# THE DYNKIN DIAGRAMS PACKAGE
# VERSION 3.14159

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## Contents



## 1. Quick introduction

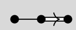
*Date*: 21 January 2019.

### Load the Dynkin diagram package (see options below)

```
\documentclass{amsart}
\usepackage{dynkin-diagrams}
\begin{document}
The Dynkin diagram of \(B_3\) is \dynkin{B}{3}.
\end{document}
```

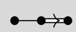
### Invoke it

```
The Dynkin diagram of \(B_3\) is \dynkin{B}{3}.
```

The Dynkin diagram of $B_3$ is .

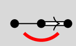
### Inside a Ti*k*Z statement

```
The Dynkin diagram of \(B_3\) is
\tikz \dynkin{B}{3};
```

The Dynkin diagram of $B_3$ is 

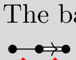
### Inside a Dynkin diagram environment

```
The Dynkin diagram of \(B_3\) is
\begin{dynkinDiagram}{B}{3}
\draw[very thick,red] (root 1) to [out=-45, in=-135] (root 3);
\end{dynkinDiagram}
```

The Dynkin diagram of $B_3$ is 

### Inside a Ti*k*Z environment

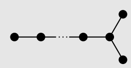
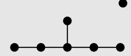
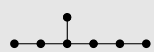
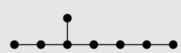
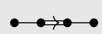
```
The baseline controls the vertical alignment:
the Dynkin diagram of \(B_3\) is
\begin{tikzpicture}[baseline=(origin.base)]
\dynkin{B}{3}
\draw[very thick,red] (root 1) to [out=-45, in=-135] (root 3);
\end{tikzpicture}
```

The baseline controls the vertical alignment: the Dynkin diagram of $B_3$ is


Indefinite rank Dynkin diagrams

```
\dynkin{B}{}
```

Table 1: The Dynkin diagrams of the reduced simple root systems [3] pp. 265–290, plates I–IX

| | | |
|---|---|---|
| $A_n$ |  | `\dynkin{A}{}` |
| $C_n$ |  | `\dynkin{C}{}` |
| $D_n$ |  | `\dynkin{D}{}` |
| $E_6$ |  | `\dynkin{E}{6}` |
| $E_7$ |  | `\dynkin{E}{7}` |
| $E_8$ |  | `\dynkin{E}{8}` |
| $F_4$ |  | `\dynkin{F}{4}` |
| $G_2$ |  | `\dynkin{G}{2}` |

## 2. Set options globally

Most options set globally . . .

```
\pgfkeys{/Dynkin diagram,edge length=.5cm,fold radius=.5cm,
indefinite edge/.style={
    draw=black,fill=white,thin,densely dashed}}
```

You can also pass options to the package in `\usepackage`. *Danger:* spaces in option names are replaced with hyphens: `edge length=1cm` is `edge-length=1cm` as a global option; moreover you should drop the extension `/.style` on any option with spaces in its name (but not otherwise). For example,

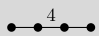
. . . or pass global options to the package

```
\usepackage[
     ordering=Kac,
     edge/.style=blue,
indefinite-edge={draw=green,fill=white,densely dashed},
indefinite-edge-ratio=5,
     mark=o,
     root-radius=.06cm]
     {dynkin-diagrams}
```

## 3. Coxeter diagrams

**Coxeter diagram option**

```
\dynkin[Coxeter]{F}{4}
```

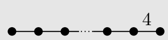
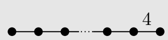
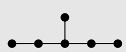
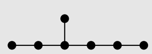
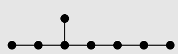
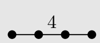
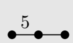
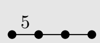


**gonality option for $G_2$ and $I_n$ Coxeter diagrams**

```
\(G_2=\dynkin[Coxeter,gonality=n]{G}{2}\), \
\(I_n=\dynkin[Coxeter,gonality=n]{I}{}\)
```

$G_2 =$ , $I_n =$ 

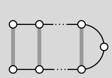
Table 2: The Coxeter diagrams of the simple reflection groups

| | | |
|---|---|---|
| $A_n$ |  | `\dynkin[Coxeter]{A}{}` |
| $B_n$ |  | `\dynkin[Coxeter]{B}{}` |
| $C_n$ |  | `\dynkin[Coxeter]{C}{}` |
| $E_6$ |  | `\dynkin[Coxeter]{E}{6}` |
| $E_7$ |  | `\dynkin[Coxeter]{E}{7}` |
| $E_8$ |  | `\dynkin[Coxeter]{E}{8}` |
| $F_4$ |  | `\dynkin[Coxeter]{F}{4}` |
| $G_2$ |  | `\dynkin[Coxeter,gonality=n]{G}{2}` |
| $H_3$ |  | `\dynkin[Coxeter]{H}{3}` |
| $H_4$ |  | `\dynkin[Coxeter]{H}{4}` |
| $I_n$ |  | `\dynkin[Coxeter,gonality=n]{I}{}` |

## 4. Satake diagrams

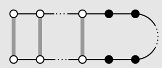
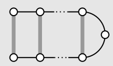
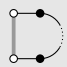
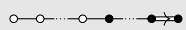
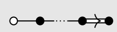
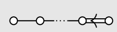
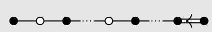
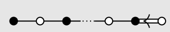
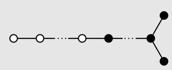
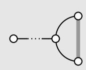
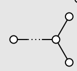
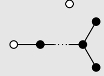
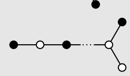
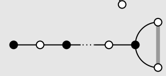
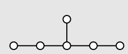
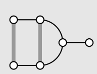
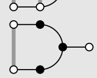
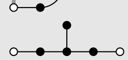
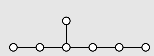
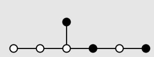
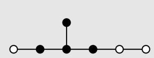
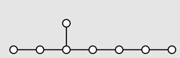
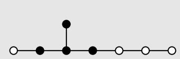
**Satake diagrams use the standard name instead of a rank**

```
\(A_{IIIb}=\dynkin{A}{IIIb}\)
```

$A_{IIIb} =$ 

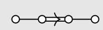
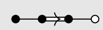
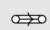
We use a solid gray bar to denote the folding of a Dynkin diagram, rather than the usual double arrow, since the diagrams turn out simpler and easier to read.

Table 3: The Satake diagrams of the real simple Lie algebras [13] p. 532–534

| | | |
|---|---|---|
| $A_{I}$ |  | `\dynkin{A}{I}` |
| $A_{II}$ |  | `\dynkin{A}{II}` |
| $A_{IIIa}$ |  | `\dynkin{A}{IIIa}` |
| $A_{IIIb}$ |  | `\dynkin{A}{IIIb}` |
| $A_{IV}$ |  | `\dynkin{A}{IV}` |
| $B_{I}$ |  | `\dynkin{B}{I}` |
| $B_{II}$ |  | `\dynkin{B}{II}` |
| $C_{I}$ |  | `\dynkin{C}{I}` |
| $C_{IIa}$ |  | `\dynkin{C}{IIa}` |
| $C_{IIb}$ |  | `\dynkin{C}{IIb}` |
| $D_{Ia}$ |  | `\dynkin{D}{Ia}` |
| $D_{Ib}$ |  | `\dynkin{D}{Ib}` |
| $D_{Ic}$ |  | `\dynkin{D}{Ic}` |
| $D_{II}$ |  | `\dynkin{D}{II}` |
| $D_{IIIa}$ |  | `\dynkin{D}{IIIa}` |
| $D_{IIIb}$ |  | `\dynkin{D}{IIIb}` |
| $E_{I}$ |  | `\dynkin{E}{I}` |
| $E_{II}$ |  | `\dynkin{E}{II}` |
| $E_{III}$ |  | `\dynkin{E}{III}` |
| $E_{IV}$ |  | `\dynkin{E}{IV}` |
| $E_{V}$ |  | `\dynkin{E}{V}` |
| $E_{VI}$ |  | `\dynkin{E}{VI}` |
| $E_{VII}$ |  | `\dynkin{E}{VII}` |
| $E_{VIII}$ |  | `\dynkin{E}{VIII}` |
| $E_{IX}$ |  | `\dynkin{E}{IX}` |

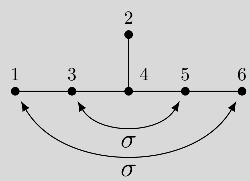
continued . . .

Table 3: . . . continued

| | | |
|---|---|---|
| $F_I$ |  | `\dynkin{F}{I}` |
| $F_{II}$ |  | `\dynkin{F}{II}` |
| $G_I$ |  | `\dynkin{G}{I}` |

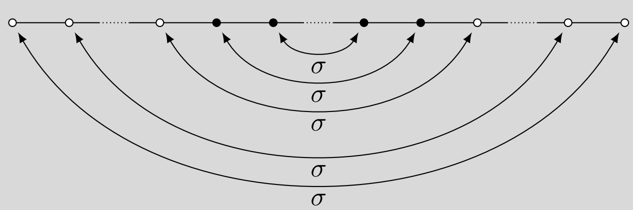
If you don't like the solid gray "folding bar", most people use arrows. Here is $E_{II}$

```
\newcommand{\invol}[2]{\draw[latex-latex] (root #1) to
[out=-60,in=-120] node[midway,below]{$\sigma$} (root #2);}
\begin{dynkinDiagram}[edge length=.75cm,labels*={1,...,6}]{E}{6}
\invol{1}{6}\invol{3}{5}
\end{dynkinDiagram}
```




The double arrows for $A_{IIIa}$ are big

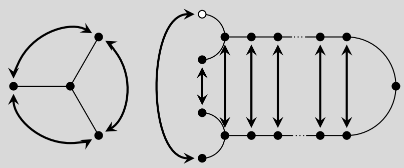
```
\newcommand{\invol}[2]{\draw[latex-latex] (root #1) to
[out=-60,in=-120] node[midway,below]{$\sigma$} (root #2);}
\begin{dynkinDiagram}[edge length=.75cm]{A}{oo.o**.**o.oo}
\invol{1}{10}\invol{2}{9}\invol{3}{8}\invol{4}{7}\invol{5}{6}
\end{dynkinDiagram}
```




If you don't like the solid gray "folding bar", most people use arrows . . .

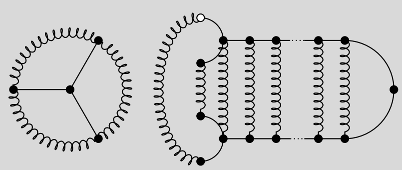
```
\tikzset{/Dynkin diagram/fold style/.style={stealth-stealth,thick,
shorten <=1mm,shorten >=1mm,}}
\dynkin[ply=3,edge length=.75cm]{D}{4}
\begin{dynkinDiagram}[ply=4]{D}[1]%
{****.*****.*****}
```

```
        \dynkinFold{1}{13}
        \dynkinFold[bend right=90]{0}{14}
\end{dynkinDiagram}
```



... but you could try springs pulling roots together

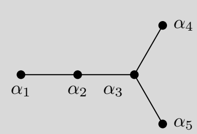
```
\tikzset{/Dynkin diagram/fold style/.style=
{decorate,decoration={name=coil,aspect=0.5,
segment length=1mm,amplitude=.6mm}}}
\dynkin[ply=3,edge length=.75cm]{D}{4}
\begin{dynkinDiagram}[ply=4]{D}[1]%
{****.*****.*****}
        \dynkinFold{1}{13}
        \dynkinFold[bend right=90]{0}{14}
\end{dynkinDiagram}
```



## 5. Labels for the roots

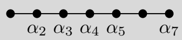
Make a macro to assign labels to roots

```
\dynkin[label,label macro/.code={\alpha_{#1}},edge
    length=.75cm]{D}{5}
```



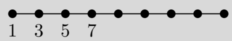

Labelling several roots

```
\dynkin[labels={,2,...,5,,7},label macro/.code={\alpha_{#1}}]{A}{7}
```



