The bodeplot package*

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1 Introduction

Generate Bode, Nyquist, and Nichols plots for transfer functions in the canonical (TF) form

$$G(s) = e^{-T_s} \frac{b_m s^m + \dots + b_1 s + b_0}{a_n s^n + \dots + a_1 s + a_0}$$
(1)

and the zero-pole-gain (ZPK) form

$$G(s) = Ke^{-Ts} \frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}.$$
 (2)

In the equations above, b_m, \dots, b_0 and a_n, \dots, a_0 are real coefficients, $T \geq 0$ is the loop delay, z_1, \dots, z_m and p_1, \dots, p_n are complex zeros and poles of the transfer function, respectively, and $K \in \Re$ is the loop gain. For transfer functions in the ZPK format in (2) with zero delay, this package also supports linear and asymptotic approximation of Bode plots.

2 Usage

2.1 Bode plots

 \BodeZPK

```
\label{eq:bodeZPK} $$ \end{array} $$ $$ \end{array} $$ \left( \begin{array}{c} \langle obj1/typ1/\{\langle opt1\rangle\}, obj2/typ2/\{\langle opt2\rangle\}, \ldots \rangle \right) \\ \left( \langle z/\{\langle zeros\rangle\}, p/\{\langle poles\rangle\}, k/\{\langle gain\rangle\}, d/\{\langle delay\rangle\} \right) \\ \left( \begin{array}{c} \langle min-freq\rangle\} \\ \left( \begin{array}{c} \langle max-freq\rangle \right) \end{aligned} $$
```

Plots the Bode plot of a transfer function given in ZPK format using the groupplot environment. The three mandatory arguments include: (1) a list of tuples, comprised of the zeros, the poles, the gain, and the transport delay of the transfer function, (2) the lower end of the frequency range for the x- axis, and (3) the higher end of the frequency range for the x-axis. The zeros and the poles are complex numbers, entered as a comma-separated

^{*}This document corresponds to bodeplot?, dated?.

list of comma-separated lists, of the form {{real part 1,imaginary part 1}, {real part 2,imaginary part 2},...}. If the imaginary part is not provided, it is assumed to be zero.

The optional argument is comprised of a comma separated list of tuples, either obj/typ/{opt}, or obj/{opt}, or just {opt}. Each tuple passes options to different pgfplots macros that generate the group, the axes, and the plots according to:

- Tuples of the form obj/typ/{opt}:
 - plot/typ/{opt}: modify plot properties by adding options {opt} to the \addplot macro for the magnitude plot if typ is mag and the phase plot if typ is ph.
 - axes/typ/{opt}: modify axis properties by adding options {opt} to
 the \nextgroupplot macro for the magnitude plot if typ is mag and
 the phase plot if typ is ph.
- Tuples of the form obj/{opt}:
 - plot/{opt}: adds options {opt} to \addplot macros for both the magnitude and the phase plots.
 - axes/{opt}: adds options {opt} to \nextgroupplot macros for both the magnitude and the phase plots.
 - group/{opt}: adds options {opt} to the groupplot environment.
 - approx/linear: plots linear approximation.
 - approx/asymptotic: plots asymptotic approximation.
- Tuples of the form {opts} add all of the supplied options to \addplot macros for both the magnitude and the phase plots.

The options {opt} can be any key=value options that are supported by the pgfplots macros they are added to. Linear or asymptotic approximation of transfer functions that include a transport delay is not supported.

For example, given a transfer function

$$G(s) = 10 \frac{s(s + 0.1 + 0.5i)(s + 0.1 - 0.5i)}{(s + 0.5 + 10i)(s + 0.5 - 10i)},$$
(3)

its Bode plot over the frequency range [0.01, 100] can be generated using \BodeZPK

$${z/{0,{-0.1,-0.5},{-0.1,0.5}},p/{{-0.5,-10},{-0.5,10}},k/10}$$

 ${0.01}{100}$

which generates the plot in Figure 1. If a delay is not specified, it is assumed to be zero. If a gain is not specified, it is assumed to be 1. By default, each of the axes, excluding ticks and labels, are 5cm wide and 2.5cm high. The width and the height, along with other properties of the plots, the axes, and the group can be customized using native pgf keys as shown in the example below.

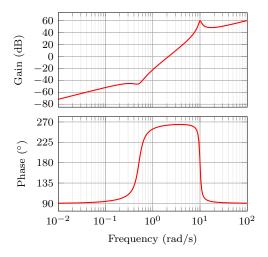


Figure 1: Output of the default \BodeZPK macro.

