

The **xint** bundle

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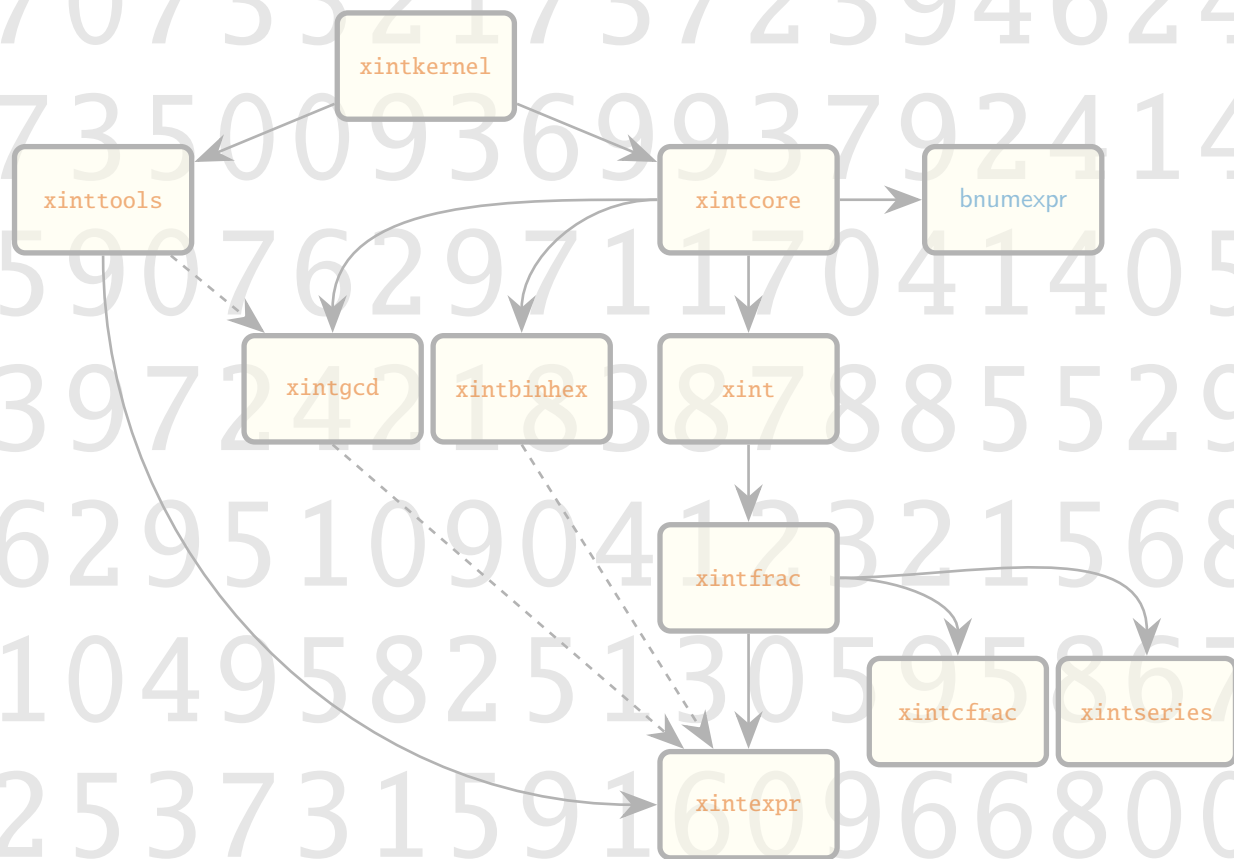
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Dependency graph for the **xint** bundle components: modules at the bottom **automatically** import modules at the top when connected by a continuous line. No module will be loaded twice, this is managed internally under Plain as well as \LaTeX . Dashed lines indicate a partial dependency, and to enable the corresponding functionalities of the lower module it is necessary to use the suitable `\usepackage` (\LaTeX) or `\input` (Plain \TeX .)

Notice that since 1.2i the macro `\xintXTrunc` does not cause a partial dependency of **xintfrac** on **xinttools** anymore.

The **bnumexpr** package is a separate package (\LaTeX only) by the author which uses (by default) **xintcore** as its mathematical engine.

1 Read this first

This section provides recommended reading on first discovering the package.

xinttools provides utilities of independent interest such as expandable and non-expandable loops. It is `\not` loaded automatically (nor needed) by the other bundle packages, apart from **xintexpr**.

xintcore provides the expandable \TeX macros doing additions, subtractions, multiplications, divisions, and powers on arbitrarily long numbers (loaded automatically by **xint**, and also by package **bnumexpr** in its default configuration).

xint extends **xintcore** with additional operations on big integers. Loads automatically **xintcore**.

xintfrac extends the scope of **xint** to decimal numbers, to numbers in scientific notation and also to fractions with arbitrarily long such numerators and denominators separated by a forward slash. Loads automatically **xint**.

xintexpr extends **xintfrac** with expandable parsers doing algebra (exact, float, or limited to integers) on comma separated expressions using standard infix notations with parentheses, numbers in decimal notation, scientific notation, comparison operators, Boolean logic, twofold and threefold way conditionals, sub-expressions, some functions with one or many arguments, user-definable variables, user-definable functions, nestable use of dummy variables for evaluation of sub-expressions, with iterations admitting omit, abort, and break instructions. Automatically loads **xinttools** and **xintfrac** (hence **xint** and **xintcore** too).

Further modules:

xintbinhex is for conversions to and from binary and hexadecimal bases.

xintseries provides some basic functionality for computing in an expandable manner partial sums of series and power series with fractional coefficients.

xintgcd implements the Euclidean algorithm and its typesetting.

xintcfrac deals with the computation of continued fractions.

All macros from the **xint** packages doing computations are *expandable*, and naturally also the parsers provided by **xintexpr**.

The reasonable range of use of the package arithmetics is with numbers of *up to a few hundred digits*. Although numbers up to about 19950 digits are acceptable inputs, the package is not at his peak efficiency when confronted with such really big numbers having thousands of digits.¹

1.1 First examples

With `\usepackage{xintexpr}` if using \TeX , or `\input xintexpr.sty\relax` for other formats, you can do computations such as the following.

with floats:

¹ The maximal handled size for inputs to multiplication is 19959 digits. This limit is observed with the current default values of some parameters of the \tex executable (input stack size at 5000, maximal expansion depth at 10000). Nesting of macros will reduce it and it is best to restrain numbers to at most 19900 digits. The output, as naturally is the case with multiplication, may exceed the bound.

1 Read this first

```
\thexintfloatexpr 3.25^100/3.2^100, 2^1000000, sqrt(1000!), 10^-3.5\relax
```

4.713443069476886, 9.900656229295898e301029, 6.343400192933548e1283, 0.0003162277660168379

with fractions:

```
\thexintexpr reduce(add((-1)^(i-1)/i**2, i=1..25))\relax
```

196669262520424458517/238898057495217120000

with integers:

```
\thexintiexpr 3^159+2^234\relax
```

7282510957931791370332035240020194155624159839406805975150495299651205982251

Float computations are done by default with 16 digits of precision. This can be changed by a prior assignment to `\xintDigits`:

```
% use braces (or a LaTeX environment) to limit the scope of the \xintDigits assignment
```

```
{\xintDigits := 88;\thexintfloatexpr 3.25^100-3.2^100\relax}\par
```

1.2155496665826543032280663867259188613692951880893931399567325222064394651297590367526e51

We can even try daring things:²

```
{\xintDigits:=500;\printnumber{\thexintfloatexpr sqrt(2)\relax}}
```

1.4142135623730950488016887242096980785696718753769480731766797379907324784621070388503875342
327641572735013846230912297024924836055850737212644121497099935831413222665927505592755799952
050115278206057147010955997160597027453459686201472851741864088919860955232923048430871432142
508397626036279952514079896872533965463318088296406206152583523950547457502877599617298355752
220337531857011354374603408498847160386899970699004815030544027790316454247823068492936918622
15805784631115966687130130156185689872372

This is release 1.2j.

1. `exp`, `cos`, `sin`, etc... are yet to be implemented,
2. `NaN`, `+Infty`, `-Infty`, etc... are yet to be implemented,
3. powers work currently only with integral or also half-integral exponents (but the latter only for float expressions),
4. `xint` can handle numbers with thousands of digits, but execution times limit the practical range to a few hundreds (if many such computations are needed),
5. computations in `\thexintexpr` and `\thexintiexpr` are exact (except if using `sqrt`, naturally),
6. fractions are not systematically reduced to smallest terms, use `reduce` function,
7. for producing fixed point numbers with `d` digits after decimal mark, use (note the extra `'i'` in the parser name!) `\thexintiexpr [d] ...\relax`. This is actually essentially synonymous with `\thexintexpr round(...,d)\relax` (for `d=0`, `\thexintiexpr [0]` is the same as `\thexintiexpr` without optional argument, and is like `\thexintexpr round(...)\relax`). If truncation rather than rounding is needed use thus `\thexintexpr trunc(...,d)\relax` (and `\thexintexpr trunc(...)\relax` for truncation to integers),
8. all three parsers allow some constructs with dummy variables as seen above; it is possible to define new functions or to declare variables for use in upcoming computations,
9. `\thexintiexpr` is slightly faster than `\thexintexpr`, but usually one can use the latter with no significant time penalty also for integer-only computations.

New with
1.2f

² The `\printnumber` is not part of the package, see subsection 1.3.

All operations executed by the parsers are based on underlying macros from packages `xintfrac` and `xint` which are loaded automatically by `xintexpr`. With extra packages `xintbinhex` and `xintgcd` the parsers can handle hexadecimal notation on input (even fractional) and compute `gcd`'s or `lcm`'s of integers.

All macros doing computations ultimately rely on (and reduce to) the `\numexpr` primitive from ε -TeX. These ε -TeX extensions date back to 1999 and are by default incorporated into the `pdfTeX` etc... executables from major modern TeX installations since more than ten years now. Only the `teTeX` binary does not benefit from them, as it has to remain the original D. K^NUTH's software, but one can then use `etex` on the command line. PDF ε TeX (in pdf or dvi output mode), LuaTeX, XeTeX all include the ε -TeX extensions.

1.2 Quick overview (expressions with `xintexpr`)

This section gives a first few examples of using the expression parsers which are provided by package `xintexpr`. Loading `xintexpr` automatically also loads packages `xinttools` and `xintfrac`. The latter loads `xint` which loads `xintcore`. All three provide the macros which ultimately do the computations associated in expressions with the various symbols like `+`, `*`, `^`, `!` and functions such as `max`, `sqrt`, `gcd` (the latter requires explicit loading of `xintgcd`). The package `xinttools` does not handle computations but provides some useful utilities.

Release 1.2h defines `\thexintexpr` as synonym to `\xinttheexpr`, `\thexintfloatexpr` as synonym of `\xintthefloatexpr`, etc...

There are three expression parsers and two subsidiary ones. They all admit comma separated expressions, and will then output a comma separated list of results.

- `\xinttheiexpr ... \relax` does exact computations *only on integers*. The forward slash `/` does the rounded integer division (`//` does truncated division, and `/:` is the associated modulo). There are two square root extractors `sqrt` and `sqrttr` for truncated and rounded square roots. Scientific notation `6.02e23` is *not* accepted on input, one needs to wrap it as `num(6.02e23)` which will convert to an integer notation `6020000000000000000000`.
- `\xintthefloatexpr ... \relax` does computations with a given precision `P`, as specified via a prior assignment `\xintDigits:=P;`. The default is `P=16` digits. An optional argument controls the precision for *formatting the output* (this is not the precision of the computations themselves). The four basic operations and the square root realize *correct rounding*.³
- `\xinttheexpr ... \relax` handles integers, decimal numbers, numbers in scientific notation and fractions. The algebraic computations are done *exactly*. The `sqrt` function is available and works according to the `\xintDigits` precision or according to its second optional argument.

Currently, the sole available non-algebraic function is the square root extraction `sqrt`. It is allowed in `\xintexpr... \relax` but naturally can't return an exact value, hence computes as if it was in `\xintfloatexpr... \relax`. The power operator `^` (equivalently `**`) works with integral exponents only in `\xintiexpr` (non-negative) and `\xintexpr` (negative exponents allowed, of course) and also with half-integral exponents in `\xintfloatexpr` (it proceeds via an integral power followed by a square-root extraction).

Two derived parsers:

- `\xinttheiexpr ... \relax` does all computations like `\xinttheexpr ... \relax` but rounds the result to the nearest integer. With an optional positive argument `[D]`, the rounding is to the nearest fixed point number with `D` digits after the decimal mark.

³ when the inputs are already floating point numbers with at most `P`-digits mantissas.

1 Read this first

- `\xinttheboolexpr ... \relax` does all computations like `\xinttheexpr ... \relax` but converts the result to 1 if it is not zero (works also on comma separated expressions). See also the booleans `\xintifboolexpr`, `\xintifboolliexpr`, `\xintifboolfloatexpr` (which do not handle comma separated expressions).

Here is a (partial) list of the recognized symbols:

- the comma (to separate distinct computations or arguments to a function),
- parentheses,
- infix operators `+`, `-`, `*`, `/`, `^` (or `**`),
- branching operators `(x)?{x non zero}{x zero}`, `(x)??{x<0}{x=0}{x>0}`,
- boolean operators `!`, `&&` or `'and'`, `||` or `'or'`,
- comparison operators `=` (or `==`), `<`, `>`, `<=`, `>=`, `!=`,
- factorial post-fix operator `!`,
- `"` for hexadecimal input (uppercase only; package `xintbinhex` must be loaded additionally to `xintexpr`),
- functions `num`, `reduce`, `abs`, `sgn`, `frac`, `floor`, `ceil`, `sqr`, `sqrt`, `sqrtr`, `float`, `round`, `trunc`, `mod`, `quo`, `rem`, `max`, `min`, ``+``, ``*``, `not`, `all`, `any`, `xor`, `if`, `ifsgn`, `even`, `odd`, `first`, `last`, `reversed`, `bool`, `togl`, `factorial`, `binomial`, `pfactorial`,
- multi-arguments `gcd` and `lcm` are available if `xintgcd` is loaded,
- functions with dummy variables `add`, `mul`, `seq`, `subs`, `rseq`, `iter`, `rrseq`, `iterr`.

See [subsection 11.1](#) for basic information and [section 2](#) for the built-in syntax elements.

The normal mode of operation of the parsers is to unveil the parsed material token by token. This means that all elements may arise from expansion of encountered macros (or active characters). For example a closing parenthesis does not have to be immediately visible, it may arise later from expansion. This general behavior has exceptions, in particular constructs with dummy variables need immediately visible balanced parentheses and commas. The expansion stops only when the ending `\relax` has been found; it is then removed from the token stream, and the final computation result is inserted.

Release 1.2 added the (pseudo) functions `qint`, `qfrac`, `qfloat` to allow swallowing in one-go all digits of a big number, fraction, or float, skipping the token by token expansion.

Here is an example of a computation:

```
\xinttheexpr (31.567^2 - 21.56*52)^3/13.52^5\relax
-1936508797861911919204831/4517347060908032[-8]
```

This illustrates that `\xinttheexpr.\relax` does its computations exactly. The same example as a floating point evaluation:

```
\xintthefloatexpr (31.567^2 - 21.56*52)^3/13.52^5\relax
-4.286827582100044
```

Again, all computations done by `\xinttheexpr.\relax` are completely exact. Thus, very quickly very big numbers are created (and computation times increase, not to say explode if one goes into handling numbers with thousands of digits). To compute something like `1.23456789^10000` it is thus better to opt for the floating point version:

```
\xintthefloatexpr 1.23456789^10000\relax
1.411795173056392e915
```

(we can deduce that the exact value has `80000+916=80916` digits). A bigger example (the scope of the assignment to `\xintDigits` is limited by the braces):

```
{\xintDigits:=24; \xintthefloatexpr 1.23456789123456789^123456789\relax }
1.90696640042856610942910e11298145 (<- notice the size of the power of ten: this surely largely exceeds your pocket calculator abilities).
```

It is also possible to do some computer algebra like evaluations (only numerically though):

```
\xinttheiexpr add(i^5, i=100..200)\relax\par
\noindent\xinttheexpr reduce(add(x/(x+1), x = 1000..1014))\relax
10665624997500
4648482709767835886400149017599415343/310206597612274815392155150733157360
```

Were it not for the `reduce` function, the latter fraction would not have been obtained in reduced terms:

By default, the basic operations on fractions are not followed in an automatic manner by reduction to smallest terms: A/B multiplied by C/D returns AC/BD , A/B added to C/D returns $(AD+BC)/BD$ except if either B divides D or D divides B .

Make sure to read [section 11](#), [section 2](#) and [subsection 3.5](#).

1.3 Printing big numbers on the page

When producing very long numbers there is the question of printing them on the page, without going beyond the page limits. In this document, I have most of the time made use of these macros (not provided by the package:)

```
\def\allowsplits #1{\ifx #1\relax \else #1\hskip 0pt plus 1pt\relax
\expandafter\allowsplits\fi}%
\def\printnumber #1{\expandafter\allowsplits \romannumeral-`0#1\relax}%
% \printnumber thus first ``fully'' expands its argument.
```

It may be used like this:

```
\printnumber {\xintiiQuo{\xintiiPow {2}{1000}}{\xintiFac{100}}}
```

or as `\printnumber\mybiginteger` or `\printnumber{\mybiginteger}` if `\mybiginteger` was previously defined via a `\newcommand`, a `\def` or an `\edef`.

An alternative is to suitably configure the thousand separator with the `numprint` package (see [footnote 7](#). This will not allow linebreaks when used in math mode; I also tried `siunitx` but even in text mode could not get it to break numbers accross lines). Recently I became aware of the `seqsplit` package⁴ which can be used to achieve this splitting accross lines, and does work in inline math mode (however it doesn't allow to separate digits by groups of three, for example).

1.4 Randomly chosen examples

Here are some examples of use of the package macros. The first one uses only the base module `xint`, the next one requires the `xintfrac` package, which deals with decimal numbers, scientific numbers (lowercase `e`), and also fractions (it loads automatically `xint`). Then some examples with expressions, which require the `xintexpr` package (it loads automatically `xintfrac`). And finally some examples using `xintseries`, `xintgcd` which are among the extra packages included in the `xint` distribution.

The printing of the outputs will either use a custom `\printnumber` macro as described in the previous section, or sometimes the `\np` macro from the `numprint` package (see [footnote 7](#)).

- 123456^{99} :
`\xintiiPow {123456}{99}`: 1147381811662665566332733300084545867470254804234261029758895452
43735908946970320276226470542663205834690270868221168133415250032403876277616895322211762
34295872033762216088606915850757168019716710712087697033536507377487778737784987816067492
99979836658125172327521549705416595667384911533326748541075607669718906235189958323778262
36999811095323939932351899922205645878127014958776791431677354372538584459487155941215192
74163986661258969837372587167573949494355201709502618658016651990307184144322311696783762
96
- 1234/56789 with 1500 digits after the decimal point:
`\xintTrunc {1500}{1234/56789}\dots`: 0.021729560302171195125816619415731919914067865255592
52737325890577400552923981757030410818996636672595044815016992727464825934600010565426402
30005810984521650319604148690767578228177992216802549789571924140238426455827713113455072
05242212400288788321682015883357692510873584673088098047157019845392593636091496592649982
50323125957491767771927662047227456021412597510081177692863054464773107467995562520910732

⁴ <http://ctan.org/pkg/seqsplit>

97559386500906865766257549877617144165243268942929088379791861099860888552360492348870372
98270791878708904893553328989769145433094437302998820194051664935110672841571431087006282
64287097853457535790381940164468471006709045765905368997517124795294863441863741217489302
25057669619116378171829051400799450597827043969782880487418338058426808008593213474440472
26267410942259944707601824296958918100336332740495518498300727253517406539998943457359692
99418901547834968039585130923242177182200778319745021042807585976157354417228688654492942
75778759971121167831798411664230748912641532691190195284298015460740636390850340735001492
67687404250823222807233795277254397858740248991882230713694553522689253200443747908926022
44061349909313423374245012238285583475673105707091162020813890013911144763950765112962012
72920812129109510644667101023085456690556269700117980594833506488932715842856891299371352
71290214654246420961805983553152899329095423409463100248287520470513655813625878251069742
94233038088362182817094859920054940217295603021711951258166194157319199140678652555952732
732589057740055292398175703041081899663667. . .

- ```
\xinttheiexpr [201] .99^100\relax: 2.731999026429026003846671721257837435505351642938572
2070833430572508246455518705343044814301378480614036805562476501925307034269685489153194
616612271015920671913840348851485747943086470963920731779793038
```

Notice that this is rounded, hence we asked `\xinttheiexpr` for one additional digit. To get a truncated result with 200 digits after the decimal mark, we should have issued `\xinttheiexpr \trunc(.99^100,200)\relax`, rather.

`\xintDigits:=20; \np{\xintthefloatexpr .7123045678952^-243\relax}`  
 $6.342,022,117.488,416,127.3 \times 10^{35}$

```
\xinttheiiexpr 200!\relax: 7886578673647905035523632139321850622951359776871732632947425
3324435944996340334292030428401198462390417721213891963883025764279024263710506192662495
2829931113462857270763317237396988943922445621451664240254033291864131227428294853277524
242407573903240321257405579568660226031904170324062351700858796178922227896237038973747
200
```

- ```
\xintthefloatexpr 2000!\relax: 3.3162750924506332411753933805763240382811172081058e5735
```

- ```
% % in the preamble:
% \usepackage[english]{babel}
% \usepackage[autolanguage,np]{numprint}
% \npthousandsep{\hskip 1pt plus .5pt minus .5pt}
% \usepackage{xintexpr}
```

8



## 1 Read this first

```
% in the body:
\np {\xinttheexpr trunc(.7^-25,300)\relax}\dots
```

7,456,739,985,837,358,837,609,119,727,341,853,488,853,339,101,579,533,584,812,792,108,394,305,337,246,328,231,852,818,407,506,767,353,741,490,769,900,570,763,145,015,081,436,139,227,188,742,972,826,645,967,904,896,381,378,616,815,228,254,509,149,848,168,782,309,405,985,245,368,923,678,816,256,779,083,136,938,645,362,240,130,036,489,416,562,067,450,212,897,407,646,036,464,074,648,484,309,937,461,948,589. . .

This computation is with `\xinttheexpr` from package `xintexpr`, which allows to use standard infix notations and function names to access the package macros, such as here `trunc` which corresponds to the `xintfrac` macro `\xintTrunc`. Regarding this computation, please keep in mind that `\xinttheexpr` computes exactly the result before truncating. As powers with fractions lead quickly to very big ones, it is good to know that `xintexpr` also provides `\xintthefloatexpr` which does computations with floating point numbers.

- Computation of a Bezout identity with  $7^{200}-3^{200}$  and  $2^{200}-1$ : (with `xintgcd`)

```
\xintAssign \xintBezout {\xinttheiexpr 7^200-3^200\relax}
{\xinttheiexpr 2^200-1\relax}\to\A\B\U\V\D
$\U\times(7^{\{200\}}-3^{\{200\}})+\xintiOpp\U\times(2^{\{200\}}-1)=\D$
```

$-220045702773594816771390169652074193009609478853 \times (7^{200} - 3^{200}) + 14325894936276369318591302683268320465474416863387714089158381672478991921132820119127462437158039177754976857191287693144240605066991456336143205677696774891 \times (2^{200} - 1) = 1803403947125$

- The Euclidean algorithm applied to 22,206,980,239,027,589,097 and 8,169,486,210,102,119,257: (with `xintgcd`)<sup>6</sup>

```
\xintTypesetEuclideanAlgorithm {22206980239027589097}{8169486210102119257}
22206980239027589097 = 2 × 8169486210102119257 + 5868007818823350583
8169486210102119257 = 1 × 5868007818823350583 + 2301478391278768674
5868007818823350583 = 2 × 2301478391278768674 + 1265051036265813235
2301478391278768674 = 1 × 1265051036265813235 + 1036427355012955439
1265051036265813235 = 1 × 1036427355012955439 + 228623681252857796
1036427355012955439 = 4 × 228623681252857796 + 121932630001524255
228623681252857796 = 1 × 121932630001524255 + 106691051251333541
121932630001524255 = 1 × 106691051251333541 + 15241578750190714
106691051251333541 = 6 × 15241578750190714 + 15241578750189257
15241578750190714 = 1 × 15241578750189257 + 1457
15241578750189257 = 10460932567048 × 1457 + 321
1457 = 4 × 321 + 173
321 = 1 × 173 + 148
173 = 1 × 148 + 25
148 = 5 × 25 + 23
25 = 1 × 23 + 2
23 = 11 × 2 + 1
2 = 2 × 1 + 0
```

- $\sum_{n=1}^{500} (4n^2 - 9)^{-2}$  with each term rounded to twelve digits, and the sum to nine digits:

```
\def\coeff #1{\xintiRound {12}{1/\xintiiSqr{\the\numexpr 4*#1*#1-9\relax }[0]}}
\xintRound {9}{\xintiSeries {1}{500}{\coeff}[-12]}
```

0.062366080

<sup>6</sup> this example is computed tremendously faster than the other ones, but we had to limit the space taken by the output hence picked up rather small big integers as input.

## 1 Read this first

The complete series, extended to infinity, has value  $\frac{\pi^2}{144} - \frac{1}{162} = 0.062,366,079,945,836,595,346,844,45\dots$ <sup>7</sup> I also used (this is a lengthier computation than the one above) `xintseries` to evaluate the sum with 100,000 terms, obtaining 16 correct decimal digits for the complete sum. The coefficient macro must be redefined to avoid a `\numexpr` overflow, as `\numexpr` inputs must not exceed  $2^{31} - 1$ ; my choice was:

```
\def\coeff #1%
{\xintiRound {22}{1/\xintiiSqr{\xintiiMul{\the\numexpr 2*#1-3\relax}
{\the\numexpr 2*#1+3\relax}}{0}}}
```

- Computation of  $2^{999,999,999}$  with 24 significant figures:

```
\numprint{\xintFloatPow [24]{2}{999999999}}
2.306,488,000,584,534,696,558,06 × 10301,029,995
```

where the `numprint` package was used (footnote 7), directly in text mode (it can also naturally be used from inside math mode). `xint` provides a simple-minded `\xintFrac` typesetting macro,<sup>8</sup> which is math-mode only:

```
$_\xintFrac{\xintFloatPow [24]{2}{999999999}}$
230648800058453469655806 · 10301029972
```

The exponent differs, but this is because `\xintFrac` does not use a decimal mark in the significant of the output. Admittedly most users will have the need of more powerful (and customizable) number formatting macros than `\xintFrac`.<sup>9</sup> We have already mentioned `\numprint` which is used above, there is also `\num` from package `siunitx`. The raw output from

```
\xintFloatPow [24]{2}{999999999}
```

is 2.30648800058453469655806e301029995.

- As an example of nesting package macros, let us consider the following code snippet within a file with filename `myfile.tex`:

```
\newwrite\outstream
\immediate\openout\outstream \jobname-out\relax
\immediate\write\outstream {\xintiiQuo{\xintiiPow{2}{1000}}{\xintiiFac{100}}}
% \immediate\closeout\outstream
```

The tex run creates a file `myfile-out.tex`, and then writes to it the quotient from the euclidean division of  $2^{1000}$  by  $100!$ . The number of digits is `\xintLen{\xintiiQuo{\xintiiPow{2}{1000}}{\xintiiFac{100}}}` which expands (in two steps) and tells us that  $[2^{1000}/100!]$  has 144 digits. This is not so many, let us print them here: 11481324964150750548227839387255106625928055177841861728836634780658265418947047379704195357988766304843582650600615037495317077293118627774829601.

## 1.5 More examples, some quite elaborate, within this document

- The utilities provided by `xinttools` (section 7), some completely expandable, others not, are of independent interest. Their use is illustrated through various examples: among those, it is shown in subsection 5.7 how to implement in a completely expandable way the `Quick Sort algorithm` and also how to illustrate it graphically. Other examples include some dynamically constructed alignments with automatically computed prime number cells: one using a completely expandable prime test and `\xintApplyUnbraced` (subsection 5.1), another one with `\xintFor*` (subsection 5.5).
- One has also a `computation of primes within an \edef` (subsection 7.14), with the help of `\xintiloop`. Also with `\xintiloop` an automatically generated table of factorizations (subsection 5.4).

<sup>7</sup> This number is typeset using the `numprint` package, with `\npthousandsep {,}\hskip 1pt plus .5pt minus .5pt`. But the breaking across lines works only in text mode. The number itself was (of course...) computed initially with `xint`, with 30 digits of  $\pi$  as input. See [how xint may compute  \$\pi\$  from scratch](#). <sup>8</sup> Plain T<sub>E</sub>X users of `xint` have `\xintFwOver`. <sup>9</sup> There should be a `\xintFloatFrac`, but it is lacking.

- The code for the title page fun with Fibonacci numbers is given in [subsection 2.9](#) with `\xint-For*` joining the game.
- The computations of  $\pi$  and  $\log 2$  ([subsection 14.11](#)) using `xint` and the computation of the [convergents of e](#) with the further help of the `xintcfrac` package are among further examples.
- Also included, an [expandable implementation of the Brent-Salamin algorithm](#) for evaluating  $\pi$ .
- The functionalities of `xintexpr` are illustrated with various examples, found in locations such as in [subsubsection 2.6.1](#) and [functions with dummy variables](#) and [subsection 2.4](#).

Almost all of the computational results interspersed throughout the documentation are not hard-coded in the source file of this document but are obtained via the expansion of the package macros during the  $\text{\TeX}$  run.<sup>10</sup>

## 1.6 Installation instructions

`xint` is made available under the [LaTeX Project Public License 1.3c](#). It is included in the major  $\text{\TeX}$  distributions, thus there is probably no need for a custom install: just use the package manager to update if necessary `xint` to the latest version available.

After installation, issuing in terminal `texdoc --list xint`, on installations with a "texdoc" or similar utility, will offer the choice to display one of the documentation files: `xint.pdf` (this file), `sourcexint.pdf` (source code), `README`, `README.pdf`, `README.html`, `CHANGES.pdf`, and `CHANGES.html`.

For manual installation, follow the instructions from the [README](#) file which is to be found on [CTAN](#); it is also available there in PDF and HTML formats. The simplest method proposed is to use the archive file `xint.tds.zip`, downloadable from the same location.

The next simplest one is to make use of the [Makefile](#), which is also downloadable from [CTAN](#). This is for GNU/Linux systems and Mac OS X, and necessitates use of the command line. If for some reason you have `xint.dtx` but no internet access, you can recreate [Makefile](#) as a file with this name and the following contents:

```
include Makefile.mk
Makefile.mk: xint.dtx ; etex xint.dtx
```

Then run `make` in a working repertory where there is `xint.dtx` and the file named [Makefile](#) and having only the two lines above. The `make` will extract the package files from `xint.dtx` and display some further instructions.

If you have `xint.dtx`, no internet access and can not use the Makefile method: `etex xint.dtx` extracts all files and among them the [README](#) as a file with name `README.md`. Further help and options will be found therein.

## 1.7 Changes

The detailed cumulative change log since the initial release is in files [CHANGES.html](#) (available on internet via [this link](#)). You can also access it as well as [CHANGES.pdf](#) locally from typing `texdoc -l xint` on the command line.

This is release [1.2j](#) of [2016/12/22](#).

Releases [1.2i](#) and [1.2j](#) brought principally across-the-board speed improvements to various token or digits handling routines, and refactored the documentation somewhat. See [CHANGES.html](#) or [CHANGES.pdf](#) for more.

The last major release was [1.2 \(2015/10/10\)](#). It came with a complete rewrite of the core arithmetic routines. The efficiency for numbers with less than 20 or 30 digits was slightly compromised

<sup>10</sup> The CPU of my computer hates me for all those re-compilations after changing a single letter in the  $\text{\LaTeX}$  source, which require each time to do all the zillions of evaluations contained in this document...

(for addition/subtraction) but it got increased for bigger numbers. For multiplication and division the gains were there for almost all sizes, and became quite noticeable for numbers with hundreds of digits. The allowable inputs are constrained to have less than about 19950 digits (19968 for addition, 19959 for multiplication).

Among subsequent evolution, we mention here some changes with 1.2f (2016/03/12) regarding floating point numbers:

1. functions `factorial`, `binomial`, `pfactorial`.
2. more precise discussion of floating point issues in subsection 3.2, but the coverage by `xintfrac` of floating point operations is yet to be substantially extended.
3. the float macros for addition, subtraction, multiplication, division now first round their two operands to P significant places (not P+2 as earlier) before doing the actual computation (P is the asked for precision or `\xinttheDigits`). The same applies to the float power and square root operations.
4. `\xintFloatSqrt` achieves correct rounding in arbitrary precision.

## 2 The syntax of `xintexpr` expressions

|    |                                          |    |    |                                                                                           |    |
|----|------------------------------------------|----|----|-------------------------------------------------------------------------------------------|----|
| .1 | Built-in operators and precedences ..... | 12 | .7 | List operations .....                                                                     | 28 |
| .2 | Built-in functions.....                  | 15 | .8 | Analogies and differences of <code>\xinttiexpr</code><br>with <code>\numexpr</code> ..... | 31 |
| .3 | Tacit multiplication.....                | 22 | .9 | Chaining expressions for expandable algo-<br>rithmics .....                               | 31 |
| .4 | More examples with dummy variables.....  | 23 |    |                                                                                           |    |
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### 2.1 Built-in operators and precedences

The Table 1 is hyperlinked to the more detailed discussion at each level. The levels are indicative and there may be some evolution in future, perhaps to distinguish some of the constructs which currently share the same precedence.

In case of equal precedence, the general rule is left-associativity: the first encountered operation is executed first. Tacit multiplication has an elevated precedence level hence seemingly breaks left-associativity:  $(1+2)/(3+4)(5+6)$  is computed as  $(1+2)/((3+4)*(5+6))$  and  $x/2y$  is interpreted as  $x/(2*y)$  when using variables.

∞ At this highest level of precedence, one finds:

- functions and variables: we approximately describe the situation as saying they have highest precedence. Functions (even the logic functions `!` and `?` which are expressed as a single character) must be used with parentheses. These parentheses may arise from expansion after the function name is parsed (there are exceptions which are documented at the relevant locations.)
- the `.` as decimal mark; the number scanner treats it as an inherent, optional and unique component of a being formed number. One can do things such as  
`\xinttheexpr 0.^2+2^.0\relax`  
which is  $0^2+2^0$  and produces 1.

Since release 1.2 an isolated decimal mark `".` is illegal input in `\xintexpr..relax`, although it remains legal as argument to the macros of `xintfrac`.

- the `e`, equivalently `E`, for scientific notation are parsed like the decimal mark is.
- the `"` for hexadecimal numbers: it is allowed only at locations where the parser expects to start forming a numeric operand, once encountered it triggers the hexadecimal scanner which looks for successive hexadecimal digits as usual skipping spaces and expanding forward everything; letters (only `ABCDEF`, not `abcdef`), an optional dot (allowed di-

| Precedence     | ``Operators'' at this level                                                                                                                                                                               |
|----------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\infty$       | functions and variables, decimal mark <code>.</code> , <code>e</code> and <code>E</code> of scientific notation, hexadecimal prefix <code>"</code> .                                                      |
| 10             | postfix <code>!</code> (factorial) and conditional branching operators <code>?</code> and <code>??</code>                                                                                                 |
| <code>=</code> | minus sign <code>-</code> as unary operator acquires the precedence level of the previous infix operator.                                                                                                 |
| 9              | <code>^</code> , <code>**</code> and list operators <code>^[</code> , <code>**[</code> , <code>]^</code> , <code>]**</code> .                                                                             |
| 8              | tacit multiplication.                                                                                                                                                                                     |
| 7              | <code>*</code> , <code>/</code> , and list operators <code>*[</code> , <code>/[</code> , <code>]*</code> , <code>]/</code> .                                                                              |
| 6              | <code>+</code> , <code>-</code> , and list operators <code>+[</code> , <code>-[</code> , <code>]+</code> , <code>]-</code> .                                                                              |
| 5              | <code>&lt;</code> , <code>&gt;</code> , <code>==</code> (or <code>=</code> ), <code>&lt;=</code> , <code>&gt;=</code> , <code>!=</code> .                                                                 |
| 4              | <code>&amp;&amp;</code> and its equivalent <code>'and'</code> .                                                                                                                                           |
| 3              | <code>  </code> , its equivalent <code>'or'</code> , and <code>'xor'</code> ; also the sequence generators <code>..</code> , <code>..[</code> , <code>]..</code> , and the Python slicer <code>:</code> . |
| 2              | the comma <code>,</code> .                                                                                                                                                                                |
| 1              | the parentheses <code>(, )</code> , list brackets <code>[, ]</code> , and semi-colon <code>;</code> in an <code>iter</code> or <code>rseq</code> .                                                        |

Table 1: Precedence levels

## 2 The syntax of `xintexpr` expressions

rectly in front) and an optional (possibly empty) fractional part. The " functionality requires to load package `xintbinhex`.

```
\xinttheiexpr "FEDCBA9876543210\relax\newline
\xinttheiexpr 16^5-("F75DE.0A8B9+"8A21.F5746+16^-5)\relax
18364758544493064720
0
```

10 The postfix operators `!` and the branching conditionals `?`, `??`.

`!` computes the factorial of an integer.

`?` is used as `(stuff)?{yes}{no}`. It evaluates `stuff` and chooses the `yes` branch if the result is non-zero, else it executes `no`. After evaluation of `stuff` it acts as a macro with two mandatory arguments within braces, chooses the correct branch *without evaluating the wrong one*. Once the braces are removed, the parser scans and expands the uncovered material so for example

```
\xinttheiexpr (3>2)?{5+6}{7-1}2^3\relax
```

is legal and computes  $5+6 \cdot 2^3 = 238333$ . It would be better practice to include here the  $2^3$  inside the branches. The contents of the branches may be arbitrary as long as once glued to what is next the syntax is respected: `\xintexpr (3>2)?{5+(6){7-(1)2^3}\relax` also works.

`??` is used as `(stuff)??{<0}{=0}{>0}`, where `stuff` is anything, its sign is evaluated and depending on the sign the correct branch is un-braced, the two others are discarded with no evaluation of their contents. The un-braced branch will then be parsed as usual.

```
\def\x{0.33}\def\y{1/3}
\xinttheiexpr (\x-\y)??{sqrt}{0}{1/}{\y-\x}\relax=5773502691896257[-17]
```

= The minus sign `-` as prefix unary operator inherits the precedence of the infix operator it follows. `\xintexpr -3-4*-5^7\relax` evaluates as  $(-3)-(4*(-(5^7)))$  and  $-3-4*-5-7$  as  $((3^4-4)*(-5))-7$ .