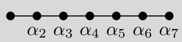

### The `foreach` notation I

```
\dynkin[labels={1,3,...,7},]{A}{9}
```



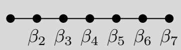

### The `foreach` notation II

```
\dynkin[labels={,\alpha_2,\alpha_...,\alpha_7},]{A}{7}
```



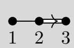

### The `foreach` notation III

```
\dynkin[label macro/.code={\beta_{#1}},labels={,2,...,7},]{A}{7}
```



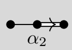

### Label the roots individually by root number

```
\dynkin[label]{B}{3}
```



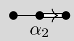

### Label a single root

```
\begin{dynkinDiagram}{B}{3}
\dynkinLabelRoot{2}{\alpha_2}
\end{dynkinDiagram}
```



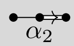

### Use a text style

```
\begin{dynkinDiagram}[text/.style={scale=1.2}]{B}{3};
\dynkinLabelRoot{2}{\alpha_2}
\end{dynkinDiagram}
```



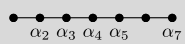

## Access root labels via TikZ

```
\begin{dynkinDiagram}{B}{3}
\node[below] at (root 2) {\(\alpha_2\)};
\end{dynkinDiagram}
```



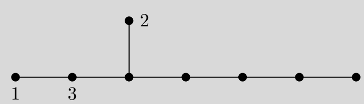

## Commands to label several roots

```
\begin{dynkinDiagram}{A}{7}
\dynkinLabelRoots{,\alpha_2,\alpha_3,\alpha_4,\alpha_5,,\alpha_7}
\end{dynkinDiagram}
```



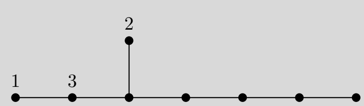

## The labels have default locations, mostly below roots

```
\dynkin[edge length=.75cm,labels={1,2,3}]{E}{8}
```



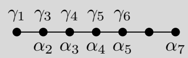

## The starred form flips labels to alternate locations, mostly above roots

```
\dynkin[edge length=.75cm,labels*={1,2,3}]{E}{8}
```



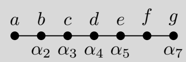

## Labelling several roots and alternates

```
\dynkin[%
label macro/.code={\alpha_{#1}},
label macro*/.code={\gamma_{#1}},
labels={,2,...,5,,7},
labels*={1,3,4,5,6}]{A}{7}
```




Commands to label several roots

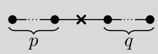
```
\begin{dynkinDiagram}{A}{7}
\dynkinLabelRoots{,\alpha_2,\alpha_3,\alpha_4,\alpha_5,,\alpha_7}
\dynkinLabelRoots*{a,b,c,d,e,f,g}
\end{dynkinDiagram}
```




## 6. Bracing roots

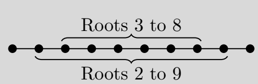
Bracing roots

```
\begin{dynkinDiagram}{A}{*.*x*.*}
\dynkinBrace[p]{1}{2}
\dynkinBrace[q]{4}{5}
\end{dynkinDiagram}
```




Bracing roots, and a starred form

```
\begin{dynkinDiagram}{A}{10}
\dynkinBrace[\text{Roots 2 to 9}]{2}{9}
\dynkinBrace*[\text{Roots 3 to 8}]{3}{8}
\end{dynkinDiagram}
```




Bracing roots

```
\newcommand\circleRoot[1]{\draw (root #1) circle (3pt);}
\begin{dynkinDiagram}{A}{**.***.***.***.***.**}
\circleRoot{4}\circleRoot{7}\circleRoot{10}\circleRoot{13}
\dynkinBrace[y-1]{1}{3}
\dynkinBrace[z-1]{5}{6}
\dynkinBrace[t-1]{11}{12}
\dynkinBrace[x-1]{14}{16}
```

```
\end{dynkinDiagram}
```

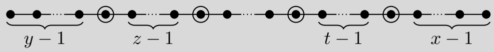
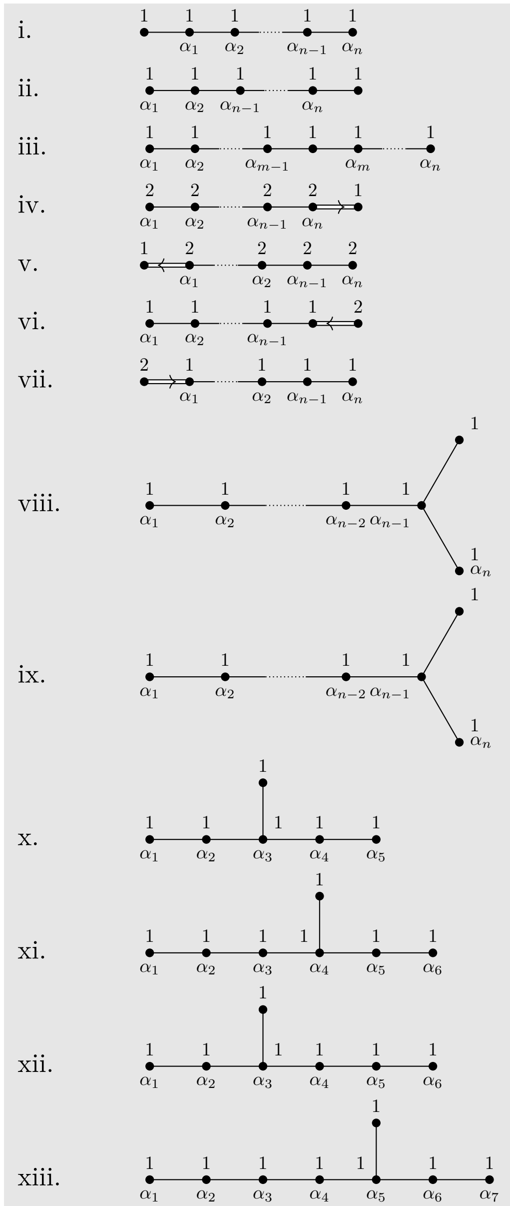
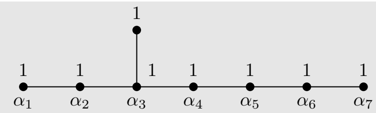
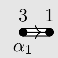
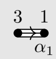
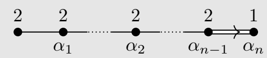
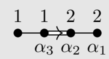
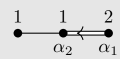
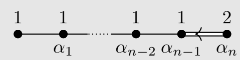
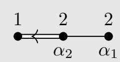
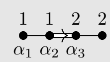
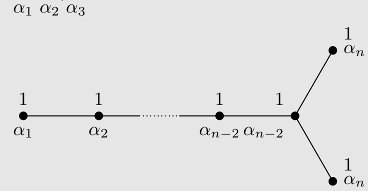
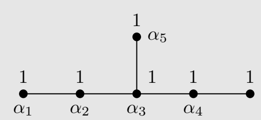
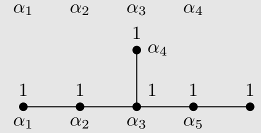
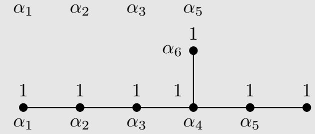
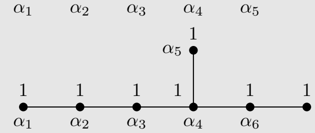
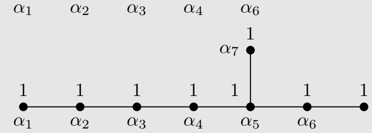
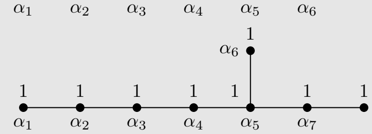



Table 4: Dynkin diagrams from Euler products [17]



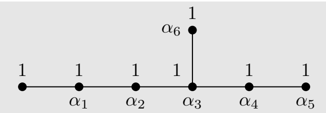
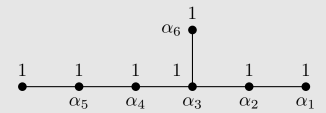
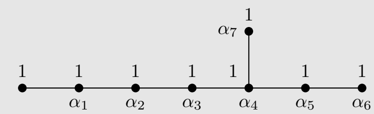

continued ...

Table 4: …continued

| | |
|---|---|
| xiv. |  |
| xv. |  |
| xvi. |  |
| xvii. |  |
| xviii. |  |
| xix. |  |
| xx. |  |
| xxi. |  |
| xxii. |  |
| xxiii. |  |
| xxiv. |  |
| xxv. |  |
| xxvi. |  |
| xxvii. |  |
| xxviii. |  |
| xxix. |  |

continued …

Table 4: . . . continued

| | |
|---|---|
| xxx. |  |
| xxxi. |  |
| xxxii. |  |

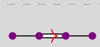
```
\tikzset{/Dynkin diagram,ordering=Dynkin,label macro/.code={\alpha_{#1}}}
\newcounter{EPNo}
\setcounter{EPNo}{0}
\NewDocumentCommand\EP{smmmm}%
{%
\stepcounter{EPNo}\roman{EPNo}. &
\def\eL{.6cm}
\IfStrEqCase{#2}%
{%
{D}{\gdef\eL{1cm}}%
{E}{\gdef\eL{.75cm}}%
{F}{\gdef\eL{.35cm}}%
{G}{\gdef\eL{.35cm}}%
}%
\tikzset{/Dynkin diagram,edge length=\eL}
\IfBooleanTF{#1}%
{\dynkin[backwards,labels*={#4},labels={#5}]{#2}{#3}}
{\dynkin[labels*={#4},labels={#5}]{#2}{#3}}
\\
}%
\begin{longtable}{MM}
\caption{Dynkin diagrams from Euler products \cite{Langlands:1967}}\\
\endfirsthead
\caption{\dots continued}\\
\endhead
\multicolumn{2}{c}{continued \dots}\\
\endfoot
\endlastfoot
\EP{A}{***.**}{1,1,1,1,1}{,1,2,n-1,n}
\EP{A}{***.**}{1,1,1,1,1}{1,2,n-1,n}
\EP{A}{**.***.*}{1,1,1,1,1,1}{1,2,m-1,,m,n}
\EP{B}{**.***}{2,2,2,2,1}{1,2,n-1,n}
\EP*{B}{***.**}{2,2,2,2,1}{n,n-1,2,1,}
\EP{C}{**.***}{1,1,1,1,2}{1,2,n-1,}
\EP*{C}{***.**}{1,1,1,1,2}{n,n-1,2,1,}
\EP{D}{**.****}{1,1,1,1,1,1}{1,2,n-2,n-1,n}
\EP{D}{**.****}{1,1,1,1,1,1}{1,2,n-2,n-1,n}
\EP{E}{6}{1,1,1,1,1,1}{1,...,5}
```

```
\EP*{E}{7}{1,1,1,1,1,1,1}{6,...,1}
\EP{E}{7}{1,1,1,1,1,1,1}{1,...,6}
\EP*{E}{8}{1,1,1,1,1,1,1,1}{7,...,1}
\EP{E}{8}{1,1,1,1,1,1,1,1}{1,...,7}
\EP{G}{2}{1,3}{,1}
\EP{G}{2}{1,3}{1}
\EP{B}{**.*.**}{2,2,2,2,1}{,1,2,n-1,n}
\EP{F}{4}{1,1,2,2}{,3,2,1}
\EP{C}{3}{1,1,2}{,2,1}
\EP{C}{**.***}{1,1,1,1,2}{,1,n-2,n-1,n}
\EP*{B}{3}{2,2,1}{1,2}
\EP{F}{4}{1,1,2,2}{1,2,3}
\EP{D}{**.****}{1,1,1,1,1,1}{1,2,n-2,n-2,n,n}
\EP{E}{6}{1,1,1,1,1,1}{1,2,3,4,,5}
\EP{E}{6}{1,1,1,1,1,1}{1,2,3,5,,4}
\EP*{E}{7}{1,1,1,1,1,1,1}{,5,...,1,6}
\EP*{E}{7}{1,1,1,1,1,1,1}{,6,4,3,2,1,5}
\EP*{E}{8}{1,1,1,1,1,1,1,1}{,6,...,1,7}
\EP*{E}{8}{1,1,1,1,1,1,1,1}{,7,5,4,3,2,1,6}
\EP*{E}{7}{1,1,1,1,1,1,1}{5,...,1,,6}
\EP*{E}{7}{1,1,1,1,1,1,1}{1,...,5,,6}
\EP*{E}{8}{1,1,1,1,1,1,1,1}{6,...,1,,7}
\end{longtable}
```