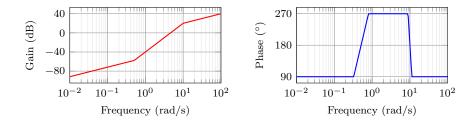


Figure 2: Customization of the default **\BodeZPK** macro.

A linear approximation of the Bode plot with customization of the plots, the axes, and the group can be generated using $\BodeZPK[plot/mag/{red,thick},plot/ph/{blue,thick}, axes/mag/{ytick distance=40,xmajorticks=true, xlabel={Frequency (rad/s)}},axes/ph/{ytick distance=90}, group/{group style={group size=2 by 1,horizontal sep=2cm, width=4cm,height=2cm}},approx/linear] {z/{0,{-0.1,-0.5},{-0.1,0.5}},p/{{-0.5,-10},{-0.5,10}},k/10} {0.01}{100} which generates the plot in Figure 2. <math display="block">\BodeTF [\langle obj1/typ1/\{\langle opt1\rangle\},obj2/typ2/\{\langle opt2\rangle\},...\rangle] {\langle num/\{\langle coeffs\rangle\},den/\{\langle coeffs\rangle\},d/\{\langle delay\rangle\}\}\}} {\langle min-freq\rangle}{\{\langle max-freq\rangle\}}$

\BodeTF

Plots the Bode plot of a transfer function given in TF format. The three mandatory arguments include: (1) a list of tuples comprised of the coefficients in the

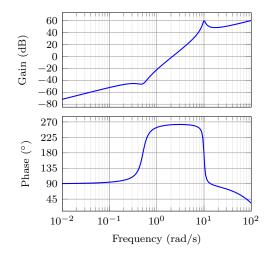


Figure 3: Output of the \BodeTF macro.

numerator and the denominator of the transfer function and the transport delay, (2) the lower end of the frequency range for the x- axis, and (3) the higher end of the frequency range for the x-axis. The coefficients are entered as a comma-separated list, in order from the highest degree of s to the lowest, with zeros for missing degrees. The optional arguments are the same as BodeZPK, except that linear/asymptotic approximation is not supported, so approx/... is ignored.

For example, given the same transfer function as (3) in TF form and with a small transport delay,

$$G(s) = e^{-0.01s} \frac{s(10s^2 + 2s + 2.6)}{(s^2 + s + 100.25)},$$
(4)

its Bode plot over the frequency range [0.01,100] can be generated using \BodeTF[blue,thick]

$$\{num/\{10,2,2.6,0\},den/\{1,0.2,100\},d/0.01\}$$
 $\{0.01\}\{100\}$

which generates the plot in Figure 3. Note the 0 added to the numerator coefficients to account for the fact that the numerator does not have a constant term in it. As demonstrated in this example, if a single comma-separated list of options is passed, it applies to both the magnitude and the phase plots.

BodePlot

 $\label{local-continuity} $$ \left(\text{min-frequency} \right) = \left(\text{min-frequency} \right) = \left(\text{max-frequency} \right) = \left(\text{min-frequency} \right) = \left(\text{min-frequen$

\end{BodePlot}

The BodePlot environment works in conjunction with the parametric function generator macros \addBodeZPKPlots, \addBodeTFPlot, and \addBodeComponentPlots. If supplied, axis-options are passed directly to the semilogaxis environment and the frequency limits are translated to the x-axis limits and the do-

main of the semilogaxis environment. Example usage in the description of \addBodeZPKPlots, \addBodeTFPlot, and \addBodeComponentPlots.

\addBodeZPKPlots

```
\addBodeZPKPlots [\langle approx1/\{\langle opt1\rangle\}, approx2/\{\langle opt2\rangle\},...\rangle] \{\langle plot\text{-}type\rangle\} \{\langle z/\{\langle zeros\rangle\}, p/\{\langle poles\rangle\}, k/\{\langle qain\rangle\}, d/\{\langle delay\rangle\}\}\}
```

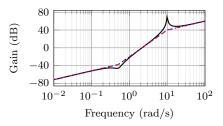
Generates the appropriate parametric functions and supplies them to multiple \addplot macros, one for each approx/{opt} pair in the optional argument. If no optional argument is supplied, then a single \addplot command corresponding to the true Bode plot is generated. This macro can be used inside any semilogaxis environment as long as a domain for the x-axis is supplied through either the approx/{opt} interface or directly in the optional argument of the semilogaxis environment. Use with the BodePlot environment supplied with this package is recommended. The second mandatory argument, plot-type is either magnitude or phase. If it is not equal to phase, it is assumed to be magnitude. The last mandatory argument is the same as \BodeZPK.