$2^4-10$  is perfectly accepted input, no need for parentheses

9 The power operator `^`, or equivalently `**`. It is left associative: `\xinttheiexpr 2^2^3\relax` evaluates to 64, not 256. Note that if the float precision is too low, iterated powers within `\xintfloatexpr.\relax` may fail: for example with the default setting  $(1+1e-8)^{(12^4)}$  will be computed with  $12^4$  approximated from its 16 most significant digits but it has 18 digits ( $=184884258895036416$ ), hence the result is wrong:

```
1.879,985,375,897,266 × 10802,942,130
```

One should code

```
\xintthe\xintfloatexpr (1+1e-8)^\xintiexpr 12^16\relax \relax
```

to obtain the correct floating point evaluation

```
1.000,000,011216 ≈ 1.879,985,676,694,948 × 10802,942,130
```

Also at this level the list operators `^`, `**`, `]`, `^`, and `]`.

8 see [Tacit multiplication](#).

7 Multiplication and division `*`, `/`. The division is left associative, too: `\xinttheiexpr 100/50/2\relax` evaluates to 1, not 4. Inside `\xintiexpr`, `/` does rounded division.

Also at this level the list operators `*`, `/`, `]`, `*` and `]`.

Also the truncated division `//` and modulo `:` (equivalently `'mod'`, quotes mandatory). Operators all at the same level of precedence are left-associative. Apply parentheses for disambiguation.

```
\xinttheiexpr 100000//13, 100000/:13, 100000 'mod' 13, trunc(100000/13,10),
trunc(100000/:13/13,10)\relax
```

```
7692, 4, 4, 7692.3076923076, 0.3076923076
```

6 Addition and subtraction `+`, `-`. According to the rule above, `-` is left associative: `\xinttheiexpr 100-50-2\relax` evaluates to 48, not 52.

Also the list operators `+`, `-`, `]`, `+`, `-` are at this precedence level.

- 5 Comparison operators `<`, `>`, `=` (same as `==`), `<=`, `>=`, `!=` all at the same level of precedence, use parentheses for disambiguation.
- 4 Conjunction (logical and) `&&` or equivalently `'and'` (quotes mandatory).<sup>11</sup>
- 3 Inclusive disjunction (logical or) `||` and equivalently `'or'` (quotes mandatory).  
Also the `'xor'` operator (quotes mandatory) is at this level.  
Also the list generation operators `..`, `..[`, `]..` are at this level.  
Also the `:` for Python slicing of lists.
- 2 The comma: with `\xinttheexpr 2^3,3^4,5^6\relax` one obtains as output 8, 81, 15625.<sup>12</sup>
- 1 The parentheses. The list outer brackets `[`, `]` share the same functional precedence as parentheses. The semi-colon `;` in an `iter` or `rseq` has the same precedence as a closing parenthesis.<sup>13</sup>

## 2.2 Built-in functions

Functions are at the same top level of priority. All functions even `?` and `!` (as prefix) require parentheses around their arguments.

|                       |                           |                          |                            |                       |                       |
|-----------------------|---------------------------|--------------------------|----------------------------|-----------------------|-----------------------|
| <a href="#">abs</a>   | <a href="#">sgn</a>       | <a href="#">num</a>      | <a href="#">reduce</a>     | <a href="#">float</a> | <a href="#">round</a> |
| <a href="#">trunc</a> | <a href="#">floor</a>     | <a href="#">ceil</a>     | <a href="#">frac</a>       | <a href="#">sqr</a>   | <a href="#">sqrt</a>  |
| <a href="#">sqrtr</a> | <a href="#">factorial</a> | <a href="#">binomial</a> | <a href="#">pfactorial</a> | <a href="#">mod</a>   | <a href="#">quo</a>   |
| <a href="#">rem</a>   | <a href="#">gcd</a>       | <a href="#">lcm</a>      | <a href="#">max</a>        | <a href="#">min</a>   | <a href="#">`+`</a>   |
| <a href="#">`*`</a>   | <a href="#">?</a>         | <a href="#">!</a>        | <a href="#">not</a>        | <a href="#">all</a>   | <a href="#">any</a>   |
| <a href="#">xor</a>   | <a href="#">if</a>        | <a href="#">ifsgn</a>    | <a href="#">even</a>       | <a href="#">odd</a>   | <a href="#">first</a> |
| <a href="#">last</a>  | <a href="#">reversed</a>  | <a href="#">len</a>      | <a href="#">subs</a>       | <a href="#">add</a>   | <a href="#">mul</a>   |
| <a href="#">seq</a>   | <a href="#">rseq</a>      | <a href="#">iter</a>     | <a href="#">rrseq</a>      | <a href="#">iterr</a> | <a href="#">bool</a>  |
| <a href="#">togl</a>  | <a href="#">qint</a>      | <a href="#">qfrac</a>    | <a href="#">qfloat</a>     |                       |                       |

Table 2: Functions

See [Table 2](#) whose elements are hyperlinked to the corresponding definitions.

Miscellaneous notes:

- `gcd` and `lcm` require explicit loading of `xintgcd`,
- `togl` is provided for the case `etoolbox` package is loaded,
- `bool`, `togl` use delimited macros to fetch their argument and the closing parenthesis must be explicit, it can not arise from on the spot expansion. The same holds for `qint`, `qfrac`, `qfloat`.
- Also [functions with dummy variables](#) use delimited macros for some tasks. See the relevant explanations there.

### functions with a single (numeric) argument:

**num** truncates to the nearest integer (truncation towards zero).

```
\xinttheexpr num(3.1415^20)\relax
```

**8764785276**

<sup>11</sup> with releases earlier than 1.1, only single character operators `&` and `|` were available, because the parser did not handle multi-character operators. Their usage in this rôle is now deprecated, and they may be assigned some new meaning in the future.

<sup>12</sup> The comma is really like a binary operator, which may be called “join”. It has lowest precedence of all (apart the parentheses) because when it is encountered all postponed operations are executed in order to finalize its *first* operand; only a new comma or a closing parenthesis or the end of the expression will finalize its *second* operand. <sup>13</sup> It is not apt to describe the opening parenthesis as an operator, but the closing parenthesis is more closely like a postfix unary operator. It has lowest precedence because when it is encountered all postponed operations are executed to finalize its operand. The start of this operand was decided by the opening parenthesis.



New with  
1.2

**qint** skips the token by token parsing of the input. The ending parenthesis must be physically present rather than arising from expansion. The **q** stands for ``quick''. This ``function'' handles the input exactly like do the **i** macros of *xintcore*, via `\xintiNum`. Hence leading signs and the leading zeroes (coming next) will be handled appropriately but spaces will not be systematically stripped. They should cause no harm and will be removed as soon as the number is used with one of the basic operators. This input mode *does not accept decimal part or scientific part*.

```
\def\x{...many many many ... digits}\def\y{...also many many many digits...}
\xinttheiexpr qint(\x)*qint(\y)+qint(\y)^2\relax
```

Testing showed that using `qint()` starts getting advantageous for inputs having more (or *f-expanding* to more) than circa 20 explicit digits. But for hundreds of digits the input gain becomes a negligible proportion of (for example) the cost of a multiplication.

New with  
1.2

**qfrac** does the same as **qint** excepts that it accepts fractions, decimal numbers, scientific numbers as they are understood by the macros of package *xintfrac*. Not to be used within an `\xintiexpr` r-ession, except if hidden inside functions such as **round** or **trunc** which produce integers from fractions.

**qfloat** does the same as **qfrac** and converts to a float with the precision given by the setting of `\xintDigits`.

**reduce** reduces a fraction to smallest terms

```
\xinttheexpr reduce(50!/20!/20!/10!)\relax
```

```
1415997888807961859400
```

Recall that this is NOT done automatically, for example when adding fractions.

**abs** absolute value

**sgn** sign

**frac** fractional part (it has nothing to do with `qfrac`... sigh...)

```
\xinttheexpr frac(-355/113), frac(-1129.218921791279)\relax
```

```
-16/113, -218921791279[-12]
```

**floor** floor function.

**ceil** ceil function.

**sqr** square.

**sqrt** in `\xintiexpr`, truncated square root; in `\xintexpr` or `\xintfloatexpr` this is the floating point square root, and there is an optional second argument for the precision.

**sqrttr** in `\xintiexpr` only, rounded square root.

**factorial** factorial function (like the post-fix `!` operator.) When used in `\xintexpr` or `\xintfloatexpr` there is an optional second argument. See discussion later.

**?(x)** is the truth value, 1 if non zero, 0 if zero. Must use parentheses.

**!(x)** is logical not, 0 if non zero, 1 if zero. Must use parentheses.

**not** logical not.

**even** evenness of the truncation.

```
\xinttheexpr seq((x,even(x)), x=-5/2..[1/3]..+5/2)\relax
```

```
-5/2, 1, -13/6, 1, -11/6, 0, -9/6, 0, -7/6, 0, -5/6, 1, -3/6, 1, -1/6, 1, 1/6, 1, 3/6, 1, 5/6,
1, 7/6, 0, 9/6, 0, 11/6, 0, 13/6, 1, 15/6, 1
```

**odd** oddness of the truncation.

```
\xinttheexpr seq((x,odd(x)), x=-5/2..[1/3]..+5/2)\relax
```

-5/2, 0, -13/6, 0, -11/6, 1, -9/6, 1, -7/6, 1, -5/6, 0, -3/6, 0, -1/6, 0, 1/6, 0, 3/6, 0, 5/6, 0, 7/6, 1, 9/6, 1, 11/6, 1, 13/6, 0, 15/6, 0

### functions with an alphabetical argument:

**bool** `bool(name)` returns 1 if the  $\TeX$  conditional `\ifname` would act as `\iftrue` and 0 otherwise. This works with conditionals defined by `\newif` (in  $\TeX$  or  $\LaTeX$ ) or with primitive conditionals such as `\ifmmode`. For example:

```
\xintifboolexpr{25*4-if(mmode),100,75}{YES}{NO}
```

will return NO if executed in math mode (the computation is then  $100 - 100 = 0$ ) and YES if not (the `if` conditional is described below; the `\xintifboolexpr` test automatically encapsulates its first argument in an `\xintexpr` and follows the first branch if the result is non-zero (see subsection 11.14)).

The alternative syntax `25*4-\ifmmode100\else75\fi` could have been used here, the usefulness of `bool(name)` lies in the availability in the `\xintexpr` syntax of the logic operators of conjunction `&`, inclusive disjunction `||`, negation `!` (or `not`), of the multi-operands functions `all`, `any`, `xor`, of the two branching operators `if` and `ifsgn` (see also `?` and `??`), which allow arbitrarily complicated combinations of various `bool(name)`.

**togl** Similarly `togl(name)` returns 1 if the  $\LaTeX$  package `etoolbox`<sup>14</sup> has been used to define a toggle named `name`, and this toggle is currently set to `true`. Using `togl` in an `\xintexpr... \relax` without having loaded `etoolbox` will result in an error from `\iftoggle` being a non-defined macro. If `etoolbox` is loaded but `togl` is used on a name not recognized by `etoolbox` the error message will be of the type `ERROR: Missing \endcsname inserted.`, with further information saying that `\protect` should have not been encountered (this `\protect` comes from the expansion of the non-expandable `etoolbox` error message).

When `bool` or `togl` is encountered by the `\xintexpr` parser, the argument enclosed in a parenthesis pair is expanded as usual from left to right, token by token, until the closing parenthesis is found, but everything is taken literally, no computations are performed. For example `togl(2+3)` will test the value of a toggle declared to `etoolbox` with name `2+3`, and not `5`. Spaces are gobbled in this process. It is impossible to use `togl` on such names containing spaces, but `\iftoggle{name with spaces}{1}{0}` will work, naturally, as its expansion will pre-empt the `\xintexpr` scanner.

There isn't in `\xintexpr...` a `test` function available analogous to the `test{\ifsometest}` construct from the `etoolbox` package; but any expandable `\ifsometest` can be inserted directly in an `\xintexpr`-ession as `\ifsometest10` (or `\ifsometest{1}{0}`), for example `if(\ifsometest{1}{0},YES,NO)` (see the `if` operator below) works.

A straight `\ifsometest{YES}{NO}` would do the same more efficiently, the point of `\ifsometest10` is to allow arbitrary boolean combinations using the (described later) `&` and `|` logic operators: `\ifsometest10 & \ifsomeothertest10 | \ifsomeotherthertest10`, etc... `YES` or `NO` above stand for material compatible with the `\xintexpr` parser syntax.

See also `\xintifboolexpr`, in this context.

### functions with one mandatory and a second but optional argument:

**round** For example `round(-2^9/3^5,12)=-2.106995884774.`

**trunc** For example `trunc(-2^9/3^5,12)=-2.106995884773.`

**float** For example `float(-2^9/3^5,12)=-210699588477[-11].`

**sqrt** in `\xintexpr... \relax` and `\xintfloatexpr... \relax` it achieves the precision given by the optional second argument. For legacy reasons the `sqrt` function in `\xintiexpr` truncates (to an integer), whereas `sqrt` in `\xintfloatexpr... \relax` (and in `\xintexpr... \relax` which borrows it) rounds (in the sense of floating numbers). There is `sqrttr` in `\xintiexpr` for rounding to nearest integer.

```
\xinttheexpr sqrt(2,31)\relax\ and \xinttheiexpr sqrt(num(2e60))\relax
```

<sup>14</sup> <http://www.ctan.org/pkg/etoolbox>

## 2 The syntax of *xintexpr* expressions

1414213562373095048801688724210[-30] and 1414213562373095048801688724209

New with  
1.2f

**factorial** when the second optional argument is made use of inside `\xintexpr...\relax`, this switches to the use of the float version, rather than the exact one.

```
\xinttheexpr factorial (100,32)\relax, {\xintDigits:=32;\xintthefloatexpr
factorial (100)\relax}\newline
\xinttheexpr factorial (50)\relax\newline
\xinttheexpr factorial (50, 32)\relax
```

93326215443944152681699238856267[126], 9.3326215443944152681699238856267e157  
30414093201713378043612608166064768844377641568960512000000000000  
30414093201713378043612608166065[33]

**functions with two arguments:**

**quo** first truncates the arguments then computes the Euclidean quotient.

**rem** first truncates the arguments then computes the Euclidean remainder.

**mod** computes the modulo associated to the truncated division, same as `/:` infix operator.

```
\xinttheexpr mod(11/7,1/13), reduce(((11/7)/(1/13))*1/13+mod(11/7,1/13)),
mod(11/7,1/13)- (11/7):(1/13), (11/7)/(1/13)\relax
```

3/91, 11/7, 0, 20

New with  
1.2f  
Changed  
(1.2h)

**binomial** computes binomial coefficients. For some obscure reason the initial version rather than returning zero for `binomial(x,y)` with `y<0` or `x<y` deliberately raised an out-of-range error. This has been fixed in 1.2h. An error is raised only for `x<0` (or if `x>99999999`.)

```
\xinttheexpr seq(binomial(20, i), i=0..20)\relax
```

1, 20, 190, 1140, 4845, 15504, 38760, 77520, 125970, 167960, 184756, 167960, 125970, 77520,  
38760, 15504, 4845, 1140, 190, 20, 1

```
\printnumber{\xintthefloatexpr seq(binomial(100, 50+i), i=-5..+5)\relax}%
```

6.144847121413618e28, 7.347099819081500e28, 8.441348728306404e28, 9.320655887504988e28, 9.2  
891308288780803e28, 1.008913445455642e29, 9.891308288780803e28, 9.320655887504988e28, 8.44  
1348728306404e28, 7.347099819081500e28, 6.144847121413618e28

The arguments must be (expand to) short integers.

New with  
1.2f

**pfactorial** computes partial factorials i.e. `pfactorial(a,b)` evaluates the product `(a+1)...b`.

```
\xinttheexpr seq(pfactorial(20, i), i=20..30)\relax
```

1, 21, 462, 10626, 255024, 6375600, 165765600, 4475671200, 125318793600, 3634245014400,  
109027350432000

Changed  
(1.2h)

The arguments must (expand to) short integers. See [subsection 9.44](#) for the behaviour if the arguments are negative.

**if** (twofold-way conditional)

`if(cond,yes,no)` checks if `cond` is true or false and takes the corresponding branch. Any non zero number or fraction is logical true. The zero value is logical false. Both ``branches'' are evaluated (they are not really branches but just numbers). See also the `?` operator.

**ifsgn** (threefold-way conditional)

`ifsgn(cond,<0,=0,>0)` checks the sign of `cond` and proceeds correspondingly. All three are evaluated. See also the `??` operator.

**functions with an arbitrary number of arguments:**

This argument may well be generated by one or many `a..b` or `a..[d]..b` constructs, separated by commas.

**all** inserts a logical **AND** in between arguments and evaluates,

**any** inserts a logical **OR** in between all arguments and evaluates,

**xor** inserts a logical **XOR** in between all arguments and evaluates,

**`+`** adds (left ticks mandatory):

```
\xinttheexpr `+(1,3,19), `(1*2,3*4,19*20)\relax
23, 394
```

**`\*`** multiplies (left ticks mandatory):

```
\xinttheexpr `(1,3,19), `(1^2,3^2,19^2), `(1*2,3*4,19*20)\relax
57, 3249, 9120
```

**max** maximum,

**min** minimum,

**gcd** first truncates to integers then computes the **GCD**, requires **xintgcd**,

**lcm** first truncates to integers then computes the **LCM**, requires **xintgcd**,

**first** first among comma separated items, **first(list)** is like **[list][:1]**.

```
\xinttheiexpr first(-7..3), [-7..3][:1]\relax
-7, -7
```

**last** last among comma separated items, **last(list)** is like **[list][-1:]**.

```
\xinttheiexpr last(-7..3), [-7..3][-1:]\relax
3, 3
```

**reversed** reverses the order

```
\xinttheiexpr reversed(123..150)\relax
150, 149, 148, 147, 146, 145, 144, 143, 142, 141, 140, 139, 138, 137, 136, 135, 134, 133, 132,
131, 130, 129, 128, 127, 126, 125, 124, 123
```

New with  
1.2g

**len** computes the number of items in a comma separated list. Earlier syntax was **[a,b,...,z][0]** but since 1.2g this now returns the first element of the list.

```
\xinttheiexpr len(1..50, 101..150, 1001..1050)\relax
150
```

**functions requiring dummy variables:**

The ``functions`` **add**, **mul**, **seq**, **subs**, **rseq**, **iter**, **rrseq**, **iterr** use delimited macros to identify the `<letter>=` part.<sup>15</sup> This is done in a way allowing nesting via correctly balanced parentheses. The `<letter>` must not have been assigned a value before via `\xintdefvar`.

This `<letter>=` must be visible when the parser has finished absorbing the function name and the opening parenthesis. For **rseq**, **iter**, **rrseq** and **iterr** this is delayed to after the parser has assimilated a starting part delimited by a semi-colon; this mandatory segment may be generated entirely by expansion and the `<letter>=` may appear during this expansion.

After `<letter>=`, the expansion and parsing will generate a list of values (for example from an **a..b** specification, there may be multiple ones themselves separated by commas). After this step is complete the parser will know the values which will be assigned to `<letter>`, with **i++** syntax offering a special variant.

**seq**, **rseq**, **iter**, **rrseq**, **iterr** but not **add**, **mul**, **subs** admit the **omit**, **abort**, and **break(..)** keywords. In the case of a potentially infinite list generated by a `<letter>++` expression, use of **abort** or **break()** is mandatory, naturally.

Dummy variables are necessarily single-character letters, and all lowercase and uppercase Latin letters are pre-configured for that usage.

**subs** for variable substitution

<sup>15</sup> In the current implementation any token can be used rather than a `=`. What is looked for is a comma followed by two tokens, the first one will be the `<letter>`.

## 2 The syntax of *xintexpr* expressions

```
\xinttheexpr subs(subs(seq(x*z,x=1..10),z=y^2),y=10)\relax\newline
```

100, 200, 300, 400, 500, 600, 700, 800, 900, 1000

Attention that `xz` generates an error, one must use explicitly `x*z`, else the parser expects a variable with name `xz`.

This is useful for example when defining macros for which some argument #1 will be used more than once but may itself be a complicated expression or macro, and should be evaluated only once, for matters of efficiency.

The substituted variable may be a comma separated list (this is impossible with `seq` which will always pick one item after the other from a list).

```
\xinttheexpr subs([x]^2,x=-123,17,32)\relax
```

15129, 289, 1024

See the examples related to the `3x3` determinant in the [subsection 11.6](#) for an illustration of list substitution.

**add** addition

```
\xinttheiexpr add(x^3,x=1..50), add(x(x+1), x=1,3,19)\relax\newline
```

1625625, 394

See ``+`` for syntax without a dummy variable.

**mul** multiplication

```
\xinttheiexpr mul(x^2, x=1,3,19), mul(2n+1,n=1..10)\relax\newline
```

3249, 13749310575

See ``*`` for syntax without a dummy variable.

**seq** comma separated values generated according to a formula

```
\xinttheiexpr seq(x(x+1)(x+2)(x+3),x=1..10), `` (seq(3x+2,x=1..10))\relax
```

24, 120, 360, 840, 1680, 3024, 5040, 7920, 11880, 17160, 1162274713600

```
\xinttheiexpr seq(seq(i^2+j^2, i=0..j), j=0..10)\relax
```

0, 1, 2, 4, 5, 8, 9, 10, 13, 18, 16, 17, 20, 25, 32, 25, 26, 29, 34, 41, 50, 36, 37, 40, 45, 52, 61, 72, 49, 50, 53, 58, 65, 74, 85, 98, 64, 65, 68, 73, 80, 89, 100, 113, 128, 81, 82, 85, 90, 97, 106, 117, 130, 145, 162, 100, 101, 104, 109, 116, 125, 136, 149, 164, 181, 200

**rseq** recursive sequence, @ for the previous value.

```
\printnumber {\xintthefloatexpr subs(rseq (1; @/2+y/2@, i=1..10),y=1000)\relax }\newline
```

1.0000000000000000, 500.50000000000000, 251.2490009990010, 127.6145581634591, 67.725327360822, 604, 41.24542607499115, 32.74526934448864, 31.64201586865079, 31.62278245070105, 31.62277660168434, 31.62277660168379



Attention: in the example above `y/2@` is interpreted as `y/(2*@)`. With versions 1.2c or earlier it would have been interpreted as `(y/2)*@`.

In case the initial stretch is a comma separated list, @ refers at the first iteration to the whole list. Use parentheses at each iteration to maintain this ``nuple''. For example:

```
\printnumber{\xintthefloatexpr rseq(1,10^6;
 (sqrt([@][0]*[@][1]),([@][0]+[@][1])/2), i=1..7)\relax }
```

1.0000000000000000, 1.0000000000000000e6, 1000.0000000000000, 500000.5000000000, 22360.6909552, 33499, 250500.2500000000, 74842.22521066670, 136430.4704776675, 101048.3052657827, 105636.32, 478441671, 103316.8617608946, 103342.3265549749, 103329.5933734841, 103329.5941579348, 103329.5937657094, 103329.5937657095

Changed  
(1.2g)

**iter** is exactly like **rseq**, except that it only prints the last iteration. Strangely it was lacking from 1.1 release, or rather what was available from 1.1 to 1.2f is what is called now **iterr** (described below).

The new **iter** is convenient to handle compactly higher order iterations. We can illustrate its use with an expandable (!) implementation of the Brent-Salamin algorithm for the computation of  $\pi$ :

```
\xintDigits:= 91;
\xintdeffloatfunc BS(a, b, t, p):= (a+b)/2, sqrt(a*b), t-p(a-b)^2, \xintiexpr 2p\relax;
\xintthefloatexpr [88] % use 3 guard digits (output value is *rounded*)
 iter(1, 1/sqrt(2), 1, 1; % initial values
 ([@][0]-[@][1]<2[-45])? % if a-b is small enough stop iterating and ...
 {break(([@][0]+[@][1])^2/[@][2])} % ... do final computation,
 {BS(@)}, % else do iteration via pre-defined (for convenience) function BS.
 i=1++) % This generates infinite iteration. The i is not used.
\relax
\xintDigits:=16;%
```

3.141592653589793238462643383279502884197169399375105820974944592307816406286208998628035  
You can try with **\xintDigits:=1001;** and **2[-501]** in place of **\xintDigits:=91;** and **2[-45]**, but don't make a final rounding to only 88 digits of course ... and better wrap the whole thing in **\message** or **\immediate\write128** because it will run in the right margin (about 7s on my laptop last time I tried).

**rrseq** recursive sequence with multiple initial terms. Say, there are **K** of them. Then **@1**, ..., **@4** and then **@@(n)** up to **n=K** refer to the last **K** values. Notice the difference with **rseq** for which **@** refers to the complete list of all initial terms if there are more than one and may thus be a ``list'' object. This is impossible with **rrseq**. This construct is effective for scalar finite order recursions, and may be perhaps a bit more efficient than using the **rseq** syntax with a ``list'' value.

```
\xinttheiexpr rrseq(0,1; @1+@2, i=2..30)\relax
```

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040

```
\xinttheiexpr rseq(1; 2@, i=1..10)\relax
```

1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024

```
\xinttheiexpr rseq(1; 2@+1, i=1..10)\relax
```

1, 3, 7, 15, 31, 63, 127, 255, 511, 1023, 2047

```
\xinttheiexpr rseq(2; @(@+1)/2, i=1..5)\relax
```

2, 3, 6, 21, 231, 26796

```
\xinttheiexpr rrseq(0,1,2,3,4,5; @1+@2+@3+@4+@@(5)+@@(6), i=1..20)\relax
```

0, 1, 2, 3, 4, 5, 15, 30, 59, 116, 229, 454, 903, 1791, 3552, 7045, 13974, 27719, 54984, 109065, 216339, 429126, 851207, 1688440, 3349161, 6643338

I implemented an **Rseq** which at all times keeps the memory of *all* previous items, but decided to drop it as the package was becoming big.

**iterr** same as **rrseq** but does not print any value until the last **K**.

```
\xinttheiexpr iterr(0,1; @1+@2, i=2..5, 6..10)\relax
% the iterated over list is allowed to have disjoint defining parts.
```

34, 55

Recursions may be nested, with **@@@(n)** giving access to the values of the outer recursion. ... and there is even **@@@@(n)** to access the outer outer recursion but I never tried it!

With **seq**, **rseq**, **iter**, **rrseq**, **iterr**, but not with **subs**, **add**, **mul**, one has:

**abort** stop here and now.

**omit** omit this value.

**break** `break(stuff)` to abort and have `stuff` as last value.

`n++` serves to generate a potentially infinite list. The `n++` construct in conjunction with an `abort` or `break` is often more efficient, because in other cases the list to iterate over is first completely constructed.

```
\xinttheiexpr iter(1; (@>10^40)?{break(@)}{2@}, i=1++)\relax
```

10889035741470030830827987437816582766592 is the smallest power of 2 with at least forty one digits.

Note that `n++` can not work in the format `i=10,17,30++`, only `<start>++` nothing before.

```
First Fibonacci number at least |2^31| and its index
% we use iterr to refer via @1 and @2 to the previous and previous to previous.
\xinttheiexpr iterr(0,1; (@1>=2^31)?{break(i)}{@2+@1}, i=1++)\relax
```

First Fibonacci number at least  $2^{31}$  and its index 2971215073, 47

Some additional examples are to be found in [subsection 2.4](#).

## 2.3 Tacit multiplication

Tacit multiplication (insertion of a `*`) applies when the parser is currently either scanning the digits of a number (or its decimal part or scientific part, or hexadecimal input), or is looking for an infix operator, and: (1.) encounters a count or *dimen* or *skip register* or variable or an  $\varepsilon$ -TeX expression, or (2.) encounters a sub-`\xintexpression`, or (3.) encounters an opening parenthesis, or (4.) encounters a letter (which is interpreted as signaling the start of either a variable or a function name).

For example, if `x`, `y`, `z` are variables all three of `(x+y)z`, `x(y+z)`, `(x+y)(x+z)` will create a tacit multiplication.

Furthermore starting with release 1.2e, whenever tacit multiplication is applied, in all cases it always ``ties'' more than normal multiplication or division, but still less than power. Thus `x/2y` is interpreted as `x/(2y)` and similarly for `x/2max(3,5)` but `x^2y` is still interpreted as `(x^2)*y` and `2n!` as `2*n!`.

```
\xintdefvar x:=30;\xintdefvar y:=5;%
\xinttheexpr (x+y)x, x/2y, x^2y, x!, 2x!, x/2max(x,y)\relax
```

1050, 30/10, 4500, 265252859812191058636308480000000, 530505719624382117272616960000000, 30/60

The ``tie more'' rule applies to all cases of tacit multiplication. It impacts only situations when a division was the last seen operator, as the normal rule for the `xintexpr` parsers is left-associativity in case of equal precedence.

```
\xinttheexpr (1+2)/(3+4)(5+6), 2/x(10), 2/10x, 3/y\xintiexpr 5+6\relax, 1/x(y)\relax
```

3/77, 2/300, 2/300, 3/55, 1/150

Note that `y\xinttheiexpr 5+6\relax` would have tried to use a variable with name `y11` rather than doing `y*11`: tacit multiplication works only in front of sub-`\xintexpressions`, not in front of `\xinttheexpressions` which are unlocked into explicit digits.

Here is an expression whose meaning is completely modified by the ``tie more'' property of tacit multiplication:

```
\xintdeffunc e(z):=1+z(1+z/2(1+z/3(1+z/4)))
```

will be parsed as `1+z*(1+z/(2*(1+z/(3*(1+z/4))))` which is not at all like the presumably hoped:

```
\xintdeffunc e(z):=1+z(1+z/2*(1+z/3*(1+z/4)))
```

This form can also be used, alternatively:

Changed →  
👉



```
\xintdeffunc e(z):=((z/4+1)z/3+1)z/2+1)z+1;
```

Attention! tacit multiplication before an opening parenthesis applies always, but tacit multiplication after a closing parenthesis *does not* apply in front of digits:  $(1+1)5$  is not legal. But  $\text{subs}((1+1)x, x=5)$  is, because in that case a variable is following the closing parenthesis.

## 2.4 More examples with dummy variables

These examples were first added to this manual at the time of the 1.1 release (2014/10/29).

Prime numbers are always cool

```
\xinttheiexpr seq((seq((subs((x/:m)?{(m*m>x)?{1}{0}}{-1},m=2n+1))
??{break(0)}{omit}{break(1)},n=1++))){x}{omit},
x=10001..[2]..10200)\relax
```

Prime numbers are always cool 10007, 10009, 10037, 10039, 10061, 10067, 10069, 10079, 10091, 10093, 10099, 10103, 10111, 10133, 10139, 10141, 10151, 10159, 10163, 10169, 10177, 10181, 10193

The syntax in this last example may look a bit involved (... and it is so I admit). First  $x/:m$  computes  $x$  modulo  $m$  (this is the modulo with respect to truncated division, which here for positive arguments is like Euclidean division; in *xintexpr*...*\relax*,  $a/:b$  is such that  $a = b \cdot (a/b) + a/:b$ , with  $a/b$  the algebraic quotient  $a/b$  truncated to an integer.). The  $(x)?\{yes\}\{no\}$  construct checks if  $x$  (which *must* be within parentheses) is true or false, i.e. non zero or zero. It then executes either the *yes* or the *no* branch, the non chosen branch is not evaluated. Thus if  $m$  divides  $x$  we are in the second ('false') branch. This gives a -1. This -1 is the argument to a  $??$  branch which is of the type  $(y)?\{y<0\}\{y=0\}\{y>0\}$ , thus here the  $y<0$ , i.e.,  $\text{break}(0)$  is chosen. This 0 is thus given to another  $?$  which consequently chooses *omit*, hence the number is not kept in the list. The numbers which survive are the prime numbers.

The first Fibonacci number beyond  $|2^{64}|$  bound is

```
\xinttheiexpr subs(iterr(0,1;(@1>N)?{break(i)}{@1+@2},i=1++),N=2^64)\relax{}
```

and the previous number was its index.

The first Fibonacci number beyond  $2^{64}$  bound is 19740274219868223167, 94 and the previous number was its index.

One more recursion:

```
\def\syr #1{\xinttheiexpr rseq(#1; (@<=1)?{break(i)}{odd(@)?{3@+1}{@//2}},i=0++)\relax}
The 3x+1 problem: \syr{231}\par
```

The 3x+1 problem: 231, 694, 347, 1042, 521, 1564, 782, 391, 1174, 587, 1762, 881, 2644, 1322, 661, 1984, 992, 496, 248, 124, 62, 31, 94, 47, 142, 71, 214, 107, 322, 161, 484, 242, 121, 364, 182, 91, 274, 137, 412, 206, 103, 310, 155, 466, 233, 700, 350, 175, 526, 263, 790, 395, 1186, 593, 1780, 890, 445, 1336, 668, 334, 167, 502, 251, 754, 377, 1132, 566, 283, 850, 425, 1276, 638, 319, 958, 479, 1438, 719, 2158, 1079, 3238, 1619, 4858, 2429, 7288, 3644, 1822, 911, 2734, 1367, 4102, 2051, 6154, 3077, 9232, 4616, 2308, 1154, 577, 1732, 866, 433, 1300, 650, 325, 976, 488, 244, 122, 61, 184, 92, 46, 23, 70, 35, 106, 53, 160, 80, 40, 20, 10, 5, 16, 8, 4, 2, 1, 127

OK, a final one:

```
\def\syrMax #1{\xinttheiexpr iterr(#1,#1;even(i)?
{(@2<=1)?{break(i/2)}{odd(@2)?{3@2+1}{@2//2}}
{(@1>@2)?{@1}{@2}},i=0++)\relax }
```

With initial value 1161, the maximal number attained is  $\text{syrMax}\{1161\}$  and that latter number is the number of steps which was needed to reach 1.\par

With initial value 1161, the maximal number attained is 190996, 181 and that latter number is the number of steps which was needed to reach 1.

Well, one more (but recall that *gcd* is already available as a multi-argument function if *xintgcd* is loaded):

```
\newcommand\GCD [2]{\xinttheiexpr rseq(#1,#2; (@1=0)?{abort}{@2/:@1}, i=1++)\relax }
\GCD {13^10*17^5*29^5}{2^5*3^6*17^2}
```

4014838863509162883616357, 6741792, 3367717, 6358, 4335, 2023, 289, 0

Look at the [Brent-Salamin algorithm implementation](#) for a more interesting recursion.

## 2.5 User defined variables

Since release 1.1 it is possible to make an assignment to a variable name and let it be known to the parsers of *xintexpr*.

```
\xintdefvar Pi:=3.141592653589793238462643;
\xintthefloatexpr Pi^100\relax
\xintdefvar x_1 := 10;\xintdefvar x_2 := 20;\xintdefvar y@3 := 30;
\quad $x_1\cdot x_2\cdot y@3+1=\xinttheiexpr x_1*x_2*y@3+1\relax$.
```

5.187848314319574e49  $x_1 \cdot x_2 \cdot y_{@3} + 1 = 6001.$

Legal variable names are composed of letters, digits, @ and \_ signs. They can not start with a digit. They may start with @ or \_. Currently @, @1, @2, @3, and @4 are reserved because they have special meanings for use in iterations. The @@, @@@, @@@@ are also reserved but are technically functions, not variables. Thus a user may possibly use @@ as a variable name, but if it is followed by parentheses, the function interpretation will be applied, rather than the variable interpretation followed by a tacit multiplication.

$x_{1x}$  is a licit variable name, as well as  $x_{1x_}$  and  $x_{1x_2}$  and  $x_{1x_2y}$  etc... hence we can not rely on tacit multiplication being applied to something like  $x_{1x_2}$ ; the parser goes not go to the effort of tracing back its steps. Hence in such cases we have to insert explicit \* infix operators (one often falls into this trap when playing with variables and counting too much on the divinary talents of *xintexpr*...).

Single letter names *a..z* and *A..Z* are pre-declared by the package for use as special type of variables called ``dummy variables''. It is allowed to overwrite their original meanings and assign them values.

The assignments are done with *\xintdefvar*, *\xintdefiivar*, or with *\xintdeffloatvar*. The variable will be computed using respectively *\xintexpr*, *\xintiexpr* or *\xintfloatexpr*. Once defined, it can be used in the other parsers, except naturally that in *\xintiexpr* only integers are accepted.

When defining a variable with *\xintdeffloatvar*, it is important that reduction to *\xinttheDigits* digits of precision happens inside *\xintfloatexpr* only if an operation is executed. Thus, for a variable declaration with no operations, the value is recorded with all its digits.

```
\xintdeffloatvar e:=2.7182818284590452353602874713526624977572470936999595749669676;%
\xinttheexpr e\relax\newline % shows the recorded value
\xintthefloatexpr e\relax\newline % output rounds
\xintthefloatexpr 1+e\relax\newline % the rounding was done by addition (trust me...)
\xintdeffloatvar e:=float(2.7182818284590452353602874713526624977572470936999595749669676);%
\xinttheexpr e\relax\par % use of float forced immediate rounding
```

27182818284590452353602874713526624977572470936999595749669676[-61]

2.718281828459045

3.718281828459045

2718281828459045[-15]

In the next examples we examine the effect of cumulated float operations on rounding errors:

```
\xintdefvar e_1:=add(1/i!, i=0..10);% exact sum
\xintdeffloatvar e_2:=add(1/i!, i=0..10);% float sum
\xintthefloatexpr e_1, e_2\relax\newline
\xintdefvar e_3:=e_1+add(1/i!, i=11..20);% exact sum
\xintdeffloatvar e_4:=e_2+add(1/i!, i=11..20);% float sum
\xintthefloatexpr e_3, e_4\relax\newline
\xintdeffloatvar e:=2.7182818284590452353602874713526624977572470936999595749669676;%
\xintDigits:=24;
\xintthefloatexpr[16] e, e^1000, e^1000000\relax (e rounded to 24 digits first)\newline
\xintDigits:=16;
\xintthefloatexpr e, e^1000, e^1000000\relax (e rounded to 16 digits first)\par
```

2.718281801146384, 2.718281801146385

2.718281828459045, 2.718281828459046

2.718281828459045, 1.970071114017047e434, 3.033215396802088e434294(e rounded to 24 digits first)

2.718281828459045, 1.970071114016876e434, 3.033215396539459e434294(e rounded to 16 digits first)

With `\xintverbosetrue` the values of the assigned variables will be written to the log. For example like this (the line numbers here are artificial):

```
Package xintexpr Info: (on line 2875)
 Variable "e" defined with value 2718281828459045235360287471352662497757247
0936999595749669676[-61].
Package xintexpr Info: (on line 2879)
 Variable "e" defined with value 2718281828459045[-15].
Package xintexpr Info: (on line 2886)
 Variable "e_1" defined with value 9864101/3628800[0].
Package xintexpr Info: (on line 2887)
 Variable "e_2" defined with value 2718281801146385[-15].
Package xintexpr Info: (on line 2889)
 Variable "e_3" defined with value 6613313319248080001/2432902008176640000[0
].
Package xintexpr Info: (on line 2890)
 Variable "e_4" defined with value 2718281828459046[-15].
Package xintexpr Info: (on line 2892)
 Variable "e" defined with value 2718281828459045235360287471352662497757247
0936999595749669676[-61].
```

Variable declarations are local. One can not really ‘unassign’ a declared variable, but `\xintunassignvar` redefines it to insert a zero and raise a TeX ‘undefined macro’ error.