## 7. Style

### Colours

```
\dynkin[
        edge/.style={blue!50,thick},
        */.style=blue!50!red,
        arrow color=red]{F}{4}
```



### Edge lengths

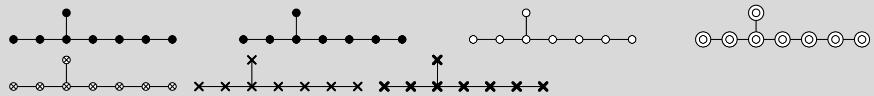
```
The Dynkin diagram of \(A_3\) is \dynkin[edge
    length=1.2,parabolic=3]{A}{3}
```

The Dynkin diagram of $A_3$ is 

### Root marks

```
\dynkin{E}{8}
\dynkin[mark=*]{E}{8}
\dynkin[mark=o]{E}{8}
\dynkin[mark=O]{E}{8}
\dynkin[mark=t]{E}{8}
```

```
\dynkin[mark=x]{E}{8}
\dynkin[mark=X]{E}{8}
```

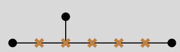


At the moment, you can only use:

- `*` solid dot
- `o` hollow circle
- `O` double hollow circle
- `t` tensor root
- `x` crossed root
- `X` thickly crossed root

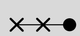
Mark styles

```
The parabolic subgroup \(E_{8,124}\) is
    \dynkin[parabolic=124,x/.style={brown,very thick}]{E}{8}
```

The parabolic subgroup $E_{8,124}$ is 

Sizes of root marks

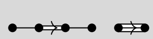
```
\(A_{3,3}\) with big root marks is \dynkin[root
    radius=.08cm,parabolic=3]{A}{3}
```

$A_{3,3}$ with big root marks is 

## 8. Suppress or reverse arrows

Some diagrams have double or triple edges

```
\dynkin{F}{4}
\dynkin{G}{2}
```



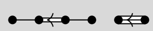
### Suppress arrows

```
\dynkin[arrows=false]{F}{4}
\dynkin[arrows=false]{G}{2}
```



### Reverse arrows

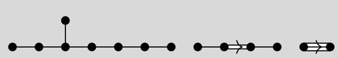
```
\dynkin[reverse arrows]{F}{4}
\dynkin[reverse arrows]{G}{2}
```



## 9. Backwards and upside down

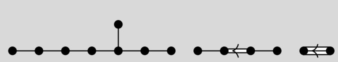
### Default

```
\dynkin{E}{8}
\dynkin{F}{4}
\dynkin{G}{2}
```



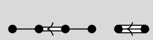
### Backwards

```
\dynkin[backwards]{E}{8}
\dynkin[backwards]{F}{4}
\dynkin[backwards]{G}{2}
```



### Reverse arrows

```
\dynkin[reverse arrows]{F}{4}
\dynkin[reverse arrows]{G}{2}
```

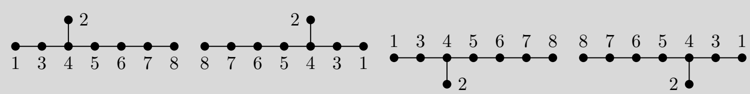


**Backwards, reverse arrows**

```
\dynkin[backwards,reverse arrows]{F}{4}
\dynkin[backwards,reverse arrows]{G}{2}
```

**Backwards versus upside down**

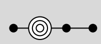
```
\dynkin[label]{E}{8}
\dynkin[label,backwards]{E}{8}
\dynkin[label,upside down]{E}{8}
\dynkin[label,backwards,upside down]{E}{8}
```



## 10. Drawing on top of a Dynkin diagram

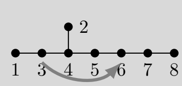
**TikZ can access the roots themselves**

```
\begin{dynkinDiagram}{A}{4}
        \fill[white,draw=black] (root 2) circle (.15cm);
        \fill[white,draw=black] (root 2) circle (.1cm);
        \draw[black] (root 2) circle (.05cm);
\end{dynkinDiagram}
```



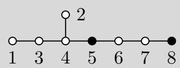
**Draw curves between the roots**

```
\begin{dynkinDiagram}[label]{E}{8}
        \draw[very thick, black!50,-latex]
                (root 3.south) to [out=-45, in=-135] (root 6.south);
\end{dynkinDiagram}
```



**Change marks**

```
\begin{dynkinDiagram}[mark=o,label]{E}{8}
        \dynkinRootMark{*}{5}
        \dynkinRootMark{*}{8}
\end{dynkinDiagram}
```

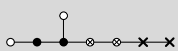



## 11. Mark lists

The package allows a list of root marks instead of a rank:

**A mark list**

```
\dynkin{E}{oo**ttxx}
```



The mark list `oo**ttxx` has one mark for each root: `o`, `o`, . . . , `x`. Roots are listed in the current default ordering. (Careful: in an affine root system, a mark list will *not* contain a mark for root zero.)

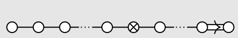
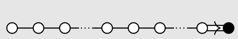
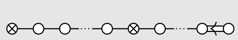
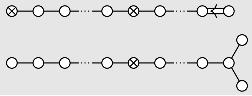
If you need to repeat a mark, you can give a *single digit* positive integer to indicate how many times to repeat it.

**A mark list with repetitions**

```
\dynkin{A}{x4o3t4}
```



Table 5: Classical Lie superalgebras [10]. We need a slightly larger root radius parameter to distinguish the tensor product symbols from the solid dots.

| | | |
|---|---|---|
| | | `\tikzset{/Dynkin diagram,root radius=.07cm}` |
| $A_{mn}$ |  | `\dynkin{A}{o3.oto.oo}` |
| $B_{mn}$ |  | `\dynkin{B}{o3.oto.oo}` |
| $B_{0n}$ |  | `\dynkin{B}{o3.o3.o*}` |
| $C_n$ |  | `\dynkin{C}{too.oto.oo}` |
| $D_{mn}$ |  | `\dynkin{D}{o3.oto.o4}` |

continued . . .

Table 5: . . . continued

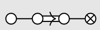
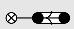
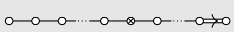
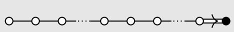
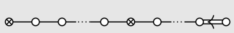
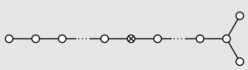
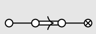
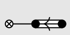
| | | |
|---|---|---|
| $D_{21\alpha}$ |  | `\dynkin{A}{oto}` |
| $F_4$ |  | `\dynkin{F}{ooot}` |
| $G_3$ |  | `\dynkin[extended,affine mark=t, reverse arrows]{G}{2}` |

Table 6: Classical Lie superalgebras [10]. Here we see the problem with using the default root radius parameter, which is too small for tensor product symbols.

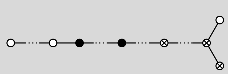
| | | |
|---|---|---|
| $A_{mn}$ |  | `\dynkin{A}{o3.oto.oo}` |
| $B_{mn}$ |  | `\dynkin{B}{o3.oto.oo}` |
| $B_{0n}$ |  | `\dynkin{B}{o3.o3.o*}` |
| $C_n$ |  | `\dynkin{C}{too.oto.oo}` |
| $D_{mn}$ |  | `\dynkin{D}{o3.oto.o4}` |
| $D_{21\alpha}$ |  | `\dynkin{A}{oto}` |
| $F_4$ |  | `\dynkin{F}{ooot}` |
| $G_3$ |  | `\dynkin[extended,affine mark=t, reverse arrows]{G}{2}` |

## 12. Indefinite edges

An *indefinite edge* is a dashed edge between two roots,  indicating that an indefinite number of roots have been omitted from the Dynkin diagram. In between any two entries in a mark list, place a period to indicate an indefinite edge:

**Indefinite edges**

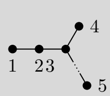
```
\dynkin{D}{o.o*.*.t.to.t}
```



In certain diagrams, roots may have an edge between them even though they are not subsequent in the ordering. For such rare situations, there is an option:

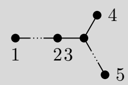
**Indefinite edge option**

```
\dynkin[make indefinite edge={3-5},label]{D}{5}
```



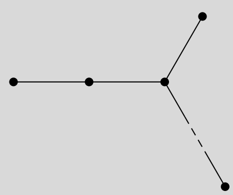
**Give a list of edges to become indefinite**

```
\dynkin[make indefinite edge/.list={1-2,3-5},label]{D}{5}
```




**Indefinite edge style**

```
\dynkin[indefinite edge/.style={draw=black,fill=white,thin,densely
    dashed},%
        edge length=1cm,%
        make indefinite edge={3-5}]
        {D}{5}
```



**The ratio of the lengths of indefinite edges to those of other edges**

```
\dynkin[edge length = .5cm,%
        indefinite edge ratio=3,%
        make indefinite edge={3-5}]
        {D}{5}
```

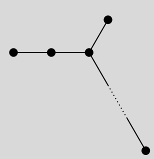
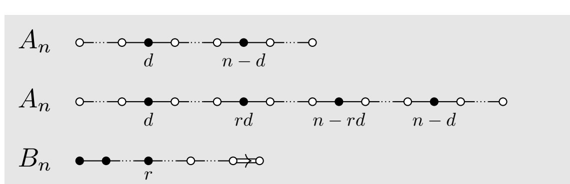
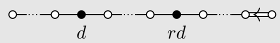
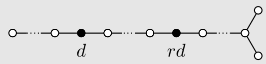
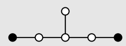
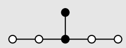
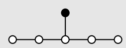
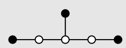
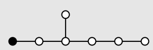
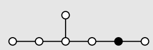
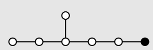
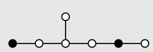
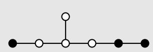
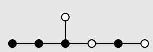
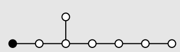
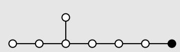
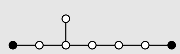
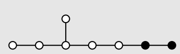
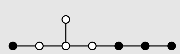
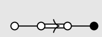
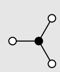


Table 7: Springer's table of indices [24], pp. 320-321, with one form of $E_7$ corrected




continued ...

Table 7: ... continued

| | | |
|---|---|---|
| $C_n$ |  | |
| $D_n$ |  | |
| $E_6$ |  | \dynkin{E}{*oooo*} |
| $E_6$ |  | \dynkin{E}{o*o*oo} |
| $E_6$ |  | \dynkin{E}{o*oooo} |
| $E_6$ |  | \dynkin{E}{**ooo*} |
| $E_7$ |  | \dynkin{E}{*oooooo} |
| $E_7$ |  | \dynkin{E}{ooooo*o} |
| $E_7$ |  | \dynkin{E}{oooooo*} |
| $E_7$ |  | \dynkin{E}{*oooo*o} |
| $E_7$ |  | \dynkin{E}{*oooo**} |
| $E_7$ |  | \dynkin{E}{*o**o*o} |
| $E_8$ |  | \dynkin{E}{*ooooooo} |
| $E_8$ |  | \dynkin{E}{ooooooo*} |
| $E_8$ |  | \dynkin{E}{*oooooo*} |
| $E_8$ |  | \dynkin{E}{oooooo**} |
| $E_8$ |  | \dynkin{E}{*oooo***} |
| $F_4$ |  | \dynkin{F}{ooo*} |
| $D_4$ |  | \dynkin{D}{o*oo} |

## 13. Parabolic subgroups

Each set of roots is assigned a number, with each binary digit zero or one to say whether the corresponding root is crossed or not:

```
The flag variety of pointed lines in
projective 3-space is associated to
the Dynkin diagram \dynkin[parabolic=3]{A}{3}.
```

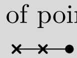
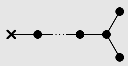
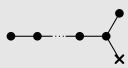
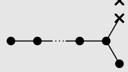
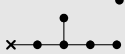
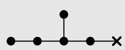
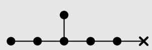
The flag variety of pointed lines in projective 3-space is associated to the Dynkin diagram .