For example, given the transfer function in (3), its linear, asymptotic, and true Bode plots can be superimposed using

```
\begin{BodePlot}[ ylabel={Gain (dB)}, ytick distance=40,
  height=2cm, width=4cm] {0.01} {100}
  \addBodeZPKPlots[
    true/{black,thick},
    linear/{red,dashed,thick},
    asymptotic/{blue,dotted,thick}]
     {magnitude}
     \{z/\{0,\{-0.1,-0.5\},\{-0.1,0.5\}\},p/\{\{-0.1,-10\},\{-0.1,10\}\},k/10\}
\end{BodePlot}
\begin{BodePlot}[ylabel={Phase ($^{\circ}$)},
  height=2cm, width=4cm, ytick distance=90,] {0.01} {100}
  \addBodeZPKPlots[
    true/{black,thick},
    linear/{red,dashed,thick},
    asymptotic/{blue,dotted,thick}]
     \{z/\{0,\{-0.1,-0.5\},\{-0.1,0.5\}\},p/\{\{-0.1,-10\},\{-0.1,10\}\},k/10\}
\end{BodePlot}
which generates the plot in Figure 4.
    \addBodeTFPlot[\langle plot\text{-}options \rangle]
     \{\langle plot\text{-}type\rangle\}
     \{\langle num/\{\langle coeffs \rangle\}, den/\{\langle coeffs \rangle\}, d/\{\langle delay \rangle\}\}\}
```

\addBodeTFPlot

Generates a single parametric function for either Bode magnitude or phase plot of a transfer function in TF form. The generated parametric function is passed to the \addplot macro. This macro can be used inside any semilogaxis environment as long as a domain for the x-axis is supplied through either the plot-options interface or directly in the optional argument of the container semilogaxis environment. Use with the BodePlot environment supplied with this package is



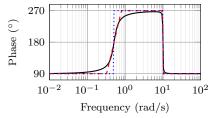


Figure 4: Superimposed approximate and true Bode plots using the BodePlot environment and the \addBodeZPKPlots macro.

recommended. The second mandatory argument, plot-type is either magnitude or phase. If it is not equal to phase, it is assumed to be magnitude. The last mandatory argument is the same as \BodeTF.

\addBodeComponentPlot

 $\addBodeComponentPlot[\langle plot-options \rangle] \{\langle plot-command \rangle\}$

Generates a single parametric function corresponding to the mandatory argument plot-command and passes it to the \addplot macro. The plot command can be any parametric function that uses t as the independent variable. The parametric function must be gnuplot compatible (or pgfplots compatible if the package is loaded using the pgf option). The intended use of this macro is to plot the parametric functions generated using the basic component macros described in Section 2.1.1 below.

2.1.1 Basic components up to first order

\TypeFeatureApprox

 $\label{thm:continuous} $$\operatorname{TypeFeatureApprox}_{\langle real-part\rangle}_{\langle imaginary-part\rangle}$$$

This entry describes 20 different macros of the form \TypeFeatureApprox that take the real part and the imaginary part of a complex number as arguments. The Type in the macro name should be replaced by either Mag or Ph to generate a parametric function corresponding to the magnitude or the phase plot, respectively. The Feature in the macro name should be replaced by one of K, Pole, Zero, or Del, to generate the Bode plot of a gain, a complex pole, a complex zero, or a transport delay, respectively. If the Feature is set to either K or Del, the imaginary-part mandatory argument is ignored. The Approx in the macro name should either be removed, or it should be replaced by Lin or Asymp to generate the true Bode plot, the linear approximation, or the asymptotic approximation, respectively. If the Feature is set to Del, then Approx has to be removed. For example,

- \MagK{k}{0} or \MagK{k}{400} generates a parametric function for the true Bode magnitude of G(s)=k
- \PhPoleLin{a}{b} generates a parametric function for the linear approximation of the Bode phase of $G(s) = \frac{1}{s-a-ib}$.