Also, using `\xintunassignvar` on a letter will let it recover fully its original meaning as dummy variable.

```
\xintFor #1 in {e_1, e_2, e_3, e_4, e} \do {\xintunassignvar {#1}}
```

## 2.6 User defined functions

### 2.6.1 `\xintdeffunc`

Since release 1.2c it is possible to declare functions:

```
\xintdeffunc
 Rump(x,y):=1335 y^6/4 + x^2 (11 x^2 y^2 - y^6 - 121 y^4 - 2) + 11 y^8/2 + x/2y;
```

(notice the numerous tacit multiplications in this expression; and that `x/2y` is interpreted as `x/(2y)`.)

The (dummy) variables used in the function declaration are necessarily single letters (lowercase or uppercase) which have not been re-declared via `\xintdefvar` as assigned variables. The choice of the letters is entirely up to the user and has nil influence on the actual function, naturally.

A function can have at most nine variables. <sup>16</sup>

A function must be defined for a specific parser, using either `\xintdeffunc`, `\xintdefiifunc` or `\xintdeffloatfunc`.

Let's try the famous RUMP test:

```
\xinttheexpr Rump(77617,33096)\relax.
```

**-54767/66192.** Nothing problematic for an exact evaluation, naturally !

A function may be declared either via `\xintdeffunc`, `\xintdefiifunc`, `\xintdeffloatfunc`. It will then be known *only* to the parser which was used for its definition.

Thus to test the RUMP polynomial (it is not quite a polynomial with its `x/2y` final term) with floats, we *must* also declare `Rump` as a function to be used there:

```
\xintdeffloatfunc
 Rump(x,y):=333.75 y^6 + x^2 (11 x^2 y^2 - y^6 - 121 y^4 - 2) + 5.5 y^8 + x/2y;
```

<sup>16</sup> with the current syntax, the `;` as used for `iterr`, `rseq`, `rrseq` must be hidden as `{;}` to not be confused with the `;` ending the declaration.

## 2 The syntax of *xintexpr* expressions

(I used coefficients 333.75 and 5.5 rather than fractions only because this is how I saw the polynomial defined in one computer class reference found on internet; and for float operations this may matter on the rounding).

The numbers are scanned with the current precision, hence as here it is 16, they are scanned exactly in this case. We can then vary the precision for the evaluation.

```
\def\CR{\cr}
\halign
{\tabskiplex
\hfil\bfseries#&\xintDigits:=\xintloopindex;\xintthefloatexpr Rump(77617,33096)#\cr
\xintloop [8+1]
\xintloopindex &\relax\CR
\ifnum\xintloopindex<40 \repeat
}
8 7.0000000e29
9 -1.0000000e28
10 5.0000000e27
11 -3.0000000e26
12 4.0000000e25
13 3.0000000e24
14 3.0000000e23
15 -2.0000000e22
16 1.0000000e21
17 -5.0000000e20
18 1.17260394005317863
19 1.0000000000000001e18
20 -9.9999999999999827e16
21 1.0000000000000011726e16
22 3.00000000000001172604e15
23 -9.99999999999827396060e13
24 -1.999999999998273960599e13
25 -1.99999999999827396059947e12
26 1.1726039400531786318588349
27 -5.999999999998273960599468214e10
28 -9.999999999998273960599468213681e8
29 2.0000000117260394005317863186e8
30 1.00000011726039400531786318588e7
31 -999998.8273960599468213681411651
32 200001.17260394005317863185883490
33 -9998.82739605994682136814116509548
34 -1998.827396059946821368141165095480
35 -198.82739605994682136814116509547982
36 21.1726039400531786318588349045201837
37 -0.8273960599468213681411650954798162920
38 -0.82739605994682136814116509547981629200
39 -0.827396059946821368141165095479816292000
40 -0.8273960599468213681411650954798162919990
```

It is licit to overload a variable name (all Latin letters are predefined as dummy variables) with a function name and vice versa. The parsers will decide from the context if the function or variable interpretation must be used (dropping various cases of tacit multiplication as normally applied).

```
\xintdefiifunc f(x):=x^3;
\xinttheiexpr add(f(f),f=100..120)\relax\newline
\xintdeffunc f(x,y):=x^2+y^2;
\xinttheexpr mul(f(f(f,f),f(f,f)),f=1..10)\relax
```

28205100

186188134867578885427848806400000000

The mechanism for functions is identical with the one underlying the `\xintNewExpr` macro. A function once declared is a first class citizen, its expression is entirely parsed and converted into a big nested `f-expandable` macro. When used its action is via this defined macro. For example

```
\xintdeffunc
```

$$e(z) := (((((((((z/10+1)z/9+1)z/8+1)z/7+1)z/6+1)z/5+1)z/4+1)z/3+1)z/2+1)z+1;$$

creates a macro whose meaning one can find in the log file, after `\xintverbosettrue`. Here it is:

```
Function e for \xintexpr parser associated to \XINT_expr_userfunc_e with me
aning macro:#1,->\xintAdd {\xintMul {\xintAdd {\xintDiv {\xintMul {\xintAdd {\x
intDiv {\xintMul {\xintAdd {\xintDiv {\xintMul {\xintAdd {\xintDiv {\xintMul {\
xintAdd {\xintDiv {\xintMul {\xintAdd {\xintDiv {\xintMul {\xintAdd {\xintDiv {\
\xintMul {\xintAdd {\xintDiv {\xintMul {\xintAdd {\xintDiv {#1}{10}}{1}}{#1}}{9
{1}}{1}}{#1}}{8}}{1}}{#1}}{7}}{1}}{#1}}{6}}{1}}{#1}}{5}}{1}}{#1}}{4}}{1}}{#1}}{3
{1}}{#1}}{2}}{1}}{#1}}{1}}
```

This has the same limitations as the `\xintNewExpr` macro. The main one is that dummy variables are usable only to the extent that their values are numerical. For example `\xintdefunc f(x):=ad d(i^2,i=1..x)`; is not possible. See [subsubsection 11.6.3](#) and the next subsection.

In this example one could use the alternative syntax with list operations:

```
\xintdefunc f(x):='`([1..x]^2);\xinttheexpr seq(f(x), x=1..20)\relax
```

1, 5, 14, 30, 55, 91, 140, 204, 285, 385, 506, 650, 819, 1015, 1240, 1496, 1785, 2109, 2470, 2870

Side remark: as the `seq(f(x), x=1..10)` does many times the same computations, an `rseq` here would be more efficient:<sup>18</sup>

```
\xinttheexpr rseq(1; (x>20)?{abort}{@+x^2}, x=2++)\relax
```

1, 5, 14, 30, 55, 91, 140, 204, 285, 385, 506, 650, 819, 1015, 1240, 1496, 1785, 2109, 2470, 2870

On the other hand a construct like the following has no issue, as the values iterated over do not depend upon the function parameters:

```
\xintdefunc f(x):=iter(1{;; @*x/i+1, i=10..1);% one must hide the first semi-colon !
```

```
\xinttheexpr e(1), f(1)\relax
```

9864101/3628800, 9864101/3628800

### 2.6.2 \ifxintverbose conditional

With `\xintverbosetrue` the meanings of the functions (or rather their associated macros) will be written to the log. For example the first `Rump` declaration above generates this in the log file:

Function Rump for \xintexpr parser associated to \XINT\_expr\_userfunc\_Rump with meaning macro:#1,#2,->\xintAdd {\xintAdd {\xintAdd {\xintDiv {\xintMul {1335}{\xintPow {#2}{6}}}{4}}{\xintMul {\xintPow {#1}{2}}{\xintSub {\xintSub {\xintSub {\xintMul {11}{\xintMul {\xintPow {#1}{2}}{\xintPow {#2}{2}}}}{\xintPow {#2}{6}}}{\xintMul {121}{\xintPow {#2}{4}}}{2}}}{\xintDiv {\xintMul {11}{\xintPow {#2}{8}}}{2}}}{\xintDiv {#1}{\xintMul {2}{#2}}}

and the declaration `\xintdefunc f(x):=iter(1{;} @*x/i+1, i=10..1);` generates:

```
Function f for \xintexpr parser associated to \XINT_expr_userfunc_f with me
aning macro:#1,->\xintAdd {\xintDiv {\xintMul {\xintAdd {\xintDiv {\xintMul {\x
intAdd {\xintDiv {\xintMul {\xintAdd {\xintDiv {\xintMul {\xintAdd {\xintDiv {\
xintMul {\xintAdd {\xintDiv {\xintMul {\xintAdd {\xintDiv {\xintMul {\xintAdd {\
\xintDiv {\xintMul {\xintAdd {\xintDiv {\xintMul {\xintAdd {\xintDiv {\xintMul
{#1}}{#1}}}{10/1[0]}}{#1}}{9/1[0]}}{#1}}{8/1[0]}}{#1}}{7/1[0]}}{#1}}{6/1[0]
}}{#1}}{5/1[0]}}{#1}}{4/1[0]}}{#1}}{3/1[0]}}{#1}}{2/1[0]}}{#1}}{1/1[0]}}
```

Starting with 1.2d the definitions made by `\xintNewExpr` have local scope, hence this is also the case with the definitions made by `\xintdeffunc`. One can not ``undeclare'' a function, but naturally one can provide a new definition for it.

It is possible to define functions which expand to comma-separated values, for example the declarations:

<sup>17</sup> It turns out ``+(seq(i^2, i=1..x))` would work here, but this isn't always the case with `seq` constructs. <sup>18</sup> Note that `omit` and `abort` are not usable in `add` or `mul` (currently).

```
\xintdeffunc f(x):= x, x^2, x^3, x^x;
\xintdeffunc g(x):= x^[0..x];% x^[1, 2, 3, x] would be like f above.
```

will generate

```
Function f for \xintexpr parser associated to \XINT_expr_userfunc_f with me
aning macro:#1,->#1,\xintPow {#1}{2},\xintPow {#1}{3},\xintPow {#1}{#1}
```

```
Function g for \xintexpr parser associated to \XINT_expr_userfunc_g with me
aning macro:#1,->\xintApply::csv {\xintPow {#1}}{\xintSeq::csv {0}{#1}}
```

and we can check that they work:

```
\xinttheexpr f(10)\relax; \xinttheexpr g(10)\relax
10, 100, 1000, 10000000000; 1, 10, 100, 1000, 10000, 100000, 1000000, 10000000, 100000000,
1000000000, 10000000000
```

N.B.: we declared in this section *e*, *f*, *g* as functions. Except naturally if the function declarations are done in a group or a  $\text{\TeX}$  environment whose scope has ended, they can not be completely undone, and if *e*, *f*, or *g* are used as dummy variables the tacit multiplication in front of parentheses will not be applied, it is their function interpretation which will prevail. However, with an explicit *\** in front of the opening parenthesis, it does work:

```
\xinttheexpr add(f*(f+f), f= 1..10)\relax % f is used as variable, not as a function.
```

770

### 2.6.3 \xintNewFunction

New with  
1.2h

The syntax is analogous to the one of `\xintNewExpr` but achieves something completely different. Here is an example:

```
\xintNewFunction {foo}[3]{add(mul(x+i, i=#1..#2),x=1..#3)}
```

We now have a genuine function `foo( , , )` of three variables which we can use fully in the three parsers, be it with numerical arguments or variables or whatever.

```
\xinttheexpr seq(foo(0, 3, j), j= 1..10)\relax
```

```
24, 144, 504, 1344, 3024, 6048, 11088, 19008, 30888, 48048
```

A notable aspect is that this syntax allows to make recursive definitions, contrarily (currently) to `\xintdeffunc`. See [subsection 5.3](#) for an example.

However this construct is only syntactic sugar to benefit from functional notation. Each time the function `foo` will be encountered the corresponding expression will be inserted as a sub-expression (of the same type as the surrounding one), the macro parameters having been replaced with the (already evaluated) function arguments, and the parser will then parse the expression. It is very much like a macro substitution, but using functional notation.

```
Package xintexpr Info: (on line 3151)
```

```
Function foo for the expression parsers is associated to \XINT_expr_macrofu
nc_foo with meaning macro:#1,#2,#3,->add(mul(x+i, i=\XINT_expr_wrapit {#1}..\XI
NT_expr_wrapit {#2}),x=1..\XINT_expr_wrapit {#3})
```

This is thus very different from a function defined via `\xintdeffunc` which expands to some (possibly very complicated) nesting of various macro calls, which were determined at the time of the function definition. But (see [subsubsection 11.6.3](#)) it is not currently possible to define a `foo` function like the one above via `\xintdeffunc`.

One can declare a function `foo` with `[0]` arguments but it must be used as `foo(nil)`, as `foo()` with no argument would generate an error.

## 2.7 List operations

By *list* we hereby mean simply comma-separated values, for example `3, -7, 1e5`. This section describes some syntax which allows to manipulate such lists, for example `[3, -7, 1e5][1]` extracts `-7` (we follow the Python convention of enumerating starting at zero; see the frame next).

In the context of dummy variables, lists can be used in substitutions:

```
\xinttheiexpr subs(`+(L), L = 1, 3, 5, 7, 9)\relax\newline
```

25

and also the `rseq` and `iter` constructs allow `@` to refer to a list:

```
\xinttheiexpr iter(0, 1; ([@][1], [@][0]+[@][1]), i=1..10)\relax\newline
```

55, 89

where each step constructs a new list with two entries.

However, despite appearances there is not really internally a notion of a *list type* and it is currently impossible to create, manipulate, or return on output a *list of lists*. There is a special reserved variable `nil` which stands for the empty list: for example `len()` is not legal but `len(nil)` works.

The syntax which is explained next includes in particular what are called *list itemwise operators* such as:

```
\xinttheiexpr 37+[13,100,1000]\relax\newline
```

50, 137, 1037

This part of the syntax is considered provisory, for the reason that its presence might make more difficult some extensions in the future. On the other hand the Python-like slicing syntax should not change.



*A backwards incompatible change.*

Up to release 1.2f inclusive, the item accessor `[list][n]` returned the *n*th element of a list. The Python-like slices `[list][a:b]` on the other hand act exactly as in Python where list items are enumerated starting at zero. For example `[list][:5]` or equivalently `[list][0:5]` have the effect to keep only the first five elements. Thus `[list][n]` which returned the *n*th element was akin to `[list][n-1:n]` whereas in Python which enumerates from zero it would be `[list][n:n+1]`.

One reason for that choice was that `[list][0]` allowed access to the length, and there was thus no need to add a new function to the list of recognized keywords.

1.2g does the backwards incompatible change to adhere more fully to Python conventions and now `[list][1]` picks the *second* element of the list and `[list][0]` the *first*. There is `len(list)` for the length.

The reason for the change is that the author has become more accustomed to Python than he was when he first introduced list operations to *xintexpr*, and the difference was becoming distracting.

Changed  
(1.2g)

- `a..b` constructs the **small** integers from the ceil `[a]` to the floor `[b]` (possibly a decreasing sequence): one has to be careful if using this for algorithms that `1..0` for example is not empty or `1` but expands to `1, 0`. Again, `a..b` can not be used with `a` and `b` greater than  $2^{31} - 1$ . Also, only about at most **5000** integers can be generated (this depends upon some `TeX` memory settings).

The `..` has lower precedence than the arithmetic operations.

```
\xinttheexpr 1.5+0.4..2.3+1.1\relax; \xinttheexpr 1.9..3.4\relax; \xinttheexpr 2..3\relax
```

2, 3; 2, 3; 2, 3

- `a..[d]..b` allows to generate big integers, or also fractions, it proceeds with step (non necessarily integral nor positive) `d`. It does not replace `a` by its ceil, nor `b` by its floor. The generated list is empty if `b-a` and `d` are of opposite signs; if `d=0` or if `a=b` the list expands to single element `a`.

```
\xinttheexpr 1.5..[1.01]..11.23\relax
```

15[-1], 251[-2], 352[-2], 453[-2], 554[-2], 655[-2], 756[-2], 857[-2], 958[-2], 1059[-2]

Changed  
(1.2g)

- `[list][n]` extracts the *n+1*th element if `n>=0`. If `n<0` it extracts from the tail. List items are numbered as in Python. `len(list)` computes the number of items of the list.

```
\xinttheiexpr \empty[0..10][6], len(0..10), [0..10][-1], [0..10][23*18-22*19]\relax\
(and 23*18-22*19 has value \the\numexpr 23*18-22*19\relax).
```



6, 11, 10, 7 (and  $23 \cdot 18 - 22 \cdot 19$  has value -4).

See the next frame for why the example above has `\empty` token at start.

As shown, it is perfectly legal to do operations in the index parameter, which will be handled by the parser as everything else. The same remark applies to the next items.

- `[list][:n]` extracts the first `n` elements if `n>0`, or suppresses the last `|n|` elements if `n<0`.  
`\xinttheiexpr [0..10][:6]\relax` and `\xinttheiexpr [0..10][: -6]\relax`  
 0, 1, 2, 3, 4, 5 and 0, 1, 2, 3, 4
- `[list][n:]` suppresses the first `n` elements if `n>0`, or extracts the last `|n|` elements if `n<0`.  
`\xinttheiexpr [0..10][6:]\relax` and `\xinttheiexpr [0..10][ -6:]\relax`  
 6, 7, 8, 9, 10 and 5, 6, 7, 8, 9, 10
- More generally, `[list][a:b]` works according to the Python ``slicing'' rules (inclusive of negative indices). Notice though that there is no optional third argument for the step, which always defaults to +1.  
`\xinttheiexpr [1..20][6:13]\relax = \xinttheiexpr [1..20][6-20:13-20]\relax`  
 7, 8, 9, 10, 11, 12, 13 = 7, 8, 9, 10, 11, 12, 13
- It is naturally possible to nest these things:  
`\xinttheiexpr [[1..50][13:37]][10:-10]\relax`  
 24, 25, 26, 27
- itemwise operations either on the left or the right are possible:  
`\xinttheiexpr 123*[1..10]^2\relax`  
 123, 492, 1107, 1968, 3075, 4428, 6027, 7872, 9963, 12300



List operations are implemented using square brackets, but the `\xintiexpr` and `\xintfloqatexpr` parsers also check to see if an optional parameter within brackets is specified before the start of the expression. To avoid the resulting confusion if this `[` actually serves to delimit comma separated values for list operations, one can either:

- insert something before the bracket such as `\empty` token,

```
\xinttheiexpr \empty [1,3,6,99,100,200][2:4]\relax
```

6, 99

- use parentheses:

```
\xinttheiexpr ([1,3,6,99,100,200][2:4])\relax
```

6, 99

Notice though that `([1,3,6,99,100,200])[2:4]` would not work: it is mandatory for `[[` and `][` not to be interspersed with parentheses. Spaces are perfectly legal:

```
\xinttheiexpr \empty[1..10] [: 7]\relax
```

1, 2, 3, 4, 5, 6, 7

Similarly all the `+[`, `*[`, `...and` `]**`, `]/`, `...` operators admit spaces but nothing else between their constituent characters.

```
\xinttheiexpr \empty [1 . . 1 0] * * 1 1 \relax
```

1, 2048, 177147, 4194304, 48828125, 362797056, 1977326743, 8589934592, 31381059609, 100000000000

In an other vein, the parser will be confused by `1..[a,b,c][1]`, and one must write `1..([a,b],c)[1]`. And things such as `[100,300,500,700][2]//11` or `[100,300,500,700][2]/11` are syntax errors and one must use parentheses, as in `([100,300,500,700][2])/11`.

## 2.8 Analogies and differences of `\xintiexpr` with `\numexpr`

`\xintiexpr...relax` is a parser of expressions knowing only (big) integers. There are, besides the enlarged range of allowable inputs, some important differences of syntax between `\numexpr` and `\xintiexpr` and variants:

- Contrarily to `\numexpr`, the `\xintiexpr` parser will stop expanding only after having encountered (and swallowed) a mandatory `relax` token.
- In particular, spaces between digits (and not only around infix operators or parentheses) do not stop `\xintiexpr`, contrarily to the situation with `numexpr`: `\the\numexpr 7 + 3 5\relax` expands (in one step) to `105\relax`, whereas `\xintthe\xintiexpr 7 + 3 5\relax` expands (in two steps) to `42`.
- Inside an `\edef`, expressions `\xintiexpr...relax` get fully evaluated, but to a private format which needs the prefix `\xintthe` to get printed or used as arguments to some macros; on the other hand expansion of `\numexpr` in an `\edef` occurs only if prefixed with `\the` or `\number` (or `\romannumeral`, or the expression is included in a bigger `\numexpr` which will be the one to have to be prefixed. . . .)
- `\the\numexpr` or `\number\numexpr` expands in one step, but `\xintthe\xintiexpr` needs two steps.
- `\numexpr -(1)\relax` is illegal. But `\xintiexpr -(1)\relax` is perfectly legal and gives the expected result (what else ?).
- `\numexpr 2\cnta\relax` is illegal, with `\cnta` a `\count`. But `\xintiexpr 2\cnta\relax` is perfectly legal and will do the tacit multiplication.

## 2.9 Chaining expressions for expandable algorithmics

We will see in this section how to chain `\xintexpr`-essions with `\expandafter`'s, like it is possible with `\numexpr`. For this it is convenient to use `\romannumeral0\xinteval` which is the once-expanded form of `\xintexpr`, as we can then chain using only one `\expandafter` each time.

For example, here is the code employed on the title page to compute (expandably, of course!) the 1250th Fibonacci number:

```
\catcode`_ 11
\def\Fibonacci #1{% \Fibonacci{N} computes F(N) with F(0)=0, F(1)=1.
 \expandafter\Fibonacci_a\expandafter
 {\the\numexpr #1\expandafter}\expandafter
 {\romannumeral0\xinteval 1\expandafter\relax\expandafter}\expandafter
 {\romannumeral0\xinteval 1\expandafter\relax\expandafter}\expandafter
 {\romannumeral0\xinteval 1\expandafter\relax\expandafter}\expandafter
 {\romannumeral0\xinteval 0\relax}}
%
\def\Fibonacci_a #1{%
 \ifcase #1
 \expandafter\Fibonacci_end_i
 \or
 \expandafter\Fibonacci_end_ii
 \else
 \ifodd #1
 \expandafter\expandafter\expandafter\Fibonacci_b_ii
 \else
```

```

\expandafter\expandafter\expandafter\Fibonacci_b_i
\fi
\fi {#1}%
}% * signs are omitted from the next macros, tacit multiplications
\def\Fibonacci_b_i #1#2#3{\expandafter\Fibonacci_a\expandafter
 {\the\numexpr #1/2\expandafter}\expandafter
 {\romannumeral0\xintiieval sqr(#2)+sqr(#3)\expandafter\relax\expandafter}\expandafter
 {\romannumeral0\xintiieval (2#2-#3)#3\relax}%
}% end of Fibonacci_b_i
\def\Fibonacci_b_ii #1#2#3#4#5{\expandafter\Fibonacci_a\expandafter
 {\the\numexpr (#1-1)/2\expandafter}\expandafter
 {\romannumeral0\xintiieval sqr(#2)+sqr(#3)\expandafter\relax\expandafter}\expandafter
 {\romannumeral0\xintiieval (2#2-#3)#3\expandafter\relax\expandafter}\expandafter
 {\romannumeral0\xintiieval #2#4+#3#5\expandafter\relax\expandafter}\expandafter
 {\romannumeral0\xintiieval #2#5+#3(#4-#5)\relax}%
}% end of Fibonacci_b_ii
%
% code as used on title page:
%\def\Fibonacci_end_i #1#2#3#4#5{\xintthe#5}
%\def\Fibonacci_end_ii #1#2#3#4#5{\xinttheiexpr #2#5+#3(#4-#5)\relax}
%
% new definitions:
\def\Fibonacci_end_i #1#2#3#4#5{{#4}{#5}}% {F(N+1)}{F(N)} in \xintexpr format
\def\Fibonacci_end_ii #1#2#3#4#5%
 {\expandafter
 {\romannumeral0\xintiieval #2#4+#3#5\expandafter\relax
 \expandafter}\expandafter
 {\romannumeral0\xintiieval #2#5+#3(#4-#5)\relax}}% idem.
% \FibonacciN returns F(N) (in encapsulated format: needs \xintthe for printing)
\def\FibonacciN {\expandafter\xint_secondoftwo\romannumeral-\`0\Fibonacci }%
\catcode`_ 8

```

The macro `\Fibonacci` produces not one specific value  $F(N)$  but a pair of successive values  $\{F(N)\}\{F(N+1)\}$  which can then serve as starting point of another routine devoted to compute a whole sequence  $F(N)$ ,  $F(N+1)$ ,  $F(N+2)$ , .... Each of  $F(N)$  and  $F(N+1)$  is kept in the encapsulated internal *xintexpr* format.

`\FibonacciN` produces the single  $F(N)$ . It also keeps it in the private format; thus printing it will need the `\xintthe` prefix.

Here a code snippet which checks the routine via a `\message` of the first 51 Fibonacci numbers (this is not an efficient way to generate a sequence of such numbers, it is only for validating `\FibonacciN`).

```

\def\Fibo #1.{\xintthe\FibonacciN {#1}}%
\message{\xintloop [0+1] \expandafter\Fibo\xintloopindex.,
 \ifnum\xintloopindex<49 \repeat \xintthe\FibonacciN{50}.}

```

The way we use `\expandafter`'s to chain successive `\xintiieval` evaluations is exactly analogous to what is possible with `\numexpr`. The various `\romannumeral0\xintiieval` could very well all have been `\xintieexpr`'s but then we would have needed `\expandafter\expandafter\expandafter` each time.

There is a difference though: `\numexpr` does *NOT* expand inside an `\edef`, and to force its expansion we must prefix it with `\the` or `\number` or `\romannumeral` or another `\numexpr` which is itself prefixed, etc. . . .

But `\xintexpr`, `\xintieexpr`, . . . , expand fully in an `\edef`, with the completely expanded result encapsulated in a private format.

Using `\xintthe` as prefix is necessary to print the result (like `\the` or `\number` in the case of `\numexpr`), but it is not necessary to get the computation done (contrarily to the situation with `\numexpr`).

Our `\Fibonacci` expands completely under *f-expansion*, so we can use `\fdef` rather than `\edef` in a situation such as

```
\fdef \X {\FibonacciN {100}}
```

but it is usually about as efficient to employ `\edef`. And if we want

```
\edef \Y {(\FibonacciN{100}, \FibonacciN{200})},
```

then `\edef` is necessary.

Allright, so let's now give the code to generate  $\{F(N)\}\{F(N+1)\}\{F(N+2)\}\dots$ , using `\Fibonacci` for the first two and then using the standard recursion  $F(N+2)=F(N+1)+F(N)$ :

```
\catcode_ 11
\def\FibonacciSeq #1#2{%#1=starting index, #2>#1=ending index
 \expandafter\Fibonacci_Seq\expandafter
 {\the\numexpr #1\expandafter}\expandafter{\the\numexpr #2-1}%
}%
\def\Fibonacci_Seq #1#2{%
 \expandafter\Fibonacci_Seq_loop\expandafter
 {\the\numexpr #1\expandafter}\romannumeral0\Fibonacci {#1}{#2}%
}%
\def\Fibonacci_Seq_loop #1#2#3#4{% standard Fibonacci recursion
 {#3}\unless\ifnum #1<#4 \Fibonacci_Seq_end\fi
 \expandafter\Fibonacci_Seq_loop\expandafter
 {\the\numexpr #1+1\expandafter}\expandafter
 {\romannumeral0\xintiieval #2+#3\relax}{#2}{#4}%
}%
\def\Fibonacci_Seq_end\fi\expandafter\Fibonacci_Seq_loop\expandafter
 #1\expandafter #2#3#4{\fi {#3}}%
\catcode_ 8
```

This `\FibonacciSeq` macro is completely expandable but it is not *f-expandable*.

This is not a problem in the next example which uses `\xintFor*` as the latter applies repeatedly full expansion to what comes next each time it fetches an item from its list argument. Thus `\xintFor*` still manages to generate the list via iterated full expansion.

```
\newcounter{index}
\tabskip 1ex
\fdef\Fibxxx{\FibonacciN {30}}%
\setcounter{index}{30}%
\ vbox{\halign{\bfseries#.\hfil&\hfil &\hfil #\cr
 \xintFor* #1 in {\FibonacciSeq {30}{59}}\do
 {\theindex &\xintthe#1 &
 \xintiRem{\xintthe#1}{\xintthe\Fibxxx}\stepcounter{index}\cr }}%
}\vrule
\ vbox{\halign{\bfseries#.\hfil&\hfil &\hfil #\cr
 \xintFor* #1 in {\FibonacciSeq {60}{89}}\do
 {\theindex &\xintthe#1 &
 \xintiRem{\xintthe#1}{\xintthe\Fibxxx}\stepcounter{index}\cr }}%
}\vrule
\ vbox{\halign{\bfseries#.\hfil&\hfil &\hfil #\cr
 \xintFor* #1 in {\FibonacciSeq {90}{119}}\do
 {\theindex &\xintthe#1 &
 \xintiRem{\xintthe#1}{\xintthe\Fibxxx}\stepcounter{index}\cr }}%
}%
}
```

This produces the Fibonacci numbers from  $F(30)$  to  $F(119)$ , and computes also all the congruence classes modulo  $F(30)$ . The output has been put in a `float`, which appears on the following page. I leave to the mathematically inclined readers the task to explain the visible patterns. . . ;-).

### 3 The *xint* bundle

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|                  |        |                         |        |                                |        |
|------------------|--------|-------------------------|--------|--------------------------------|--------|
| 30. 832040       | 0      | 60. 1548008755920       | 0      | 90. 2880067194370816120        | 0      |
| 31. 1346269      | 514229 | 61. 2504730781961       | 1      | 91. 4660046610375530309        | 514229 |
| 32. 2178309      | 514229 | 62. 4052739537881       | 1      | 92. 7540113804746346429        | 514229 |
| 33. 3524578      | 196418 | 63. 6557470319842       | 2      | 93. 12200160415121876738       | 196418 |
| 34. 5702887      | 710647 | 64. 10610209857723      | 3      | 94. 19740274219868223167       | 710647 |
| 35. 9227465      | 75025  | 65. 17167680177565      | 5      | 95. 31940434634990099905       | 75025  |
| 36. 14930352     | 785672 | 66. 27777890035288      | 8      | 96. 51680708854858323072       | 785672 |
| 37. 24157817     | 28657  | 67. 44945570212853      | 13     | 97. 83621143489848422977       | 28657  |
| 38. 39088169     | 814329 | 68. 72723460248141      | 21     | 98. 135301852344706746049      | 814329 |
| 39. 63245986     | 10946  | 69. 117669030460994     | 34     | 99. 218922995834555169026      | 10946  |
| 40. 102334155    | 825275 | 70. 190392490709135     | 55     | 100. 354224848179261915075     | 825275 |
| 41. 165580141    | 4181   | 71. 308061521170129     | 89     | 101. 573147844013817084101     | 4181   |
| 42. 267914296    | 829456 | 72. 498454011879264     | 144    | 102. 927372692193078999176     | 829456 |
| 43. 433494437    | 1597   | 73. 806515533049393     | 233    | 103. 1500520536206896083277    | 1597   |
| 44. 701408733    | 831053 | 74. 1304969544928657    | 377    | 104. 2427893228399975082453    | 831053 |
| 45. 1134903170   | 610    | 75. 2111485077978050    | 610    | 105. 3928413764606871165730    | 610    |
| 46. 1836311903   | 831663 | 76. 3416454622906707    | 987    | 106. 6356306993006846248183    | 831663 |
| 47. 2971215073   | 233    | 77. 5527939700884757    | 1597   | 107. 10284720757613717413913   | 233    |
| 48. 4807526976   | 831896 | 78. 8944394323791464    | 2584   | 108. 16641027750620563662096   | 831896 |
| 49. 7778742049   | 89     | 79. 14472334024676221   | 4181   | 109. 26925748508234281076009   | 89     |
| 50. 12586269025  | 831985 | 80. 23416728348467685   | 6765   | 110. 43566776258854844738105   | 831985 |
| 51. 20365011074  | 34     | 81. 37889062373143906   | 10946  | 111. 70492524767089125814114   | 34     |
| 52. 32951280099  | 832019 | 82. 61305790721611591   | 17711  | 112. 114059301025943970552219  | 832019 |
| 53. 53316291173  | 13     | 83. 99194853094755497   | 28657  | 113. 184551825793033096366333  | 13     |
| 54. 86267571272  | 832032 | 84. 160500643816367088  | 46368  | 114. 298611126818977066918552  | 832032 |
| 55. 139583862445 | 5      | 85. 259695496911122585  | 75025  | 115. 483162952612010163284885  | 5      |
| 56. 225851433717 | 832037 | 86. 420196140727489673  | 121393 | 116. 781774079430987230203437  | 832037 |
| 57. 365435296162 | 2      | 87. 679891637638612258  | 196418 | 117. 1264937032042997393488322 | 2      |
| 58. 591286729879 | 832039 | 88. 1100087778366101931 | 317811 | 118. 2046711111473984623691759 | 832039 |
| 59. 956722026041 | 1      | 89. 1779979416004714189 | 514229 | 119. 3311648143516982017180081 | 1      |

Some Fibonacci numbers together with their residues modulo  $F(30)=832040$

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### 3.1 Characteristics

The main characteristics are:

1. exact algebra on ``big numbers'', integers as well as fractions,
2. floating point variants with user-chosen precision,
3. the computational macros are compatible with expansion-only context,
4. the bundle comes with parsers (integer-only, or handling fractions, or doing floating point computations) of infix operations implementing beyond infix operations extra features such as dummy variables.

Since 1.2 ``big numbers'' must have less than about 19950 digits: the maximal number of digits for addition is at 19968 digits, and it is 19959 for multiplication. The reasonable range of use of the package is with numbers of up to a few hundred digits.<sup>19</sup>

$\TeX$  does not know off-hand how to print on the page such very long numbers, see subsection 1.3.

Integers with only 10 digits and starting with a 3 already exceed the  $\TeX$  bound; and  $\TeX$  does not have a native processing of floating point numbers (multiplication by a decimal number of a dimension register is allowed --- this is used for example by the `pgf` basic math engine.)

$\TeX$  elementary operations on numbers are done via the non-expandable `\advance`, `\multiply`, and `\divide` assignments. This was changed with  $\epsilon$ - $\TeX$ 's `\numexpr` which does expandable computations using standard infix notations with  $\TeX$  integers. But  $\epsilon$ - $\TeX$  did not modify the  $\TeX$  bound on acceptable integers, and did not add floating point support.

The `bigintcalc` package by HEIKO OBERDIEK provided expandable macros (using some of `\numexpr` possibilities, when available) on arbitrarily big integers, beyond the  $\TeX$  bound. It does not provide an expression parser.<sup>20</sup> `xint` did it again using more of `\numexpr` for higher speed, and in a later evolution added handling of exact fractions, of scientific numbers, and an expression parser. Arbitrary precision floating points operations were added as a derivative, and not part of the initial design goal. Currently (1.2j), the only non-elementary operation implemented for floating point numbers is the square-root extraction; no signed infinities, signed zeroes, NaN's, error traps... have been implemented, only the notion of 'scientific notation with a given number of significant figures'.<sup>21</sup>

The  $\mathbb{M}\TeX$  project has implemented expandably floating-point computations with 16 significant figures (`l3fp`), including functions such as exp, log, sine and cosine.<sup>22</sup>

More directly related to the `xint` bundle there is the `l3bigint` package, also devoted to big integers and in development a.t.t.o.w (2015/10/09, no division yet). It is part of the experimental trunk of the  $\mathbb{M}\TeX$  Project and provides an expression parser for expandable arithmetic with big integers. Its author Bruno LE FLOCH succeeded brilliantly into implementing expandably the Karatsuba multiplication algorithm and he achieves *sub-quadratic growth for the computation time*. This shows up very clearly with numbers having thousands of digits, up to the maximum which a.t.t.o.w is at 8192 digits.

The `l3bigint` multiplication from late 2015 is observed to be roughly 3x--4x faster than the one from `\xintiexpr` in the range of 4000 to 5000 digits integers, and isn't far from being 9x faster at 8000 digits. On the other hand `\xintiexpr`'s multiplication is found to be on average roughly 2.5x faster than `l3bigint`'s for numbers up to 100 digits and the two packages achieve about the same speed at 900 digits: but each such multiplication of numbers of 900 digits costs about one or two tenths of a second on a 2012 desktop computer, whereas the order of magnitude is rather the ms for numbers with 50--100 digits.<sup>23</sup>

Even with the superior `l3bigint` Karatsuba multiplication it takes about 3.5s on this 2012 desktop computer for a single multiplication of two 5000-digits numbers. Hence it is not possible to do routinely such computations in a document. I have long been thinking that without the expandability constraint much higher speeds could be achieved, but perhaps I have not given enough thought

<sup>19</sup> For example multiplication of integers having from 50 to 100 digits takes roughly of the order of the millisecond on a 2012 desktop computer. This is, I guess, at least about 1000 times slower than what can be expected with any reasonable programming language, nevertheless as compilation of a typical  $\LaTeX$  document already takes of the order of seconds if not dozens of seconds, this leaves room for reasonably many computations via `xintexpr` or via direct use of the macros of `xint/xintfrac`.<sup>20</sup> One can currently use package `bnumexpr` to associate the `bigintcalc` macros with an expression parser. This may be unavailable in future if `bnumexpr` becomes more tightly associated with future evolutions or variants of `xintcore`.<sup>21</sup> multiplication of two floats with  $P=Q$  `\xinttheDigits` digits is first done exactly then rounded to  $P$  digits, rather than using a specially tailored multiplication for floating point numbers which would be more efficient (it is a waste to evaluate fully the multiplication result with  $2P$  or  $2P-1$  digits.)<sup>22</sup> at the time of writing (2014/10/28) the `l3fp` (exactly represented) floating point numbers have their exponents limited to  $\pm 9999$ .

