet, following the conventions of [6].

	<code>1 \dynkin{A}{I}</code>
	<code>1 \dynkin{A}{II}</code>
	<code>1 \dynkin{E}{I}</code>
	<code>1 \dynkin{E}{II}</code>

	<code>1 \dynkin{E}{III}</code>
	<code>1 \dynkin{E}{IV}</code>
	<code>1 \dynkin{E}{V}</code>
	<code>1 \dynkin{E}{VI}</code>
	<code>1 \dynkin{E}{VII}</code>
	<code>1 \dynkin{E}{VIII}</code>
	<code>1 \dynkin{E}{XI}</code>
	<code>1 \dynkin{F}{I}</code>
	<code>1 \dynkin{F}{II}</code>
	<code>1 \dynkin{G}{I}</code>
	<pre> 1 \begin{tikzpicture} 2 \dynkin[open]{E}{6} 3 \draw[\dynkinfoldarrowstyle,\dynkinfoldarrowcolor] 4 (root 1.south) to [out=-45, in=-135] (root 6.south); 5 \draw[\dynkinfoldarrowstyle,\dynkinfoldarrowcolor] 6 (root 3.south) to [out=-45, in=-135] (root 5.south); 7 \end{tikzpicture} </pre>
	<pre> 1 \begin{tikzpicture} 2 \dynkin[open]{E}{6} 3 \dynkincloseddot{3} 4 \dynkincloseddot{4} 5 \dynkincloseddot{5} 6 \draw[\dynkinfoldarrowstyle,\dynkinfoldarrowcolor] 7 (root 1.south) to [out=-45, in=-135] (root 6.south); 8 \end{tikzpicture} </pre>

1. Nicolas Bourbaki, *Lie groups and Lie algebras. Chapters 4–6*, Elements of Mathematics (Berlin), Springer-Verlag, Berlin, 2002, Translated from the 1968 French original by Andrew Pressley. MR 1890629
2. R. W. Carter, *Lie algebras of finite and affine type*, Cambridge Studies in Advanced Mathematics, vol. 96, Cambridge University Press, Cambridge, 2005. MR 2188930
3. E. B. Dynkin, *Semisimple subalgebras of semisimple Lie algebras*, Mat. Sbornik N.S. **30(72)** (1952), 349–462 (3 plates), Reprinted in English translation in [4]. MR 0047629
4. ———, *Selected papers of E. B. Dynkin with commentary*, American Mathematical Society, Providence, RI; International Press, Cambridge, MA, 2000, Edited by A. A. Yushkevich, G. M. Seitz and A. L. Onishchik. MR 1757976
5. L. C. Grove and C. T. Benson, *Finite reflection groups*, second ed., Graduate Texts in Mathematics, vol. 99, Springer-Verlag, New York, 1985. MR 777684
6. Sigurdur Helgason, *Differential geometry, Lie groups, and symmetric spaces*, Graduate Studies in Mathematics, vol. 34, American Mathematical Society, Providence, RI, 2001, Corrected reprint of the 1978 original. MR 1834454
7. James E. Humphreys, *Reflection groups and Coxeter groups*, Cambridge Studies in Advanced Mathematics, vol. 29, Cambridge University Press, Cambridge, 1990. MR 1066460
8. Victor G. Kac, *Infinite-dimensional Lie algebras*, third ed., Cambridge University Press, Cambridge, 1990. MR 1104219
9. A. L. Onishchik and E. B. Vinberg, *Lie groups and algebraic groups*, Springer Series in Soviet Mathematics, Springer-Verlag, Berlin, 1990, Translated from the Russian and with a preface by D. A. Leites. MR 91g:22001
10. A. L. Onishchik and E. B. Vinberg, *Lie groups and algebraic groups*, Springer Series in Soviet Mathematics, Springer-Verlag, Berlin, 1990, Translated from the Russian and with a preface by D. A. Leites. MR 1064110
11. Ichirō Satake, *Algebraic structures of symmetric domains*, Kanō Memorial Lectures, vol. 4, Iwanami Shoten, Tokyo; Princeton University Press, Princeton, N.J., 1980. MR 591460
12. È. B. Vinberg (ed.), *Lie groups and Lie algebras, III*, Encyclopaedia of Mathematical Sciences, vol. 41, Springer-Verlag, Berlin, 1994. Structure of Lie groups and Lie algebras. A translation

of it Current problems in mathematics. Fundamental directions. Vol. 41 (Russian), Akad. Nauk SSSR, Vsesoyuz. Inst. Nauchn. i Tekhn. Inform., Moscow, 1990 [MR1056485 (91b:22001)], Translation by V. Minachin [V. V. Minakhin], Translation edited by A. L. Onishchik and È. B. Vinberg. MR 1349140