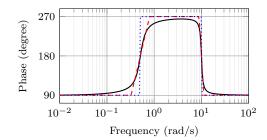


Figure 5: Superimposed approximate and true Bode Phase plot using the BodePlot environment, the \addBodeComponentPlot macro, and several macros of the \TypeFeatureApprox form.

• \PhDel{T}{200} or \PhDel{T}{0} generates a parametric function for the Bode phase of $G(s) = e^{-Ts}$.

All 20 of the macros defined by combinations of Type, Feature, and Approx, and any gnuplot (or pgfplot if the pgf class option is loaded) compatible function of the 20 macros can be used as plot-command in the addBodeComponentPlot macro. This is sufficient to generate the Bode plot of any rational transfer function with delay. For example, the Bode phase plot in Figure 4 can also be generated using:

```
\begin{BodePlot}[ylabel={Phase (degree)},ytick distance=90]{0.01}{100}
\addBodeComponentPlot[black,thick]{\PhZero{0}{0} + \PhZero{-0.1}{-0.5} +
\PhZero{-0.1}{0.5} + \PhPole{-0.5}{-10} + \PhPole{-0.5}{10} +
\PhK{10}{0}}
\addBodeComponentPlot[red,dashed,thick] {\PhZeroLin{0}{0} +
\PhZeroLin{-0.1}{-0.5} + \PhZeroLin{-0.1}{0.5} +
\PhPoleLin{-0.5}{-10} + \PhPoleLin{-0.5}{10} + \PhKLin{10}{20}}
\addBodeComponentPlot[blue,dotted,thick] {\PhZeroAsymp{0}{0} +
\PhZeroAsymp{-0.1}{-0.5} + \PhZeroAsymp{-0.1}{0.5} +
\PhPoleAsymp{-0.5}{-10} + \PhPoleAsymp{-0.5}{10} + \PhKAsymp{10}{40}}
\end{BodePlot}
```

which gives us the plot in Figure 5.

2.1.2 Basic components of the second order

\TypeSOFeatureApprox

\TypeSOFeatureApprox $\{\langle a1 \rangle\}\{\langle a0 \rangle\}$

This entry describes 12 different macros of the form \TypeSOFeatureApprox that take the coefficients a_1 and a_0 of a general second order system as inputs. The Feature in the macro name should be replaced by either Poles or Zeros to generate the Bode plot of $G(s) = \frac{1}{s^2 + a_1 s + a_0}$ or $G(s) = s^2 + a_1 s + a_0$, respectively. The Type in the macro name should be replaced by either Mag or Ph to generate a parametric function corresponding to the magnitude or the phase plot, respectively. The Approx in the macro name should either be removed, or it should be replaced

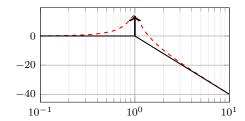


Figure 6: Resonant peak in asymptotic Bode plot using \MagSOPolesPeak.

by Lin or Asymp to generate the true Bode plot, the linear approximation, or the asymptotic approximation, respectively.

\MagSOFeaturePeak

 $\MagSOFeaturePeak[\langle draw-options \rangle] \{\langle a1 \rangle\} \{\langle a0 \rangle\}$

This entry describes 2 different macros of the form $\mbox{MagS0FeaturePeak}$ that take the the coefficients a_1 and a_0 of a general second order system as inputs, and draw a resonant peak using the \mbox{draw} TikZ macro. The Feature in the macro name should be replaced by either Poles or Zeros to generate a peak for poles and a valley for zeros, respectively. For example, the command

```
\begin{BodePlot}[xlabel={}]{0.1}{10}
   \addBodeComponentPlot[red,dashed,thick]{\MagSOPoles{0.2}{1}}
   \addBodeComponentPlot[black,thick]{\MagSOPolesLin{0.2}{1}}
   \MagSOPolesPeak[thick]{0.2}{1}
\end{BodePlot}
```

generates the plot in Figure 6.

\TypeCSFeatureApprox

 $\TypeCSFeatureApprox{\langle zeta \rangle}{\langle omega-n \rangle}$

This entry describes 12 different macros of the form \TypeCSFeatureApprox that take the damping ratio, ζ , and the natural frequency, ω_n of a canonical second order system as inputs. The Type in the macro name should be replaced by either Mag or Ph to generate a parametric function corresponding to the magnitude or the phase plot, respectively. The Feature in the macro name should be replaced by either Poles or Zeros to generate the Bode plot of $G(s) = \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}$ or $G(s) = s^2 + 2\zeta\omega_n s + \omega_n^2$, respectively. The Approx in the macro name should either be removed, or it should be replaced by Lin or Asymp to generate the true Bode plot, the linear approximation, or the asymptotic approximation, respectively.