<sup>23</sup> I have tested this again on 2016/12/19, but the macros have not changed on the `l3bigint` side and barely on the `xintcore` side, hence I got again the same results...

to sustain that optimistic stance.<sup>24</sup>

I remain of the opinion that if one really wants to do computations with *thousands* of digits, one should drop the expandability requirement. Indeed, as clearly demonstrated long ago by the `pi computing` file by D. ROEGEL one can program  $\TeX$  to compute with many digits at a much higher speed than what `xint` achieves: but, direct access to memory storage in one form or another seems a necessity for this kind of speed and one has to renounce at the complete expandability.<sup>25</sup>

### 3.2 Floating point evaluations

Floating point macros are provided by package `xintfrac` to work with a given arbitrary precision `P`. The default value is `P = 16` meaning that the significands of the produced (non-zero) numbers have 16 decimal digits.<sup>26</sup> The syntax to set the precision to `P` is

```
\xintDigits:=P;
```

The value is local to the group or environment (if using  $\LaTeX$ ). To query the current value use `\xinttheDigits`.

Most floating point macros accept an optional first argument `[P]` which then sets the target precision and replaces the `\xintDigits` assigned value.<sup>27</sup>

`\xintfloatexpr[Q]...\relax` also admits an optional argument `[Q]` but it has an altogether different meaning: the computations are always done with the prevailing `\xinttheDigits` precision and the optional argument says to round the final result to `Q` digits of precision. This makes sense only if `Q < \xinttheDigits` and is intended to clean up the result from dubious last digits.

More reasonably, working with significands of 24, 32, 48, 64, or even 80 digits is well within the reach of the package.

The IEEE 754<sup>28</sup> requirement of *correct rounding* for addition, subtraction, multiplication, division and square root is achieved (in arbitrary precision) by the macros of `xintfrac` hence also by the infix operators `+`, `-`, `*`, `/` and `^` (which is synonym of `**`): this means that for operands given with at most `P` significant digits (and arbitrary exponents) the output coincides exactly with the rounding of an exact evaluation (barring overflow or underflow).

The rounding mode is `round to nearest, ties away from zero`. It is not customizable.

Currently `xintfrac` has no notion of NaNs or signed infinities or signed zeroes, but this is intended for the future.

Currently, the only non-elementary operation is the square root.<sup>29</sup> The elementary transcendental functions are not yet implemented. The power function accepts integer exponents (in all parsers, naturally `\xintiexpr` requires non-negative exponents) and half-integer exponents in `\xintfloatexpr`.<sup>30</sup>

The maximal floating point decimal exponent is currently 2147483647 which is the maximal number handled by  $\TeX$ . The minimal exponent is its opposite. But this means that overflow or underflow are detected only via low-level `\numexpr` arithmetic overflows which are basically un-recoverable. Besides there are some border effects as the routines need to add or subtract lengths of numbers from exponents, possibly triggering the low-level overflows. In the future not only the Precision but also the maximal and minimal exponents `Emin` and `Emax` will be specifiable by the user.

<sup>24</sup> The `apnum` package implements (non-expandably) arbitrary precision fixed point algebra and (v1.6) functions `exp`, `log`, `sqrt`, the trigonometrical direct and inverse functions. <sup>25</sup> The Lua $\TeX$  project possibly makes endeavours such as `xint` appear even more insane that they are, in truth: `xint` is able to handle fast enough computations involving numbers with less than one hundred digits and brings this to all engines. <sup>26</sup> Currently in the cases when the rounding to nearest goes to the next power of ten, the result is `10.0...0eN` with `P-1` zeroes after decimal mark, hence a total of `P+1`, not `P`, digits. <sup>27</sup> the `[P]` must be repeated

inside the arguments, if the latter are also `xintfrac` macros with arguments of their own. <sup>28</sup> The IEEE 754-1985 standard was for hardware implementations of binary floating-point arithmetic with a specific value for the precision (24 bits for single precision, 53 bits for double precision). The newer IEEE 754-2008 ([https://en.wikipedia.org/wiki/IEEE\\_floating\\_point](https://en.wikipedia.org/wiki/IEEE_floating_point)) normalizes five basic formats, three binaries and two decimals (16 and 34 decimal digits) and discusses extended formats with higher precision. These standards are only indirectly relevant to libraries like `xint` dealing with arbitrary precision. <sup>29</sup> Since 1.2f square root extraction achieves *correct rounding* in arbitrary precision. <sup>30</sup> Half-integer exponents work inside expressions, but not via the `\xintFloatPower` macro.



Another point is that `xintfrac` float macros have to handle inputs which not only may have much more digits than the target float precision, but may even be fractions, which in a way means infinite precision. As fractions should be handled as first class citizens for `xintfrac`, the float macros should give the same result independently of whether an argument is given as  $A/B$  or as  $C/D$  as long as  $A/B=C/D$  as rational numbers. This was briefly the case when `xintfrac` was first released, but very shortly thereafter the fraction-to-float parsing (via `\xintFloat`) was modified for speed and as a result, currently this is not the case.<sup>31</sup> The current situation is that for a fraction  $A/B \cdot 10^N$  its numerator and denominator are each first truncated (not rounded, but that would not change the issue) to  $P+2$  digits of precision if  $P$  is the asked-for precision. Then the new fraction is correctly rounded to  $P$  digits. This is not like correctly rounding the initial fraction, except of course if its numerator and denominator already had at most  $P+2$  digits.<sup>32</sup>

Next major release the `xint` bundle will provide floating point macros handling intrinsically their inputs, if expressed as fractions.

Also, probably, these macros will first correctly round the inputs to the target precision before doing the actual computation.

### 3.3 Expansion matters

#### 3.3.1 Full expansion of the first token

The whole business of `xint` is to build upon `\numexpr` and handle arbitrarily large numbers. Each basic operation is thus done via a macro: `\xintiiAdd`, `\xintiiSub`, `\xintiiMul`, `\xintiiDivision`. In order to handle more complex operations, it must be possible to nest these macros. An expandable macro can not execute a `\def` or an `\edef`. But the macro must expand its arguments to find the digits it is supposed to manipulate.  $\TeX$  provides a tool to do the job of (expandable !) repeated expansion of the first token found until hitting something non expandable, such as a digit, a `\def` token, a brace, a `\count` token, etc... is found. A space token also will stop the expansion (and be swallowed, contrarily to the non-expandable tokens).

By convention in this manual `f-expansion` (``full expansion'` or ``full first expansion'`) will be this  $\TeX$  process of expanding repeatedly the first token seen. For those familiar with  $\TeX$ 3 (which is not used by `xint`) this is what is called in its documentation full expansion (whereas expansion inside `\edef` would be described I think as ``exhaustive'` expansion).

Most of the package macros, and all those dealing with computations<sup>33</sup>, are expandable in the strong sense that they expand to their final result via this `f-expansion`. This will be signaled in their descriptions via a star in the margin.

These macros not only have this property of `f-expandability`, they all begin by first applying `f-expansion` to their arguments. Again from  $\TeX$ 3's conventions this will be signaled by a margin annotation next to the description of the arguments.

#### 3.3.2 Summary of important expandability aspects

1. the macros `f-expand` their arguments, this means that they expand the first token seen (for each argument), then expand, etc..., until something un-expandable such as a digit or a brace is hit against. This example

```
\def\x{98765}\def\y{43210} \xintiiAdd {\x}{\x\y}
```

is not a legal construct, as the `\y` will remain untouched by expansion and not get converted into the digits which are expected by the sub-routines of `\xintiiAdd`. It is a `\numexpr` which will expand it and an arithmetic overflow will arise as `9876543210` exceeds the  $\TeX$  bounds. The same would hold for `\xintAdd`.

<sup>31</sup> The author had forgotten that the initial situation had been quickly modified, and for some time this documentation wrongly asserted that the `xintfrac` basic floating point operations produced an output depending solely on the inputs as abstract fractions. Future version of `xintfrac` will make this valid again. <sup>32</sup> If  $B = 1$  there is no issue because truncating to  $P+2$  digits then rounding to nearest  $P$ -float is like rounding directly to nearest  $P$ -float. But for  $A = 1$  for example the issue is there. <sup>33</sup> except `\xintXTrunc`.

To the contrary `\xinttheiexpr` and others have no issues with things such as `\xinttheiexpr 2 \x+\x\y\relax`.

2. using `\if...\fi` constructs inside the package macro arguments requires suitably mastering TeXniques (`\expandafter`'s and/or swapping techniques) to ensure that the *f-expansion* will indeed absorb the `\else` or closing `\fi`, else some error will arise in further processing. Therefore it is highly recommended to use the package provided conditionals such as `\xintifEq`, `\xintifGt`, `\xintifSgn`, `\xintifOdd...`, or, for TeX users and when dealing with short integers the `etoolbox`<sup>34</sup> expandable conditionals (for small integers only) such as `\ifnumequal`, `\ifnumgreater`, .... Use of non-expandable things such as `\ifthenelse` is impossible inside the arguments of `xint` macros.

One can use naive `\if...\fi` things inside an `\xinttheexpr`-ession and cousins, as long as the test is expandable, for example

```
\xinttheiexpr\ifnum3>2 143\else 33\fi 0^2\relax→2044900=1430^2
```

3. after the definition `\def\x {12}`, one can not use `-x` as input to one of the package macros: the *f-expansion* will act only on the minus sign, hence do nothing. The only way is to use the `\xintOpp` macro, or perhaps here rather `\xintiOpp` which does maintains integer format on output, as they replace a number with its opposite.

Again, this is otherwise inside an `\xinttheexpr`-ession or `\xintthefloatexpr`-ession. There, the minus sign may prefix macros which will expand to numbers (or parentheses etc...)

4. With the definition

```
\def\AplusBC #1#2#3{\xintAdd {#1}{\xintMul {#2}{#3}}}
```

one obtains an expandable macro producing the expected result, not in two, but rather in three steps: a first expansion is consumed by the macro expanding to its definition. As the package macros expand their arguments until no more is possible (regarding what comes first), this `\AplusBC` may be used inside them: `\xintAdd {\AplusBC {1}{2}{3}}{4}` does work and returns `11/1[0]`.

If, for some reason, it is important to create a macro expanding in two steps to its final value, one may either do:

```
\def\AplusBC #1#2#3{\romannumeral-\`0\xintAdd {#1}{\xintMul {#2}{#3}}}
```

or use the lowercase form of `\xintAdd`:

```
\def\AplusBC #1#2#3{\romannumeral0\xintadd {#1}{\xintMul {#2}{#3}}}
```

and then `\AplusBC` will share the same properties as do the other `xint` 'primitive' macros.

5. The `\romannumeral0` and `\romannumeral-\`0` things above look like an invitation to hacker's territory; if it is not important that the macro expands in two steps only, there is no reason to follow these guidelines. Just chain arbitrarily the package macros, and the new ones will be completely expandable and usable one within the other.

Since release 1.07 the `\xintNewExpr` macro automatizes the creation of such expandable macros:

```
\xintNewExpr\AplusBC[3]{#1+#2*#3}
```

creates the `\AplusBC` macro doing the above and expanding in two expansion steps.

6. In the expression parsers of `xintexpr` such as `\xintexpr...\relax`, `\xintfloatexpr...\relax` the contents are expanded completely from left to right until the ending `\relax` is found and swallowed, and spaces and even (to some extent) catcodes do not matter.
7. For all variants, prefixing with `\xintthe` allows to print the result or use it in other contexts. Shortcuts `\xinttheexpr`, `\xintthefloatexpr`, `\xinttheiexpr`, ... are available.

<sup>34</sup> <http://www.ctan.org/pkg/etoolbox>

### 3.4 Input formats for macros

Macros can have different types of arguments. In the description of the macro, a margin annotation signals what is the argument type.

`num`  
`x`

1.  $\TeX$  integers are handled inside a `\numexpr...\relax` hence may be count registers or variables. Beware that `-(1+1)` is not legal (but `0-(1+1)` is). Such integers must be less than `2147483647` in absolute value.

`f`

2. the strict format applies to macros handling big integers but only `f-expansion` their arguments. After this `f-expansion` the input should be a string of digits, optionally preceded by a unique minus sign. The first digit can be zero only if it is the only digit. A plus sign is not accepted. `-0` is not legal in the strict format. Macros of `xint` with a double `ii` require this 'strict' format for the inputs.

`Num`  
`f`

3. the extended integer format applies when the macro parses its arguments via `\xintNum`. The input may then have arbitrarily many leading minus and plus signs, followed by leading zeroes, and further digits. Macros with a single `i` in their names always filter their arguments via `\xintNum`. When `xintfrac` is loaded `\xintNum` accepts fractions and truncates them to integers.

`Frac`  
`f`

4. the fraction input format applies to the arguments of `xintfrac` macros handling genuine fractions. It allows two types of inputs: general and restricted. The restricted type is parsed faster, but... is restricted.

**general:** inputs of the shape `A.BeC/D.EeF`. Example:

```
\noindent\xintRow{---0367.8920280e17/---+278.289287e-15}\newline
\xintRow{+++1253.2782e++-3/---0087.123e---5}\par
```

`-3678920280/278289287[31]`

`-12532782/87123[7]`

The input parser does not reduce fractions to smallest terms. Here are the rules of this general fraction format:

- everything is optional, absent numbers are treated as zero, here are some extreme cases:

```
\xintRow{}, \xintRow{.}, \xintRow{./1.e}, \xintRow{-.e}, \xintRow{e/-1}
```

`0/1[0], 0/1[0], 0/1[0], 0/1[0], 0/1[0]`

- `AB` and `DE` may start with pluses and minuses, then leading zeroes, then digits.
- `C` and `F` will be given to `\numexpr` and can be anything recognized as such and not provoking arithmetic overflow (the lengths of `B` and `E` will also intervene to build the final exponent naturally which must obey the  $\TeX$  bound).
- the `/`, `.` (numerator and/or denominator) and `e` (numerator and/or denominator) are all optional components.
- each of `A`, `B`, `C`, `D`, `E` and `F` may arise from `f-expansion` of a macro.
- the whole thing may arise from `f-expansion`, however the `/`, `.`, and `e` should all come from this initial expansion. The `e` of scientific notation is mandatorily lowercased.

**restricted:** inputs either of the shape `A[N]` or `A/B[N]`, which represents the fraction `A/B` times `10^N`. The whole thing or each of `A`, `B`, `N` (but then not `/` or `[]`) may arise from `f-expansion`, `A` (after expansion) *must* have a unique optional minus sign and no leading zeroes, `B` (after expansion) if present *must* be a positive integer with no signs and no leading zeroes, `[N]` if present will be given to `\numexpr`. Any deviation from the rules above will result in errors.

`Frac`  
`f`

Notice that `*`, `+` and `-` contrarily to the `/` (which is treated simply as a kind of delimiter) are not acceptable within arguments of this type (see [subsection 3.6](#) for some exceptions to this.)

Generally speaking, there should be no spaces among the digits in the inputs (in arguments to the package macros). Although most would be harmless in most macros, there are some cases where spaces could break havoc.<sup>35</sup> So the best is to avoid them entirely.

This is entirely otherwise inside an `\xintexpr`-ession, where spaces are ignored (except when they occur inside arguments to some macros, thus escaping the `\xintexpr` parser). See the [section 11](#).

There are also some slightly more obscure expansion types: in particular, the `\xintApplyInline` and `\xintFor*` macros from `xinttools` apply a special iterated *f-expansion*, which gobbles spaces, to the non-braced items (braced items are submitted to no expansion because the opening brace stops it) coming from their list argument; this is denoted by a special symbol in the margin. Some other macros such as `\xintSum` from `xintfrac` first do an *f-expansion*, then treat each found (braced or not) item (skipping spaces between such items) via the general fraction input parsing, this is signaled as here in the margin where the signification of the `*` is thus a bit different from the previous case.

A few macros from `xinttools` do not expand, or expand only once their argument. This is also signaled in the margin with notations à la  $\TeX$ 3.

### 3.5 Output formats of macros

We do not consider here the `\xintexpr`-parsers but only the macros as described in the documentation of `xint` and `xintfrac`. Macros of other components of the bundle have their own output formats (for example for continuous fractions with `xintcfrac`). There are mainly three types of output formats:<sup>36</sup>

- macros from `xint` with `i` or `ii` in their names produce on output integers in the strict format described in the previous section.
- fraction handling macros from `xintfrac` produce on output the strict fraction format `A/B[N]` (which stands for  $(A/B) \times 10^N$ ) where `A` and `B` are integers, with `B` positive, and `N` is a ``short'' integer. The output is not reduced to smallest terms. The `A` and `B` may end with zeroes (i.e., `N` does not represent all powers of ten). The denominator `B` is always strictly positive. There is no `+` sign. The `-` is always first if present (i.e. the denominator on output is always positive.) The output will be expressed as such a fraction even if the inputs are both integers and the mathematical result is an integer. The `B=1` is not removed.<sup>37</sup>
- macros with `Float` in their names produce on output scientific format with `P=\xinttheDigits` digits, a lowercase `e` and an exponent `N`. The first digit is not zero, it is preceded by an optional minus sign and is followed by a dot and `P-1` digits. Trailing zeroes are not trimmed. There are two exceptional cases:
  - if the rounding went to the next power of ten, the output is `10.0...0eN` (with `P+1` digits; and possibly a minus sign).
  - if the value is mathematically zero, it is output as `0.e0`, i.e. zeros after the decimal mark are removed and the exponent is always `0`.

Future versions of the package may modify these rules.

<sup>35</sup> The `\xintNum` macro does not remove spaces between digits beyond the first non zero ones; however this should not really alter the subsequent functioning of the arithmetic macros, and besides, since `xintcore` 1.2 there is an initial parsing of the entire number, during which spaces will be gobbled. However I have not done a complete review of the legacy code to be certain of all possibilities after 1.2 release. One thing to be aware of is that `\numexpr` stops on spaces between digits (although it provokes an expansion to see if an infix operator follows); the exponent for `\xintiiPow` or the argument of the factorial `\xintiFac` are only subjected to such a `\numexpr` (there are a few other macros with such input types in `xint`). If the input is given as, say `1 2\x` where `\x` is a macro, the macro `\x` will not be expanded by the `\numexpr`, and this will surely cause problems afterwards. Perhaps a later `xint` will force `\numexpr` to expand beyond spaces, but I decided that was not really worth the effort. Another immediate cause of problems is an input of the type `\xintiiAdd {<space>\x }{\y }`, because the space will stop the initial expansion; this will most certainly cause an arithmetic overflow later when the `\x` will be expanded in a `\numexpr`. Thus in conclusion, damages due to spaces are unlikely if only explicit digits are involved in the inputs, or arguments are single macros with no preceding space.

<sup>36</sup> There are further cases like `\xintiiDivision` which outputs a token list of two braced items. <sup>37</sup> refer to the documentation of `\xintPRaw` for an alternative.

### 3.6 Count registers and variables

Inside `\xintexpr...\relax` and its variants, a count register or count control sequence is automatically unpacked using `\number`, with tacit multiplication: `1.23\countA` is like `1.23*\number\countA`. There is a subtle difference between count *registers* and count *variables*. In `1.23*\countA` the unpacked `\countA` variable defines a complete operand thus `1.23*\countA 7` is a syntax error. But `1.23*\count0` just replaces `\count0` by `\number\count0` hence `1.23*\count0 7` is like `1.23*57` if `\count0` contains the integer value 5.

Regarding now the package macros, there is first the case of arguments having to be short integers: this means that they are fed to a `\numexpr...\relax`, hence submitted to a *complete expansion* which must deliver an integer, and count registers and even algebraic expressions with them like `\mycountA+\mycountB*17-\mycountC/12+\mycountD` are admissible arguments (the slash stands here for the rounded integer division done by `\numexpr`). This applies in particular to the number of digits to truncate or round with, to the indices of a series partial sum, ...

The macros allowing the extended format for long numbers or dealing with fractions will to some extent allow the direct use of count registers and even infix algebra inside their arguments: a count register `\mycountA` or `\count 255` is admissible as numerator or also as denominator, with no need to be prefixed by `\the` or `\number`. It is possible to have as argument an algebraic expression as would be acceptable by a `\numexpr...\relax`, under this condition: *each of the numerator and denominator is expressed with at most eight tokens*.<sup>38</sup> The slash for rounded division in a `\numexpr` should be written with braces `{/}` to not be confused with the `xintfrac` delimiter between numerator and denominator (braces will be removed internally). Example: `\mycountA+\mycountB{/}17/1+\mycountA*\mycountB`, or `\count 0+\count 2{/}17/1+\count 0*\count 2`, but in the latter case the numerator has the maximal allowed number of tokens (the braced slash counts for only one).

```
\cnta 10 \cntb 35 \xintRaw {\cnta+\cntb{/}17/1+\cnta*\cntb}->12/351[0]
```

For longer algebraic expressions using count registers, there are two possibilities:

1. let the numerator and the denominator be presented as `\the\numexpr...\relax`,
2. or as `\numexpr {...}\relax` (the braces are removed during processing; they are not legal for `\numexpr...\relax` syntax.)

```
\cnta 100 \cntb 10 \cntc 1
\xintPRaw {\numexpr {\cnta*\cnta+\cntb*\cntb+\cntc*\cntc+
2*\cnta*\cntb+2*\cnta*\cntc+2*\cntb*\cntc}\relax/%
\numexpr {\cnta*\cnta+\cntb*\cntb+\cntc*\cntc}\relax }
12321/10101
```

### 3.7 Dimension registers and variables

`\dimen` variables can be converted into (short) integers suitable for the `xint` macros by prefixing them with `\number`. This transforms a dimension into an explicit short integer which is its value in terms of the `sp` unit (1/65536pt). When `\number` is applied to a `\glue` variable, the stretch and shrink components are lost.

For  $\TeX$  users: a length is a `\glue` variable, prefixing a length macro defined by `\newlength` with `\number` will thus discard the `plus` and `minus` glue components and return the dimension component as described above, and usable in the `xint` bundle macros.

This conversion is done automatically inside an `\xintexpr`-essions, with tacit multiplication implied if prefixed by some (integral or decimal) number.

One may thus compute areas or volumes with no limitations, in units of `sp^2` respectively `sp^3`, do arithmetic with them, compare them, etc..., and possibly express some final result back in another unit, with the suitable conversion factor and a rounding to a given number of decimal places.

A [table of dimensions](#) illustrates that the internal values used by  $\TeX$  do not correspond always to the closest rounding. For example a millimeter exact value in terms of `sp` units is

<sup>38</sup> Attention! there is no problem with a  $\LaTeX$  `\value{countname}` if it comes first, but if it comes later in the input it will not get expanded, and braces around the name will be removed and chaos will ensue inside a `\numexpr`. One should enclose the whole input in `\the\numexpr...\relax` in such cases.



$72.27/10/2.54*65536=186467.981\dots$  and  $\text{\TeX}$  uses internally  $186467\text{sp}$  ( $\text{\TeX}$  truncates to get an integral multiple of the `sp` unit; see at the end of this section the exact rules applied internally by  $\text{\TeX}$ ).

| Unit                                       | definition        | Exact value in <code>sp</code> units | $\text{\TeX}$ 's value in <code>sp</code> units | Relative error |
|--------------------------------------------|-------------------|--------------------------------------|-------------------------------------------------|----------------|
| <b>cm</b>                                  | 0.01 m            | $236814336/127 = 1864679.811\dots$   | 1864679                                         | -0.0000%       |
| <b>mm</b>                                  | 0.001 m           | $118407168/635 = 186467.981\dots$    | 186467                                          | -0.0005%       |
| <b>in</b>                                  | 2.54 cm           | $118407168/25 = 4736286.720\dots$    | 4736286                                         | -0.0000%       |
| <b>pc</b>                                  | 12 pt             | $786432 = 786432.000\dots$           | 786432                                          | 0%             |
| <b>pt</b>                                  | $1/72.27$ in      | $65536 = 65536.000\dots$             | 65536                                           | 0%             |
| <b>bp</b>                                  | $1/72$ in         | $1644544/25 = 65781.760\dots$        | 65781                                           | -0.0012%       |
| 3bp                                        | $1/24$ in         | $4933632/25 = 197345.280\dots$       | 197345                                          | -0.0001%       |
| 12bp                                       | $1/6$ in          | $19734528/25 = 789381.120\dots$      | 789381                                          | -0.0000%       |
| 72bp                                       | 1 in              | $118407168/25 = 4736286.720\dots$    | 4736286                                         | -0.0000%       |
| <b>dd</b>                                  | $1238/1157$ pt    | $81133568/1157 = 70124.086\dots$     | 70124                                           | -0.0001%       |
| 11dd                                       | $11*1238/1157$ pt | $892469248/1157 = 771364.950\dots$   | 771364                                          | -0.0001%       |
| 12dd                                       | $12*1238/1157$ pt | $973602816/1157 = 841489.037\dots$   | 841489                                          | -0.0000%       |
| <b>sp</b>                                  | $1/65536$ pt      | $1 = 1.000\dots$                     | 1                                               | 0%             |
| <b><math>\text{\TeX}</math> dimensions</b> |                   |                                      |                                                 |                |

There is something quite amusing with the Didot point. According to the  $\text{\TeX}$ Book,  $1157\text{dd}=1238\text{pt}$ . The actual internal value of  $1\text{dd}$  in  $\text{\TeX}$  is  $70124\text{sp}$ . We can use `xintcfac` to display the list of centered convergents of the fraction  $70124/65536$ :

```
\xintListWithSep{, }\{\xintFtoCCv{70124/65536}\}
```

$1/1$ ,  $15/14$ ,  $61/57$ ,  $107/100$ ,  $1452/1357$ ,  $17531/16384$ , and we don't find  $1238/1157$  therein, but another approximant  $1452/1357$ !

And indeed multiplying  $70124/65536$  by  $1157$ , and respectively  $1357$ , we find the approximations (wait for more, later):

```
`1157 dd'='1237.998474121093...pt
`1357 dd'='1451.999938964843...pt
```

and we seemingly discover that  $1357\text{dd}=1452\text{pt}$  is *far more accurate* than the  $\text{\TeX}$ Book formula  $1157\text{dd}=1238\text{pt}$ ! The formula to compute  $N\text{dd}$  was

```
\xinttheexpr trunc(N\dimexpr 1dd\relax/\dimexpr 1pt\relax,12)\relax}
```

What's the catch? The catch is that  $\text{\TeX}$  does not compute  $1157\text{dd}$  like we just did:

```
1157 dd=\number\dimexpr 1157dd\relax/65536=1238.000000000000...pt
1357 dd=\number\dimexpr 1357dd\relax/65536=1452.001724243164...pt
```

We thus discover that  $\text{\TeX}$  (or rather here,  $\text{\eTeX}$ , but one can check that this works the same in  $\text{\TeX82}$ ), uses  $1238/1157$  as a conversion factor (and necessarily intermediate computations simulate higher precision than a priori available with integers less than  $2^{31}$  or rather  $2^{30}$  for dimensions). Hence the  $1452/1357$  ratio is irrelevant, an artefact of the rounding (or rather, as we see, truncating) for one `dd` to be expressed as an integral number of `sp`'s.

Let us now use `\xintexpr` to compute the value of the Didot point in millimeters, if the above rule is exactly verified:

```
\xinttheexpr trunc(1238/1157*25.4/72.27,12)\relax=0.376065027442...mm
```

This fits very well with the possible values of the Didot point as listed in the [Wikipedia Article](#). The value  $0.376065\text{mm}$  is said to be *the traditional value in European printers' offices*. So the  $1157\text{dd}=1238\text{pt}$  rule refers to this Didot point, or more precisely to the *conversion factor* to be used between this Didot and  $\text{\TeX}$  points.



The actual value in millimeters of exactly one Didot point as implemented in  $\TeX$  is

```
\xinttheexpr trunc(\dimexpr 1dd\relax/65536/72.27*25.4,12)\relax
=0.376064563929...mm
```

The difference of circa 5Å is arguably tiny!

By the way the *European printers' offices* (dixit Wikipedia) *Didot* is thus exactly

```
\xinttheexpr reduce(.376065/(25.4/72.27))\relax=543564351/508000000pt
```

and the centered convergents of this fraction are  $1/1$ ,  $15/14$ ,  $61/57$ ,  $107/100$ ,  $1238/1157$ ,  $11249/10513$ ,  $23736/22183$ ,  $296081/276709$ ,  $615898/575601$ ,  $11382245/10637527$ ,  $22148592/20699453$ ,  $1885709281/176233151$ ,  $543564351/508000000$ . We do recover the  $1238/1157$  therein!

Here is how  $\TeX$  converts `abc.xyz...<unit>`. First the decimal is *rounded* to the nearest integral multiple of  $1/65536$ , say  $X/65536$ . The `<unit>` is associated to a ratio  $N/D$ , which represents `<unit>/pt`. For the Didot point the ratio is indeed  $1238/1157$ .  $\TeX$  truncates the fraction  $XN/D$  to an integer  $M$ . The dimension is represented by  $M\ sp$ .

For more details refer to:

<http://tex.stackexchange.com/questions/338297/why-pdf-file-cannot-be-reproduced/338510#338510>.

### 3.8 `\ifcase`, `\ifnum`, ... constructs

When using things such as `\ifcase \xintSgn{\A}` one has to make sure to leave a space after the closing brace for  $\TeX$  to stop its scanning for a number: once  $\TeX$  has finished expanding `\xintSgn{\A}` and has so far obtained either  $1$ ,  $0$ , or  $-1$ , a space (or something 'unexpandable') must stop it looking for more digits. Using `\ifcase\xintSgn\A` without the braces is very dangerous, because the blanks (including the end of line) following `\A` will be skipped and not serve to stop the number which `\ifcase` is looking for.

```
\begin{enumerate}[nosep]\def\A{1}
\item \ifcase \xintSgn\A 0\or OK\else ERROR\fi
\item \ifcase \xintSgn\A\space 0\or OK\else ERROR\fi
\item \ifcase \xintSgn{\A} 0\or OK\else ERROR\fi
\end{enumerate}
```

1. **ERROR**
2. **OK**
3. **OK**

In order to use successfully `\if...\fi` constructions either as arguments to the `xint` bundle expandable macros, or when building up a completely expandable macro of one's own, one needs some  $\TeX$  technical expertise (see also [item 2](#) on page 38).

It is thus much to be recommended to opt rather for already existing expandable branching macros, such as the ones which are provided by `xint/xintfrac`: among them `\xintSgnFork`, `\xintifSgn`, `\xintifZero`, `\xintifOne`, `\xintifNotZero`, `\xintifTrueAelseB`, `\xintifCmp`, `\xintifGt`, `\xintifLt`, `\xintifEq`, `\xintifOdd`, and `\xintifInt`. See their respective documentations. All these conditionals always have either two or three branches, and empty brace pairs `{}` for unused branches should not be forgotten.

If these tests are to be applied to standard  $\TeX$  short integers, it is more efficient to use (under  $\LaTeX$ ) the equivalent conditional tests from the [etoolbox](#)<sup>39</sup> package.

### 3.9 No variable declarations are needed

There is no notion of a *declaration of a variable*.

To do a computation and assign its result to some macro `\z`, the user will employ the `\def`, `\edef`, or `\newcommand` (in  $\LaTeX$ ) as usual, keeping in mind that two expansion steps are needed, thus `\edef` is initially the main tool:

<sup>39</sup> <http://www.ctan.org/pkg/etoolbox>



```
\def\x{1729728} \def\y{352827927} \edef\z{\xintiiMul {\x}{\y}}
\meaning\z
```

```
macro:->610296344513856
```

As an alternative to `\edef` the package provides `\oodef` which expands exactly twice the replacement text, and `\fdef` which applies *f-expansion* to the replacement text during the definition.

```
\def\x{1729728} \def\y{352827927} \oodef\w {\xintiiMul\x\y} \fdef\z{\xintiiMul {\x}{\y}}
\meaning\w, \meaning\z
```

```
macro:->610296344513856, macro:->610296344513856
```



In practice `\oodef` is slower than `\edef`, except for computations ending in very big final replacement texts (thousands of digits). On the other hand `\fdef` appears to be slightly faster than `\edef` already in the case of expansions leading to only a few dozen digits.

`xintexpr` does provide an interface to declare and assign values to identifiers which can then be used in expressions: [subsection 2.5](#).

### 3.10 When expandability is too much

Let's use the macros of [subsection 2.9](#) related to Fibonacci numbers. Notice that the 47th Fibonacci number is **2971215073** thus already too big for  $\text{\TeX}$  and  $\varepsilon\text{-}\text{\TeX}$ .

The `\FibonacciN` macro found in [subsection 2.9](#) is completely expandable, it is even *f-expandable*. We need a wrapper with `\xintthe` prefix

```
\def\xinttheFibonacciN{\xintthe\FibonacciN}
```

to print in the document or to use within `\message` (or `\typeout`) to write to the log and terminal.

The `\xintthe` prefix also allows its use it as argument to the `xint` macros: for example if we are interested in knowing how many digits  $F(1250)$  has, it suffices to issue `\xintLen {\theFibonacciN {1250}}` (which expands to **261**). Or if we want to check the formula  $\gcd(F(1859), F(1573)) = F(\gcd(1859, 1573)) = F(143)$ , we only need<sup>40</sup>

```
$\xintiiGCD{\theFibonacciN{1859}}{\theFibonacciN{1573}}=%
\theFibonacciN{\xintiiGCD{1859}{1573}}$
```

which produces:

```
343358302784187294870275058337 = 343358302784187294870275058337
```

The `\theFibonacciN` macro expanded its `\xintiiGCD{1859}{1573}` argument via the services of `\nu\mexpr`: this step allows only things obeying the  $\text{\TeX}$  bound, naturally! (but **F(2147483648)** would be rather big anyhow...).

This is very convenient but of course it repeats the complete evaluation each time it is done. In practice, it is often useful to store the result of such evaluations in macros. Any `\edef` will break expandability, but if the goal is at some point to print something to the `dvi` or `pdf` output, and not only to the `log` file, then expandability has to be broken one day or another!

Hence, in practice, if we want to print in the document some computation results, we can proceed like this and avoid having to repeat identical evaluations:

```
\begingroup
\def\A {1859} \def\B {1573}
\edef\X {\theFibonacciN\A} \edef\Y {\theFibonacciN\B}
\edef\GCDAB {\xintiiGCD\A\B} \edef\Z {\theFibonacciN\GCDAB}
\edef\GCDXY{\xintiiGCD\X\Y}
```

The identity  $\gcd(F(\text{\A}), F(\text{\B})) = F(\gcd(\text{\A}, \text{\B}))$  can be checked via evaluation of both sides:  $\gcd(F(\text{\A}), F(\text{\B})) = \gcd(\text{\printnumber\X}, \text{\printnumber\Y}) = \text{\printnumber\GCDXY} = F(\gcd(\text{\A}, \text{\B})) = F(\text{\GCDAB}) = \text{\printnumber\Z}$ .`\par`

```
% some further computations involving \A, \B, \X, \Y
```

```
\endgroup % closing the group removes assignments to \A, \B, ...
```

```
% or choose longer names less susceptible to overwrite something.
```

```
% Note: there is no LaTeX \newcommand which would be to \edef like \newcommand is to \def
```

The identity  $\gcd(F(1859), F(1573)) = F(\gcd(1859, 1573))$  can be checked via evaluation of both sides:  $\gcd(F(1859), F(1573)) = \gcd(14405827913044251198771689151504042869913161495023481014226)68636701088272597575494722482437753529619459794869227357628882216309358018264080851775319974$

<sup>40</sup> The `\xintiiGCD` macro is provided by the `xintgcd` package.

```
256956055294350288615852451737250886736422228492908228952455838894954421926557604129992902552
659797113378761054522176234908415299798114131996600875176897034109975200799936107075760195202
876324584695551467505894985013610208598628752325727241, 24438419251951185733282794597776261992
853990248157061923260536090078401339403674321244522327895990951586958110318917797690580327412
516325953076166866610137252008667540965698889510100228880168314593473101315665177215932493442
798634399479371195758766544765827958909282390070313197135548122004938644531329524847747273162
6471511289078393) = 343358302784187294870275058337 = F(gcd(1859, 1573)) = F(143) = 34335830278412
87294870275058337.
```

One may legitimately ask the author: why expandability to such extremes, for things such as big fractions or floating point numbers (even continued fractions...) which anyhow can not be used directly within  $\TeX$ 's primitives such as `\ifnum`? Why insist on a concept which is foreign to the vast majority of  $\TeX$  users and even programmers?

I have no answer: it made definitely sense at the start of `xint` (see subsection 3.14) and once started I could not stop.

### 3.11 Possible syntax errors to avoid

Here is a list of imaginable input errors. Some will cause compilation errors, others are more annoying as they may pass through unsigned.

- using `-` to prefix some macro: `-\xintiSqr{35}/271`.<sup>41</sup>
- using one pair of braces too many `\xintIrr{\{\xintiPow {3}{13}\}/243}` (the computation goes through with no error signaled, but the result is completely wrong).
- things like `\xintiiAdd { \x}{\y}` as the space will cause `\x` to be expanded later, most probably within a `\numexpr` thus provoking possibly an arithmetic overflow.
- using `[]` and decimal points at the same time `1.5/3.5[2]`, or with a sign in the denominator `3/-5[7]`. The scientific notation has no such restriction, the two inputs `1.5/-3.5e-2` and `-1.25e2/3.5` are equivalent: `\xintRaw{1.5/-3.5e-2}=-15/35[2]`, `\xintRaw{-1.5e2/3.5}=-15/35[2]`.
- generally speaking, using in a context expecting an integer (possibly restricted to the  $\TeX$  bound) a macro or expression which returns a fraction: `\xinttheexpr 4/2\relax` outputs `4/2`, not `2`. Use `\xintNum {\xinttheexpr 4/2\relax}` or `\xinttheiexpr 4/2\relax` (which rounds the result to the nearest integer, here, the result is already an integer) or `\xinttheiiexpr 4/2\relax`. Or, divide in your head `4` by `2` and insert the result directly in the  $\TeX$  source.