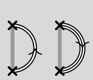
Table 8: The Hermitian symmetric spaces

| | | |
|---|---|---|
| $A_n$ |  | Grassmannian of $k$-planes in $\mathbb{C}^{n+1}$ |
| $B_n$ |  | $(2n-1)$-dimensional hyperquadric, i.e. the variety of null lines in $\mathbb{C}^{2n+1}$ |
| $C_n$ |  | space of Lagrangian $n$-planes in $\mathbb{C}^{2n}$ |
| $D_n$ |  | $(2n-2)$-dimensional hyperquadric, i.e. the variety of null lines in $\mathbb{C}^{2n}$ |
| $D_n$ |  | one component of the variety of maximal dimension null subspaces of $\mathbb{C}^{2n}$ |
| $D_n$ |  | the other component |
| $E_6$ |  | complexified octave projective plane |
| $E_6$ |  | its dual plane |
| $E_7$ |  | the space of null octave 3-planes in octave 6-space |

```
\NewDocumentCommand\HSS{mommm}
{#1&\IfNoValueTF{#2}{\dynkin{#3}{#4}}{\dynkin[parabolic=#2]{#3}{#4}}&#5\\}
\renewcommand*{\arraystretch}{1.5}
\begin{longtable}
{>{\columncolor[gray]{.9}}>$l<$>{\columncolor[gray]{.9}}>$l<$>{\columncolor[gray]{.9}}l}
\caption{The Hermitian symmetric spaces}\endfirsthead
\caption{\dots continued}\\ \endhead
\caption{continued \dots}\\ \endfoot
\endlastfoot
\HSS{A_n}{A}{**.*x*.**}{Grassmannian of $k$-planes in $\C{n+1}$}
\HSS{B_n}[1]{B}{}{$(2n-1)$-dimensional hyperquadric, i.e. the variety of null lines in $\C{2n+1}$}
\HSS{C_n}[16]{C}{}{space of Lagrangian $n$-planes in $\C{2n}$}
\HSS{D_n}[1]{D}{}{$(2n-2)$-dimensional hyperquadric, i.e. the variety of null lines in $\C{2n}$}
\HSS{D_n}[32]{D}{}{one component of the variety of maximal dimension null subspaces of $\C{2n}$}
\HSS{D_n}[16]{D}{}{the other component}
\HSS{E_6}[1]{E}{6}{complexified octave projective plane}
\HSS{E_6}[32]{E}{6}{its dual plane}
\HSS{E_7}[64]{E}{7}{the space of null octave 3-planes in octave 6-space}
\end{longtable}
```

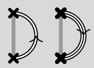
Folded parabolics look bad (zoom in on a root)

```
\dynkin[fold,parabolic=3]{C}{2}
\dynkin[fold,parabolic=3]{G}{2}
```



Folded parabolics: you can try using thicker crosses

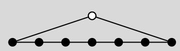
```
\dynkin[fold,x/.style={very thick,line cap=round},parabolic=3]{C}{2}
\dynkin[fold,x/.style={ultra thick,line
    cap=round},parabolic=3]{G}{2}
```



## 14. Extended Dynkin diagrams

Extended Dynkin diagrams

```
\dynkin[extended]{A}{7}
```



The extended Dynkin diagrams are also described in the notation of Kac [15] p. 55 as affine untwisted Dynkin diagrams: we extend `\dynkin{A}{7}` to become `\dynkin{A}[1]{7}`:

Extended Dynkin diagrams

```
\dynkin{A}[1]{7}
```

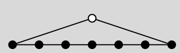
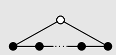
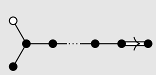
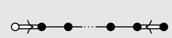
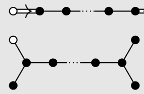
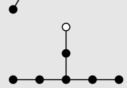
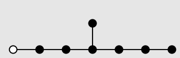
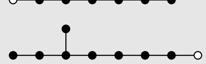
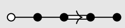
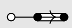


Table 9: The Dynkin diagrams of the extended simple root systems

| | | |
|---|---|---|
| $A_1^1$ |  | `\dynkin[extended]{A}{1}` |
| $A_n^1$ |  | `\dynkin[extended]{A}{}` |
| $B_n^1$ |  | `\dynkin[extended]{B}{}` |
| $C_n^1$ |  | `\dynkin[extended]{C}{}` |
| $D_n^1$ |  | `\dynkin[extended]{D}{}` |
| $E_6^1$ |  | `\dynkin[extended]{E}{6}` |
| $E_7^1$ |  | `\dynkin[extended]{E}{7}` |
| $E_8^1$ |  | `\dynkin[extended]{E}{8}` |

continued . . .

Table 9: . . . continued

| | | |
|---|---|---|
| $F^1_4$ |  | `\dynkin[extended]{F}{4}` |
| $G^1_2$ |  | `\dynkin[extended]{G}{2}` |

## 15. Affine twisted and untwisted Dynkin diagrams

The affine Dynkin diagrams are described in the notation of Kac [15] p. 55:

Affine Dynkin diagrams

```
\(A^{(1)}_7=\dynkin{A}[1]{7}, \
E^{(2)}_6=\dynkin{E}[2]{6}, \
D^{(3)}_4=\dynkin{D}[3]{4}\)
```

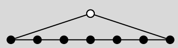
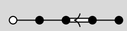
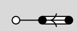
$A^{(1)}_7 =$ , $E^{(2)}_6 =$ , $D^{(3)}_4 =$ 

Table 10: The affine Dynkin diagrams

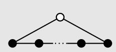
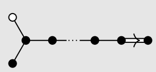
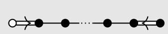
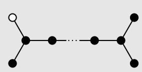
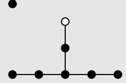
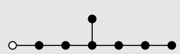
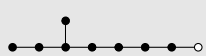
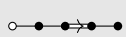
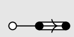
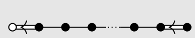
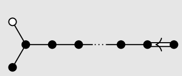
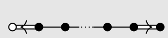
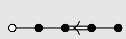
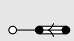
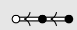
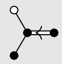
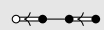
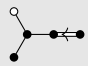
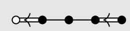
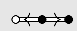
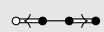
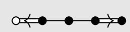
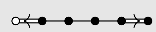
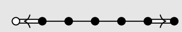
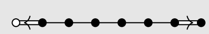
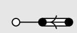
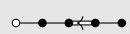
| | | |
|---|---|---|
| $A^1_1$ |  | `\dynkin{A}[1]{1}` |
| $A^1_n$ |  | `\dynkin{A}[1]{}` |
| $B^1_n$ |  | `\dynkin{B}[1]{}` |
| $C^1_n$ |  | `\dynkin{C}[1]{}` |
| $D^1_n$ |  | `\dynkin{D}[1]{}` |
| $E^1_6$ |  | `\dynkin{E}[1]{6}` |
| $E^1_7$ |  | `\dynkin{E}[1]{7}` |
| $E^1_8$ |  | `\dynkin{E}[1]{8}` |
| $F^1_4$ |  | `\dynkin{F}[1]{4}` |
| $G^1_2$ |  | `\dynkin{G}[1]{2}` |
| $A^2_2$ |  | `\dynkin{A}[2]{2}` |
| $A^2_{ev}$ |  | `\dynkin{A}[2]{even}` |
| $A^2_{od}$ |  | `\dynkin{A}[2]{odd}` |
| $D^2_n$ |  | `\dynkin{D}[2]{}` |
| $E^2_6$ |  | `\dynkin{E}[2]{6}` |
| $D^3_4$ |  | `\dynkin{D}[3]{4}` |

Table 11: Some more affine Dynkin diagrams

| | | |
|---|---|---|
| $A_4^2$ |  | `\dynkin{A}[2]{4}` |
| $A_5^2$ |  | `\dynkin{A}[2]{5}` |
| $A_6^2$ |  | `\dynkin{A}[2]{6}` |
| $A_7^2$ |  | `\dynkin{A}[2]{7}` |
| $A_8^2$ |  | `\dynkin{A}[2]{8}` |
| $D_3^2$ |  | `\dynkin{D}[2]{3}` |
| $D_4^2$ |  | `\dynkin{D}[2]{4}` |
| $D_5^2$ |  | `\dynkin{D}[2]{5}` |
| $D_6^2$ |  | `\dynkin{D}[2]{6}` |
| $D_7^2$ |  | `\dynkin{D}[2]{7}` |
| $D_8^2$ |  | `\dynkin{D}[2]{8}` |
| $D_4^3$ |  | `\dynkin{D}[3]{4}` |
| $E_6^2$ |  | `\dynkin{E}[2]{6}` |

## 16. Extended Coxeter diagrams

Extended and Coxeter options together

```
\dynkin[extended,Coxeter]{F}{4}
```

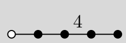
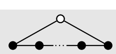
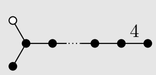
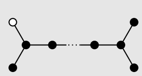
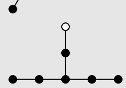
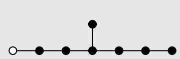
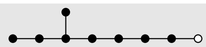
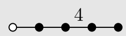
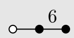
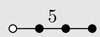
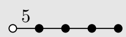



Table 12: The extended (affine) Coxeter diagrams

| | | |
|---|---|---|
| $A_n$ |  | `\dynkin[extended,Coxeter]{A}{}` |
| $B_n$ |  | `\dynkin[extended,Coxeter]{B}{}` |
| $C_n$ |  | `\dynkin[extended,Coxeter]{C}{}` |
| $D_n$ |  | `\dynkin[extended,Coxeter]{D}{}` |
| $E_6$ |  | `\dynkin[extended,Coxeter]{E}{6}` |
| $E_7$ |  | `\dynkin[extended,Coxeter]{E}{7}` |

continued . . .

Table 12: . . . continued

| | | |
|---|---|---|
| $E_8$ |  | `\dynkin[extended,Coxeter]{E}{8}` |
| $F_4$ |  | `\dynkin[extended,Coxeter]{F}{4}` |
| $G_2$ |  | `\dynkin[extended,Coxeter]{G}{2}` |
| $H_3$ |  | `\dynkin[extended,Coxeter]{H}{3}` |
| $H_4$ |  | `\dynkin[extended,Coxeter]{H}{4}` |
| $I_1$ |  | `\dynkin[extended,Coxeter]{I}{1}` |

## 17. Kac style

We include a style called `Kac` which tries to imitate the style of [15].