\MagCSFeaturePeak

 $\verb|\MagCSFeaturePeak[| \langle draw-options \rangle] = \{\langle zeta \rangle\} = \{\langle omega-n \rangle\}$

This entry describes 2 different macros of the form $\mbox{MagCSFeaturePeak}$ that take the damping ratio, ζ , and the natural frequency, ω_n of a canonical second order system as inputs, and draw a resonant peak using the \mbox{draw} TikZ macro. The Feature in the macro name should be replaced by either Poles or Zeros to generate a peak for poles and a valley for zeros, respectively.

\MagCCFeaturePeak

 $\MagCCFeaturePeak[\langle draw-options \rangle] \{\langle real-part \rangle\} \{\langle imaginary-part \rangle\}$ This entry describes 2 different macros of the form \MagCCFeaturePeak that take the real and imaginary parts of a pair of complex conjugate poles or zeros as

inputs, and draw a resonant peak using the \draw TikZ macro. The Feature in the macro name should be replaced by either Poles or Zeros to generate a peak for poles and a valley for zeros, respectively.

2.2 Nyquist plots

\NyquistZPK

```
\label{eq:local_system} $$ \begin{aligned} & \langle plot/\{\langle opt \rangle\}, axes/\{\langle opt \rangle\} \rangle \\ & \{ \langle z/\{\langle zeros \rangle\}, p/\{\langle poles \rangle\}, k/\{\langle gain \rangle\}, d/\{\langle delay \rangle\} \} \\ & \{ \langle min\text{-}freq \rangle\} \{ \langle max\text{-}freq \rangle \} \end{aligned}
```

Plots the Nyquist plot of a transfer function given in ZPK format with a thick red + marking the critical point (-1,0). The mandatory arguments are the same as \BodeZPK . Since there is only one plot in a Nyquist diagram, the \typ specifier in the optional argument tuples is not needed. As such, the supported optional argument tuples are $\typeprox beta = \typeprox beta =$

```
plot/{postaction=decorate,decoration={markings,
  mark=between positions 0.1 and 0.9 step 5em with
  {\arrow{Stealth [length=2mm, blue]}}}}
```

Caution: with a high number of samples, adding arrows in this way may cause the error message! Dimension too big.

```
For example, the command
```

generates the Nyquist plot in Figure 7.

\NyquistTF

```
\NyquistTF [\langle plot/\{\langle opt\rangle \}, axes/\{\langle opt\rangle \}\rangle \] \{\langle num/\{\langle coeffs\rangle \}, den/\{\langle coeffs\rangle \}, d/\{\langle delay\rangle \}\}\} \{\langle min-freq\rangle \}\{\langle max-freq\rangle \}}
```

Nyquist plot of a transfer function given in TF format. Same mandatory arguments as \BodeTF and same optional arguments as \NyquistZPK. For example, the command

```
\NyquistTF[plot/{green,thick,samples=500,postaction=decorate,
  decoration={markings,
  mark=between positions 0.1 and 0.9 step 5em
  with{\arrow{Stealth[length=2mm, blue]}}}}
  {num/{10,2,2.6,0},den/{1,1,100.25}}
  {-30}{30}
```

NyquistPlot

```
generates the Nyquist plot in Figure 8. 
 \begin{NyquistPlot}[\langle axis-options\rangle] {\langle min-frequency\rangle} {\langle max-frequency\rangle} \\ \addNyquist... \\ \end{NyquistPlot}
```

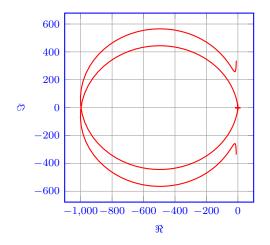


Figure 7: Output of the \NyquistZPK macro.

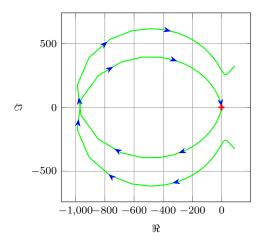


Figure 8: Output of the \NyquistTF macro with direction arrows. Increasing the number of samples can cause decorations.markings to throw errors.