### 3.12 Error messages

In situations such as division by zero, the package will insert in the  $\TeX$  processing an undefined control sequence (we copy this method from the `bigintcalc` package). This will trigger the writing to the log of a message signaling an undefined control sequence. The name of the control sequence is the message. The error is raised *before* the end of the expansion so as to not disturb further processing of the token stream, after completion of the operation. Generally the problematic operation will output a zero. Possible such error message control sequences:

```
\xintError:ArrayIndexIsNegative
\xintError:ArrayIndexBeyondLimit
\xintError:FactorialOfNegative
\xintError:TooBigFactorial
\xintError:DivisionByZero
\xintError:NaN
\xintError:FractionRoundedToZero
```

```
\xintError:NotAnInteger
\xintError:ExponentTooBig
\xintError:RootOfNegative
\xintError:NoBezoutForZeros
\xintError:ignored
\xintError:removed
\xintError:inserted
```

<sup>41</sup> to the contrary, this *is* allowed inside an `\xintexpr`-ession.

```
\xintError:unknownfunction
\xintError:we_are_doomed
```

```
\xintError:missing_xintthe!
```

There are now a few more if for example one attempts to use `\xintAdd` without having loaded *xintfrac* (with only *xint* loaded, only `\xintiAdd` and `\xintiiAdd` are legal) or to use deprecated macros.

```
\Did_you_mean_iiAbs?or_load_xintfrac
\Did_you_mean_iiOpp?or_load_xintfrac
\Did_you_mean_iiAdd?or_load_xintfrac
\Did_you_mean_iiSub?or_load_xintfrac
\Did_you_mean_iiMul?or_load_xintfrac
\Did_you_mean_iiPow?or_load_xintfrac
\Did_you_mean_iiSqr?or_load_xintfrac
\Did_you_mean_iiMax?or_load_xintfrac
\Did_you_mean_iiMin?or_load_xintfrac
```

```
\Did_you_mean_iMaxof?or_load_xintfrac
\Did_you_mean_iMinof?or_load_xintfrac
\Did_you_mean_iiSum?or_load_xintfrac
\Did_you_mean_iiPrd?or_load_xintfrac
\Did_you_mean_iiPrdExpr?or_load_xintfrac
\Did_you_mean_iiSumExpr?or_load_xintfrac
\Removed!use_xintiQuo_or_xintiiQuo!
\Removed!use_xintiRem_or_xintiiRem!
```

One should set `\errorcontextlines` to at least 2 to get from  $\TeX$  more meaningful error messages. Errors occurring during the parsing of `\xintexpr`-essions try to provide helpful information about the offending token.

Release 1.1 employs in some situations delimited macros and there is the possibility in case of an ill-formed expression to end up beyond the `\relax` end-marker. The errors inevitably arising could then lead to very cryptic messages; but nothing unusual or especially traumatizing for the daring experienced  $\TeX$ / $\LaTeX$  user.

### 3.13 Package namespace, catcodes

The bundle packages needs that the `\space` and `\empty` control sequences are pre-defined with the identical meanings as in Plain  $\TeX$  or  $\LaTeX$ 2 $\epsilon$ .

Private macros of *xintkernel*, *xintcore*, *xinttools*, *xint*, *xintfrac*, *xintexpr*, *xintbinhex*, *xintgcd*, *xintseries*, and *xintcfrac* use one or more underscores `_` as private letter, to reduce the risk of getting overwritten. They almost all begin either with `\XINT_` or with `\xint_`, a handful of these private macros such as `\XINTsetupcatcodes`, `\XINTdigits` and those with names such as `\XINTinFloat` ... or `\XINTinfloat` ... do not have any underscore in their names (for obscure legacy reasons).

*xinttools* provides `\odef`, `\oodef`, `\fdef` (if macros with these names already exist *xinttools* will not overwrite them but provide `\xintodef` etc...) but all other public macros from the *xint* bundle packages start with `\xint`.

For the good functioning of the macros, standard catcodes are assumed for the minus sign, the forward slash, the square brackets, the letter ``e'`. These requirements are dropped inside an `\xintexpr`-ession: spaces are gobbled, catcodes mostly do not matter, the `e` of scientific notation may be `E` (on input) ...

If a character used in the `\xintexpr` syntax is made active, this will surely cause problems; prefixing it with `\string` is one option. There is `\xintexprSafeCatcodes` and `\xintexprRestoreCatcodes` to temporarily turn off potentially active characters (but setting catcodes is an un-expandable action).

For advanced  $\TeX$  users. At loading time of the packages the catcode configuration may be arbitrary as long as it satisfies the following requirements: the percent is of category code comment character, the backslash is of category code escape character, digits have category code other and letters have category code letter. Nothing else is assumed.

### 3.14 Origins of the package

2013/03/28. Package `bigintcalc` by HEIKO OBERDIEK already provides expandable arithmetic operations on ``big integers'', exceeding the  $\TeX$  limits (of  $2^{31} - 1$ ), so why another<sup>42</sup> one?

I got started on this in early March 2013, via a thread on the `c.t.tex` usenet group, where ULRICH DIEZ used the previously cited package together with a macro (`\ReverseOrder`) which I had contributed to another thread.<sup>43</sup> What I had learned in this other thread thanks to interaction with ULRICH DIEZ and GL on expandable manipulations of tokens motivated me to try my hands at addition and multiplication.

I wrote macros `\bigMul` and `\bigAdd` which I posted to the newsgroup; they appeared to work comparatively fast. These first versions did not use the  $\varepsilon$ - $\TeX$  `\numexpr` primitive, they worked one digit at a time, having previously stored carry-arithmetic in 1200 macros.

I noticed that the `bigintcalc` package used `\numexpr` if available, but (as far as I could tell) not to do computations many digits at a time. Using `\numexpr` for one digit at a time for `\bigAdd` and `\bigMul` slowed them a tiny bit but avoided cluttering  $\TeX$  memory with the 1200 macros storing pre-computed digit arithmetic. I wondered if some speed could be gained by using `\numexpr` to do four digits at a time for elementary multiplications (as the maximal admissible number for `\numexpr` has ten digits).

2013/04/14. This initial `xint` was followed by `xintfrac` which handled exactly fractions and decimal numbers.

2013/05/25. Later came `xintexpr` and at the same time `xintfrac` got extended to handle floating point numbers.

2013/11/22. Later, `xinttools` was detached.

2014/10/28. Release 1.1 significantly extended the `xintexpr` parsers.

2015/10/10. Release 1.2 rewrote the core integer routines which had remained essentially unmodified, apart from a slight improvement of division early 2014.

This 1.2 release also got its impulse from a fast ``reversing'' macro, which I wrote after my interest got awakened again as a result of correspondence with Bruno LE FLOCH during September 2015: this new reverse uses a  $\TeX$  technique which *requires* the tokens to be digits. I wrote a routine which works (expandably) in quasi-linear time, but a less fancy  $O(N^2)$  variant which I developed concurrently proved to be faster all the way up to perhaps 7000 digits, thus I dropped the quasi-linear one. The less fancy variant has the advantage that `xint` can handle numbers with more than 19900 digits (but not much more than 19950). This is with the current common values of the input save stack and maximal expansion depth: 5000 and 10000 respectively.

## 4 Some utilities from the `xinttools` package

This is a first overview. Many examples combining these utilities with the arithmetic macros of `xint` are to be found in section 7. See also section 5.

### 4.1 Assignments

It might not be necessary to maintain at all times complete expandability. A devoted syntax is provided to make these things more efficient, for example when using the `\xintiDivision` macro which computes both quotient and remainder at the same time:

```
\xintAssign \xintiiDivision{\xintiiPow {2}{1000}}{\xintiiFac{100}}\to\A\B
give: \meaning\A: macro:->114813249641507505482278393872551066259805517784186172883663478065\
826541894704737970419535798876630484358265060061503749531707793118627774829601 and \meaning\B:
macro:->549362945213398322513812878622391280734105004984760505953218996123132766490228838\
8132878702444582075129603152041054804964625083138567652624386837205668069376. Another example
```

<sup>42</sup> this section was written before the `xintfrac` package; the author is not aware of another package allowing expandable computations with arbitrarily big fractions. <sup>43</sup> the `\ReverseOrder` could be avoided in that circumstance, but it does play a crucial rôle here.

(which uses `\xintBezout` from the *xintgcd* package):

```
\xintAssign \xintBezout{357}{323}\to\A\B\U\V\D
```

is equivalent to setting `\A` to 357, `\B` to 323, `\U` to -9, `\V` to -10, and `\D` to 17. And indeed  $(-9) \times 357 - (-10) \times 323 = 17$  is a Bezout Identity.

Thus, what `\xintAssign` does is to first apply an *f-expansion* to what comes next; it then defines one after the other (using `\def`; an optional argument allows to modify the expansion type, see subsection 7.21 for details), the macros found after `\to` to correspond to the successive braced contents (or single tokens) located prior to `\to`. In case the first token (after the optional parameter within brackets, cf. the `\xintAssign` detailed document) is not an opening brace `{`, `\xintAssign` consider that there is only one macro to define, and that its replacement text should be all that follows until the `\to`.

```
\xintAssign\xintBezout{3570902836026}{200467139463}\to\A\B\U\V\D
```

gives then `\U: 5812117166`, `\V: 103530711951` and `\D=3`.

In situations when one does not know in advance the number of items, one has `\xintAssignArray` or its synonym `\xintDigitsOf`:

```
\xintDigitsOf\xintiPow{2}{100}\to\DIGITS
```

This defines `\DIGITS` to be macro with one parameter, `\DIGITS{0}` gives the size `N` of the array and `\DIGITS{n}`, for `n` from 1 to `N` then gives the `n`th element of the array, here the `n`th digit of  $2^{100}$ , from the most significant to the least significant. As usual, the generated macro `\DIGITS` is completely expandable (in two steps). As it wouldn't make much sense to allow indices exceeding the  $\text{\TeX}$  bounds, the macros created by `\xintAssignArray` put their argument inside a `\numexpr`, so it is completely expanded and may be a count register, not necessarily prefixed by `\the` or `\number`. Consider the following code snippet:

```
% \newcount\cnta
% \newcount\cntb
\begingroup
\xintDigitsOf\xintiPow{2}{100}\to\DIGITS
\cnta = 1
\cntb = 0
\loop
\advance \cntb \xintiSqr{\DIGITS{\cnta}}
\ifnum \cnta < \DIGITS{0}
\advance\cnta 1
\repeat
```

```
|2^{100}| (= \xintiPow {2}{100}) has \DIGITS{0} digits and the sum of their squares is \the\cntb.
These digits are, from the least to the most significant: \cnta = \DIGITS{0} \loop
\dig\cnta{\cnta}\ifnum \cnta > 1 \advance\cnta -1 , \repeat.\endgroup
```

$2^{100}$  (=267650600228229401496703205376) has 31 digits and the sum of their squares is 679. These digits are, from the least to the most significant: 6, 7, 3, 5, 0, 2, 3, 0, 7, 6, 9, 4, 1, 0, 4, 9, 2, 2, 8, 2, 2, 0, 0, 6, 0, 5, 6, 7, 6, 2, 1.

Warning: `\xintAssign`, `\xintAssignArray` and `\xintDigitsOf` do not do any check on whether the macros they define are already defined.

## 4.2 Utilities for expandable manipulations

The package now has more utilities to deal expandably with 'lists of things', which were treated un-expandably in the previous section with `\xintAssign` and `\xintAssignArray`: `\xintReverseOrder` and `\xintLength` since the first release, `\xintApply` and `\xintListWithSep` since 1.04, `\xintRevWithBraces`, `\xintCSVtoList`, `\xintNthElt` since 1.06, `\xintApplyUnbraced`, since 1.06b, `\xintloop` and `\xintiloop` since 1.09g.<sup>44</sup>

As an example the following code uses only expandable operations:

```
2^{100} (= \xintiPow {2}{100}) has \xintLen{\xintiPow {2}{100}} digits and the sum of their
```

<sup>44</sup> All these utilities, as well as `\xintAssign`, `\xintAssignArray` and the `\xintFor` loops are now available from the *xinttools* package, independently of the big integers facilities of *xint*.

squares is `\xintiSum{\xintApply {\xintiSqr}{\xintiPow {2}{100}}}`. These digits are, from the least to the most significant: `\xintListWithSep {, }{\xintRev{\xintiPow {2}{100}}}`. The thirteenth most significant digit is `\xintNthElt{13}{\xintiPow {2}{100}}`. The seventh least significant one is `\xintNthElt{7}{\xintRev{\xintiPow {2}{100}}}`.

$2^{100}$  (=1267650600228229401496703205376) has 31 digits and the sum of their squares is 679. These digits are, from the least to the most significant: 6, 7, 3, 5, 0, 2, 3, 0, 7, 6, 9, 4, 1, 0, 4, 9, 2, 2, 8, 2, 2, 0, 0, 6, 0, 5, 6, 7, 6, 2, 1. The thirteenth most significant digit is 8. The seventh least significant one is 3.

It would be more efficient to do once and for all `\edef\z{\xintiPow {2}{100}}`, and then use `\z` in place of `\xintiPow {2}{100}` everywhere as this would spare the CPU some repetitions.

Expandably computing primes is done in [subsection 7.12](#).

### 4.3 A new kind of for loop

As part of the [utilities](#) coming with the `xinttools` package, there is a new kind of for loop, `\xintFor`. Check it out ([subsection 7.16](#) and also in next section).

### 4.4 A new kind of expandable loop

Also included in `xinttools`, `\xintiloop` is an expandable loop giving access to an iteration index, without using count registers which would break expandability. Check it out ([subsection 7.14](#) and also in next section).

## 5 Additional examples using `xinttools` or `xintexpr` or both

Actually, recall that `xintexpr.sty` automatically loads `xinttools.sty`.

### 5.1 Completely expandable prime test

Let us now construct a completely expandable macro which returns 1 if its given input is prime and 0 if not:

```
\def\remainder #1#2{\the\numexpr #1-(#1/#2)*#2\relax }
\def\IsPrime #1%
{\xintANDof {\xintApply {\remainder {#1}}{\xintSeq {2}{\xintiSqrt{#1}}}}
```

This uses `\xintiSqrt` and assumes its input is at least 5. Rather than `xint`'s own `\xintiRem` we used a quicker `\numexpr` expression as we are dealing with short integers. Also we used `\xintANDof` which will return 1 only if all the items are non-zero. The macro is a bit silly with an even input, ok, let's enhance it to detect an even input:

```
\def\IsPrime #1%
{\xintifOdd {#1}
{\xintANDof % odd case
{\xintApply {\remainder {#1}}
{\xintSeq [2]{3}{\xintiSqrt{#1}}}%
}%
}
{\xintifEq {#1}{2}{1}{0}}%
}
```

We used the `xint` expandable tests (on big integers or fractions) in order for `\IsPrime` to be *f-expandable*.

Our integers are short, but without `\expandafter`'s with `\@firstoftwo`, or some other related techniques, direct use of `\ifnum... \fi` tests is dangerous. So to make the macro more efficient we are going to use the expandable tests provided by the package `etoolbox`<sup>45</sup>. The macro becomes:

```
\def\IsPrime #1%
```

<sup>45</sup> <http://ctan.org/pkg/etoolbox>



```

{\ifnumodd {#1}
 {\xintANDof % odd case
 {\xintApply {\remainder {#1}}{\xintSeq [2]{3}{\xintiSqrt{#1}}}}
 {\ifnumequal {#1}{2}{1}{0}}}}

```

In the odd case however we have to assume the integer is at least 7, as `\xintSeq` generates an empty list if `#1=3` or `5`, and `\xintANDof` returns 1 when supplied an empty list. Let us ease up a bit `\xintANDof`'s work by letting it work on only 0's and 1's. We could use:

```

\def\IsNotDivisibleBy #1#2%
 {\ifnum\numexpr #1-(#1/#2)*#2=0 \expandafter 0\else \expandafter 1\fi}

```

where the `\expandafter`'s are crucial for this macro to be *f-expandable* and hence work within the applied `\xintANDof`. Anyhow, now that we have loaded `etoolbox`, we might as well use:

```

\newcommand{\IsNotDivisibleBy}[2]{\ifnumequal{#1-(#1/#2)*#2}{0}{0}{1}}

```

Let us enhance our prime macro to work also on the small primes:

```

\newcommand{\IsPrime}[1] % returns 1 if #1 is prime, and 0 if not
 {\ifnumodd {#1}
 {\ifnumless {#1}{8}
 {\ifnumequal{#1}{1}{0}{1}}% 3,5,7 are primes
 {\xintANDof
 {\xintApply
 {\IsNotDivisibleBy {#1}}{\xintSeq [2]{3}{\xintiSqrt{#1}}}}%
 }}% END OF THE ODD BRANCH
 {\ifnumequal {#1}{2}{1}{0}}% EVEN BRANCH
 }
 }

```

The input is still assumed positive. There is a deliberate blank before `\IsNotDivisibleBy` to use this feature of `\xintApply`: a space stops the expansion of the applied macro (and disappears). This expansion will be done by `\xintANDof`, which has been designed to skip everything as soon as it finds a false (i.e. zero) input. This way, the efficiency is considerably improved.

We did generate via the `\xintSeq` too many potential divisors though. Later sections give two variants: one with `\xintiloop` (subsection 5.2) which is still expandable and another one (subsection 5.5) which is a close variant of the `\IsPrime` code above but with the `\xintFor` loop, thus breaking expandability. The *xintiloop* variant does not first evaluate the integer square root, the *xintFor* variant still does. I did not compare their efficiencies.

Let us construct with this expandable primality test a table of the prime numbers up to 1000. We need to count how many we have in order to know how many tab stops one should add in the last row.<sup>46</sup> There is some subtlety for this last row. Turns out to be better to insert a `\\` only when we know for sure we are starting a new row; this is how we have designed the `\OneCell` macro. And for the last row, there are many ways, we use again `\xintApplyUnbraced` but with a macro which gobbles its argument and replaces it with a tabulation character. The `\xintFor*` macro would be more elegant here.

```

\newcounter{primecount}
\newcounter{cellcount}
\newcommand{\NbOfColumns}{13}
\newcommand{\OneCell}[1]{%
 \ifnumequal{\IsPrime{#1}}{1}
 {\stepcounter{primecount}
 \ifnumequal{\value{cellcount}}{\NbOfColumns}
 {\setcounter{cellcount}{1}#1}
 {\&\stepcounter{cellcount}#1}%
 } % was prime
 {}% not a prime, nothing to do
 }
\newcommand{\OneTab}[1]{&}
\begin{tabular}{|*{\NbOfColumns}{r}|}
\hline
2 \setcounter{cellcount}{1}\setcounter{primecount}{1}%

```

<sup>46</sup> although a tabular row may have less tabs than in the preamble, there is a problem with the | vertical rule, if one does that.



```

\intApplyUnbraced \OneCell {\xintSeq [2]{3}{999}}%
\intApplyUnbraced \OneTab
 {\xintSeq [1]{1}{\the\numexpr\NbOfColumns-\value{cellcount}\relax}}%
 \\
\hline
\end{tabular}
There are \arabic{primecount} prime numbers up to 1000.

```

The table has been put in `float` which appears on the current page. We had to be careful to use in the last row `\xintSeq` with its optional argument `[1]` so as to not generate a decreasing sequence from 1 to 0, but really an empty sequence in case the row turns out to already have all its cells (which doesn't happen here but would with a number of columns dividing 168).

|     |     |     |     |     |     |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 2   | 3   | 5   | 7   | 11  | 13  | 17  | 19  | 23  | 29  | 31  | 37  | 41  |
| 43  | 47  | 53  | 59  | 61  | 67  | 71  | 73  | 79  | 83  | 89  | 97  | 101 |
| 103 | 107 | 109 | 113 | 127 | 131 | 137 | 139 | 149 | 151 | 157 | 163 | 167 |
| 173 | 179 | 181 | 191 | 193 | 197 | 199 | 211 | 223 | 227 | 229 | 233 | 239 |
| 241 | 251 | 257 | 263 | 269 | 271 | 277 | 281 | 283 | 293 | 307 | 311 | 313 |
| 317 | 331 | 337 | 347 | 349 | 353 | 359 | 367 | 373 | 379 | 383 | 389 | 397 |
| 401 | 409 | 419 | 421 | 431 | 433 | 439 | 443 | 449 | 457 | 461 | 463 | 467 |
| 479 | 487 | 491 | 499 | 503 | 509 | 521 | 523 | 541 | 547 | 557 | 563 | 569 |
| 571 | 577 | 587 | 593 | 599 | 601 | 607 | 613 | 617 | 619 | 631 | 641 | 643 |
| 647 | 653 | 659 | 661 | 673 | 677 | 683 | 691 | 701 | 709 | 719 | 727 | 733 |
| 739 | 743 | 751 | 757 | 761 | 769 | 773 | 787 | 797 | 809 | 811 | 821 | 823 |
| 827 | 829 | 839 | 853 | 857 | 859 | 863 | 877 | 881 | 883 | 887 | 907 | 911 |
| 919 | 929 | 937 | 941 | 947 | 953 | 967 | 971 | 977 | 983 | 991 | 997 |     |

There are 168 prime numbers up to 1000.

## 5.2 Another completely expandable prime test

The `\IsPrime` macro from [subsection 5.1](#) checked expandably if a (short) integer was prime, here is a partial rewrite using `\xintloop`. We use the `etoolbox` expandable conditionals for convenience, but not everywhere as `\xintloopindex` can not be evaluated while being braced. This is also the reason why `\xintbreakloopanddo` is delimited, and the next macro `\SmallestFactor` which returns the smallest prime factor exemplifies that. One could write more efficient completely expandable routines, the aim here was only to illustrate use of the general purpose `\xintloop`. A little table giving the first values of `\SmallestFactor` follows, its coding uses `\xintFor`, which is described later; none of this uses count registers.

```

\let\IsPrime\undefined \let\SmallestFactor\undefined % clean up possible previous mess
\newcommand{\IsPrime}[1] % returns 1 if #1 is prime, and 0 if not
 {\ifnumodd {#1}
 {\ifnumless {#1}{8}
 {\ifnumequal{#1}{1}{0}{1}}% 3,5,7 are primes
 {\if
 \xintloop [3+2]
 \ifnum#1<\numexpr\xintloopindex*\xintloopindex\relax
 \expandafter\xintbreakloopanddo\expandafter1\expandafter.%
 \fi

```

## 5 Additional examples using *xinttools* or *xintexpr* or both

```

\ifnum#1=\numexpr (#1/\xintloopindex)*\xintloopindex\relax
\else
\repeat 00\expandafter0\else\expandafter1\fi
}%
}% END OF THE ODD BRANCH
{\ifnumequal {#1}{2}{1}{0}}% EVEN BRANCH
}%
\catcode_ 11
\newcommand{\SmallestFactor}[1] % returns the smallest prime factor of #1>1
{\ifnumodd {#1}
{\ifnumless {#1}{8}
{#1}% 3,5,7 are primes
{\xintloop [3+2]
\ifnum#1<\numexpr\xintloopindex*\xintloopindex\relax
\xint_afterfi{\xintbreakloopanddo#1.}%
\fi
\ifnum#1=\numexpr (#1/\xintloopindex)*\xintloopindex\relax
\xint_afterfi{\expandafter\xintbreakloopanddo\xintloopindex.}%
\fi
\iftrue\repeat
}%
}% END OF THE ODD BRANCH
{2}% EVEN BRANCH
}%
\catcode_ 8
{\centering
\begin{tabular}{|c|*{10}c|}
\hline
\xintFor #1 in {0,1,2,3,4,5,6,7,8,9}\do {\&\bfseries #1}\\
\hline
\bfseries 0&--&--&2&3&2&5&2&7&2&3\\
\xintFor #1 in {1,2,3,4,5,6,7,8,9}\do
{\bfseries #1%
\xintFor #2 in {0,1,2,3,4,5,6,7,8,9}\do
{\&\SmallestFactor{#1#2}}\\}%
\hline
\end{tabular}\par
}

```

|   | 0  | 1  | 2 | 3  | 4 | 5 | 6 | 7  | 8 | 9  |
|---|----|----|---|----|---|---|---|----|---|----|
| 0 | -- | -- | 2 | 3  | 2 | 5 | 2 | 7  | 2 | 3  |
| 1 | 2  | 11 | 2 | 13 | 2 | 3 | 2 | 17 | 2 | 19 |
| 2 | 2  | 3  | 2 | 23 | 2 | 5 | 2 | 3  | 2 | 29 |
| 3 | 2  | 31 | 2 | 3  | 2 | 5 | 2 | 37 | 2 | 3  |
| 4 | 2  | 41 | 2 | 43 | 2 | 3 | 2 | 47 | 2 | 7  |
| 5 | 2  | 3  | 2 | 53 | 2 | 5 | 2 | 3  | 2 | 59 |
| 6 | 2  | 61 | 2 | 3  | 2 | 5 | 2 | 67 | 2 | 3  |
| 7 | 2  | 71 | 2 | 73 | 2 | 3 | 2 | 7  | 2 | 79 |
| 8 | 2  | 3  | 2 | 83 | 2 | 5 | 2 | 3  | 2 | 89 |
| 9 | 2  | 7  | 2 | 3  | 2 | 5 | 2 | 97 | 2 | 3  |

### 5.3 Miller-Rabin Pseudo-Primality expandably

This section is based on my <http://tex.stackexchange.com/a/165008> post. But I have modified it to use `\xintNewFunction` which is available since 1.2i. This is good opportunity to illustrate how `\xintNewFunction` can be used to define a recursive function (here modular exponentiation.)

The `isPseudoPrime(n)` is usable in `\xintiexpr`-essions and establishes if its (positive) argument is a Miller-Rabin PseudoPrime to the bases 2, 3, 5, 7, 11, 13, 17. If this is true and  $n < 341550071728321$  (which has 15 digits) then  $n$  really is a prime number.

Similarly  $n = 3825123056546413051$  (19 digits) is the smallest composite number which is a strong pseudo prime for bases 2, 3, 5, 7, 11, 13, 17, 19 and 23. It is easy to extend the code below to include these additional tests (we could make the list of tested bases an argument too, now that I think about it.)

For more information see

[https://en.wikipedia.org/wiki/Miller%E2%80%93Rabin\\_primality\\_test#Deterministic\\_variants\\_of\\_the\\_test](https://en.wikipedia.org/wiki/Miller%E2%80%93Rabin_primality_test#Deterministic_variants_of_the_test) and

[http://primes.utm.edu/prove/prove2\\_3.html](http://primes.utm.edu/prove/prove2_3.html)

In particular, according to JÄESCHKE *On strong pseudoprimes to several bases*, Math. Comp., 61 (1993) 915-926, if  $n < 4,759,123,141$  it is enough to establish Rabin-Miller pseudo-primality to bases  $a = 2, 7, 61$  to prove that  $n$  is prime. This range is enough for  $\text{\TeX}$  numbers and we could then write a very fast expandable primality test for such numbers using only `\numexpr`. Left as an exercise...

```
% I ----- Modular Exponentiation
% #1=x, #2=m, #3=N, compute x^m modulo N (with m non negative)
% We will always use it with 1< x < N hence we skip an initial reduction modulo N.

% We can not use \xintdefiifunction for such recursive definition but
% \xintNewFunction succeeds!
\xintNewFunction{powmod}[3]{% x = #1, m = #2, n = #3
 (#2)?
 % m non zero (assume positive), and look if m=1
 {(#2=1)?{#1/:#3}
 {odd(#2)?{(#1*sqr(powmod(#1,#2//2,#3)))/:#3}
 {sqr(powmod(#1,#2//2,#3))/:#3}}}
 % m is zero, return 1
 {1}}

% See http://tex.stackexchange.com/a/165008 for a (probably faster) macro-only approach
% not using \xintexpr.

% II ----- Miller-Rabin compositeness witness

% n=2^k m + 1 with m odd and k at least 1

% Choose 1<x<n.
% compute y=x^m modulo n
% if equals 1 we can't say anything
% if equals n-1 we can't say anything
% else put j=1, and
% compute repeatedly the square, incrementing j by 1 each time,
% thus always we have y^{2^{j-1}}
% -> if at some point n-1 mod n found, we can't say anything and break out
% -> if however we never find n-1 mod n before reaching
% z=y^{2^{k-1}} with j=k
% we then have z^2=x^{n-1}.
% Suppose z is not -1 mod n. If z^2 is 1 mod n, then n can be prime only if
% z is 1 mod n, and we can go back up, until initial y, and we have already
```

## 5 Additional examples using *xinttools* or *xintexpr* or both

```

% excluded y=1. Thus if z is not -1 mod n and z^2 is 1 then n is not prime.
% But if z^2 is not 1, then n is not prime by Fermat. Hence (z not -1 mod n)
% implies (n is composite). (Miller test)

% let's use again xintexpr indecipherable (except to author) syntax. Of course
% doing it with macros only would be faster.

% Here \xintdefiifunction is not usable because not compatible with iter, break, ...
% but \xintNewFunction comes to the rescue.

\xintNewFunction{isCompositeWitness}[4]{% x=#1, n=#2, m=#3, k=#4
 subs((y==1)?{0}
 {iter(y;(j=#4)?{break(!(@==#2-1))}
 {(@==#2-1)?{break(0)}{sqr(@)/:#2}},j=1++)}
 ,y=powmod(#1,#3,#2))}

% III ----- Strong Pseudo Primes

% cf
% http://oeis.org/A014233
% http://mathworld.wolfram.com/Rabin-MillerStrongPseudoprimeTest.html
% http://mathworld.wolfram.com/StrongPseudoprime.html

% check if positive integer <49 si a prime.
% 2,3,5,7,11,13,17,19,23,29,31,37,41,43,47
\def\IsVerySmallPrime #1%
{ \ifnum#1=1 \xintdothis0\fi
 \ifnum#1=2 \xintdothis1\fi
 \ifnum#1=3 \xintdothis1\fi
 \ifnum#1=5 \xintdothis1\fi
 \ifnum#1=\numexpr (#1/2)*2\relax\xintdothis0\fi
 \ifnum#1=\numexpr (#1/3)*3\relax\xintdothis0\fi
 \ifnum#1=\numexpr (#1/5)*5\relax\xintdothis0\fi
 \xintorthat 1}

\xintNewFunction{isPseudoPrime}[1]{% n = #1
 (#1<49)?% use ? syntax to evaluate only what is needed
 {\IsVerySmallPrime{\xintthe#1}}% macro needs to be fed with #1 unlocked.
 {(even(#1))}?
 {0}
 {subs(%
 % L expands to two values m, k hence isCompositeWitness does get
 % its four variables x, n, m, k
 isCompositeWitness(2, #1, L)?
 {0}%
 {isCompositeWitness(3, #1, L)?
 {0}%
 {isCompositeWitness(5, #1, L)?
 {0}%
 {isCompositeWitness(7, #1, L)?
 {0}%
 % above enough for N<3215031751 hence all TeX numbers
 {isCompositeWitness(11, #1, L)?
 {0}%
 % above enough for N<2152302898747, hence all 12-digits numbers
 {isCompositeWitness(13, #1, L)?
 {0}%

```

```

% above enough for N<3474749660383
 {isCompositeWitness(17, #1, L)?
 {0}%
% above enough for N<341550071728321
 {1}%
 }% not needed to comment-out end of lines spaces inside
 }% \xintexpr but this is too much of a habit for me with TeX!
 }% I left some after the ? characters.
 }%
 }%
 }% this computes (m, k) such that $n = 2^k m + 1$, m odd, $k \geq 1$
 , L=iter(#1//2;(even(@))?{@//2}{break(@,k)},k=1++)}%
 }%
 }%
}

% if needed:
%\def\IsPseudoPrime #1{\xinttheiiexpr isPseudoPrime(#1)\relax}

\noindent The smallest prime number at least equal to 3141592653589 is
\xinttheiiexpr
 seq(isPseudoPrime(3141592653589+n)?
 {break(3141592653589+n)}{omit}, n=0++)\relax.
% we could not use 3141592653589++ syntax because it works only with TeX numbers
\par

```

The smallest prime number at least equal to 3141592653589 is 3141592653601.

## 5.4 A table of factorizations

As one more example with `\xintloop` let us use an alignment to display the factorization of some numbers. The loop will actually only play a minor rôle here, just handling the row index, the row contents being almost entirely produced via a macro `\factorize`. The factorizing macro does not use `\xintloop` as it didn't appear to be the convenient tool. As `\factorize` will have to be used on `\xintloopindex`, it has been defined as a delimited macro.

To spare some fractions of a second in the compilation time of this document (which has many many other things to do), 2147483629 and 2147483647, which turn out to be prime numbers, are not given to `factorize` but just typeset directly; this illustrates use of `\xintloopskipnext`.

The code next generates a `table` which has been made into a float appearing on page 57. Here is now the code for factorization; the conditionals use the package provided `\xint_firstoftwo` and `\xint_secondoftwo`, one could have employed rather  $\TeX$ 's own `\@firstoftwo` and `\@secondoftwo`, or, simpler still in  $\TeX$  context, the `\ifnumequal`, `\ifnumless` . . . , utilities from the package `etoolbox` which do exactly that under the hood. Only  $\TeX$  acceptable numbers are treated here, but it would be easy to make a translation and use the `xint` macros, thus extending the scope to big numbers; naturally up to a cost in speed.