Kac style

```
\dynkin[Kac]{F}{4}
```

Table 13: The Dynkin diagrams of the simple root systems in Kac style

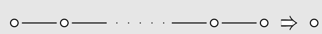
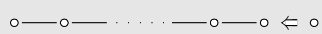
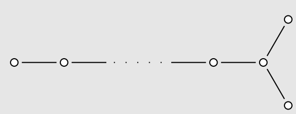
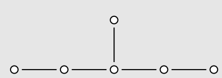
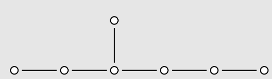
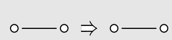
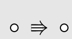
| | | |
|---|---|---|
| $A_n$ |  | `\dynkin{A}{}` |
| $B_n$ |  | `\dynkin{B}{}` |
| $C_n$ |  | `\dynkin{C}{}` |
| $D_n$ |  | `\dynkin{D}{}` |
| $E_6$ |  | `\dynkin{E}{6}` |
| $E_7$ |  | `\dynkin{E}{7}` |
| $E_8$ |  | `\dynkin{E}{8}` |
| $F_4$ |  | `\dynkin{F}{4}` |
| $G_2$ |  | `\dynkin{G}{2}` |

Table 14: The Dynkin diagrams of the extended simple root systems in Kac style

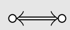
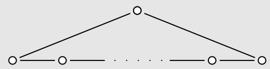
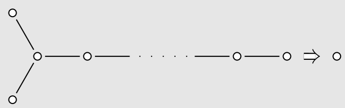
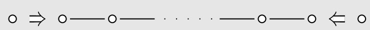
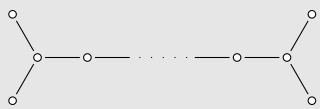
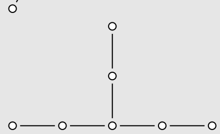
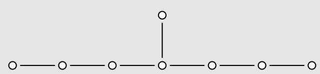
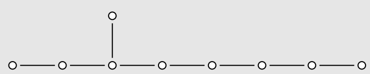
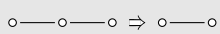
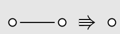
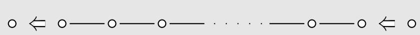
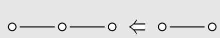
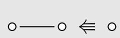
| | | |
|---|---|---|
| $A_1^1$ |  | `\dynkin[extended]{A}{1}` |
| $A_n^1$ |  | `\dynkin[extended]{A}{}` |
| $B_n^1$ |  | `\dynkin[extended]{B}{}` |
| $C_n^1$ |  | `\dynkin[extended]{C}{}` |
| $D_n^1$ |  | `\dynkin[extended]{D}{}` |
| $E_6^1$ |  | `\dynkin[extended]{E}{6}` |
| $E_7^1$ |  | `\dynkin[extended]{E}{7}` |
| $E_8^1$ |  | `\dynkin[extended]{E}{8}` |
| $F_4^1$ |  | `\dynkin[extended]{F}{4}` |
| $G_2^1$ |  | `\dynkin[extended]{G}{2}` |

Table 15: The Dynkin diagrams of the twisted simple root systems in Kac style

| | | |
|---|---|---|
| $A_2^2$ |  | `\dynkin{A}[2]{2}` |
| $A_{ev}^2$ |  | `\dynkin{A}[2]{even}` |
| $A_{od}^2$ |  | `\dynkin{A}[2]{odd}` |
| $D_n^2$ |  | `\dynkin{D}[2]{}` |
| $E_6^2$ |  | `\dynkin{E}[2]{6}` |
| $D_4^3$ |  | `\dynkin{D}[3]{4}` |

## 18. Ceref style

We include a style called `ceref` which shapes the root markers more oblongly and with shadows. The word “ceref” is an old form of the word “serif”.

### Ceref style

```
\dynkin[ceref]{F}{4}
```

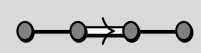


Table 16: The Dynkin diagrams of the simple root systems in ceref style

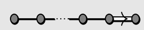
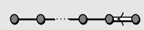
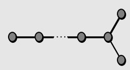
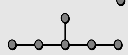
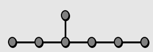
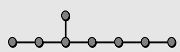
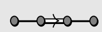
| | | |
|---|---|---|
| $A_n$ |  | `\dynkin{A}{}` |
| $B_n$ |  | `\dynkin{B}{}` |
| $C_n$ |  | `\dynkin{C}{}` |
| $D_n$ |  | `\dynkin{D}{}` |
| $E_6$ |  | `\dynkin{E}{6}` |
| $E_7$ |  | `\dynkin{E}{7}` |
| $E_8$ |  | `\dynkin{E}{8}` |
| $F_4$ |  | `\dynkin{F}{4}` |
| $G_2$ |  | `\dynkin{G}{2}` |

Table 17: The Dynkin diagrams of the extended simple root systems in ceref style

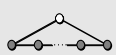
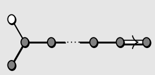
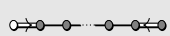
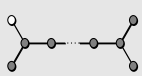
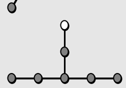
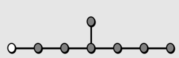
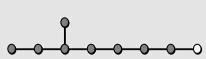
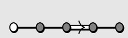
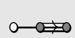
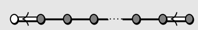
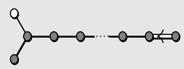
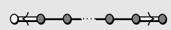
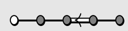
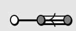
| | | |
|---|---|---|
| $A_1^1$ |  | `\dynkin[extended]{A}{1}` |
| $A_n^1$ |  | `\dynkin[extended]{A}{}` |
| $B_n^1$ |  | `\dynkin[extended]{B}{}` |
| $C_n^1$ |  | `\dynkin[extended]{C}{}` |
| $D_n^1$ |  | `\dynkin[extended]{D}{}` |
| $E_6^1$ |  | `\dynkin[extended]{E}{6}` |
| $E_7^1$ |  | `\dynkin[extended]{E}{7}` |
| $E_8^1$ |  | `\dynkin[extended]{E}{8}` |
| $F_4^1$ |  | `\dynkin[extended]{F}{4}` |
| $G_2^1$ |  | `\dynkin[extended]{G}{2}` |

Table 18: The Dynkin diagrams of the twisted simple root systems in ceref style

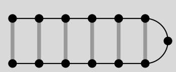
| | | |
|---|---|---|
| $A_2^2$ |  | `\dynkin{A}[2]{2}` |
| $A_{ev}^2$ |  | `\dynkin{A}[2]{even}` |
| $A_{od}^2$ |  | `\dynkin{A}[2]{odd}` |
| $D_n^2$ |  | `\dynkin{D}[2]{}` |
| $E_6^2$ |  | `\dynkin{E}[2]{6}` |
| $D_4^3$ |  | `\dynkin{D}[3]{4}` |

## 19. Folded Dynkin diagrams

The Dynkin diagrams package has limited support for folding Dynkin diagrams.

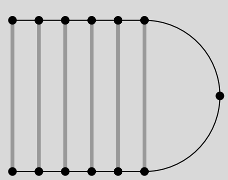
**Folding**

```
\dynkin[fold]{A}{13}
```



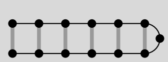
**Big fold radius**

```
\dynkin[fold,fold radius=1cm]{A}{13}
```



**Small fold radius**

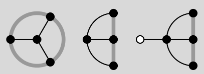
```
\dynkin[fold,fold radius=.2cm]{A}{13}
```



Some Dynkin diagrams have multiple foldings, which we attempt to distinguish (not entirely successfully) by their *ply*: the maximum number of roots folded together. Most diagrams can only allow a 2-ply folding, so `fold` is a synonym for `ply=2`.

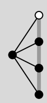
3-ply

```
\dynkin[ply=3]{D}{4}
\dynkin[ply=3,fold right]{D}{4}
\dynkin[ply=3]{D}[1]{4}
```



4-ply

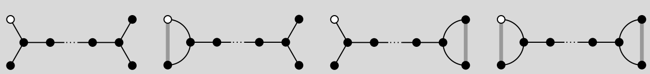
```
\dynkin[ply=4]{D}[1]{4}
```



The $D_\ell^{(1)}$ diagrams can be folded on their left end and separately on their right end:

Left, right and both

```
\dynkin{D}[1]{} \
\dynkin[fold left]{D}[1]{} \
\dynkin[fold right]{D}[1]{} \
\dynkin[fold]{D}[1]{}
```



We have to be careful about the 4-ply foldings of $D_{2\ell}^{(1)}$, for which we can have two different patterns, so by default, the package only draws as much as it can without distinguishing the two:

Default $D_{2\ell}^{(1)}$ and the two ways to finish it

```
        \dynkin[ply=4]{D}[1]{****.*****.*****}%
 \
\begin{dynkinDiagram}[ply=4]{D}[1]{****.*****.*****}%
        \dynkinFold[bend right=90]{1}{13}%
        \dynkinFold[bend right=90]{0}{14}%
\end{dynkinDiagram} \
\begin{dynkinDiagram}[ply=4]{D}[1]{****.*****.*****}%
        \dynkinFold{0}{1}%
        \dynkinFold{1}{13}%
        \dynkinFold{13}{14}%
```

```
\end{dynkinDiagram}
```

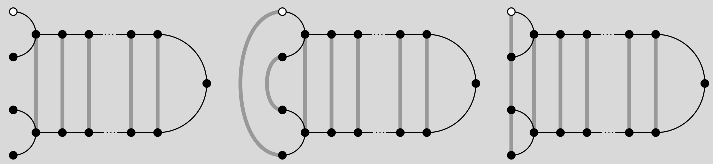
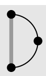
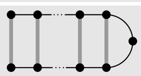
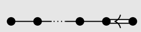
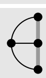
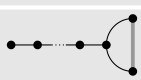
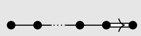
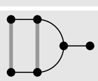
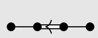
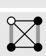
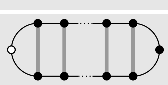
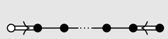
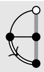
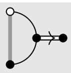
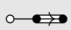
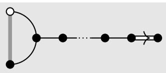
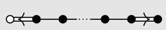
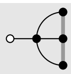
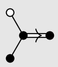
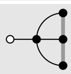
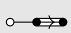
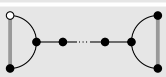
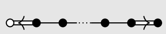
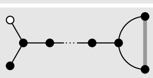
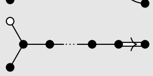
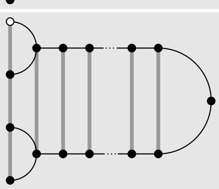
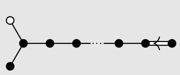


Table 19: Some foldings of Dynkin diagrams. For these diagrams, we want to compare a folding diagram with the diagram that results when we fold it, so it looks best to set `fold radius` and `edge length` to equal lengths.

| | | |
|---|---|---|
| $A_3$ |  | `\dynkin[fold]{A}[0]{3}` |
| $C_2$ |  | `\dynkin{C}[0]{2}` |
| $A_{2\ell-1}$ |  | `\dynkin[fold]{A}{**.*****.**}` |
| $C_\ell$ |  | `\dynkin{C}{}` |
| $B_3$ |  | `\dynkin[fold]{B}[0]{3}` |
| $G_2$ |  | `\dynkin[reverse arrows]{G}[0]{2}` |
| $D_4$ |  | `\dynkin[ply=3,fold right]{D}{4}` |
| $G_2$ |  | `\dynkin{G}{2}` |
| $D_{\ell+1}$ |  | `\dynkin[fold]{D}{}` |
| $B_\ell$ |  | `\dynkin{B}{}` |
| $E_6$ |  | `\dynkin[fold]{E}[0]{6}` |
| $F_4$ |  | `\dynkin[reverse arrows]{F}[0]{4}` |
| $A_3^1$ |  | `\dynkin[ply=4]{A}[1]{3}` |
| $A_1^1$ |  | `\dynkin{A}[1]{1}` |
| $A_{2\ell-1}^1$ |  | `\dynkin[fold]{A}[1]{**.*****.**}` |
| $C_\ell^1$ |  | `\dynkin{C}[1]{}` |

continued . . .