The NyquistPlot environment works in conjunction with the parametric function generator macros \addNyquistZPKPlot and \addNyquistTFPlot. If supplied, axis-options are passed directly to the axis environment and the frequency limits are translated to the x-axis limits and the domain of the axis environment.

\addNyquistZPKPlot

```
\addNyquistZPKPlot[\langle plot-options \rangle]
```

```
\{\langle z/\{\langle zeros \rangle\}, p/\{\langle poles \rangle\}, k/\{\langle gain \rangle\}, d/\{\langle delay \rangle\}\}\}
```

Generates a twp parametric functions for the magnitude and the phase a transfer function in ZPK form. The generated magnitude and phase parametric functions are converted to real and imaginary part parametric functions and passed to the \addplot macro. This macro can be used inside any axis environment as long as a domain for the x-axis is supplied through either the plot-options interface or directly in the optional argument of the container axis environment. Use with the NyquistPlot environment supplied with this package is recommended. The mandatory argument is the same as \BodeZPK.

\addNyquistTFPlot

```
\addNyquistTFPlot[\langle plot-options \rangle]
```

```
\{\langle num/\{\langle coeffs \rangle\}, den/\{\langle coeffs \rangle\}, d/\{\langle delay \rangle\}\}\}
```

Similar to \addNyquistZPKPlot, with a transfer function input in the TF form.

2.3 Nichols charts

\NicholsZPK

```
\label{eq:nicholszpk} $$ \left( \left\langle \frac{\langle plot/\{\langle opt \rangle\}, axes/\{\langle opt \rangle\} \rangle}{\{\langle z/\{\langle zeros \rangle\}, p/\{\langle poles \rangle\}, k/\{\langle gain \rangle\}, d/\{\langle delay \rangle\} \rangle} {\{\langle min-freq \rangle\} \{\langle max-freq \rangle\}} $$
```

Nichols chart of a transfer function given in ZPK format. Same arguments as \NyquistZPK.

\NicholsTF

```
\label{eq:linear_cont} $$ \left( \left( \frac{dot}{dot} \right), axes / \left( \frac{dot}{dot} \right) \right) $$ $$ \left( \frac{dot}{dot} \right), den / \left( \frac{dot}{dot} \right) \right) $$ $$ \left( \frac{dot}{dot} \right) \left( \frac{dot}{dot} \right) \right) $$ $$ \left( \frac{dot}{dot} \right) \left( \frac{dot}{dot} \right) \right) $$ $$
```

Nichols chart of a transfer function given in TF format. Same arguments as \NyquistTF. For example, the command

```
\NicholsTF[plot/{green,thick,samples=2000}] {num/{10,2,2.6,0},den/{1,1,100.25},d/0.01} {0.001}{100}
```

generates the Nichols chart in Figure 9.

NicholsChart

\end{NicholsChart}

The NicholsChart environment works in conjunction with the parametric function generator macros \addNicholsZPKChart and \addNicholsTFChart. If supplied, axis-options are passed directly to the axis environment and the frequency limits are translated to the x-axis limits and the domain of the axis environment.

\addNicholsZPKChart

```
\label{localizero} $$ \addNicholsZPKChart[\langle plot\text{-}options\rangle] $$ $ \{\langle z/\{\langle zeros\rangle\}, p/\{\langle poles\rangle\}, k/\{\langle gain\rangle\}, d/\{\langle delay\rangle\}\} \}$$
```

Generates a two parametric functions for the magnitude and the phase a transfer function in ZPK form. The generated magnitude and phase parametric functions

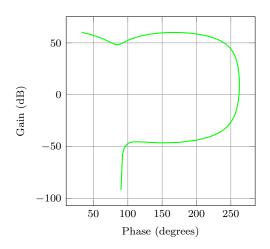


Figure 9: Output of the \NyquistZPK macro.

are passed to the \addplot macro. This macro can be used inside any axis environment as long as a domain for the x-axis is supplied through either the plot-options interface or directly in the optional argument of the container axis environment. Use with the NicholsChart environment supplied with this package is recommended. The mandatory argument is the same as \BodeZPK.

\addNicholsTFChart

```
\addNicholsTFChart[\langle plot-options \rangle]
```

 $\{\langle num/\{\langle coeffs \rangle\}, den/\{\langle coeffs \rangle\}, d/\{\langle delay \rangle\}\}\}$

Similar to \addNicholsZPKChart, with a transfer function input in the TF form.

Change History

```
v1.0 General: Initial release . . . . . . . . 1
```