The reason for some strange looking expressions is to avoid arithmetic overflow.

```

\catcode`_ 11
\def\abortfactorize #1\xint_secondoftwo\fi #2#3{\fi}

\def\factorize #1.{\ifnum#1=1 \abortfactorize\fi
 \ifnum\numexpr #1-2=\numexpr ((#1/2)-1)*2\relax
 \expandafter\xint_firstoftwo
 \else\expandafter\xint_secondoftwo
 \fi
 {2&\expandafter\factorize\the\numexpr#1/2.}%
 {\factorize_b #1.3.}}%

```

```

\def\factorize_b #1.#2.{\ifnum#1=1 \abortfactorize\fi
 \ifnum\numexpr #1-(#2-1)*#2<#2
 #1\abortfactorize
 \fi
 \ifnum \numexpr #1-#2=\numexpr ((#1/#2)-1)*#2\relax
 \expandafter\xint_firstoftwo
 \else\expandafter\xint_secondoftwo
 \fi
 {#2&\expandafter\factorize_b\the\numexpr#1/#2.#2.}%
 {\expandafter\factorize_b\the\numexpr #1\expandafter.%
 \the\numexpr #2+2.}%}%

\catcode_ 8
\begin{figure*}[ht!]
\centering\phantomsection\label{floatfactorize}\normalcolor
\tabskip1ex
\centeredline{\vbox{\halign {\hfil\strut#\hfil&\hfil#\hfil\cr\noalign{\hrule}
 \xintilooop ["7FFFFFFE0+1]
 \expandafter\bfseries\xintilooopindex &
 \ifnum\xintilooopindex="7FFFFFFD
 \number"7FFFFFFED\cr\noalign{\hrule}
 \expandafter\xintilooopskiptonext
 \fi
 \expandafter\factorize\xintilooopindex.\cr\noalign{\hrule}
 \ifnum\xintilooopindex<"7FFFFFFE
 \repeat
 \bfseries \number"7FFFFFFF&\number "7FFFFFFF\cr\noalign{\hrule}
}}}
\centeredline{A table of factorizations}
\end{figure*}

```

## 5.5 Another table of primes

As a further example, let us dynamically generate a tabular with the first 50 prime numbers after 12345. First we need a macro to test if a (short) number is prime. Such a completely expandable macro was given in [subsection 5.1](#), here we consider a variant which will be slightly more efficient. This new `\IsPrime` has two parameters. The first one is a macro which it redefines to expand to the result of the primality test applied to the second argument. For convenience we use the `etoolbox` wrappers to various `\ifnum` tests, although here there isn't anymore the constraint of complete expandability (but using explicit `\if.. \fi` in tabulars has its quirks); equivalent tests are provided by `xint`, but they have some overhead as they are able to deal with arbitrarily big integers.

```

\def\IsPrime #1#2% #1=\Result, #2=tested number (assumed >0).
{\edef\TheNumber {\the\numexpr #2}% hence #2 may be a count or \numexpr.
 \ifnumodd {\TheNumber}
 {\ifnumgreater {\TheNumber}{1}
 {\edef\ItsSquareRoot{\xintiSqrt \TheNumber}%
 \xintFor ##1 in {\xintintegers [3+2]}\do
 {\ifnumgreater {##1}{\ItsSquareRoot} % ##1 is a \numexpr.
 {\def#1{1}\xintBreakFor}
 }%
 \ifnumequal {\TheNumber}{(\TheNumber/##1)*##1}
 {\def#1{0}\xintBreakFor }
 }%
 }}
{\def#1{0}}% 1 is not prime
\ifnumequal {\TheNumber}{2}{\def#1{1}}{\def#1{0}}%

```

5 Additional examples using `xinttools` or `xintexpr` or both

|            |            |           |           |         |          |         |        |         |
|------------|------------|-----------|-----------|---------|----------|---------|--------|---------|
| 2147483616 | 2          | 2         | 2         | 2       | 2        | 3       | 2731   | 8191    |
| 2147483617 | 6733       | 318949    |           |         |          |         |        |         |
| 2147483618 | 2          | 7         | 367       | 417961  |          |         |        |         |
| 2147483619 | 3          | 3         | 23        | 353     | 29389    |         |        |         |
| 2147483620 | 2          | 2         | 5         | 4603    | 23327    |         |        |         |
| 2147483621 | 14741      | 145681    |           |         |          |         |        |         |
| 2147483622 | 2          | 3         | 17        | 467     | 45083    |         |        |         |
| 2147483623 | 79         | 967       | 28111     |         |          |         |        |         |
| 2147483624 | 2          | 2         | 2         | 11      | 13       | 1877171 |        |         |
| 2147483625 | 3          | 5         | 5         | 5       | 7        | 199     | 4111   |         |
| 2147483626 | 2          | 19        | 37        | 1527371 |          |         |        |         |
| 2147483627 | 47         | 53        | 862097    |         |          |         |        |         |
| 2147483628 | 2          | 2         | 3         | 3       | 59652323 |         |        |         |
| 2147483629 | 2147483629 |           |           |         |          |         |        |         |
| 2147483630 | 2          | 5         | 6553      | 32771   |          |         |        |         |
| 2147483631 | 3          | 137       | 263       | 19867   |          |         |        |         |
| 2147483632 | 2          | 2         | 2         | 2       | 7        | 73      | 262657 |         |
| 2147483633 | 5843       | 367531    |           |         |          |         |        |         |
| 2147483634 | 2          | 3         | 12097     | 29587   |          |         |        |         |
| 2147483635 | 5          | 11        | 337       | 115861  |          |         |        |         |
| 2147483636 | 2          | 2         | 536870909 |         |          |         |        |         |
| 2147483637 | 3          | 3         | 3         | 13      | 6118187  |         |        |         |
| 2147483638 | 2          | 2969      | 361651    |         |          |         |        |         |
| 2147483639 | 7          | 17        | 18046081  |         |          |         |        |         |
| 2147483640 | 2          | 2         | 2         | 3       | 5        | 29      | 43     | 113 127 |
| 2147483641 | 2699       | 795659    |           |         |          |         |        |         |
| 2147483642 | 2          | 23        | 46684427  |         |          |         |        |         |
| 2147483643 | 3          | 715827881 |           |         |          |         |        |         |
| 2147483644 | 2          | 2         | 233       | 1103    | 2089     |         |        |         |
| 2147483645 | 5          | 19        | 22605091  |         |          |         |        |         |
| 2147483646 | 2          | 3         | 3         | 7       | 11       | 31      | 151    | 331     |
| 2147483647 | 2147483647 |           |           |         |          |         |        |         |

A table of factorizations

}

As we used `\xintFor` inside a macro we had to double the `#` in its `#1` parameter. Here is now the code which creates the prime table (the table has been put in a `float`, which should be found on page 58):

```

\newcounter{primecount}
\newcounter{cellcount}
\begin{figure*}[ht!]
\centering
\begin{tabular}{|*{7}c|}
\hline
\setcounter{primecount}{0}\setcounter{cellcount}{0}%
\xintFor #1 in {\xintintegers [12345+2]} \do
% #1 is a \numexpr.
{\IsPrime\Result{#1}%
\ifnumgreater{\Result}{0}
{\stepcounter{primecount}%
\stepcounter{cellcount}%
\ifnumequal {\value{cellcount}}{7}

```



```

{\the#1 \\setcounter{cellcount}{0}}
{\the#1 &}}
{}%
\ifnumequal {\value{primecount}}{50}
{\xintBreakForAndDo
{\multicolumn {6}{l|}{These are the first 50 primes after 12345.}}\}}
{}%
}\hline
\end{tabular}
\end{figure*}

```

|       |                                            |       |       |       |       |       |
|-------|--------------------------------------------|-------|-------|-------|-------|-------|
| 12347 | 12373                                      | 12377 | 12379 | 12391 | 12401 | 12409 |
| 12413 | 12421                                      | 12433 | 12437 | 12451 | 12457 | 12473 |
| 12479 | 12487                                      | 12491 | 12497 | 12503 | 12511 | 12517 |
| 12527 | 12539                                      | 12541 | 12547 | 12553 | 12569 | 12577 |
| 12583 | 12589                                      | 12601 | 12611 | 12613 | 12619 | 12637 |
| 12641 | 12647                                      | 12653 | 12659 | 12671 | 12689 | 12697 |
| 12703 | 12713                                      | 12721 | 12739 | 12743 | 12757 | 12763 |
| 12781 | These are the first 50 primes after 12345. |       |       |       |       |       |

## 5.6 Factorizing again

Here is an *f-expandable* macro which computes the factors of an integer. It uses the *xint* macros only.

```

\catcode\@ 11
\let\factorize\relax
\newcommand\Factorize [1]
{\romannumeral0\expandafter\factorize\expandafter{\romannumeral-`0#1}}%
\newcommand\factorize [1]{\xintiifOne{#1}{ 1}{\factors@a #1.#1};}%
\def\factors@a #1.{\xintiifOdd{#1}
{\factors@c 3.#1.}%
{\expandafter\factors@b \expandafter1\expandafter.\romannumeral0\xinthalff{#1}.}}%
\def\factors@b #1.#2.{\xintiifOne{#2}
{\factors@end {2, #1}}%
{\xintiifOdd{#2}{\factors@c 3.#2.{2, #1}}%
{\expandafter\factors@b \the\numexpr #1+\@ne\expandafter.%
\romannumeral0\xinthalff{#2}.}}%
}%
\def\factors@c #1.#2.{%
\expandafter\factors@d\romannumeral0\xintiividivision {#2}{#1}{#1}{#2}%
}%
\def\factors@d #1#2#3#4{\xintiifNotZero{#2}
{\xintiifGt{#3}{#1}
{\factors@end {#4, 1}}% ultimate quotient is a prime with power 1
{\expandafter\factors@c\the\numexpr #3+\tw@.#4.}}%
{\factors@e 1.#3.#1.}%
}%
\def\factors@e #1.#2.#3.{\xintiifOne{#3}
{\factors@end {#2, #1}}%
{\expandafter\factors@f\romannumeral0\xintiividivision {#3}{#2}{#1}{#2}{#3}}%
}%
\def\factors@f #1#2#3#4#5{\xintiifNotZero{#2}

```

```

{\expandafter\factor@c\the\numexpr #4+\tw@.#5.{#4, #3}}%
{\expandafter\factor@e\the\numexpr #3+\@ne.#4.#1.}%
}%
\def\factor@end #1;{\xintlistwithsep{,}{\xintRevWithBraces {#1}}}%
\catcode\@ 12

```

The macro will be acceptably efficient only with numbers having somewhat small prime factors.

```
\Factorize{16246355912554185673266068721806243461403654781833}
```

```
16246355912554185673266068721806243461403654781833, 13, 5, 17, 8, 29, 5, 37, 6, 41, 4, 59, 6
```

It puts a little stress on the input save stack in order not be bothered with previously gathered things.<sup>47</sup>

Its output is a comma separated list with the number first, then its prime factors with multiplicity. Let's produce something prettier:

```

\catcode_ 11
\def\ShowFactors #1{\expandafter\ShowFactors_a\romannumeral-_0\Factorize{#1},\relax,\relax,}
\def\ShowFactors_a #1,{#1=\ShowFactors_b}
\def\ShowFactors_b #1,#2,{\if\relax#1\else#1^{#2}\expandafter\ShowFactors_b\fi}
\catcode_ 8
$$\ShowFactors{16246355912554185673266068721806243461403654781833}$$
16246355912554185673266068721806243461403654781833 = 135178295376414596

```

If we only considered small integers, we could write pure `\numexpr` methods which would be very much faster (especially if we had a table of small primes prepared first) but still ridiculously slow compared to any non expandable implementation, not to mention use of programming languages directly accessing the CPU registers. . .

## 5.7 The Quick Sort algorithm illustrated

First a completely expandable macro which sorts a comma separated list of numbers.<sup>48</sup>

The `\QSx` macro expands its list argument, which may thus be a macro; its comma separated items must expand to integers or decimal numbers or fractions or scientific notation as acceptable to `xintfrac`, but if an item is itself some (expandable) macro, this macro will be expanded each time the item is considered in a comparison test! This is actually good if the macro expands in one step to the digits, and there are many many digits, but bad if the macro needs to do many computations. Thus `\QSx` should be used with either explicit numbers or with items being macros expanding in one step to the numbers (particularly if these numbers are very big).

If the interest is only in  $\text{\TeX}$  integers, then one should replace the `\xintifCmp` macro with a suitable conditional, possibly helped by tools such as `\ifnumgreater`, `\ifnumequal` and `\ifnumless` from `etoolbox` ( $\text{\TeX}$  only; I didn't see a direct equivalent to `\xintifCmp`.) Or, if we are dealing with decimal numbers with at most four+four digits, then one should use suitable `\ifdim` tests. Naturally this will boost consequently the speed, from having skipped all the overhead in parsing fractions and scientific numbers as are acceptable by `xintfrac` macros, and subsequent treatment.

```

% THE QUICK SORT ALGORITHM EXPANDABLY
% \usepackage{xintfrac} in the preamble (latex)
\makeatletter
% use extra safe delimiters
\catcode\! 3 \catcode\? 3
\def\QSx {\romannumeral0\qsx }%
% first we check if empty list (else \qsx@finish will not find a comma)

```

<sup>47</sup> 2015/11/18 I have not revisited this code for a long time, and perhaps I could improve it now with some new techniques.

<sup>48</sup> The code in earlier versions of this manual handled inputs composed of braced items. I have switched to comma separated inputs on the occasion of <http://tex.stackexchange.com/a/273084>. The version here is like `code 3` on <http://tex.stackexchange.com> (which is about 3x faster than the earlier code it replaced in this manual) with a modification to make it more efficient if the data has many repeated values. A faster routine (for sorting hundreds of values) is provided as `code 6` at the link mentioned in the footnote, it is based on Merge Sort, but limited to inputs which one can handle as  $\text{\TeX}$  dimensions. This `code 6` could be extended to handle more general numbers, as acceptable by `xintfrac`. I have also written a non expandable version, which is even faster, but this matters really only when handling hundreds or rather thousands of values.

## 5 Additional examples using *xinttools* or *xintexpr* or both

```

\def\qxsx #1{\expandafter\qxs@a\romannumeral-`0#1,!,%}%
\def\qxs@a #1{\ifx,#1\expandafter\qxs@abort\else
\expandafter\qxs@start\fi #1}%
\def\qxs@abort #1{ }%
\def\qxs@start {\expandafter\qxs@finish\romannumeral0\qxs@b,%}%
\def\qxs@finish ,#1{ #1}%
%
% we check if empty of single and if not pick up the first as Pivot:
\def\qxs@b ,#1#2,#3{\ifx?#3\xintdothis\qxs@empty\fi
\ifx!#3\xintdothis\qxs@single\fi
\xintorthat\qxs@separate {#1#2}{#3}{#1#2#3}%
\def\qxs@empty #1#2#3#4#5{ }%
\def\qxs@single #1#2#3#4#5?{, #4}%
\def\qxs@separate #1#2#3#4#5#6,%
{
\ifx!#5\expandafter\qxs@separate@done\fi
\xintifCmp {#5#6}{#4}%
\qxs@separate@appendtosmaller
\qxs@separate@appendtoequal
\qxs@separate@appendtogreater {#5#6}{#1}{#2}{#3}{#4}%
}%
%
\def\qxs@separate@appendtoequal #1#2{\qxs@separate {#2,#1}}%
\def\qxs@separate@appendtogreater #1#2#3{\qxs@separate {#2}{#3,#1}}%
\def\qxs@separate@appendtosmaller #1#2#3#4{\qxs@separate {#2}{#3}{#4,#1}}%
%
\def\qxs@separate@done\xintifCmp #1%
\qxs@separate@appendtosmaller
\qxs@separate@appendtoequal
\qxs@separate@appendtogreater #2#3#4#5#6#7?%
{
\expandafter\qxs@f\expandafter {\romannumeral0\qxs@b #4,!,%}{\qxs@b #5,!,%}{#3}%
}%
%
\def\qxs@f #1#2#3{#2, #3#1}%
%
\catcode`! 12 \catcode`? 12
\makeatother

% EXAMPLE
\begingroup
\edef\z {\Qsx {1.0, 0.5, 0.3, 1.5, 1.8, 2.0, 1.7, 0.4, 1.2, 1.4,
1.3, 1.1, 0.7, 1.6, 0.6, 0.9, 0.8, 0.2, 0.1, 1.9}}
\meaning\z

\def\ a {3.123456789123456789}\def\ b {3.123456789123456788}
\def\ c {3.123456789123456790}\def\ d {3.123456789123456787}
\oodef\z {\Qsx { \ a, \ b, \ c, \ d}}%
% The space before \ a to let it not be expanded during the conversion from CSV
% values to List. The \oodef expands exactly twice (via a bunch of \expandafter's)
\meaning\z
\endgroup

```

macro:->0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0

macro:->\d , \b , \ a , \ c (the spaces after \d, etc... come from the use of the \meaning primitive.)

The choice of pivot as first element is bad if the list is already almost sorted. Let's add a

variant which will pick up the pivot index randomly. The previous routine worked also internally with comma separated lists, but for a change this one will use internally lists of braced items (the initial conversion via `\xintCSVtoList` handles all potential spurious space problems).

```
% QuickSort expandably on comma separated values with random choice of pivots
% ===== Requires availability of \pdfuniformdeviate =====
% \usepackage{xintfrac, xinttools} in preamble
\makeatletter
\def\QSx {\romannumeral0\qsx }% This is a f-expandable macro.
% This converts from comma separated values on input and back on output.
% **** NOTE: these steps (and the other ones too, actually) are costly if input
% has thousands of items.
\def\qsx #1{\xintlistwithsep{, }%
 {\expandafter\qsx@sort@a\expandafter{\romannumeral0\xintcsvtolist{#1}}}%
%
% we check if empty or single or double and if not pick up the first as Pivot:
\def\qsx@sort@a #1%
 {\expandafter\qsx@sort@b\expandafter{\romannumeral0\xintlength{#1}}{#1}}%
\def\qsx@sort@b #1{\ifcase #1
 \expandafter\qsx@sort@empty
 \or\expandafter\qsx@sort@single
 \or\expandafter\qsx@sort@double
 \else\expandafter\qsx@sort@c\fi {#1}}%
\def\qsx@sort@empty #1#2{ }%
\def\qsx@sort@single #1#2{#2}%
\catcode_ 11
\def\qsx@sort@double #1#2{\xintifGt #2{\xint_exchangetwo_keepbraces}{#2}}%
\catcode_ 8
\def\qsx@sort@c #1#2{%
 \expandafter\qsx@sort@sep@a\expandafter
 {\romannumeral0\xintnthelt{\pdfuniformdeviate #1+\@ne}{#2}}{#2}}%
\def\qsx@sort@sep@a #1{\qsx@sort@sep@loop {}{}{}{#1}}%
\def\qsx@sort@sep@loop #1#2#3#4#5%
{%
 \ifx?#5\expandafter\qsx@sort@sep@done\fi
 \xintifCmp {#5}{#4}%
 \qsx@sort@sep@appendtosmaller
 \qsx@sort@sep@appendtoequal
 \qsx@sort@sep@appendtogreater {#5}{#1}{#2}{#3}{#4}%
}%
%
\def\qsx@sort@sep@appendtoequal #1#2{\qsx@sort@sep@loop {#2}{#1}}%
\def\qsx@sort@sep@appendtogreater #1#2#3{\qsx@sort@sep@loop {#2}{#3}{#1}}%
\def\qsx@sort@sep@appendtosmaller #1#2#3#4{\qsx@sort@sep@loop {#2}{#3}{#4}{#1}}%
%
\def\qsx@sort@sep@done\xintifCmp #1%
 \qsx@sort@sep@appendtosmaller
 \qsx@sort@sep@appendtoequal
 \qsx@sort@sep@appendtogreater #2#3#4#5#6%
{%
 \expandafter\qsx@sort@recurse\expandafter
 {\romannumeral0\qsx@sort@a {#4}}{\qsx@sort@a {#5}}{#3}%
}%
%
\def\qsx@sort@recurse #1#2#3{#2#3#1}%
%
\makeatother
```

## 5 Additional examples using `xinttools` or `xintexpr` or both

```
% EXAMPLES
\begingroup
\edef\z {\QSx {1.0, 0.5, 0.3, 1.5, 1.8, 2.0, 1.7, 0.4, 1.2, 1.4,
 1.3, 1.1, 0.7, 1.6, 0.6, 0.9, 0.8, 0.2, 0.1, 1.9}}
\meaning\z

\def\ a {3.123456789123456789}\def\ b {3.123456789123456788}
\def\ c {3.123456789123456790}\def\ d {3.123456789123456787}
\oodef\z {\QSx { \a, \b, \c, \d}}%
% The space before \a to let it not be expanded during the conversion from CSV
% values to List. The \oodef expands exactly twice (via a bunch of \expandafter's)
\meaning\z
```

```
\def\somenumbers{%
3997.6421, 8809.9358, 1805.4976, 5673.6478, 3179.1328, 1425.4503, 4417.7691,
2166.9040, 9279.7159, 3797.6992, 8057.1926, 2971.9166, 9372.2699, 9128.4052,
1228.0931, 3859.5459, 8561.7670, 2949.6929, 3512.1873, 1698.3952, 5282.9359,
1055.2154, 8760.8428, 7543.6015, 4934.4302, 7526.2729, 6246.0052, 9512.4667,
7423.1124, 5601.8436, 4433.5361, 9970.4849, 1519.3302, 7944.4953, 4910.7662,
3679.1515, 8167.6824, 2644.4325, 8239.4799, 4595.1908, 1560.2458, 6098.9677,
3116.3850, 9130.5298, 3236.2895, 3177.6830, 5373.1193, 5118.4922, 2743.8513,
8008.5975, 4189.2614, 1883.2764, 9090.9641, 2625.5400, 2899.3257, 9157.1094,
8048.4216, 3875.6233, 5684.3375, 8399.4277, 4528.5308, 6926.7729, 6941.6278,
9745.4137, 1875.1205, 2755.0443, 9161.1524, 9491.1593, 8857.3519, 4290.0451,
2382.4218, 3678.2963, 5647.0379, 1528.7301, 2627.8957, 9007.9860, 1988.5417,
2405.1911, 5065.8063, 5856.2141, 8989.8105, 9349.7840, 9970.3013, 8105.4062,
3041.7779, 5058.0480, 8165.0721, 9637.7196, 1795.0894, 7275.3838, 5997.0429,
7562.6481, 8084.0163, 3481.6319, 8078.8512, 2983.7624, 3925.4026, 4931.5812,
1323.1517, 6253.0945}%
```

```
\oodef\z {\QSx \somenumbers}%
\hsize 87\fontcharwd\font`0 \setbox0\hbox{\kern\fontcharwd\font`0}%
\lccode`~\32 \lowercase{\def~}{\discretionary}{~}{\copy0}}\catcode32 13
\noindent\ \ \ \scantokens\expandafter{\meaning\z}\par
\endgroup
```

macro:->0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0

macro:->\d , \b , \a , \c

macro:->1055.2154, 1228.0931, 1323.1517, 1425.4503, 1519.3302, 1528.7301, 1560.2458, 1698.3952, 1795.0894, 1805.4976, 1875.1205, 1883.2764, 1988.5417, 2166.9040, 2382.4218, 2405.1911, 2625.5400, 2627.8957, 2644.4325, 2743.8513, 2755.0443, 2899.3257, 2949.6929, 2971.9166, 2983.7624, 3041.7779, 3116.3850, 3177.6830, 3179.1328, 3236.2895, 3481.6319, 3512.1873, 3678.2963, 3679.1515, 3797.6992, 3859.5459, 3875.6233, 3925.4026, 3997.6421, 4189.2614, 4290.0451, 4417.7691, 4433.5361, 4528.5308, 4595.1908, 4910.7662, 4931.5812, 4934.4302, 5058.0480, 5065.8063, 5118.4922, 5282.9359, 5373.1193, 5601.8436, 5647.0379, 5673.6478, 5684.3375, 5856.2141, 5997.0429, 6098.9677, 6246.0052, 6253.0945, 6926.7729, 6941.6278, 7275.3838, 7423.1124, 7526.2729, 7543.6015, 7562.6481, 7944.4953, 8008.5975, 8048.4216, 8057.1926, 8078.8512, 8084.0163, 8105.4062, 8165.0721, 8167.6824, 8239.4799, 8399.4277, 8561.7670, 8760.8428, 8809.9358, 8857.3519, 8989.8105, 9007.9860, 9090.9641, 9128.4052, 9130.5298, 9157.1094, 9161.1524, 9279.7159, 9349.7840, 9372.2699, 9491.1593, 9512.4667, 9637.7196, 9745.4137, 9970.3013, 9970.4849

All these examples were with numbers which may have been handled via `\ifdim` tests rather than `\xintifCmp` from `xintfrac`; naturally that would have been faster. For a yet faster routine (based however on the Merge Sort and using the `\pdfescapestring` PDF<sub>TeX</sub> primitive) see code 6 at <http://tex.stackexchange.com/a/273084>.

We then turn to a graphical illustration of the algorithm.<sup>49</sup> For simplicity the pivot is always chosen as the first list item. Then we also give a variant which picks up the last item as pivot.

```
% in LaTeX preamble:
% \usepackage{xintfrac, xinttools}
% \usepackage{color}
% or, when using Plain TeX:
% \input xintfrac.sty \input xinttools.sty
% \input color.tex
%
% Color definitions
\definecolor{LEFT}{RGB}{216,195,88}
\definecolor{RIGHT}{RGB}{208,231,153}
\definecolor{INERT}{RGB}{199,200,194}
\definecolor{INERTpivot}{RGB}{237,237,237}
\definecolor{PIVOT}{RGB}{109,8,57}
% Start of macro definitions
\makeatletter
% \catcode`? 3 % a bit too paranoid. Normal ? will do.
%
% argument will never be empty
\def\QS@cmp@a #1{\QS@cmp@b #1??}%
\def\QS@cmp@b #1{\noexpand\QS@sep@a\@ne{#1}\QS@cmp@d {#1}}%
\def\QS@cmp@d #1#2{\ifx ?#2\expandafter\QS@cmp@done\fi
\XintifCmp {#1}{#2}\tw@\@ne\z@{#2}\QS@cmp@d {#1}}%
\def\QS@cmp@done #1{??}%
%
\def\QS@sep@a #1{\QSLr\QS@sep@L #1\thr@@?#1\thr@@?#1\thr@@?}%
\def\QS@sep@L #1#2{\ifcase #1{#2}\or\or\else\expandafter\QS@sep@I@start\fi \QS@sep@L}%
\def\QS@sep@I@start\QS@sep@L {\noexpand\empty?\QSIr\QS@sep@I}%
\def\QS@sep@I #1#2{\ifcase#1\or{#2}\or\else\expandafter\QS@sep@R@start\fi\QS@sep@I}%
\def\QS@sep@R@start\QS@sep@I {\noexpand\empty?\QSRr\QS@sep@R}%
\def\QS@sep@R #1#2{\ifcase#1\or\or{#2}\else\expandafter\QS@sep@done\fi\QS@sep@R}%
\def\QS@sep@done\QS@sep@R {\noexpand\empty?}%
%
\def\QS@loop {%
\Xintloop
% pivot phase
\def\QS@pivotcount{0}%
\let\QSLr\DecoLEFTwithPivot \let\QSIr \DecoINERT
\let\QSRr\DecoRIGHTwithPivot \let\QSIrr\DecoINERT
\centerline{\QS@list}%
% sorting phase
\ifnum\QS@pivotcount>\z@
\def\QSLr {\QS@cmp@a}\def\QSRr {\QS@cmp@a}%
\def\QSIr {\QSIrr}\let\QSIrr\relax
\edef\QS@list{\QS@list}% compare
\let\QSLr\relax\let\QSRr\relax\let\QSIr\relax
\edef\QS@list{\QS@list}% separate
\def\QSLr ##1##2?{\ifx\empty##1\else\noexpand \QSLr {{##1}##2}\fi}%
\def\QSIr ##1##2?{\ifx\empty##1\else\noexpand \QSIr {{##1}##2}\fi}%
\def\QSRr ##1##2?{\ifx\empty##1\else\noexpand \QSRr {{##1}##2}\fi}%
\edef\QS@list{\QS@list}% gather
\let\QSLr\DecoLEFT \let\QSRr\DecoRIGHT
```

<sup>49</sup> I have rewritten the routine to do only once (and not thrice) the needed calls to `\XintifCmp`, up to the price of one additional `\edef`, although due to the context execution time on our side is not an issue and moreover is anyhow overwhelmed by the TikZ's activities. Simultaneously I have updated the code <http://tex.stackexchange.com/a/142634/4686>. The variant with the choice of pivot on the right has more overhead: the reason is simply that we do not convert the data into an array, but maintain a list of tokens with self-reorganizing delimiters.

## 5 Additional examples using *xinttools* or *xintexpr* or both

```

\let\QSIr\DecoINERTwithPivot \let\QSIrr\DecoINERT
\centerline{\QS@list}%
\repeat }%
%
% \xintFor* loops handle gracefully empty lists.
\def\DecoLEFT #1{\xintFor* ##1 in {#1} \do {\colorbox{LEFT}{##1}}}%
\def\DecoINERT #1{\xintFor* ##1 in {#1} \do {\colorbox{INERT}{##1}}}%
\def\DecoRIGHT #1{\xintFor* ##1 in {#1} \do {\colorbox{RIGHT}{##1}}}%
\def\DecoPivot #1{\begingroup\color{PIVOT}\advance\fbboxsep-\fbboxrule\fbbox{#1}\endgroup}%
%
\def\DecoLEFTwithPivot #1{\xdef\QS@pivotcount{\the\numexpr\QS@pivotcount+\@ne}%
\xintFor* ##1 in {#1} \do
{\xintifForFirst {\DecoPivot {##1}}{\colorbox{LEFT}{##1}}}%
\def\DecoINERTwithPivot #1{\xdef\QS@pivotcount{\the\numexpr\QS@pivotcount+\@ne}%
\xintFor* ##1 in {#1} \do
{\xintifForFirst {\colorbox{INERTpiv}{##1}}{\colorbox{INERT}{##1}}}%
\def\DecoRIGHTwithPivot #1{\xdef\QS@pivotcount{\the\numexpr\QS@pivotcount+\@ne}%
\xintFor* ##1 in {#1} \do
{\xintifForFirst {\DecoPivot {##1}}{\colorbox{RIGHT}{##1}}}%
%
\def\QuickSort #1{% warning: not compatible with empty #1.
% initialize, doing conversion from comma separated values to a list of braced items
\edef\QS@list{\noexpand\QSRr{\xintCSVtoList{#1}}}% many \edef's are to follow anyhow
% earlier I did a first drawing of the list, here with the color of RIGHT elements,
% but the color should have been for example white, anyway I drop this first line
%\let\QSRr\DecoRIGHT
%\par\centerline{\QS@list}%
%
% loop as many times as needed
\QS@loop }%
%
% \catcode`? 12 % in case we had used a funny ? as delimiter.
\makeatother
%% End of macro definitions.
%% Start of Example
\begingroup\offinterlineskip
\small
% \QuickSort {1.0, 0.5, 0.3, 1.5, 1.8, 2.0, 1.7, 0.4, 1.2, 1.4,
% 1.3, 1.1, 0.7, 1.6, 0.6, 0.9, 0.8, 0.2, 0.1, 1.9}
% \medskip
% with repeated values
\QuickSort {1.0, 0.5, 0.3, 0.8, 1.5, 1.8, 2.0, 1.7, 0.4, 1.2, 1.4,
1.3, 1.1, 0.7, 0.3, 1.6, 0.6, 0.3, 0.8, 0.2, 0.8, 0.7, 1.2}
\endgroup

```

|     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1.0 | 0.5 | 0.3 | 0.8 | 1.5 | 1.8 | 2.0 | 1.7 | 0.4 | 1.2 | 1.4 | 1.3 | 1.1 | 0.7 | 0.3 | 1.6 | 0.6 | 0.3 | 0.8 | 0.2 | 0.8 | 0.7 | 1.2 |
| 0.5 | 0.3 | 0.8 | 0.4 | 0.7 | 0.3 | 0.6 | 0.3 | 0.8 | 0.2 | 0.8 | 0.7 | 1.0 | 1.5 | 1.8 | 2.0 | 1.7 | 1.2 | 1.4 | 1.3 | 1.1 | 1.6 | 1.2 |
| 0.5 | 0.3 | 0.8 | 0.4 | 0.7 | 0.3 | 0.6 | 0.3 | 0.8 | 0.2 | 0.8 | 0.7 | 1.0 | 1.5 | 1.8 | 2.0 | 1.7 | 1.2 | 1.4 | 1.3 | 1.1 | 1.6 | 1.2 |
| 0.3 | 0.4 | 0.3 | 0.3 | 0.2 | 0.5 | 0.8 | 0.7 | 0.6 | 0.8 | 0.8 | 0.7 | 1.0 | 1.2 | 1.4 | 1.3 | 1.1 | 1.2 | 1.5 | 1.8 | 2.0 | 1.7 | 1.6 |
| 0.3 | 0.4 | 0.3 | 0.3 | 0.2 | 0.5 | 0.8 | 0.7 | 0.6 | 0.8 | 0.8 | 0.7 | 1.0 | 1.2 | 1.4 | 1.3 | 1.1 | 1.2 | 1.5 | 1.8 | 2.0 | 1.7 | 1.6 |
| 0.2 | 0.3 | 0.3 | 0.3 | 0.4 | 0.5 | 0.7 | 0.6 | 0.7 | 0.8 | 0.8 | 0.8 | 1.0 | 1.1 | 1.2 | 1.2 | 1.4 | 1.3 | 1.5 | 1.7 | 1.6 | 1.8 | 2.0 |
| 0.2 | 0.3 | 0.3 | 0.3 | 0.4 | 0.5 | 0.7 | 0.6 | 0.7 | 0.8 | 0.8 | 0.8 | 1.0 | 1.1 | 1.2 | 1.2 | 1.4 | 1.3 | 1.5 | 1.7 | 1.6 | 1.8 | 2.0 |
| 0.2 | 0.3 | 0.3 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.7 | 0.8 | 0.8 | 0.8 | 1.0 | 1.1 | 1.2 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 | 2.0 |
| 0.2 | 0.3 | 0.3 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.7 | 0.8 | 0.8 | 0.8 | 1.0 | 1.1 | 1.2 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 | 2.0 |
| 0.2 | 0.3 | 0.3 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.7 | 0.8 | 0.8 | 0.8 | 1.0 | 1.1 | 1.2 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 | 2.0 |

Here is the variant which always picks the pivot as the rightmost element.

```
\makeatletter
```



## 5 Additional examples using `xinttools` or `xintexpr` or both

```
%
\def\QS@cmp@a #1{\noexpand\QS@sep@a\expandafter\QS@cmp@d\expandafter
 {\romannumeral0\xintnthelt{-1}{#1}}#1??}%

%
\def\DecoLEFTwithPivot #1{\xdef\QS@pivotcount{\the\numexpr\QS@pivotcount+\@ne}%
 \xintFor* ##1 in {#1} \do
 {\xintifForLast {\DecoPivot {##1}}{\colorbox{LEFT}{##1}}}}
\def\DecoINERTwithPivot #1{\xdef\QS@pivotcount{\the\numexpr\QS@pivotcount+\@ne}%
 \xintFor* ##1 in {#1} \do
 {\xintifForLast {\colorbox{INERTpiv}{##1}}{\colorbox{INERT}{##1}}}}
\def\DecoRIGHTwithPivot #1{\xdef\QS@pivotcount{\the\numexpr\QS@pivotcount+\@ne}%
 \xintFor* ##1 in {#1} \do
 {\xintifForLast {\DecoPivot {##1}}{\colorbox{RIGHT}{##1}}}}
\def\QuickSort #1{%
 % initialize, doing conversion from comma separated values to a list of braced items
 \edef\QS@list{\noexpand\QSLr {\xintCSVtoList{#1}}}% many \edef's are to follow anyhow
 %
 % loop as many times as needed
 \QS@loop}%
\makeatother
\begingroup\offinterlineskip
\small
% \QuickSort {1.0, 0.5, 0.3, 1.5, 1.8, 2.0, 1.7, 0.4, 1.2, 1.4,
% 1.3, 1.1, 0.7, 1.6, 0.6, 0.9, 0.8, 0.2, 0.1, 1.9}
% \medskip
% with repeated values
\QuickSort {1.0, 0.5, 0.3, 0.8, 1.5, 1.8, 2.0, 1.7, 0.4, 1.2, 1.4,
 1.3, 1.1, 0.7, 0.3, 1.6, 0.6, 0.3, 0.8, 0.2, 0.8, 0.7, 1.2}
\endgroup
```

|     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1.0 | 0.5 | 0.3 | 0.8 | 1.5 | 1.8 | 2.0 | 1.7 | 0.4 | 1.2 | 1.4 | 1.3 | 1.1 | 0.7 | 0.3 | 1.6 | 0.6 | 0.3 | 0.8 | 0.2 | 0.8 | 0.7 | 1.2 |
| 1.0 | 0.5 | 0.3 | 0.8 | 0.4 | 1.1 | 0.7 | 0.3 | 0.6 | 0.3 | 0.8 | 0.2 | 0.8 | 0.7 | 1.2 | 1.2 | 1.5 | 1.8 | 2.0 | 1.7 | 1.4 | 1.3 | 1.6 |
| 1.0 | 0.5 | 0.3 | 0.8 | 0.4 | 1.1 | 0.7 | 0.3 | 0.6 | 0.3 | 0.8 | 0.2 | 0.8 | 0.7 | 1.2 | 1.2 | 1.5 | 1.8 | 2.0 | 1.7 | 1.4 | 1.3 | 1.6 |
| 0.5 | 0.3 | 0.4 | 0.3 | 0.6 | 0.3 | 0.2 | 0.7 | 0.7 | 1.0 | 0.8 | 1.1 | 0.8 | 0.8 | 1.2 | 1.2 | 1.5 | 1.4 | 1.3 | 1.6 | 1.8 | 2.0 | 1.7 |
| 0.5 | 0.3 | 0.4 | 0.3 | 0.6 | 0.3 | 0.2 | 0.7 | 0.7 | 1.0 | 0.8 | 1.1 | 0.8 | 0.8 | 1.2 | 1.2 | 1.5 | 1.4 | 1.3 | 1.6 | 1.8 | 2.0 | 1.7 |
| 0.2 | 0.5 | 0.3 | 0.4 | 0.3 | 0.6 | 0.3 | 0.7 | 0.7 | 0.8 | 0.8 | 0.8 | 1.0 | 1.1 | 1.2 | 1.2 | 1.3 | 1.5 | 1.4 | 1.6 | 1.7 | 1.8 | 2.0 |
| 0.2 | 0.5 | 0.3 | 0.4 | 0.3 | 0.6 | 0.3 | 0.7 | 0.7 | 0.8 | 0.8 | 0.8 | 1.0 | 1.1 | 1.2 | 1.2 | 1.3 | 1.5 | 1.4 | 1.6 | 1.7 | 1.8 | 2.0 |
| 0.2 | 0.3 | 0.3 | 0.3 | 0.5 | 0.4 | 0.6 | 0.7 | 0.7 | 0.8 | 0.8 | 0.8 | 1.0 | 1.1 | 1.2 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 | 2.0 |
| 0.2 | 0.3 | 0.3 | 0.3 | 0.5 | 0.4 | 0.6 | 0.7 | 0.7 | 0.8 | 0.8 | 0.8 | 1.0 | 1.1 | 1.2 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 | 2.0 |
| 0.2 | 0.3 | 0.3 | 0.3 | 0.5 | 0.4 | 0.6 | 0.7 | 0.7 | 0.8 | 0.8 | 0.8 | 1.0 | 1.1 | 1.2 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 | 2.0 |
| 0.2 | 0.3 | 0.3 | 0.3 | 0.5 | 0.4 | 0.6 | 0.7 | 0.7 | 0.8 | 0.8 | 0.8 | 1.0 | 1.1 | 1.2 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 | 2.0 |
| 0.2 | 0.3 | 0.3 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.7 | 0.8 | 0.8 | 0.8 | 1.0 | 1.1 | 1.2 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 | 2.0 |
| 0.2 | 0.3 | 0.3 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.7 | 0.8 | 0.8 | 0.8 | 1.0 | 1.1 | 1.2 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 | 2.0 |
| 0.2 | 0.3 | 0.3 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.7 | 0.8 | 0.8 | 0.8 | 1.0 | 1.1 | 1.2 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 | 2.0 |

The choice of the first or last item as pivot is not a good one as nearly ordered lists will take quadratic time. But for explaining the algorithm via a graphical interpretation, it is not that bad. If one wanted to pick up the pivot randomly, the routine would have to be substantially rewritten: in particular the `\Deco.withPivot` macros need to know where the pivot is, and currently this is implemented by using either `\xintifForFirst` or `\xintifForLast`.

## 6 Macros of the *xintkernel* package

|    |                                         |    |    |                                   |    |
|----|-----------------------------------------|----|----|-----------------------------------|----|
| .1 | <code>\odef, \oodef, \fdef</code> ..... | 66 | .4 | <code>\xintLastItem</code> .....  | 67 |
| .2 | <code>\xintReverseOrder</code> .....    | 66 | .5 | <code>\xintreplicate</code> ..... | 67 |
| .3 | <code>\xintLength</code> .....          | 66 | .6 | <code>\xintgobble</code> .....    | 67 |

The *xintkernel* package contains mainly the common code base for handling the load-order of the bundle packages, the management of catcodes at loading time, definition of common constants and macro utilities which are used throughout the code etc ... it is automatically loaded by all packages of the bundle.

It provides a few macros possibly useful in other contexts.

### 6.1 `\odef, \oodef, \fdef`

`\oodef\controlsequence {<stuff>}` does

```
\expandafter\expandafter\expandafter\def
\expandafter\expandafter\expandafter\controlsequence
\expandafter\expandafter\expandafter{<stuff>}
```

This works only for a single `\controlsequence`, with no parameter text, even without parameters. An alternative would be:

```
\def\oodef #1#{\def\oodefparametertext{#1}%
\expandafter\expandafter\expandafter\expandafter
\expandafter\expandafter\expandafter\def
\expandafter\expandafter\expandafter\oodefparametertext
\expandafter\expandafter\expandafter }
```

but it does not allow `\global` as prefix, and, besides, would have anyhow its use (almost) limited to parameter texts without macro parameter tokens (except if the expanded thing does not see them, or is designed to deal with them).

There is a similar macro `\odef` with only one expansion of the replacement text `<stuff>`, and `\fdef` which expands fully `<stuff>` using `\romannumeral-`0`.

They can be prefixed with `\global`. It appears than `\fdef` is generally a bit faster than `\edef` when expanding macros from the *xint* bundle, when the result has a few dozens of digits. `\oodef` needs thousands of digits it seems to become competitive.