Table 19: … continued

| | | |
|---|---|---|
| $B_3^1$ |  | `\dynkin[ply=3]{B}[1]{3}` |
| $A_2^2$ |  | `\dynkin{A}[2]{2}` |
| $B_3^1$ |  | `\dynkin[ply=2]{B}[1]{3}` |
| $G_2^1$ |  | `\dynkin{G}[1]{2}` |
| $B_\ell^1$ |  | `\dynkin[fold]{B}[1]{}` |
| $D_\ell^2$ |  | `\dynkin{D}[2]{}` |
| $D_4^1$ |  | `\dynkin[ply=3]{D}[1]{4}` |
| $B_3^1$ |  | `\dynkin{B}[1]{3}` |
| $D_4^1$ |  | `\dynkin[ply=3]{D}[1]{4}` |
| $G_2^1$ |  | `\dynkin{G}[1]{2}` |
| $D_{\ell+1}^1$ |  | `\dynkin[fold]{D}[1]{}` |
| $D_\ell^2$ |  | `\dynkin{D}[2]{}` |
| $D_{\ell+1}^1$ |  | `\dynkin[fold right]{D}[1]{}` |
| $B_\ell^1$ |  | `\dynkin{B}[1]{}` |
| $D_{2\ell}^1$ |  | `\begin{dynkinDiagram}[ply=4]{D}[1]%`<br>`{****.*****.*****}`<br>` \dynkinFold{0}{1}`<br>` \dynkinFold{1}{13}`<br>` \dynkinFold{13}{14}`<br>`\end{dynkinDiagram}` |
| $A_{\text{odd}}^2$ |  | `\dynkin{A}[2]{odd}` |

continued …

Table 19: . . . continued

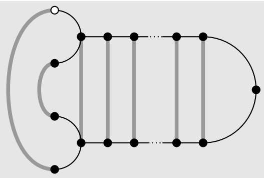
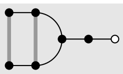
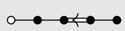
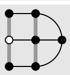
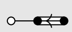
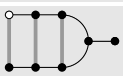
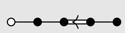
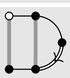
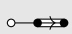
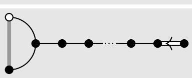
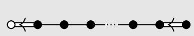
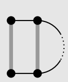
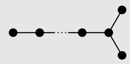
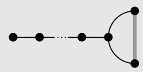
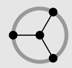
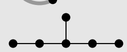
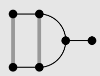
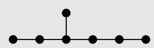
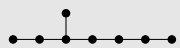
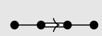
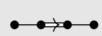
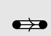
| | | |
|---|---|---|
| $D^1_{2\ell}$ |  | `\begin{dynkinDiagram}[ply=4]{D}[1]%`<br>`{****.*****.*****}`<br>` \dynkinFold[bend right=90]{1}{13}`<br>` \dynkinFold[bend right=90]{0}{14}`<br>`\end{dynkinDiagram}` |
| $A^2_{\text{even}}$ |  | `\dynkin{A}[2]{even}` |
| $E^1_6$ |  | `\dynkin[fold]{E}[1]{6}` |
| $F^1_4$ |  | `\dynkin[reverse arrows]{F}[1]{4}` |
| $E^1_6$ |  | `\dynkin[ply=3]{E}[1]{6}` |
| $D^3_4$ |  | `\dynkin{D}[3]{4}` |
| $E^1_7$ |  | `\dynkin[fold]{E}[1]{7}` |
| $E^2_6$ |  | `\dynkin{E}[2]{6}` |
| $F^1_4$ |  | `\dynkin[fold]{F}[1]{4}` |
| $G^1_2$ |  | `\dynkin{G}[1]{2}` |
| $A^2_{\text{odd}}$ |  | `\dynkin[odd,fold]{A}[2]{****.***}` |
| $A^2_{\text{even}}$ |  | `\dynkin{A}[2]{even}` |
| $D^2_3$ |  | `\dynkin[fold]{D}[2]{3}` |
| $A^2_2$ |  | `\dynkin{A}[2]{2}` |

Table 20: Frobenius fixed point subgroups of finite simple groups of Lie type [4] p. 15

| | | |
|---|---|---|
| $A_{\ell\ge1}$ |  | `\dynkin{A}{}` |
| ${}^2A_{\ell\ge2}$ |  | `\dynkin[fold]{A}{}` |
| $B_{\ell\ge2}$ |  | `\dynkin{B}{}` |
| ${}^2B_2$ |  | `\dynkin[fold]{B}{2}` |

continued . . .

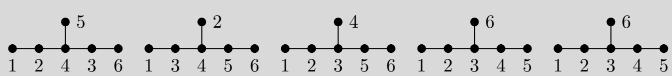
Table 20: . . . continued

| | | |
|---|---|---|
| $C_{\ell\geq 3}$ |  | `\dynkin{C}{}` |
| $D_{\ell\geq 4}$ |  | `\dynkin{D}{}` |
| ${}^2D_{\ell\geq 4}$ |  | `\dynkin[fold]{D}{}` |
| ${}^3D_4$ |  | `\dynkin[ply=3]{D}{4}` |
| $E_6$ |  | `\dynkin{E}{6}` |
| ${}^2E_6$ |  | `\dynkin[fold]{E}{6}` |
| $E_7$ |  | `\dynkin{E}{7}` |
| $E_8$ |  | `\dynkin{E}{8}` |
| $F_4$ |  | `\dynkin{F}{4}` |
| ${}^2F_4$ |  | `\dynkin[fold]{F}{4}` |
| $G_2$ |  | `\dynkin{G}{2}` |
| ${}^2G_2$ |  | `\dynkin[fold]{G}{2}` |

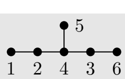
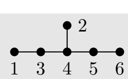
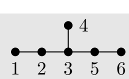
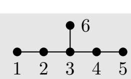
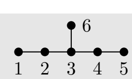
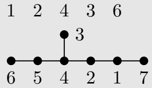
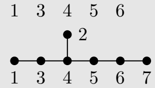
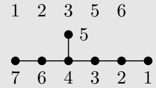
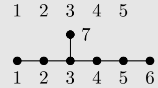
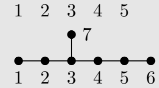
## 20. Root ordering

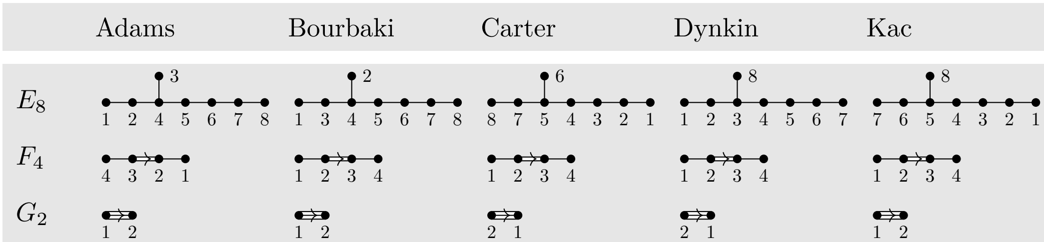
Root ordering

```
\dynkin[label,ordering=Adams]{E}{6}
\dynkin[label,ordering=Bourbaki]{E}{6}
\dynkin[label,ordering=Carter]{E}{6}
\dynkin[label,ordering=Dynkin]{E}{6}
\dynkin[label,ordering=Kac]{E}{6}
```



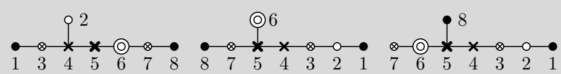

Default is Bourbaki. Sources are Adams [1] p. 56–57, Bourbaki [3] p. pp. 265–290 plates I-IX, Carter [5] p. 540–609, Dynkin [8], Kac [15] p. 43.

| | Adams | Bourbaki | Carter | Dynkin | Kac |
|---|---|---|---|---|---|
| $E_6$ |   |   |   |   |   |
| $E_7$ |   |   |   |   |   |




The marks are set down in order according to the current root ordering:

```
\dynkin[label]{E}{*otxXOt*}
\dynkin[label,ordering=Carter]{E}{*otxXOt*}
\dynkin[label,ordering=Kac]{E}{*otxXOt*}
```

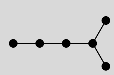


## 21. Connecting Dynkin diagrams

We can make some sophisticated folded diagrams by drawing multiple diagrams, each with a name:

Name a diagram

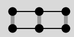
```
\dynkin[name=Bob]{D}{6}
```



We can then connect the two with folding edges:

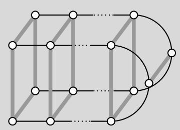
Connect diagrams

```
\begin{dynkinDiagram}[name=upper]{A}{3}
        \node (current) at ($(upper root 1)+(0,-.3cm)$) {};
        \dynkin[at=(current),name=lower]{A}{3}
        \begin{scope}[on background layer]
                \foreach \i in {1,...,3}%
                {%
                        \draw[/Dynkin diagram/fold style]
                                ($(upper root \i)$)
                                -- ($(lower root \i)$);%
                }%
        \end{scope}
\end{dynkinDiagram}
```



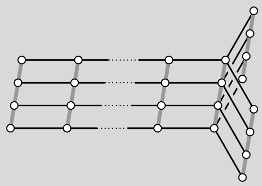
The following diagrams arise in the Satake diagrams of the pseudo-Riemannian symmetric spaces [2].

```
\pgfkeys{/Dynkin diagram,edge length=.5cm,fold radius=.5cm}
\begin{tikzpicture}
        \dynkin[name=1]{A}{IIIb}
        \node (a) at (-.3,-.4){};
        \dynkin[name=2,at=(a)]{A}{IIIb}
        \begin{scope}[on background layer]
                \foreach \i in {1,...,7}%
                {%
                        \draw[/Dynkin diagram/fold style]
                                ($(1 root \i)$)
                                --
                                ($(2 root \i)$);%
                }%
        \end{scope}
\end{tikzpicture}
```



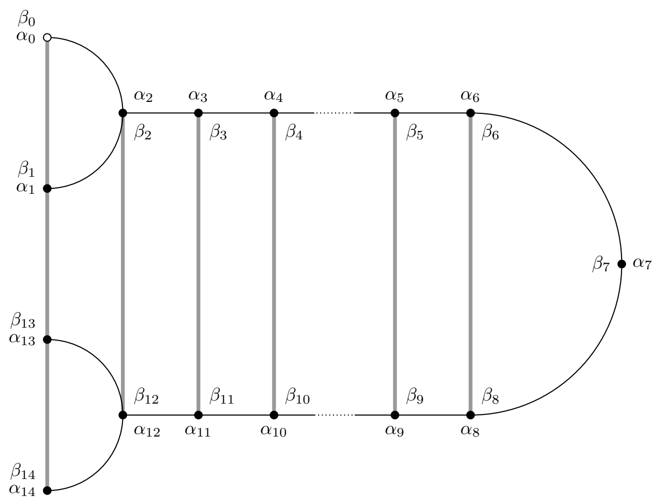
```
\pgfkeys{/Dynkin diagram,
edge length=.75cm,
edge/.style={draw=example-color,double=black,very thick}}
\begin{tikzpicture}
        \foreach \d in {1,...,4}
        {
                \node (current) at ($(\d*.05,\d*.3)$){};
                \dynkin[name=\d,at=(current)]{D}{oo.oooo}
        }
        \begin{scope}[on background layer]
                \foreach \i in {1,...,6}%
                {%
                        \draw[/Dynkin diagram/fold style] ($(1 root
    \i)$) -- ($(2 root \i)$);%
                        \draw[/Dynkin diagram/fold style] ($(2 root
    \i)$) -- ($(3 root \i)$);%
                        \draw[/Dynkin diagram/fold style] ($(3 root
    \i)$) -- ($(4 root \i)$);%
                }%
```

```
        \end{scope}
\end{tikzpicture}
```

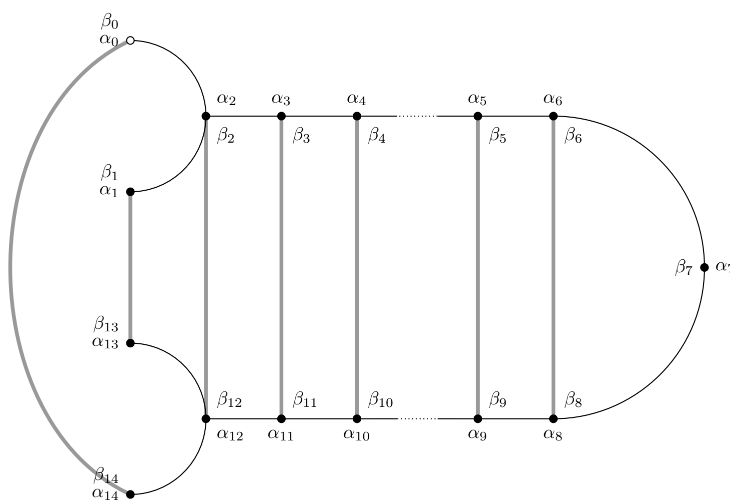


## 22. Other examples

$^1D_4$ 4-ply tied straight:




$^1D_4$ 4-ply tied bending:



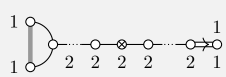

```
\tikzset{/Dynkin diagram,edge length=1cm,fold radius=1cm}
\tikzset{/Dynkin diagram,label macro/.code={\alpha_{#1}},label macro*/.code={\beta_{#1}}}
\({}^1 D_4\) 4-ply tied straight:
\begin{dynkinDiagram}[ply=4]{D}[1]%
{****.*****.*****}
```

```
 \dynkinFold{0}{1}
 \dynkinFold{1}{13}
 \dynkinFold{13}{14}
\dynkinLabelRoots{0,...,14}
\dynkinLabelRoots*{0,...,14}
\end{dynkinDiagram}
\({}^1 D_4\) 4-ply tied bending:
\begin{dynkinDiagram}[ply=4]{D}[1]%
{****.*****.*****}
 \dynkinFold{1}{13}
 \dynkinFold[bend right=65]{0}{14}
\dynkinLabelRoots{0,...,14}
\dynkinLabelRoots*{0,...,14}
\end{dynkinDiagram}
```

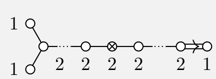
Below we draw the Vogan diagrams of some affine Lie superalgebras [21, 20].