### 6.2 `\xintReverseOrder`

*n* ★ `\xintReverseOrder{<list>}` does not do any expansion of its argument and just reverses the order of the tokens in the `<list>`. Braces are removed once and the enclosed material, now unbraced, does not get reversed. Unprotected spaces (of any character code) are gobbled.

```
\xintReverseOrder{\xintDigitsOf\xintiPow {2}{100}\to\Stuff}
gives: \Stuff\to1002\xintiPow\xintDigitsOf
```

### 6.3 `\xintLength`

*n* ★ `\xintLength{<list>}` counts how many tokens (or braced items) there are (possibly none). It does no expansion of its argument, so to use it to count things in the replacement text of a macro `\x` one should do `\expandafter\xintLength\expandafter{x}`. Blanks between items are not counted. See also `\xintNthElt{0}` (from *xinttools*) which first *f*-expands its argument and then applies the same code.

```
\xintLength {\xintiPow {2}{100}}=3
≠ \xintLen {\xintiPow {2}{100}}=31
```

## 6.4 `\xintLastItem`

`\xintLastItem{<list>}` returns the last item (unbraced) of its argument. If the list has no items the output is empty.

It does no expansion, which should be obtained via suitable `\expandafter`'s. See also `\xintNthElt{-1}` from *xinttools* which obtains the same result (but with another code) after having however *f-expanded* its argument first.

## 6.5 `\xintreplicate`

`\romannumeral\xintreplicate{x}{<stuff>}` is simply copied over from  $\TeX$ 's `\prg_replicate:nn` with some minor changes.<sup>50</sup> It does not do any expansion of its second argument but inserts it in the upcoming token stream precisely *x* times. Using it with a negative *x* raises no error and does nothing.<sup>51</sup>

Note that expansion must be triggered by a `\romannumeral`.

## 6.6 `\xintgobble`

`\romannumeral\xintgobble{x}` is a Gobbling macro written in the spirit of  $\TeX$ 's `\prg_replicate:nn` (which I cloned as `\xintreplicate`.) It gobbles *x* tokens upstream, with *x* allowed to be as large as 531440. Don't use it with *x* < 0.

Note that expansion must be triggered by a `\romannumeral`.

`\xintgobble` looks as if it must be related to `\xintTrim` from *xinttools*, but the latter uses different code (using directly `\xintgobble` is not possible because one must make sure not to gobble more than the number of available items; and counting available items first is an overhead which `\xintTrim` avoids.) It is rather `\xintKeep` with a negative first argument which hands over to `\xintgobble` (because in that case it is needed to count anyhow beforehand the number of items, hence `\xintgobble` can then be used safely.)

I wrote an `\xintcount` in the same spirit as `\xintreplicate` and `\xintgobble`. But it needs to be counting hundreds of tokens to be worth its salt compared to `\xintLength`.

<sup>50</sup> I started with the code from Joseph Wright's answer to <http://tex.stackexchange.com/questions/16189/repeat-command-n-times>.

<sup>51</sup> This behavior may change in future.

## 7 Macros of the `xinttools` package

|     |                                                                 |    |     |                                                                      |    |
|-----|-----------------------------------------------------------------|----|-----|----------------------------------------------------------------------|----|
| .1  | <code>\xintRevWithBraces</code> .....                           | 68 | .14 | <code>\xintilooop</code> , <code>\xintilooopindex</code> ,           |    |
| .2  | <code>\xintZapFirstSpaces</code> ,                              |    |     | <code>\xintouterilooopindex</code> ,                                 |    |
|     | <code>\xintZapLastSpaces</code> , <code>\xintZapSpaces</code> , |    |     | <code>\xintbreakilooop</code> , <code>\xintbreakilooopanddo</code> , |    |
|     | <code>\xintZapSpacesB</code> .....                              | 68 |     | <code>\xintilooopskiptonext</code> ,                                 |    |
| .3  | <code>\xintCSVtoList</code> .....                               | 69 |     | <code>\xintilooopskipandredo</code> .....                            | 77 |
| .4  | <code>\xintNthElt</code> .....                                  | 70 | .15 | <code>\xintApplyInline</code> .....                                  | 79 |
| .5  | <code>\xintKeep</code> .....                                    | 71 | .16 | <code>\xintFor</code> , <code>\xintFor*</code> .....                 | 81 |
| .6  | <code>\xintKeepUnbraced</code> .....                            | 71 | .17 | <code>\xintifForFirst</code> , <code>\xintifForLast</code> .....     | 83 |
| .7  | <code>\xintTrim</code> .....                                    | 72 | .18 | <code>\xintBreakFor</code> , <code>\xintBreakForAndDo</code> ....    | 83 |
| .8  | <code>\xintTrimUnbraced</code> .....                            | 72 | .19 | <code>\xintintegers</code> , <code>\xintdimensions</code> ,          |    |
| .9  | <code>\xintListWithSep</code> .....                             | 72 |     | <code>\xinrationals</code> .....                                     | 84 |
| .10 | <code>\xintApply</code> .....                                   | 73 | .20 | <code>\xintForpair</code> , <code>\xintForthree</code> ,             |    |
| .11 | <code>\xintApplyUnbraced</code> .....                           | 73 |     | <code>\xintForfour</code> .....                                      | 85 |
| .12 | <code>\xintSeq</code> .....                                     | 74 | .21 | <code>\xintAssign</code> .....                                       | 85 |
| .13 | <code>\xintloop</code> , <code>\xintbreakloop</code> ,          |    | .22 | <code>\xintAssignArray</code> .....                                  | 86 |
|     | <code>\xintbreakloopanddo</code> ,                              |    | .23 | <code>\xintDigitsOf</code> .....                                     | 86 |
|     | <code>\xintloopskiptonext</code> .....                          | 74 | .24 | <code>\xintRelaxArray</code> .....                                   | 87 |

These utilities used to be provided within the `xint` package; since 1.09g (2013/11/22) they have been moved to an independently usable package `xinttools`, which has none of the `xint` facilities regarding big numbers. Whenever relevant release 1.09h has made the macros `\long` so they accept `\par` tokens on input.

First the completely expandable utilities up to `\xintilooop`, then the non expandable utilities.

This section contains various concrete examples and ends with a [completely expandable implementation of the Quick Sort algorithm](#) together with a graphical illustration of its action.

See also 6.2 and 6.3 which come with package `xintkernel`, automatically loaded by `xinttools`.

### 7.1 `\xintRevWithBraces`

*f* ★ `\xintRevWithBraces{⟨list⟩}` first does the *f*-expansion of its argument then it reverses the order of the tokens, or braced material, it encounters, maintaining existing braces and adding a brace pair around each naked token encountered. Space tokens (in-between top level braces or naked tokens) are gobbled. This macro is mainly thought out for use on a `⟨list⟩` of such braced material; with such a list as argument the *f*-expansion will only hit against the first opening brace, hence do nothing, and the braced stuff may thus be macros one does not want to expand.

```
\edef\x{\xintRevWithBraces{12345}}
\meaning\x:macro->{5}{4}{3}{2}{1}
\edef\y{\xintRevWithBraces\x}
\meaning\y:macro->{1}{2}{3}{4}{5}
```

The examples above could be defined with `\edef`'s because the braced material did not contain macros. Alternatively:

```
\expandafter\def\expandafter\w\expandafter
{\romannumeral0\xintrevwithbraces{{\A}{\B}{\C}{\D}{\E}}}
\meaning\w:macro->{\E }{\D }{\C }{\B }{\A }
```

*n* ★ The macro `\xintReverseWithBracesNoExpand` does the same job without the initial expansion of its argument.

### 7.2 `\xintZapFirstSpaces`, `\xintZapLastSpaces`, `\xintZapSpaces`, `\xintZapSpacesB`

*n* ★ `\xintZapFirstSpaces{⟨stuff⟩}` does not do any expansion of its argument, nor brace removal of any

sort, nor does it alter  $\langle stuff \rangle$  in anyway apart from stripping away all *leading* spaces.

This macro will be mostly of interest to programmers who will know what I will now be talking about. *The essential points, naturally, are the complete expandability and the fact that no brace removal nor any other alteration is done to the input.*

$\text{\TeX}$ 's input scanner already converts consecutive blanks into single space tokens, but `\xintZapFirstSpaces` handles successfully also inputs with consecutive multiple space tokens. However, it is assumed that  $\langle stuff \rangle$  does not contain (except inside braced sub-material) space tokens of character code distinct from 32.

It expands in two steps, and if the goal is to apply it to the expansion text of `\x` to define `\y`, then one should do: `\expandafter\def\expandafter\y\expandafter {\romannumeral0\expandafter\xintZapFirstSpaces\expandafter{x}}`.

Other use case: inside a macro as `\edef\x{\xintZapFirstSpaces {#1}}` assuming naturally that `#1` is compatible with such an `\edef` once the leading spaces have been stripped.

```
\xintZapFirstSpaces { \a { \X } { \b \Y } }->\a { \X } { \b \Y }+++
```

- $n \star$  `\xintZapLastSpaces` $\{\langle stuff \rangle\}$  does not do any expansion of its argument, nor brace removal of any sort, nor does it alter  $\langle stuff \rangle$  in anyway apart from stripping away all *ending* spaces. The same remarks as for `\xintZapFirstSpaces` apply.

```
\xintZapLastSpaces { \a { \X } { \b \Y } }->\a { \X } { \b \Y }+++
```

- $n \star$  `\xintZapSpaces` $\{\langle stuff \rangle\}$  does not do any expansion of its argument, nor brace removal of any sort, nor does it alter  $\langle stuff \rangle$  in anyway apart from stripping away all *leading* and all *ending* spaces. The same remarks as for `\xintZapFirstSpaces` apply.

```
\xintZapSpaces { \a { \X } { \b \Y } }->\a { \X } { \b \Y }+++
```

- $n \star$  `\xintZapSpacesB` $\{\langle stuff \rangle\}$  does not do any expansion of its argument, nor does it alter  $\langle stuff \rangle$  in anyway apart from stripping away all leading and all ending spaces and possibly removing one level of braces if  $\langle stuff \rangle$  had the shape `<spaces>\{braced\}<spaces>`. The same remarks as for `\xintZapFirstSpaces` apply.

```
\xintZapSpacesB { \a { \X } { \b \Y } }->\a { \X } { \b \Y }+++
```

```
\xintZapSpacesB { { \a { \X } { \b \Y } } }->\a { \X } { \b \Y }+++
```

The spaces here at the start and end of the output come from the braced material, and are not removed (one would need a second application for that; recall though that the `xint` zapping macros do not expand their argument).

### 7.3 `\xintCSVtoList`

- $f \star$  `\xintCSVtoList` $\{a,b,c,\dots,z\}$  returns  $\{a\}\{b\}\{c\}\dots\{z\}$ . A *list* is by convention in this manual simply a succession of tokens, where each braced thing will count as one item ('items' are defined according to the rules of  $\text{\TeX}$  for fetching undelimited parameters of a macro, which are exactly the same rules as for  $\text{\LaTeX}$  and macro arguments [they are the same things]). The word 'list' in 'comma separated list of items' has its usual linguistic meaning, and then an 'item' is what is delimited by commas.

So `\xintCSVtoList` takes on input a 'comma separated list of items' and converts it into a ' $\text{\TeX}$  list of braced items'. The argument to `\xintCSVtoList` may be a macro: it will first be *f-expanded*. Hence the item before the first comma, if it is itself a macro, will be expanded which may or may not be a good thing. A space inserted at the start of the first item serves to stop that expansion (and disappears). The macro `\xintCSVtoListNoExpand` does the same job without the initial expansion of the list argument.

Apart from that no expansion of the items is done and the list items may thus be completely arbitrary (and even contain perilous stuff such as unmatched `\if` and `\fi` tokens).

Contiguous spaces and tab characters, are collapsed by  $\text{\TeX}$  into single spaces. All such spaces around commas<sup>52</sup> are removed, as well as the spaces at the start and the spaces at the end of the

<sup>52</sup> and multiple space tokens are not a problem; but those at the top level (not hidden inside braces) *must* be of character code 32.

list.<sup>53</sup> The items may contain explicit `\par`'s or empty lines (converted by the  $\TeX$  input parsing into `\par` tokens).

```
\xintCSVtoList { 1 , { 2 , 3 , 4 , 5 } , a , {b,T} U , { c , d } , { {x , y} } }
->{1}{2 , 3 , 4 , 5}{a}{b,T} U { c , d } { {x , y} }
```

One sees on this example how braces protect commas from sub-lists to be perceived as delimiters of the top list. Braces around an entire item are removed, even when surrounded by spaces before and/or after. Braces for sub-parts of an item are not removed.

We observe also that there is a slight difference regarding the brace stripping of an item: if the braces were not surrounded by spaces, also the initial and final (but no other) spaces of the enclosed material are removed. This is the only situation where spaces protected by braces are nevertheless removed.

From the rules above: for an empty argument (only spaces, no braces, no comma) the output is `{ }` (a list with one empty item), for `<opt. spaces>{ }<opt. spaces>` the output is `{ }` (again a list with one empty item, the braces were removed), for `{ }` the output is `{ }` (again a list with one empty item, the braces were removed and then the inner space was removed), for `{ }` the output is `{ }` (again a list with one empty item, the initial space served only to stop the expansion, so this was like `{ }` as input, the braces were removed and the inner space was stripped), for `{ }` the output is `{ }` (this time the ending space of the first item meant that after brace removal the inner spaces were kept; recall though that  $\TeX$  collapses on input consecutive blanks into one space token), for `,,` the output consists of two consecutive empty items `{ }{ }`. Recall that on output everything is braced, a `{ }` is an 'empty' item. Most of the above is mainly irrelevant for every day use, apart perhaps from the fact to be noted that an empty input does not give an empty output but a one-empty-item list (it is as if an ending comma was always added at the end of the input).

```
\def\y{ \a,\b,\c,\d,\e} \xintCSVtoList\y->{\a }{\b }{\c }{\d }{\e }
\def\t {{\if},\ifnum,\ifx,\ifdim,\ifcat,\ifmmode}
\xintCSVtoList\t->{\if }{\ifnum }{\ifx }{\ifdim }{\ifcat }{\ifmmode }
```

The results above were automatically displayed using  $\TeX$ 's primitive `\meaning`, which adds a space after each control sequence name. These spaces are not in the actual braced items of the produced lists. The first items `\a` and `\if` were either preceded by a space or braced to prevent expansion. The macro `\xintCSVtoListNoExpand` would have done the same job without the initial expansion of the list argument, hence no need for such protection but if `\y` is defined as `\def\y{\a,\b,\c,\d,\e}` we then must do:

```
\expandafter\xintCSVtoListNoExpand\expandafter {\y}
```

Else, we may have direct use:

```
\xintCSVtoListNoExpand {\if,\ifnum,\ifx,\ifdim,\ifcat,\ifmmode}
->{\if }{\ifnum }{\ifx }{\ifdim }{\ifcat }{\ifmmode }
```

Again these spaces are an artefact from the use in the source of the document of `\meaning` (or rather here, `\detokenize`) to display the result of using `\xintCSVtoListNoExpand` (which is done for real in this document source).

For the similar conversion from comma separated list to braced items list, but without removal of spaces around the commas, there is `\xintCSVtoListNonStripped` and `\xintCSVtoListNonStrippedNoExpand`.

## 7.4 `\xintNthElt`

`\xintNthElt{x}{<list>}` gets (expandably) the *x*th item of the *<list>*. A braced item will lose one level of brace pairs. The token list is first *f-expanded*.

Items are counted starting at one.

```
\xintNthElt {3}{agh}\u{zzz}\v{Z} is zzz
\xintNthElt {3}{agh}\u{zzz}\v{Z} is {zzz}
```

<sup>53</sup> let us recall that this is all done completely expandably... There is absolutely no alteration of any sort of the item apart from the stripping of initial and final space tokens (of character code 32) and brace removal if and only if the item apart from initial and final spaces (or more generally multiple `char 32` space tokens) is braced.

```

\intNthElt {2}{{agh}\u{zzz}\v{Z}} is \u
\intNthElt {37}{{\xintiiFac {100}}}=9 is the thirty-seventh digit of 100!.
\intNthElt {10}{{\xintFtoCv {566827/208524}}}=1457/536
is the tenth convergent of 566827/208524 (uses xintcfrac package).
\intNthElt {7}{{\xintCSVtoList {1,2,3,4,5,6,7,8,9}}}=7
\intNthElt {0}{{\xintCSVtoList {1,2,3,4,5,6,7,8,9}}}=9
\intNthElt {-3}{{\xintCSVtoList {1,2,3,4,5,6,7,8,9}}}=7

```

If  $x=0$ , the macro returns the *length* of the expanded list: this is not equivalent to `\xintLength` which does no pre-expansion. And it is different from `\xintLen` which is to be used only on integers or fractions.

If  $x<0$ , the macro returns the  $|x|$ th element from the end of the list. Thus for example  $x=-1$  will fetch the last item of the list.

```

\intNthElt {-5}{{agh}\u{zzz}\v{Z}} is {agh}

```

The macro `\xintNthEltNoExpand` does the same job but without first expanding the list argument:  
`\xintNthEltNoExpand {-4}{{u\v{w} T\x{y}\z}}` is T.

If  $x$  is strictly larger (in absolute value) than the length of the list then `\xintNthElt` produces empty contents.

## 7.5 `\xintKeep`

`\xintKeep{x}{{list}}` expands the token list argument  $L$  and produces a new list, depending on the value of  $x$ :

- if  $x>0$ , the new list contains the first  $x$  items from  $L$  (counting starts at one.) *Each such item will be output within a brace pair.* Use `\xintKeepUnbraced` if this is not desired. This means that if the list item was braced to start with, there is no modification, but if it was a token without braces, then it acquires them.
- if  $x=\text{length}(L)$ , the new list is the old one with all its items now braced.
- if  $x=0$  the empty list is returned.
- if  $x<0$  the last  $|x|$  elements compose the output in the same order as in the initial list; as the macro proceeds by removing head items the kept items end up in output as they were in input: no added braces.
- if  $x=-\text{length}(L)$  the output is identical with the input.

`\xintKeepNoExpand` does the same without first *expanding* its list argument.

```

\def\test {\xintKeep {17}{{\xintKeep {-69}{{\xintSeq {1}{100}}}}}\meaning\test\par
\noindent\def\test {\xintKeep {7}{{1}{2}{3}{4}{5}{6}{7}{8}{9}}}\meaning\test\par
\noindent\def\test {\xintKeep {-7}{{1}{2}{3}{4}{5}{6}{7}{8}{9}}}\meaning\test\par
\noindent\def\test {\xintKeep {7}{{123456789}}}\meaning\test\par
\noindent\def\test {\xintKeep {-7}{{123456789}}}\meaning\test\par

```

```
macro:->{32}{33}{34}{35}{36}{37}{38}{39}{40}{41}{42}{43}{44}{45}{46}{47}{48}
```

```
macro:->{1}{2}{3}{4}{5}{6}{7}
```

```
macro:->{3}{4}{5}{6}{7}{8}{9}
```

```
macro:->{1}{2}{3}{4}{5}{6}{7}
```

```
macro:->3456789
```

## 7.6 `\xintKeepUnbraced`

Same as `\xintKeep` but no brace pairs are added around the kept items from the head of the list in the case  $x>0$ : each such item will lose one level of braces. Thus, to remove braces from all items of the list, one can use `\xintKeepUnbraced` with its first argument larger than the length of the list; the same is obtained from `\xintListWithSep{{list}}`. But the new list will then have generally many more items than the original ones, corresponding to the unbraced original items.

For  $x<0$  the macro is no different from `\xintKeep`. Hence the name is a bit misleading because brace removal will happen only if  $x>0$ .



`\xintKeepUnbracedNoExpand` does the same without first *f-expanding* its list argument.

```
\fdef\test {\xintKeepUnbraced {10}{\xintSeq {1}{100}}}\meaning\test\par
\noindent\fdef\test {\xintKeepUnbraced {7}{\{1\}{2\}{3\}{4\}{5\}{6\}{7\}{8\}{9\}}}\meaning\test\par
\noindent\fdef\test {\xintKeepUnbraced {-7}{\{1\}{2\}{3\}{4\}{5\}{6\}{7\}{8\}{9\}}}\meaning\test\par
\noindent\fdef\test {\xintKeepUnbraced {7}{123456789}}\meaning\test\par
\noindent\fdef\test {\xintKeepUnbraced {-7}{123456789}}\meaning\test\par
```

```
macro:->12345678910
```

```
macro:->1234567
```

```
macro:->{3}{4}{5}{6}{7}{8}{9}
```

```
macro:->1234567
```

```
macro:->3456789
```

## 7.7 `\xintTrim`

num *x* *f* ★ `\xintTrim{x}{\langle list \rangle}` expands the list argument and gobbles its first *x* elements.

- if *x*>0, the first *x* items from *L* are gobbled. The remaining items are not modified.
- if *x*≥length(*L*), the returned list is empty.
- if *x*=0 the original list is returned (with no added braces.)
- if *x*<0 the last *|x|* items of the list are removed. *The head items end up braced in the output.* Use `\xintTrimUnbraced` if this is not desired.
- if *x*≤-length(*L*) the output is empty.

`\xintTrimNoExpand` does the same without first *f-expanding* its list argument.

```
\fdef\test {\xintTrim {17}{\xintTrim {-69}{\xintSeq {1}{100}}}\meaning\test\par
\noindent\fdef\test {\xintTrim {7}{\{1\}{2\}{3\}{4\}{5\}{6\}{7\}{8\}{9\}}}\meaning\test\par
\noindent\fdef\test {\xintTrim {-7}{\{1\}{2\}{3\}{4\}{5\}{6\}{7\}{8\}{9\}}}\meaning\test\par
\noindent\fdef\test {\xintTrim {7}{123456789}}\meaning\test\par
\noindent\fdef\test {\xintTrim {-7}{123456789}}\meaning\test\par
```

```
macro:->{18}{19}{20}{21}{22}{23}{24}{25}{26}{27}{28}{29}{30}{31}
```

```
macro:->{8}{9}
```

```
macro:->{1}{2}
```

```
macro:->89
```

```
macro:->{1}{2}
```

## 7.8 `\xintTrimUnbraced`

Same as `\xintTrim` but in case of a negative *x* (cutting items from the tail), the kept items from the head are not enclosed in brace pairs. They will lose one level of braces. The name is a bit misleading because when *x*>0 there is no brace-stripping done on the kept items, because the macro works simply by gobbling the head ones.

`\xintTrimUnbracedNoExpand` does the same without first *f-expanding* its list argument.

```
\fdef\test {\xintTrimUnbraced {-90}{\xintSeq {1}{100}}}\meaning\test\par
\noindent\fdef\test {\xintTrimUnbraced {7}{\{1\}{2\}{3\}{4\}{5\}{6\}{7\}{8\}{9\}}}\meaning\test\par
\noindent\fdef\test {\xintTrimUnbraced {-7}{\{1\}{2\}{3\}{4\}{5\}{6\}{7\}{8\}{9\}}}\meaning\test\par
\noindent\fdef\test {\xintTrimUnbraced {7}{123456789}}\meaning\test\par
\noindent\fdef\test {\xintTrimUnbraced {-7}{123456789}}\meaning\test\par
```

```
macro:->12345678910
```

```
macro:->{8}{9}
```

```
macro:->12
```

```
macro:->89
```

```
macro:->12
```

## 7.9 `\xintListWithSep`

`\xintListWithSep{sep}{\langle list \rangle}` inserts the separator *sep* in-between all items of the given list.

*nf* ★

The items will be unbraced. The separator may be a macro but will not be pre-expanded. The list argument is *f*-expanded.

```
\edef\foo {\xintListWithSep{,}{\{1\}{2\}{3\}}}\meaning\foo\newline
\edef\foo {\xintListWithSep{:}{\xintiiFac{20}}}\meaning\foo\par
```

```
macro:->1,2,3
```

```
macro:->2:4:3:2:9:0:2:0:0:8:1:7:6:6:4:0:0:0:0
```

An empty input gives an empty output, a singleton gives a singleton, and the separator is used starting with at least two elements. Using an empty separator has the net effect of unbracing the braced items constituting the  $\langle list \rangle$  (then the new list will generally have many more ``items'' than the original one).

*nn* ★

The macro `\xintListWithSepNoExpand` does the same job without the initial expansion.

## 7.10 `\xintApply`

*ff* ★

`\xintApply{\macro}{\langle list \rangle}` expandably applies the one parameter macro `\macro` to each item in the  $\langle list \rangle$  given as second argument and returns a new list with these outputs: each item is given one after the other as parameter to `\macro` which is expanded at that time (as usual, i.e. fully for what comes first), the results are braced and output together as a succession of braced items (if `\macro` is defined to start with a space, the space will be gobbled and the `\macro` will not be expanded; it is allowed to have its own arguments, the list items serve as last arguments to `\macro`). Hence `\xintApply{\macro}{\{1\}{2\}{3\}}` returns `\macro{1}\macro{2}\macro{3}` where all instances of `\macro` have been already *f*-expanded.

Being expandable, `\xintApply` is useful for example inside alignments where implicit groups make standard loops constructs usually fail. In such situation it is often not wished that the new list elements be braced, see `\xintApplyUnbraced`. The `\macro` does not have to be expandable: `\xintApply` will try to expand it, the expansion may remain partial.

The  $\langle list \rangle$  may itself be some macro expanding (in the previously described way) to the list of tokens to which the macro `\macro` will be applied. For example, if the  $\langle list \rangle$  expands to some positive number, then each digit will be replaced by the result of applying `\macro` on it.

```
\def\macro #1{\the\numexpr 9-#1\relax}
\xintApply\macro{\xintiiFac {20}}=7567097991823359999
```

*fn* ★

The macro `\xintApplyNoExpand` does the same job without the first initial expansion which gave the  $\langle list \rangle$  of braced tokens to which `\macro` is applied.

## 7.11 `\xintApplyUnbraced`

*ff* ★

`\xintApplyUnbraced{\macro}{\langle list \rangle}` is like `\xintApply`. The difference is that after having expanded its list argument, and applied `\macro` in turn to each item from the list, it reassembles the outputs without enclosing them in braces. The net effect is the same as doing

```
\xintListWithSep {}{\xintApply {\macro}{\langle list \rangle}}
```

This is useful for preparing a macro which will itself define some other macros or make assignments, as the scope will not be limited by brace pairs.

```
\def\macro #1{\expandafter\def\csname myself#1\endcsname {#1}}
\xintApplyUnbraced\macro{\elta}{eltb}{eltc}
\begin{enumerate}[nosep,label=(\arabic{*})]
\item \meaning\myselfelta
\item \meaning\myselfeltb
\item \meaning\myselfeltc
\end{enumerate}
```

```
(1) macro:->elta
```


```
(2) macro:->eltb
```

```
(3) macro:->eltc
```

*fn* ★


The macro `\xintApplyUnbracedNoExpand` does the same job without the first initial expansion which gave the  $\langle list \rangle$  of braced tokens to which `\macro` is applied.

## 7.12 `\xintSeq`

 `\xintSeq[d]{x}{y}` generates expandably `{x}{x+d}`... up to and possibly including `{y}` if `d>0` or down to and including `{y}` if `d<0`. Naturally `{y}` is omitted if `y-x` is not a multiple of `d`. If `d=0` the macro returns `{x}`. If `y-x` and `d` have opposite signs, the macro returns nothing. If the optional argument `d` is omitted it is taken to be the sign of `y-x`. Hence `\xintSeq {1}{0}` is not empty but `{1}{0}`. But `\xintSeq [1]{1}{0}` is empty.

The arguments `x` and `y` are expanded inside a `\numexpr` so they may be count registers or a  $\TeX$  `\value{counternam}`, or arithmetic with such things.

```
\xintListWithSep{\hskip2pt plus 1pt minus 1pt }{\xintSeq {12}{-25}}
12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0, -1, -2, -3, -4, -5, -6, -7, -8, -9, -10, -11, -12, -13, -14, -15,
-16, -17, -18, -19, -20, -21, -22, -23, -24, -25
\xintiSum{\xintSeq [3]{1}{1000}}
167167
```

 When the macro is used without the optional argument `d`, it can only generate up to about 5000 numbers, the precise value depends upon some  $\TeX$  memory parameter (input save stack).

With the optional argument `d` the macro proceeds differently (but less efficiently) and does not stress the input save stack.

## 7.13 `\xintloop`, `\xintbreakloop`, `\xintbreakloopanddo`, `\xintloopskiptonext`

☆ `\xintloop<stuff>\if<test>...\repeat` is an expandable loop compatible with nesting. However to break out of the loop one almost always need some un-expandable step. The cousin `\xintilloop` is `\xintloop` with an embedded expandable mechanism allowing to exit from the loop. The iterated macros may contain `\par` tokens or empty lines.

If a sub-loop is to be used all the material from the start of the main loop and up to the end of the entire subloop should be braced; these braces will be removed and do not create a group. The simplest to allow the nesting of one or more sub-loops is to brace everything between `\xintloop` and `\repeat`, being careful not to leave a space between the closing brace and `\repeat`.

As this loop and `\xintilloop` will primarily be of interest to experienced  $\TeX$  macro programmers, my description will assume that the user is knowledgeable enough. Some examples in this document will be perhaps more illustrative than my attempts at explanation of use.

One can abort the loop with `\xintbreakloop`; this should not be used inside the final test, and one should expand the `\fi` from the corresponding test before. One has also `\xintbreakloopanddo` whose first argument will be inserted in the token stream after the loop; one may need a macro such as `\xint_afterfi` to move the whole thing after the `\fi`, as a simple `\expandafter` will not be enough.

One will usually employ some count registers to manage the exit test from the loop; this breaks expandability, see `\xintilloop` for an expandable integer indexed loop. Use in alignments will be complicated by the fact that cells create groups, and also from the fact that any encountered un-expandable material will cause the  $\TeX$  input scanner to insert `\endtemplate` on each encountered `&` or `\cr`; thus `\xintbreakloop` may not work as expected, but the situation can be resolved via `\xint_t_firstofone{&}` or use of `\TAB` with `\def\tab{&}`. It is thus simpler for alignments to use rather than `\xintloop` either the expandable `\xintApplyUnbraced` or the non-expandable but alignment compatible `\xintApplyInline`, `\xintFor` or `\xintFor*`.

As an example, let us suppose we have two macros `\A{<i>}{<j>}` and `\B{<i>}{<j>}` behaving like (small) integer valued matrix entries, and we want to define a macro `\C{<i>}{<j>}` giving the matrix product (`i` and `j` may be count registers). We will assume that `\A[I]` expands to the number of rows, `\A[J]` to the number of columns and want the produced `\C` to act in the same manner. The code is very dispendious in use of `\count` registers, not optimized in any way, not made very robust (the defined macro can not have the same name as the first two matrices for example), we just wanted to

quickly illustrate use of the nesting capabilities of `\xintloop`.<sup>54</sup>

```

\newcount\rowmax \newcount\colmax \newcount\summax
\newcount\rowindex \newcount\colindex \newcount\sumindex
\newcount\tmpcount
\makeatletter
\def\MatrixMultiplication #1#2#3{%
 \rowmax #1[I]\relax
 \colmax #2[J]\relax
 \summax #1[J]\relax
 \rowindex 1
 \xintloop % loop over row index i
 {\colindex 1
 \xintloop % loop over col index k
 {\tmpcount 0
 \sumindex 1
 \xintloop % loop over intermediate index j
 \advance\tmpcount \numexpr #1\rowindex\sumindex*#2\sumindex\colindex\relax
 \ifnum\sumindex<\summax
 \advance\sumindex 1
 \repeat }%
 \expandafter\edef\csname\string#3{\the\rowindex.\the\colindex}\endcsname
 {\the\tmpcount}%
 \ifnum\colindex<\colmax
 \advance\colindex 1
 \repeat }%
 \ifnum\rowindex<\rowmax
 \advance\rowindex 1
 \repeat
 \expandafter\edef\csname\string#3{I}\endcsname{\the\rowmax}%
 \expandafter\edef\csname\string#3{J}\endcsname{\the\colmax}%
 \def #3##1{\ifx[##1\expandafter\Matrix@helper@size
 \else\expandafter\Matrix@helper@entry\fi #3{##1}}%
 }%
 \def\Matrix@helper@size #1#2#3{\csname\string#1{#3}\endcsname }%
 \def\Matrix@helper@entry #1#2#3%
 {\csname\string#1{\the\numexpr#2.\the\numexpr#3}\endcsname }%
 \def\A #1{\ifx[#1\expandafter\A@size
 \else\expandafter\A@entry\fi {#1}}%
 \def\A@size #1#2{\ifx I#2\else4\fi}% 3rows, 4columns
 \def\A@entry #1#2{\the\numexpr #1+#2-1\relax}% not pre-computed...
 \def\B #1{\ifx[#1\expandafter\B@size
 \else\expandafter\B@entry\fi {#1}}%
 \def\B@size #1#2{\ifx I#2\else3\fi}% 4rows, 3columns
 \def\B@entry #1#2{\the\numexpr #1-#2\relax}% not pre-computed...
 \makeatother
 \MatrixMultiplication\A\B\C \MatrixMultiplication\C\C\D
 \MatrixMultiplication\C\D\E \MatrixMultiplication\C\E\F
 \begin{multicols}2
 \[\begin{pmatrix}
 \A11&\A12&\A13&\A14\\
 \A21&\A22&\A23&\A24\\
 \A31&\A32&\A33&\A34
 \end{pmatrix}
 \times
 \begin{pmatrix}

```

<sup>54</sup> for a more sophisticated implementation of matrix multiplication, inclusive of determinants, inverses, and display utilities, with entries big integers or decimal numbers or even fractions see <http://tex.stackexchange.com/a/143035/4686> from November 11, 2013.

```

\B11&\B12&\B13\\
\B21&\B22&\B23\\
\B31&\B32&\B33\\
\B41&\B42&\B43
\end{pmatrix}
=
\begin{pmatrix}
\C11&\C12&\C13\\
\C21&\C22&\C23\\
\C31&\C32&\C33
\end{pmatrix}\]
\[\begin{pmatrix}
\C11&\C12&\C13\\
\C21&\C22&\C23\\
\C31&\C32&\C33
\end{pmatrix}^2 = \begin{pmatrix}
\D11&\D12&\D13\\
\D21&\D22&\D23\\
\D31&\D32&\D33
\end{pmatrix}\]
\[\begin{pmatrix}
\C11&\C12&\C13\\
\C21&\C22&\C23\\
\C31&\C32&\C33
\end{pmatrix}^3 = \begin{pmatrix}
\E11&\E12&\E13\\
\E21&\E22&\E23\\
\E31&\E32&\E33
\end{pmatrix}\]
\[\begin{pmatrix}
\C11&\C12&\C13\\
\C21&\C22&\C23\\
\C31&\C32&\C33
\end{pmatrix}^4 = \begin{pmatrix}
\F11&\F12&\F13\\
\F21&\F22&\F23\\
\F31&\F32&\F33
\end{pmatrix}\]
\end{multicols}

```

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \end{pmatrix} \times \begin{pmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 20 & 10 & 0 \\ 26 & 12 & -2 \\ 32 & 14 & -4 \end{pmatrix}$$

$$\begin{pmatrix} 20 & 10 & 0 \\ 26 & 12 & -2 \\ 32 & 14 & -4 \end{pmatrix}^3 = \begin{pmatrix} 20880 & 10160 & -560 \\ 24624 & 11968 & -688 \\ 28368 & 13776 & -816 \end{pmatrix}$$

$$\begin{pmatrix} 20 & 10 & 0 \\ 26 & 12 & -2 \\ 32 & 14 & -4 \end{pmatrix}^2 = \begin{pmatrix} 660 & 320 & -20 \\ 768 & 376 & -16 \\ 876 & 432 & -12 \end{pmatrix}$$

$$\begin{pmatrix} 20 & 10 & 0 \\ 26 & 12 & -2 \\ 32 & 14 & -4 \end{pmatrix}^4 = \begin{pmatrix} 663840 & 322880 & -18080 \\ 781632 & 380224 & -21184 \\ 899424 & 437568 & -24288 \end{pmatrix}$$

## 7.14 `\xintloop`, `\xintloopindex`, `\xintouterloopindex`, `\xintbreakloop`, `\xintbreakloopanddo`, `\xintloopskiptonext`, `\xintloopskipandredo`

☆ `\xintloop[start+delta](stuff)\if<test> ... \repeat` is a completely expandable nestable loop. complete expandability depends naturally on the actual iterated contents, and complete expansion will not be achievable under a sole *f-expansion*, as is indicated by the hollow star in the margin; thus the loop can be used inside an `\edef` but not inside arguments to the package macros. It can be used inside an `\xintexpr...\relax`. The `[start+delta]` is mandatory, not optional.

This loop benefits via `\xintloopindex` to (a limited access to) the integer index of the iteration. The starting value `start` (which may be a `\count`) and increment `delta` (*id.*) are mandatory arguments. A space after the closing square bracket is not significant, it will be ignored. Spaces inside the square brackets will also be ignored as the two arguments are first given to a `\numexpr...\relax`. Empty lines and explicit `\par` tokens are accepted.

As with `\xintloop`, this tool will mostly be of interest to advanced users. For nesting, one puts inside braces all the material from the start (immediately after `[start+delta]`) and up to and inclusive of the inner loop, these braces will be removed and do not create a loop. In case of nesting, `\xintouterloopindex` gives access to the index of the outer loop. If needed one could write on its model a macro giving access to the index of the outer outer loop (or even to the *n*th outer loop).

The `\xintloopindex` and `\xintouterloopindex` can not be used inside braces, and generally speaking this means they should be expanded first when given as argument to a macro, and that this macro receives them as delimited arguments, not braced ones. Or, but naturally this will break expandability, one can assign the value of `\xintloopindex` to some `\count`. Both `\xintloopindex` and `\xintouterloopindex` extend to the literal representation of the index, thus in `\ifnum` tests, if it comes last one has to correctly end the macro with a `\space`, or encapsulate it in a `\numexpr...\relax`.

When the repeat-test of the loop is, for example, `\ifnum\xintloopindex<10 \repeat`, this means that the last iteration will be with `\xintloopindex=10` (assuming `delta=1`). There is also `\ifnum\xintloopindex=10 \else\repeat` to get the last iteration to be the one with `\xintloopindex=10`.

One has `\xintbreakloop` and `\xintbreakloopanddo` to abort the loop. The syntax of `\xintbreakloopanddo` is a bit surprising, the sequence of tokens to be executed after breaking the loop is not within braces but is delimited by a dot as in:

```
\xintbreakloopanddo <afterloop>.etc.. etc... \repeat
```

The reason is that one may wish to use the then current value of `\xintloopindex` in `<afterloop>` but it can't be within braces at the time it is evaluated. However, it is not that easy as `\xintloopindex` must be expanded before, so one ends up with code like this:

```
\expandafter\xintbreakloopanddo\expandafter\macro\xintloopindex.%
etc.. etc.. \repeat
```

As moreover the `\fi` from the test leading to the decision of breaking out of the loop must be cleared out of the way, the above should be a branch of an expandable conditional test, else one needs something such as:

```
\xint_afterfi{\expandafter\xintbreakloopanddo\expandafter\macro\xintloopindex.}%
\fi etc..etc.. \repeat
```

There is `\xintloopskiptonext` to abort the current iteration and skip to the next, `\xintloopskipandredo` to skip to the end of the current iteration and redo it with the same value of the index (something else will have to change for this not to become an eternal loop...).