$\mathfrak{sl}(2m|2n)^{(2)}$

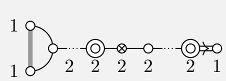
```
\begin{dynkinDiagram}[ply=2,label]{B}[1]{oo.oto.oo}
        \dynkinLabelRoot*{7}{1}
\end{dynkinDiagram}
```



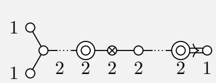

```
\dynkin[label]{B}[1]{oo.oto.oo}
```



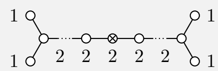

```
\dynkin[ply=2,label]{B}[1]{oo.Oto.Oo}
```



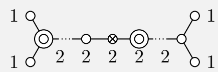

```
\dynkin[label]{B}[1]{oo.Oto.Oo}
```



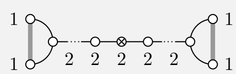
```
\dynkin[label]{D}[1]{oo.oto.ooo}
```




```
\dynkin[label]{D}[1]{oO.otO.ooo}
```

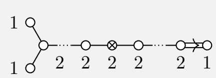



```
\dynkin[label,fold]{D}[1]{oo.oto.ooo}
```

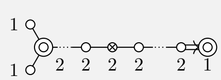



## $\mathfrak{sl}(2m+1|2n)^2$

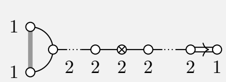
```
\dynkin[label]{B}[1]{oo.oto.oo}
```




```
\dynkin[label]{B}[1]{oO.oto.oO}
```

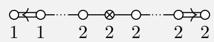



```
\dynkin[label,fold]{B}[1]{oo.oto.oo}
```

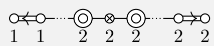


### $\mathfrak{sl}\left(2m+1|2n+1\right)^2$

```
\dynkin[label]{D}[2]{o.oto.oo}
```

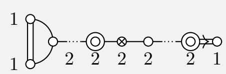



```
\dynkin[label]{D}[2]{o.OtO.oo}
```

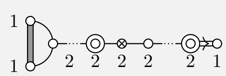



### $\mathfrak{sl}\left(2|2n+1\right)^{(2)}$

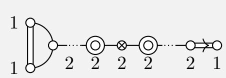
```
\dynkin[ply=2,label,double edges]{B}[1]{oo.Oto.Oo}
```



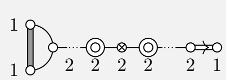

```
\dynkin[ply=2,label,double fold]{B}[1]{oo.Oto.Oo}
```




```
\dynkin[ply=2,label,double edges]{B}[1]{oo.OtO.oo}
```

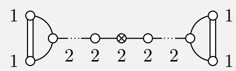



```
\dynkin[ply=2,label,double fold]{B}[1]{oo.OtO.oo}
```

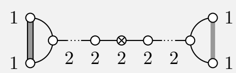


## $\mathfrak{sl}\,(2|2n)^{(2)}$

```
\dynkin[ply=2,label,double edges]{D}[1]{oo.oto.ooo}
```

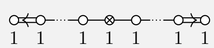



```
\dynkin[ply=2,label,double fold
    left]{D}[1]{oo.oto.ooo}
```

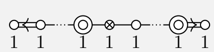



## $\mathfrak{osp}\,(2m|2n)^{(2)}$

```
\dynkin[label,label macro/.code={1}]{D}[2]{o.oto.oo}
```

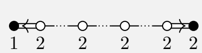



```
\dynkin[label,label macro/.code={1}]{D}[2]{o.Oto.Oo}
```

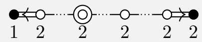



## $\mathfrak{osp}\,(2|2n)^{(2)}$

```
\dynkin[label,label macro/.code=\lablIt{#1},
        affine mark=*]
        {D}[2]{o.o.o.o*}
```

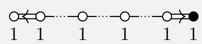


```
\dynkin[label,label macro/.code=\lablIt{#1},
        affine mark=*]
        {D}[2]{o.O.o.o*}
```

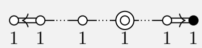



## $\mathfrak{sl}\left(1|2n+1\right)^4$

```
\dynkin[label,label macro/.code={1}]{D}[2]{o.o.o.o*}
```

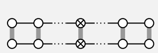



```
\dynkin[label,label macro/.code={1}]{D}[2]{o.o.O.o*}
```

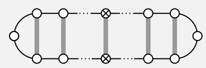



## $A^1$

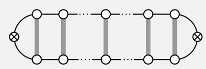
```
\begin{tikzpicture}
        \dynkin[name=upper]{A}{oo.t.oo}
        \node (Dynkin current) at (upper root 1){};
        \dynkinSouth
        \dynkin[at=(Dynkin
    current),name=lower]{A}{oo.t.oo}
        \begin{scope}[on background layer]
        \foreach \i in {1,...,5}{
                \draw[/Dynkin diagram/fold style]
                        ($(upper root \i)$) --
    ($(lower root \i)$);
        }
        \end{scope}
\end{tikzpicture}
```



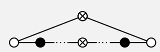
```
\dynkin[fold]{A}[1]{oo.t.ooooo.t.oo}
```



```
\dynkin[fold,affine mark=t]{A}[1]{oo.o.ootoo.o.oo}
```

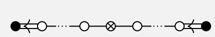


```
\dynkin[affine mark=t]{A}[1]{o*.t.*o}
```

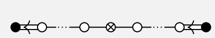


## $B^1$

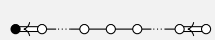
```
\dynkin[affine mark=*]{A}[2]{o.oto.o*}
```



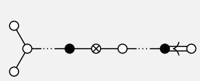
```
\dynkin[affine mark=*]{A}[2]{o.oto.o*}
```



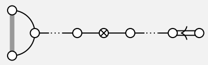
```
\dynkin[affine mark=*]{A}[2]{o.ooo.oo}
```



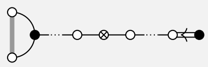
```
\dynkin[odd]{A}[2]{oo.*to.*o}
```



```
\dynkin[odd,fold]{A}[2]{oo.oto.oo}
```

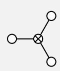


```
\dynkin[odd,fold]{A}[2]{o*.oto.o*}
```

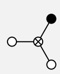


## $D^1$

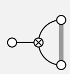
```
\dynkin{D}{otoo}
```



```
\dynkin{D}{ot*o}
```

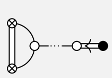


```
\dynkin[fold]{D}{otoo}
```

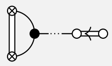


## $C^1$

```
\dynkin[double edges,fold,affine
    mark=t,odd]{A}[2]{to.o*}
```

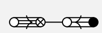


```
\dynkin[double edges,fold,affine
    mark=t,odd]{A}[2]{t*.oo}
```

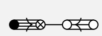


$F^1$

```
\begin{dynkinDiagram}{A}{oto*}%
        \dynkinQuadrupleEdge{1}{2}%
        \dynkinTripleEdge{4}{3}%
\end{dynkinDiagram}%
```

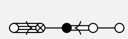


```
\begin{dynkinDiagram}{A}{*too}%
        \dynkinQuadrupleEdge{1}{2}%
        \dynkinTripleEdge{4}{3}%
\end{dynkinDiagram}%
```

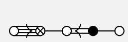


$G^1$

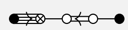
```
\begin{dynkinDiagram}{A}{ot*oo}%
        \dynkinQuadrupleEdge{1}{2}%
        \dynkinDefiniteDoubleEdge{4}{3}%
\end{dynkinDiagram}%
```



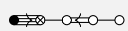
```
\begin{dynkinDiagram}{A}{oto*o}%
        \dynkinQuadrupleEdge{1}{2}%
        \dynkinDefiniteDoubleEdge{4}{3}%
\end{dynkinDiagram}%
```



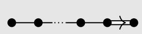
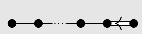
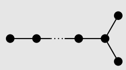
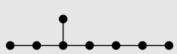
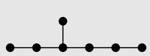
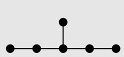
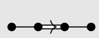
```
\begin{dynkinDiagram}{A}{*too*}%
        \dynkinQuadrupleEdge{1}{2}%
        \dynkinDefiniteDoubleEdge{4}{3}%
\end{dynkinDiagram}%
```



```
\begin{dynkinDiagram}{A}{*tooo}%
        \dynkinQuadrupleEdge{1}{2}%
        \dynkinDefiniteDoubleEdge{4}{3}%
\end{dynkinDiagram}%
```



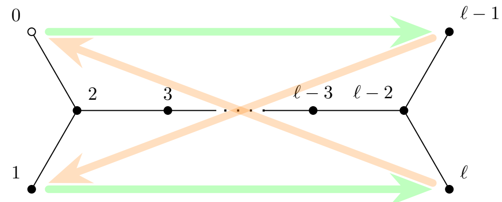
## 23. Example: the complex simple Lie algebras

| $\mathfrak{g}$ | Diagram | Weights | Roots | Simple roots |
|---|---|---|---|---|
| $A_n$ |  | $\frac{1}{r+1}\mathbb{Z}^{n+1}/\left\langle \sum e_j \right\rangle$ | $e_i - e_j$ | $e_i - e_{i+1}$ |
| $B_n$ |  | $\frac{1}{2}\mathbb{Z}^n$ | $\pm e_i, \pm e_i \pm e_j, i \neq j$ | $e_i - e_{i+1}, e_n$ |
| $C_n$ |  | $\mathbb{Z}^n$ | $\pm 2e_i, \pm e_i \pm e_j, i \neq j$ | $e_i - e_{i+1}, 2e_n$ |
| $D_n$ |  | $\frac{1}{2}\mathbb{Z}^n$ | $\pm e_i \pm e_j, i \neq j$ | $e_i - e_{i+1}, \quad i \leq n-1$<br>$e_{n-1} + e_n$ |
| $E_8$ |  | $\frac{1}{2}\mathbb{Z}^8$ | $\pm 2e_i \pm 2e_j, \quad i \neq j,$<br>$\sum_i (-1)^{m_i} e_i, \quad \sum m_i$ even | $2e_1 - 2e_2,$<br>$2e_2 - 2e_3,$<br>$2e_3 - 2e_4,$<br>$2e_4 - 2e_5,$<br>$2e_5 - 2e_6,$<br>$2e_6 + 2e_7,$<br>$-\sum e_j,$<br>$2e_6 - 2e_7$ |
| $E_7$ |  | $\frac{1}{2}\mathbb{Z}^8/\langle e_1 - e_2 \rangle$ | quotient of $E_8$ | quotient of $E_8$ |
| $E_6$ |  | $\frac{1}{3}\mathbb{Z}^8/\langle e_1 - e_2, e_2 - e_3 \rangle$ | quotient of $E_8$ | quotient of $E_8$ |
| $F_4$ |  | $\mathbb{Z}^4$ | $\pm 2e_i,$<br>$\pm 2e_i \pm 2e_j, \quad i \neq j,$<br>$\pm e_1 \pm e_2 \pm e_3 \pm e_4$ | $2e_2 - 2e_3,$<br>$2e_3 - 2e_4,$<br>$2e_4,$<br>$e_1 - e_2 - e_3 - e_4$ |

| $\mathfrak{g}$ | Diagram | Weights | Roots | Simple roots |
|---|---|---|---|---|
| $G_2$ | | $\mathbb{Z}^3/\left\langle\sum e_j\right\rangle$ | $\pm(1,-1,0)$,<br>$\pm(-1,0,1)$,<br>$\pm(0,-1,1)$,<br>$\pm(2,-1,-1)$,<br>$\pm(1,-2,1)$,<br>$\pm(-1,-1,2)$ | $(-1,0,1)$,<br>$(2,-1,-1)$ |

```
\NewDocumentEnvironment{bunch}{}%
{\renewcommand*{\arraystretch}{1}\begin{array}{@{}ll@{}}\\ \midrule}{\\ \midrule\end{array}}
\small
\NewDocumentCommand\nct{mm}{\newcolumntype{#1}{>{\columncolor[gray]{.9}}>{$}m{#2cm}<{$}}}
\nct{G}{.3}\nct{D}{2.1}\nct{W}{3}\nct{R}{3.7}\nct{S}{3}
\NewDocumentCommand\LieG{}{\mathfrak{g}}
\NewDocumentCommand\W{om}{\ensuremath{\mathbb{Z}^{#2}\IfValueT{#1}{/\left<#1\right>}}}
\renewcommand*{\arraystretch}{1.5}
\NewDocumentCommand\quo{}{\text{quotient of } E_8}
\begin{longtable}{@{}GDWRS@{}}
\LieG&\text{Diagram}&\text{Weights}&\text{Roots}&\text{Simple roots}\\ \midrule\endfirsthead
\LieG&\text{Diagram}&\text{Weights}&\text{Roots}&\text{Simple roots}\\ \midrule\endhead
A_n&\dynkin{A}{}&\frac{1}{r+1}\W[\sum e_j]{n+1}&e_i-e_j&e_i-e_{i+1}\\
B_n&\dynkin{B}{}&\frac{1}{2}\W{n}& \pm e_i, \pm e_i \pm e_j, i\ne j&e_i-e_{i+1}, e_n\\
C_n&\dynkin{C}{}&\W{n}& \pm 2 e_i, \pm e_i \pm e_j, i\ne j&e_i-e_{i+1}, 2e_n\\
D_n&\dynkin{D}{}&\frac{1}{2}\W{n}& \pm e_i \pm e_j, i\ne j &
\begin{bunch}e_i-e_{i+1},&i\le n-1\\e_{n-1}+e_n\end{bunch}\\
E_8&\dynkin{E}{8}&\frac{1}{2}\W{8}&
\begin{bunch}\pm2e_i\pm2e_j,&i\ne j,\\ \sum_i(-1)^{m_i}e_i,&\sum m_i \text{ even}\end{bunch}&
\begin{bunch}
2e_1-2e_2,\\2e_2-2e_3,\\2e_3-2e_4,\\2e_4-2e_5,\\2e_5-2e_6,\\2e_6+2e_7,\\
-\sum e_j,\\2e_6-2e_7
\end{bunch}\\
E_7&\dynkin{E}{7}&\frac{1}{2}\W[e_1-e_2]{8}&\quo&\quo\\
E_6&\dynkin{E}{6}&\frac{1}{3}\W[e_1-e_2,e_2-e_3]{8}&\quo&\quo\\
F_4& \dynkin{F}{4}&\W{4}&
\begin{bunch}\pm 2e_i,\\ \pm 2e_i \pm 2e_j, \quad i \ne j,\\ \pm e_1 \pm e_2 \pm e_3 \pm e_4
\end{bunch}&
\begin{bunch}2e_2-2e_3,\\2e_3-2e_4,\\2e_4,\\e_1-e_2-e_3-e_4\end{bunch}\\
G_2&\dynkin{G}{2}&\W[\sum e_j]{3}&
\begin{bunch}
\pm(1,-1,0),\\ \pm(-1,0,1),\\ \pm(0,-1,1),\\ \pm(2,-1,-1),\\ \pm(1,-2,1),\\ \pm(-1,-1,2)
\end{bunch}&
\begin{bunch}(-1,0,1),\\(2,-1,-1)\end{bunch}
\end{longtable}
```

## 24. An example of Mikhail Borovoi




```
\tikzset{big arrow/.style={
 -Stealth,line cap=round,line width=1mm,
 shorten <=1mm,shorten >=1mm}}
\newcommand\catholic[2]{\draw[big arrow,green!25!white]
(root #1) to (root #2);}
\newcommand\protestant[2]{
\begin{scope}[transparency group, opacity=.25]
\draw[big arrow,orange] (root #1) to (root #2);
\end{scope}}
\begin{dynkinDiagram}[edge length=1.2cm,
indefinite edge/.style={thick,loosely dotted},
labels*={0,1,2,3,\ell-3,\ell-2,\ell-1,\ell}]{D}[1]{}
\catholic{0}{6}\catholic{1}{7}
\protestant{7}{0}\protestant{6}{1}
\end{dynkinDiagram}
```

## 25. Syntax

The syntax is \dynkin[<options>]{<letter>}[<twisted rank>]{<rank>} where <letter> is A, B, C, D, E, F or G, the family of root system for the Dynkin diagram, <twisted rank> is 0, 1, 2, 3 (default is 0) representing:

| | |
|---|---|
| 0 | finite root system |
| 1 | affine extended root system, i.e. of type $^{(1)}$ |
| 2 | affine twisted root system of type $^{(2)}$ |
| 3 | affine twisted root system of type $^{(3)}$ |

and <rank> is

(1) an integer representing the rank or
(2) blank to represent an indefinite rank or
(3) the name of a Satake diagram as in section 4.

The environment syntax is \begin{dynkinDiagram} followed by the same parameters as \dynkin, then various Dynkin diagram and Ti*k*Z commands, and then \end{dynkinDiagram}.

## 26. Options

ceref = ⟨true or false⟩,
default : false
  whether to draw roots in a "ceref" style.

edge length = ⟨number⟩cm,
default : .35cm
  distance between nodes in the Dynkin diagram

edge/.style = TikZ style data,
default : solid,draw=black,fill=white,thin
  style of edges in the Dynkin diagram

Kac = ⟨true or false⟩,
default : false
  whether to draw in the style of [15]

name = ⟨string⟩,
default : anonymous
  A name for the Dynkin diagram, with anonymous treated as a blank; see section 21.

parabolic = ⟨integer⟩,
default : 0
  A parabolic subgroup with specified integer, where the integer is computed as $n = \sum 2^{i-1} a_i$, $a_i = 0$ or 1, to say that root $i$ is crossed, i.e. a noncompact root.

root radius = ⟨number⟩cm,
default : .05cm
  size of the dots and of the crosses in the Dynkin diagram

text/.style = ⟨TikZ style data⟩,
default : scale=.7
  Style for any labels on the roots.

mark = ⟨o,O,t,x,X,*⟩,

continued . . .

Table 23: . . . continued

default : *
    default root mark
affine mark = o,O,t,x,X,*,
default : *
    default root mark for root zero in an affine Dynkin diagram
label = true or false,
default : false
    whether to label the roots according to the current labelling scheme.
label macro = ⟨1-parameter TEX macro⟩,
default : #1
    the current labelling scheme for roots.
label macro* = ⟨1-parameter TEX macro⟩,
default : #1
    the current labelling scheme for alternate roots.
make indefinite edge = ⟨edge pair $i$-$j$ or list of such⟩,
default : {}
    edge pair or list of edge pairs to treat as having indefinitely many roots on them.
indefinite edge ratio = ⟨float⟩,
default : 1.6
    ratio of indefinite edge lengths to other edge lengths.
indefinite edge/.style = ⟨TikZ style data⟩,
default : solid,draw=black,fill=white,thin,densely dotted
    style of the dotted or dashed middle third of each indefinite edge.
backwards = ⟨true or false⟩,
default : false
    whether to reverse right to left.
upside down = ⟨true or false⟩,
default : false
    whether to reverse up to down.
arrows = ⟨true or false⟩,
default : true
    whether to draw the arrows that arise along the edges.
reverse arrows = ⟨true or false⟩,
default : true
    whether to reverse the direction of the arrows that arise along the edges.
fold = ⟨true or false⟩,
default : true
    whether, when drawing Dynkin diagrams, to draw them 2-ply.
ply = ⟨0,1,2,3,4⟩,
default : 0
    how many roots get folded together, at most.
fold left = ⟨true or false⟩,
default : true
    whether to fold the roots on the left side of a Dynkin diagram.

continued . . .

Table 23: ... continued

`fold right` = ⟨`true or false`⟩,
default : `true`
 whether to fold the roots on the right side of a Dynkin diagram.
`fold radius` = ⟨`length`⟩,
default : `.3cm`
 the radius of circular arcs used in curved edges of folded Dynkin diagrams.
`fold style/.style` = ⟨`TikZ style data`⟩,
default : `solid,draw=black!40,fill=none,line width=radius`
 when drawing folded diagrams, style for the fold indicators.
`*/.style` = ⟨`TikZ style data`⟩,
default : `solid,draw=black,fill=black`
 style for roots like •
`o/.style` = ⟨`TikZ style data`⟩,
default : `solid,draw=black,fill=black`
 style for roots like ∘
`O/.style` = ⟨`TikZ style data`⟩,
default : `solid,draw=black,fill=black`
 style for roots like ⊚
`t/.style` = ⟨`TikZ style data`⟩,
default : `solid,draw=black,fill=black`
 style for roots like ⊗
`x/.style` = ⟨`TikZ style data`⟩,
default : `solid,draw=black,line cap=round`
 style for roots like ×
`X/.style` = ⟨`TikZ style data`⟩,
default : `solid,draw=black,thick,line cap=round`
 style for roots like **×**
`fold left style/.style` = ⟨`TikZ style data`⟩,
default :
 style to override the `fold` style when folding roots together on the left half of a Dynkin diagram
`fold right style/.style` = ⟨`TikZ style data`⟩,
default :
 style to override the `fold` style when folding roots together on the right half of a Dynkin diagram
`double edges` = ⟨⟩,
default : `not set`
 set to override the `fold` style when folding roots together in a Dynkin diagram, so that the foldings are indicated with double edges (like those of an $F_4$ Dynkin diagram without arrows).
`double fold` = ⟨⟩,
default : `not set`

continued ...

Table 23: ...continued

set to override the `fold` style when folding roots together in a Dynkin diagram, so that the foldings are indicated with double edges (like those of an $F_4$ Dynkin diagram without arrows), but filled in solidly.

`double left` = ⟨⟩,
default : `not set`
set to override the `fold` style when folding roots together at the left side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an $F_4$ Dynkin diagram without arrows).

`double fold left` = ⟨⟩,
default : `not set`
set to override the `fold` style when folding roots together at the left side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an $F_4$ Dynkin diagram without arrows), but filled in solidly.

`double right` = ⟨⟩,
default : `not set`
set to override the `fold` style when folding roots together at the right side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an $F_4$ Dynkin diagram without arrows).

`double fold right` = ⟨⟩,
default : `not set`
set to override the `fold` style when folding roots together at the right side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an $F_4$ Dynkin diagram without arrows), but filled in solidly.

`arrow color` = ⟨⟩,
default : `black`
set to override the default color for the arrows in nonsimply laced Dynkin diagrams.

`Coxeter` = ⟨`true or false`⟩,
default : `false`
whether to draw a Coxeter diagram, rather than a Dynkin diagram.

`ordering` = ⟨`Adams, Bourbaki, Carter, Dynkin, Kac`⟩,
default : `Bourbaki`
which ordering of the roots to use in exceptional root systems as in section 20.

All other options are passed to TikZ.

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