Inside alignments, if the looped-over text contains a `&` or a `\cr`, any un-expandable material before a `\xintloopindex` will make it fail because of `\endtemplate`; in such cases one can always either replace `&` by a macro expanding to it or replace it by a suitable `\firstofone{&}`, and similarly for `\cr`.

As an example, let us construct an `\edef\z{...}` which will define `\z` to be a list of prime numbers:

```
\begingroup
```

```

\edef\z
{\xintilooop [10001+2]
{\xintilooop [3+2]
\ifnum\xintouterilooopindex<\numexpr\xintilooopindex*\xintilooopindex\relax
\xintouterilooopindex,
\expandafter\xintbreakilooop
\fi
\ifnum\xintouterilooopindex=\numexpr
(\xintouterilooopindex/\xintilooopindex)*\xintilooopindex\relax
\else
\repeat
}% no space here
\ifnum \xintilooopindex < 10999 \repeat }%
\meaning\z\endgroup

```

macro:->10007, 10009, 10037, 10039, 10061, 10067, 10069, 10079, 10091, 10093, 10099, 10103, 10111, 10133, 10139, 10141, 10151, 10159, 10163, 10169, 10177, 10181, 10193, 10211, 10223, 10243, 10247, 10253, 10259, 10267, 10271, 10273, 10289, 10301, 10303, 10313, 10321, 10331, 10333, 10337, 10343, 10357, 10369, 10391, 10399, 10427, 10429, 10433, 10453, 10457, 10459, 10463, 10477, 10487, 10499, 10501, 10513, 10529, 10531, 10559, 10567, 10589, 10597, 10601, 10607, 10613, 10627, 10631, 10639, 10651, 10657, 10663, 10667, 10687, 10691, 10709, 10711, 10723, 10729, 10733, 10739, 10753, 10771, 10781, 10789, 10799, 10831, 10837, 10847, 10853, 10859, 10861, 10867, 10883, 10889, 10891, 10903, 10909, 10937, 10939, 10949, 10957, 10973, 10979, 10987, 10993, and we should have taken some steps to not have a trailing comma, but the point was to show that one can do that in an `\edef`! See also [subsection 5.2](#) which extracts from this code its way of testing primality.

Let us create an alignment where each row will contain all divisors of its first entry. Here is the output, thus obtained without any count register:

```

\begin{multicols}2
\tabskiplex \normalcolor
\halign{\&\hfil#\hfil\cr
\xintilooop [1+1]
{\expandafter\bfseries\xintilooopindex &
\xintilooop [1+1]
\ifnum\xintouterilooopindex=\numexpr
(\xintouterilooopindex/\xintilooopindex)*\xintilooopindex\relax
\xintilooopindex&\fi
\ifnum\xintilooopindex<\xintouterilooopindex\space % CRUCIAL \space HERE
\repeat \cr }%
\ifnum\xintilooopindex<30
\repeat
}
\end{multicols}

```

|    |   |    |    |    |    |    |    |    |    |    |    |
|----|---|----|----|----|----|----|----|----|----|----|----|
| 1  | 1 | 14 | 1  | 2  | 7  | 14 |    |    |    |    |    |
| 2  | 1 | 2  | 15 | 1  | 3  | 5  | 15 |    |    |    |    |
| 3  | 1 | 3  | 16 | 1  | 2  | 4  | 8  | 16 |    |    |    |
| 4  | 1 | 2  | 4  | 17 | 1  | 17 |    |    |    |    |    |
| 5  | 1 | 5  | 18 | 1  | 2  | 3  | 6  | 9  | 18 |    |    |
| 6  | 1 | 2  | 3  | 6  | 19 | 1  | 19 |    |    |    |    |
| 7  | 1 | 7  | 20 | 1  | 2  | 4  | 5  | 10 | 20 |    |    |
| 8  | 1 | 2  | 4  | 8  | 21 | 1  | 3  | 7  | 21 |    |    |
| 9  | 1 | 3  | 9  | 22 | 1  | 2  | 11 | 22 |    |    |    |
| 10 | 1 | 2  | 5  | 10 | 23 | 1  | 23 |    |    |    |    |
| 11 | 1 | 11 | 24 | 1  | 2  | 3  | 4  | 6  | 8  | 12 | 24 |
| 12 | 1 | 2  | 3  | 4  | 6  | 12 | 25 | 1  | 5  | 25 |    |
| 13 | 1 | 13 | 26 | 1  | 2  | 13 | 26 |    |    |    |    |



```
27 1 3 9 27
28 1 2 4 7 14 28
```

```
29 1 29
30 1 2 3 5 6 10 15 30
```

We wanted this first entry in bold face, but `\bfseries` leads to unexpandable tokens, so the `\expandafter` was necessary for `\xintloopindex` and `\xintouterloopindex` not to be confronted with a hard to digest `\endtemplate`. An alternative way of coding:

```
\tabskiplex
\def\firstofone #1{#1}%
\halign{&\hfil#\hfil\cr
\xintloop [1+1]
{\bfseries\xintloopindex\firstofone{&}}%
\xintloop [1+1] \ifnum\xintouterloopindex=\numexpr
(\xintouterloopindex/\xintloopindex)*\xintloopindex\relax
\xintloopindex\firstofone{&}\fi
\ifnum\xintloopindex<\xintouterloopindex\space % \space is CRUCIAL
\repeat \firstofone{\cr}}%
\ifnum\xintloopindex<30 \repeat }
```

The next utilities are not compatible with expansion-only context.

## 7.15 `\xintApplyInline`

*o\*f* `\xintApplyInline{\macro}{\list}` works non expandably. It applies the one-parameter `\macro` to the first element of the expanded list (`\macro` may have itself some arguments, the list item will be appended as last argument), and is then re-inserted in the input stream after the tokens resulting from this first expansion of `\macro`. The next item is then handled.

This is to be used in situations where one needs to do some repetitive things. It is not expandable and can not be completely expanded inside a macro definition, to prepare material for later execution, contrarily to what `\xintApply` or `\xintApplyUnbraced` achieve.

```
\def\Macro #1{\advance\cnta #1 , \the\cnta}
\cnta 0
0\xintApplyInline\Macro {3141592653}.
```

0, 3, 4, 8, 9, 14, 23, 25, 31, 36, 39. The first argument `\macro` does not have to be an expandable macro.

`\xintApplyInline` submits its second, token list parameter to an *f-expansion*. Then, each *unbraced* item will also be *f-expanded*. This provides an easy way to insert one list inside another. *Braced* items are not expanded. Spaces in-between items are gobbled (as well as those at the start or the end of the list), but not the spaces *inside* the braced items.

`\xintApplyInline`, despite being non-expandable, does survive to contexts where the executed `\macro` closes groups, as happens inside alignments with the tabulation character `&`. This tabular provides an example:

```
\centerline{\normalcolor\begin{tabular}{ccc}
N & N^2 & N^3 \\ \hline
\def\Row #1{ #1 & \xintiiSqr {#1} & \xintiiPow {#1}{3} \\ \hline }%
\xintApplyInline \Row {\xintCSVtoList{17,28,39,50,61}}
\end{tabular}}\medskip
```

| N  | N <sup>2</sup> | N <sup>3</sup> |
|----|----------------|----------------|
| 17 | 289            | 4913           |
| 28 | 784            | 21952          |
| 39 | 1521           | 59319          |
| 50 | 2500           | 125000         |
| 61 | 3721           | 226981         |

We see that despite the fact that the first encountered tabulation character in the first row close a group and thus erases `\Row` from  $\text{\TeX}$ 's memory, `\xintApplyInline` knows how to deal with this.

Using `\xintApplyUnbraced` is an alternative: the difference is that this would have prepared all rows first and only put them back into the token stream once they are all assembled, whereas with `\xintApplyInline` each row is constructed and immediately fed back into the token stream: when one does things with numbers having hundreds of digits, one learns that keeping on hold and shuffling around hundreds of tokens has an impact on  $\text{\TeX}$ 's speed (make this ``thousands of tokens'' for the impact to be noticeable).

One may nest various `\xintApplyInline`'s. For example (see the [table](#) on the current page):

```
\begin{figure*}[ht!]
\centering\phantomsection\label{float}
\def\Row #1{#1:\xintApplyInline {\Item {#1}}{0123456789}\ }%
\def\Item #1#2{&\xintiPow {#1}{#2}}%
\centeredline {\begin{tabular}{ccccccccc} &0&1&2&3&4&5&6&7&8&9\\ \hline
\xintApplyInline \Row {0123456789}
\end{tabular}}
\end{figure*}
```

|    | 0 | 1 | 2  | 3   | 4    | 5     | 6      | 7       | 8        | 9         |
|----|---|---|----|-----|------|-------|--------|---------|----------|-----------|
| 0: | 1 | 0 | 0  | 0   | 0    | 0     | 0      | 0       | 0        | 0         |
| 1: | 1 | 1 | 1  | 1   | 1    | 1     | 1      | 1       | 1        | 1         |
| 2: | 1 | 2 | 4  | 8   | 16   | 32    | 64     | 128     | 256      | 512       |
| 3: | 1 | 3 | 9  | 27  | 81   | 243   | 729    | 2187    | 6561     | 19683     |
| 4: | 1 | 4 | 16 | 64  | 256  | 1024  | 4096   | 16384   | 65536    | 262144    |
| 5: | 1 | 5 | 25 | 125 | 625  | 3125  | 15625  | 78125   | 390625   | 1953125   |
| 6: | 1 | 6 | 36 | 216 | 1296 | 7776  | 46656  | 279936  | 1679616  | 10077696  |
| 7: | 1 | 7 | 49 | 343 | 2401 | 16807 | 117649 | 823543  | 5764801  | 40353607  |
| 8: | 1 | 8 | 64 | 512 | 4096 | 32768 | 262144 | 2097152 | 16777216 | 134217728 |
| 9: | 1 | 9 | 81 | 729 | 6561 | 59049 | 531441 | 4782969 | 43046721 | 387420489 |

One could not move the definition of `\Item` inside the tabular, as it would get lost after the first `&`. But this works:

```
\begin{tabular}{ccccccccc}
&0&1&2&3&4&5&6&7&8&9\\ \hline
\def\Row #1{#1:\xintApplyInline {\&\xintiPow {#1}}{0123456789}\ }%
\xintApplyInline \Row {0123456789}
\end{tabular}
```

A limitation is that, contrarily to what one may have expected, the `\macro` for an `\xintApplyInline` can not be used to define the `\macro` for a nested sub-`\xintApplyInline`. For example, this does not work:

```
\def\Row #1{#1:\def\Item ##1{&\xintiPow {#1}{##1}}%
```

```
\xintApplyInline \Item {0123456789}\ \ }%
\xintApplyInline \Row {0123456789} % does not work
```

But see `\xintFor`.

## 7.16 `\xintFor`, `\xintFor*`

*on* `\xintFor` is a new kind of for loop.<sup>55</sup> Rather than using macros for encapsulating list items, its behavior is like a macro with parameters: `#1`, `#2`, ..., `#9` are used to represent the items for up to nine levels of nested loops. Here is an example:

```
\xintFor #9 in {1,2,3} \do {%
 \xintFor #1 in {4,5,6} \do {%
 \xintFor #3 in {7,8,9} \do {%
 \xintFor #2 in {10,11,12} \do {%
 $$#9\times#1\times#3\times#2=\xintiiPrd{{#1}{#2}{#3}{#9}}$$$$}}
```

This example illustrates that one does not have to use `#1` as the first one: the order is arbitrary. But each level of nesting should have its specific macro parameter. Nine levels of nesting is presumably overkill, but I did not know where it was reasonable to stop. `\par` tokens are accepted in both the comma separated list and the replacement text.

Technical notes:

- The `#1` is replaced in the iterated-over text exactly as in general  $\TeX$  macros or  $\LaTeX$  commands. This spares the user quite a few `\expandafter`'s or other tricks needed with loops which have the values encapsulated in macros, like  $\LaTeX$ 's `\@for` and `\@tfor`.
- `\xintFor` (and `\xintFor*`) isn't purely expandable: one can not use it inside an `\edef`. But it may be used, as will be shown in examples, in some contexts such as  $\LaTeX$ 's `tabular` which are usually hostile to non-expandable loops.
- `\xintFor` (and `\xintFor*`) does some assignments prior to executing each iteration of the replacement text, but it acts purely expandably after the last iteration, hence if for example the replacement text ends with a `\\`, the loop can be used inside a `tabular` and be followed by a `\hline` without creating the dreaded `“Misplaced \noalign”` error.
- It does not create groups.
- It makes no global assignments.
- The iterated replacement text may close a group which was opened even before the start of the loop (typical example being with `&` in alignments).

```
\begin{tabular}{rcccc}
 \hline
 \xintFor #1 in {A, B, C} \do {%
 #1:\xintFor #2 in {a, b, c, d, e} \do {&($ #2 \to #1 $)}\\
 \hline
\end{tabular}
```

|    |         |         |         |         |         |
|----|---------|---------|---------|---------|---------|
| A: | (a → A) | (b → A) | (c → A) | (d → A) | (e → A) |
| B: | (a → B) | (b → B) | (c → B) | (d → B) | (e → B) |
| C: | (a → C) | (b → C) | (c → C) | (d → C) | (e → C) |

- There is no facility provided which would give access to a count of the number of iterations as it is technically not easy to do so it in a way working with nested loops while

<sup>55</sup> first introduced with *xint* 1.09c of 2013/10/09.

maintaining the ``expandable after done'' property; something in the spirit of `\xint-loopindex` is possible but this approach would bring its own limitations and complications. Hence the user is invited to update her own count or  $\TeX$  counter or macro at each iteration, if needed.

- A `\macro` whose definition uses internally an `\xintFor` loop may be used inside another `\xintFor` loop even if the two loops both use the same macro parameter. The loop definition inside `\macro` must use `##` as is the general rule for definitions done inside macros.
- `\xintFor` is for comma separated values and `\xintFor*` for lists of braced items; their respective expansion policies differ. They are described later.

Regarding `\xintFor`:

- the spaces between the various declarative elements are all optional,
- in the list of comma separated values, spaces around the commas or at the start and end are ignored,
- if an item must contain itself its own commas, then it should be braced, and the braces will be removed before feeding the iterated-over text,
- the list may be a macro, it is expanded only once,
- items are not pre-expanded. The first item should be braced or start with a space if the list is explicit and the item should not be pre-expanded,
- empty items give empty `#1`'s in the replacement text, they are not skipped,
- an empty list executes once the replacement text with an empty parameter value,
- the list, if not a macro, must be braced.

*\*fn*

Regarding `\xintFor*`:

- it handles lists of braced items (or naked tokens),
- it *f-expands* the list,
- and more generally it *f-expands* each naked token encountered before assigning the `#1` values (gobbling spaces in the process); this makes it easy to simulate concatenation of multiple lists `\x, \y`: if `\x` expands to `{1}{2}{3}` and `\y` expands to `{4}{5}{6}` then `{\x\y}` as argument to `\xintFor*` has the same effect as `{{1}{2}{3}{4}{5}{6}}`.

For a further illustration see the use of `\xintFor*` at the end of [subsection 2.9](#).

- spaces at the start, end, or in-between items are gobbled (but naturally not the spaces inside braced items),
- except if the list argument is a macro (with no parameters), it must be braced.,
- an empty list leads to an empty result.

The macro `\xintSeq` which generates arithmetic sequences is to be used with `\xintFor*` as its output consists of successive braced numbers (given as digit tokens).

```
\xintFor* #1 in {\xintSeq [+2]{-7}{+2}}\do {stuff
 with #1\xintifForLast{\par}{\newline}}
```

stuff with -7

stuff with -5

stuff with -3

stuff with -1

stuff with 1

When nesting `\xintFor*` loops, using `\xintSeq` in the inner loops is inefficient, as the arithmetic sequence will be re-created each time. A more efficient style is:

```
\edef\innersequence {\xintSeq[+2]{-50}{50}}%
\xintFor* #1 in {\xintSeq {13}{27}} \do
 {\xintFor* #2 in \innersequence \do {stuff with #1 and #2}%
 .. some other macros .. }
```

This is a general remark applying for any nesting of loops, one should avoid recreating the inner lists of arguments at each iteration of the outer loop.

When the loop is defined inside a macro for later execution the # characters must be doubled.<sup>56</sup> For example:

```
\def\T{\def\z {}%
 \xintFor* ##1 in {{u}{v}{w}} \do {%
 \xintFor ##2 in {x,y,z} \do {%
 \expandafter\def\expandafter\z\expandafter {\z\sep (##1,##2)} }%
 }%
 }%
\T\def\sep {\def\sep{, }}\z
```

(u,x), (u,y), (u,z), (v,x), (v,y), (v,z), (w,x), (w,y), (w,z)

Similarly when the replacement text of `\xintFor` defines a macro with parameters, the macro character # must be doubled.

The iterated macros as well as the list items are allowed to contain explicit `\par` tokens.

### 7.17 `\xintifForFirst`, `\xintifForLast`

**nn ★** `\xintifForFirst` {YES branch}{NO branch} and `\xintifForLast` {YES branch}{NO branch} execute the YES or NO branch if the `\xintFor` or `\xintFor*` loop is currently in its first, respectively last, iteration.

Designed to work as expected under nesting (but see frame next.) Don't forget an empty brace pair {} if a branch is to do nothing. May be used multiple times in the replacement text of the loop.

Pay attention to these implementation features:

- if an inner `\xintFor` loop is positioned before the `\xintifForFirst` or `\xintifForLast` of the outer loop it will contaminate their settings. This applies also naturally if the inner loop arises from the expansion of some macro located before the outer conditionals.

One fix is to make sure that the outer conditionals are expanded before the inner loop is executed, e.g. this will be the case if the inner loop is located inside one of the branches of the conditional.

Another approach is to enclose, if feasible, the inner loop in a group of its own.

- if the replacement text closes a group (e.g. from a & inside an alignment), the conditionals will lose their ascribed meanings and end up possibly undefined, depending whether there is some outer loop whose execution started before the opening of the group.

The fix is to arrange things so that the conditionals are expanded before  $\TeX$  encounters the closing-group token.

### 7.18 `\xintBreakFor`, `\xintBreakForAndDo`

One may immediately terminate an `\xintFor` or `\xintFor*` loop with `\xintBreakFor`.

As it acts by clearing up all the rest of the replacement text when encountered, it will not work from inside some `\if...\fi` without suitable `\expandafter` or swapping technique.

Also it can't be used from inside braces as from there it can't see the end of the replacement text.

There is also `\xintBreakForAndDo`. Both are illustrated by various examples in the next section which is devoted to ``forever'' loops.

<sup>56</sup> sometimes what seems to be a macro argument isn't really; in `\raisebox{1cm}{\xintFor #1 in {a,b,c}\do {#1}}` no doubling should be done.

## 7.19 `\xintintegers`, `\xintdimensions`, `\xinrationals`

If the list argument to `\xintFor` (or `\xintFor*`, both are equivalent in this context) is `\xintintegers` (equivalently `\xintegers`) or more generally `\xintintegers[start+delta]` (*the whole within braces!*)<sup>57</sup>, then `\xintFor` does an infinite iteration where #1 (or #2, . . . , #9) will run through the arithmetic sequence of (short) integers with initial value `start` and increment `delta` (default values: `start=1`, `delta=1`; if the optional argument is present it must contain both of them, and they may be explicit integers, or macros or count registers). The #1 (or #2, . . . , #9) will stand for `\numexpr <opt sign><digits>\relax`, and the literal representation as a string of digits can thus be obtained as `\the#1` or `\number#1`. Such a #1 can be used in an `\ifnum` test with no need to be postfixed with a space or a `\relax` and one should not add them.

If the list argument is `\xintdimensions` or more generally `\xintdimensions[start+delta]` (*within braces!*), then `\xintFor` does an infinite iteration where #1 (or #2, . . . , #9) will run through the arithmetic sequence of dimensions with initial value `start` and increment `delta`. Default values: `start=0pt`, `delta=1pt`; if the optional argument is present it must contain both of them, and they may be explicit specifications, or macros, or dimen registers, or length macros in  $\TeX$  (the stretch and shrink components will be discarded). The #1 will be `\dimexpr <opt sign><digits>sp\relax`, from which one can get the literal (approximate) representation in points via `\the#1`. So #1 can be used anywhere  $\TeX$  expects a dimension (and there is no need in conditionals to insert a `\relax`), and to print its value one uses `\the#1`. The chosen representation guarantees exact incrementation with no rounding errors accumulating from converting into points at each step.

If the list argument to `\xintFor` (or `\xintFor*`) is `\xinrationals` or more generally `\xinrationals[start+delta]` (*within braces!*), then `\xintFor` does an infinite iteration where #1 (or #2, . . . , #9) will run through the arithmetic sequence of `xintfrac` fractions with initial value `start` and increment `delta` (default values: `start=1/1`, `delta=1/1`). This loop works *only with xintfrac loaded*. If the optional argument is present it must contain both of them, and they may be given in any of the formats recognized by `xintfrac` (fractions, decimal numbers, numbers in scientific notations, numerators and denominators in scientific notation, etc. . . .), or as macros or count registers (if they are short integers). The #1 (or #2, . . . , #9) will be an `a/b` fraction (without a `[n]` part), where the denominator `b` is the product of the denominators of `start` and `delta` (for reasons of speed #1 is not reduced to irreducible form, and for another reason explained later `start` and `delta` are not put either into irreducible form; the input may use explicitly `\xintIrr` to achieve that).

```
\begingroup\small
\noindent\parbox{\dimexpr\linewidth-3em}{\color[named]{OrangeRed}%
\xintFor #1 in {\xinrationals [10/21+1/21]} \do
{#1=\xintifInt {#1}
{\textcolor{blue}{\xintTrunc{10}{#1}}}
{\xintTrunc{10}{#1}}}% display in blue if an integer
\xintifGt {#1}{1.123}{\xintBreakFor}{, }%
}}
\endgroup\smallskip
```

|                     |                     |                     |                     |
|---------------------|---------------------|---------------------|---------------------|
| 10/21=0.4761904761, | 11/21=0.5238095238, | 12/21=0.5714285714, | 13/21=0.6190476190, |
| 14/21=0.6666666666, | 15/21=0.7142857142, | 16/21=0.7619047619, | 17/21=0.8095238095, |
| 18/21=0.8571428571, | 19/21=0.9047619047, | 20/21=0.9523809523, | 21/21=1.0000000000, |
| 22/21=1.0476190476, | 23/21=1.0952380952, | 24/21=1.1428571428  |                     |

The example above confirms that computations are done exactly, and illustrates that the two initial (reduced) denominators are not multiplied when they are found to be equal. It is thus recommended to input `start` and `delta` with a common smallest possible denominator, or as fixed point numbers with the same numbers of digits after the decimal mark; and this is also the reason why `start` and `delta` are not by default made irreducible. As internally the computations are done with numerators and denominators completely expanded, one should be careful not to input numbers

<sup>57</sup> the `start+delta` optional specification may have extra spaces around the plus sign of near the square brackets, such spaces are removed. The same applies with `\xintdimensions` and `\xinrationals`.

in scientific notation with exponents in the hundreds, as they will get converted into as many zeroes.

```
\noindent\parbox{\dimexpr.7\linewidth}{\raggedright
\intFor #1 in {\xintrationals [0.000+0.125]} \do
{\edef\tmp{\xintTrunc{3}{#1}}%
\xintifInt {#1}
{\textcolor{blue}{\tmp}}
{\tmp}%
\xintifGt {#1}{2}{\xintBreakFor}{, }%
}}\smallskip
0, 0.125, 0.250, 0.375, 0.500, 0.625, 0.750, 0.875, 1.000, 1.125,
1.250, 1.375, 1.500, 1.625, 1.750, 1.875, 2.000, 2.125
```

We see here that `\xintTrunc` outputs (deliberately) zero as 0, not (here) 0.000, the idea being not to lose the information that the truncated thing was truly zero. Perhaps this behavior should be changed? or made optional? Anyhow printing of fixed points numbers should be dealt with via dedicated packages such as `numprint` or `siunitx`.

## 7.20 `\xintForpair`, `\xintForthree`, `\xintForfour`

*on* The syntax is illustrated in this example. The notation is the usual one for *n*-uples, with parentheses and commas. Spaces around commas and parentheses are ignored.

```
{\centering\begin{tabular}{cccc}
\xintForpair #1#2 in { (A , a) , (B , b) , (C , c) } \do {%
\xintForpair #3#4 in { (X , x) , (Y , y) , (Z , z) } \do {%
$\Biggl(\begin{tabular}{cc}
-#1- & -#3-\\
-#4- & -#2-\\
\end{tabular}\Biggr)\$&\\noalign{\vskip1\jot}}%
\end{tabular}}\}
```

$$\begin{pmatrix} -A- & -X- \\ -x- & -a- \end{pmatrix} \begin{pmatrix} -A- & -Y- \\ -y- & -a- \end{pmatrix} \begin{pmatrix} -A- & -Z- \\ -z- & -a- \end{pmatrix}$$

$$\begin{pmatrix} -B- & -X- \\ -x- & -b- \end{pmatrix} \begin{pmatrix} -B- & -Y- \\ -y- & -b- \end{pmatrix} \begin{pmatrix} -B- & -Z- \\ -z- & -b- \end{pmatrix}$$

$$\begin{pmatrix} -C- & -X- \\ -x- & -c- \end{pmatrix} \begin{pmatrix} -C- & -Y- \\ -y- & -c- \end{pmatrix} \begin{pmatrix} -C- & -Z- \\ -z- & -c- \end{pmatrix}$$

`\xintForpair` must be followed by either `#1#2`, `#2#3`, `#3#4`, ..., or `#8#9` with `#1` usable as an alias for `#1#2`, `#2` as alias for `#2#3`, etc ... and similarly for `\xintForthree` (using `#1#2#3` or simply `#1`, `#2#3#4` or simply `#2`, ...) and `\xintForfour` (with `#1#2#3#4` etc...).

Nesting works as long as the macro parameters are distinct among `#1`, `#2`, ..., `#9`. A macro which expands to an `\xintFor` or a `\xintFor(pair,three,four)` can be used in another one with no constraint about using distinct macro parameters.

`\par` tokens are accepted in both the comma separated list and the replacement text.

## 7.21 `\xintAssign`

`\xintAssign`*(braced things)*`\to`*(as many cs as they are things)* defines (without checking if something gets overwritten) the control sequences on the right of `\to` to expand to the successive tokens or braced items found one after the other on the left of `\to`. It is not expandable.

A 'full' expansion is first applied to the material in front of `\xintAssign`, which may thus be a macro expanding to a list of braced items.



Special case: if after this initial expansion no brace is found immediately after `\xintAssign`, it is assumed that there is only one control sequence following `\to`, and this control sequence is then defined via `\def` to expand to the material between `\xintAssign` and `\to`. Other types of expansions are specified through an optional parameter to `\xintAssign`, see *infra*.

```
\xintAssign \xintiDivision{1000000000000}{133333333}\to\Q\R
\meaning\Q:macro:->7500, \meaning\R: macro:->2500
\xintAssign \xintiPow {7}{13}\to\SevenToThePowerThirteen
\SevenToThePowerThirteen=96889010407
(same as \edef\SevenToThePowerThirteen{\xintiPow {7}{13}})
```

`\xintAssign` admits since 1.09i an optional parameter, for example `\xintAssign [e]...` or `\xintAssign [oo] ...`. With `[f]` for example the definitions of the macros initially on the right of `\to` will be made with `\fdef` which *f-expands* the replacement text. The default is simply to make the definitions with `\def`, corresponding to an empty optional parameter `[]`. Possibilities: `[]`, `[g]`, `[Q]`, `[e]`, `[x]`, `[o]`, `[go]`, `[oo]`, `[goo]`, `[f]`, `[gf]`.

In all cases, recall that `\xintAssign` starts with an *f-expansion* of what comes next; this produces some list of tokens or braced items, and the optional parameter only intervenes to decide the expansion type to be applied then to each one of these items.

Note: prior to release 1.09j, `\xintAssign` did an `\edef` by default, but it now does `\def`. Use the optional parameter `[e]` to force use of `\edef`.

Remark: since *xinttools* 1.1c, `\xintAssign` is less picky and a stray space right before the `\to` causes no surprises, and the successive braced items may be separated by spaces, which will get discarded. In case the contents up to `\to` did not start with a brace a single macro is defined and it will contain the spaces. Contrarily to the earlier version, there is no problem if such contents do contain braces after the first non-brace token.

## 7.22 `\xintAssignArray`

`\xintAssignArray<braced things>\to\myArray` first expands fully what comes immediately after `\xintAssignArray` and expects to find a list of braced things `{A}{B}...` (or tokens). It then defines `\myArray` as a macro with one parameter, such that `\myArray{x}` expands to give the *x*th braced thing of this original list (the argument `{x}` itself is fed to a `\numexpr` by `\myArray`, and `\myArray` expands in two steps to its output). With `0` as parameter, `\myArray{0}` returns the number *M* of elements of the array so that the successive elements are `\myArray{1}`, ..., `\myArray{M}`.

```
\xintAssignArray \xintBezout {1000}{113}\to\Bez
```

will set `\Bez{0}` to 5, `\Bez{1}` to 1000, `\Bez{2}` to 113, `\Bez{3}` to -20, `\Bez{4}` to -177, and `\Bez{5}` to 1:  $(-20) \times 1000 - (-177) \times 113 = 1$ . This macro is incompatible with expansion-only contexts.

`\xintAssignArray` admits now an optional parameter, for example `\xintAssignArray [e]...`. This means that the definitions of the macros will be made with `\edef`. The default is `[]`, which makes the definitions with `\def`. Other possibilities: `[]`, `[o]`, `[oo]`, `[f]`. Contrarily to `\xintAssign` one can not use the *g* here to make the definitions global. For this, one should rather do `\xintAssignArray` within a group starting with `\globaldefs` 1.

Note that prior to release 1.09j each item (token or braced material) was submitted to an `\edef`, but the default is now to use `\def`.

## 7.23 `\xintDigitsOf`

*fN* This is a synonym for `\xintAssignArray`, to be used to define an array giving all the digits of a given (positive, else the minus sign will be treated as first item) number.

```
\xintDigitsOf\xintiPow {7}{500}\to\digits
```

$7^{500}$  has `\digits{0}`=423 digits, and the 123rd among them (starting from the most significant) is `\digits{123}`=3.

### 7.24 `\xintRelaxArray`

`\xintRelaxArray\myArray` (globally) sets to `\relax` all macros which were defined by the previous `\xintAssignArray` with `\myArray` as array macro.

## 8 Macros of the *xintcore* package

|     |                                                        |    |     |                                                                  |    |
|-----|--------------------------------------------------------|----|-----|------------------------------------------------------------------|----|
| .1  | <code>\xintNum</code> , <code>\xintiNum</code> .....   | 89 | .13 | <code>\xintiDivision</code> , <code>\xintiiDivision</code> ..... | 90 |
| .2  | <code>\xintSgn</code> , <code>\xintiiSgn</code> .....  | 89 | .14 | <code>\xintiQuo</code> , <code>\xintiiQuo</code> .....           | 91 |
| .3  | <code>\xintiOpp</code> , <code>\xintiiOpp</code> ..... | 89 | .15 | <code>\xintiRem</code> , <code>\xintiiRem</code> .....           | 91 |
| .4  | <code>\xintiAbs</code> , <code>\xintiiAbs</code> ..... | 89 | .16 | <code>\xintiDivRound</code> , <code>\xintiiDivRound</code> ..... | 91 |
| .5  | <code>\xintiiFDg</code> .....                          | 89 | .17 | <code>\xintiDivTrunc</code> , <code>\xintiiDivTrunc</code> ..... | 91 |
| .6  | <code>\xintiiLDg</code> .....                          | 89 | .18 | <code>\xintiMod</code> , <code>\xintiiMod</code> .....           | 91 |
| .7  | <code>\xintiAdd</code> , <code>\xintiiAdd</code> ..... | 89 | .19 | <code>\xintDouble</code> , <code>\xintHalf</code> .....          | 91 |
| .8  | <code>\xintiSub</code> , <code>\xintiiSub</code> ..... | 89 | .20 | <code>\xintInc</code> , <code>\xintDec</code> .....              | 91 |
| .9  | <code>\xintiMul</code> , <code>\xintiiMul</code> ..... | 89 | .21 | <code>\xintDSL</code> .....                                      | 91 |
| .10 | <code>\xintiSqr</code> , <code>\xintiiSqr</code> ..... | 89 | .22 | <code>\xintDSR</code> .....                                      | 92 |
| .11 | <code>\xintiPow</code> , <code>\xintiiPow</code> ..... | 90 | .23 | <code>\xintDSRr</code> .....                                     | 92 |
| .12 | <code>\xintiFac</code> , <code>\xintiiFac</code> ..... | 90 |     |                                                                  |    |



Prior to release 1.1 the macros which are now included in the separate package *xintcore* were part of *xint*. Package *xintcore* is automatically loaded by *xint*.

*xintcore* provides the five basic arithmetic operations on big integers: addition, subtraction, multiplication, division and powers. Division may be either rounded (`\xintiiDivRound`) (the rounding of 0.5 is 1 and the one of -0.5 is -1) or Euclidean (`\xintiiQuo`) (which for positive operands is the same as truncated division), or truncated (`\xintiiDivTrunc`).

In the description of the macros the `{N}` and `{M}` symbols stand for explicit (big) integers within braces or more generally any control sequence (possibly within braces) *f-expanding* to such a big integer.

The macros with a single *i* in their names parse their arguments automatically through `\xintNum`. This type of expansion applied to an argument is signaled by a <sup>Num</sup>*f* in the margin. The accepted input format is then a sequence of plus and minus signs, followed by some string of zeroes, followed by digits.

If *xintfrac* additionally to *xintcore* is loaded, `\xintNum` becomes a synonym to `\xintTTrunc`; this means that arbitrary fractions will be accepted as arguments of the macros with a single *i* in their names, but get truncated to integers before further processing. The format of the output will be as with only *xint* loaded. The only extension is in allowing a wider variety of inputs.

The macros with *ii* in their names have arguments which will only be *f-expanded*, but will not be parsed via `\xintNum`. Arguments of this type are signaled by the margin annotation *f*. For such big integers only one minus sign and no plus sign, nor leading zeros, are accepted. -0 is not valid in this strict input format. Loading *xintfrac* does not bring any modification to these macros whether for input or output.

The letter *x* (with margin annotation <sup>num</sup>*x*) stands for something which will be inserted in-between a `\numexpr` and a `\relax`. It will thus be completely expanded and must give an integer obeying the  $\TeX$  bounds. Thus, it may be for example a count register, or itself a `\numexpr` expression, or just a number written explicitly with digits or something like `4*\count 255 + 17`, etc...

For the rules regarding direct use of count registers or `\numexpr` expression, in the arguments to the package macros, see the subsection 3.6 section.

The macros `\xintiAdd`, `\xintiMul`, ..., respectively `\xintiiAdd`, `\xintiiMul`, ... from *xintcore* are guaranteed to always output an integer without a trailing /B or [N]. The *ii* macros have the lesser overhead; the *i* macros can be used (if *xintfrac* is loaded), with fractions, as they will truncate their arguments to integers. But their output format remains unmodified: integers with no fraction slash nor [N].

The  $\star$ 's in the margin are there to remind of the complete expandability, even *f-expandability* of the macros, as discussed in [subsubsection 3.3.1](#).

### 8.1 $\backslash\text{xintNum}$ , $\backslash\text{xintiNum}$

$f \star$   $\backslash\text{xintNum}\{N\}$  removes chains of plus or minus signs, followed by zeroes.

$\backslash\text{xintNum}\{+---+-----000000000367941789479\}=-367941789479$

All *xint* macros with a single *i* in their names, such as  $\backslash\text{xintiAdd}$ ,  $\backslash\text{xintiMul}$  apply  $\backslash\text{xintNum}$  to their arguments.

When *xintfrac* is loaded,  $\backslash\text{xintNum}$  becomes a synonym to  $\backslash\text{xintTTrunc}$ . And  $\backslash\text{xintiNum}$  preserved the original integer only meaning.

### 8.2 $\backslash\text{xintSgn}$ , $\backslash\text{xintiiSgn}$

$f \star$   $\backslash\text{xintiiSgn}\{N\}$  returns 1 if the number is positive, 0 if it is zero and -1 if it is negative. It skips the  $\backslash\text{xintNum}$  overhead.

$\text{Num } f \star$   $\backslash\text{xintSgn}$  is the variant using  $\backslash\text{xintNum}$  and getting extended by *xintfrac* to fractions.

### 8.3 $\backslash\text{xintiOpp}$ , $\backslash\text{xintiiOpp}$

$\text{Num } f \star$   $\backslash\text{xintiOpp}\{N\}$  return the opposite  $-N$  of the number  $N$ .  $\backslash\text{xintiiOpp}$  is the strict integer-only variant  $f \star$  which skips the  $\backslash\text{xintNum}$  overhead.

### 8.4 $\backslash\text{xintiAbs}$ , $\backslash\text{xintiiAbs}$

$\text{Num } ff \star$   $\backslash\text{xintiAbs}\{N\}$  returns the absolute value of the number.  $\backslash\text{xintiiAbs}$  skips the  $\backslash\text{xintNum}$  overhead.

### 8.5 $\backslash\text{xintiiFDg}$

$\text{Num } f \star$   $\backslash\text{xintiiFDg}\{N\}$  returns the first digit (most significant) of the decimal expansion. It skips the overhead of parsing via  $\backslash\text{xintNum}$ . The variant  $\backslash\text{xintFDg}$  uses  $\backslash\text{xintNum}$  and gets extended by *xintfrac*.  $f \star$

### 8.6 $\backslash\text{xintiiLDg}$

$f \star$   $\backslash\text{xintiiLDg}\{N\}$  returns the least significant digit. When the number is positive, this is the same as the remainder in the euclidean division by ten. It skips the overhead of parsing via  $\backslash\text{xintNum}$ . Rewritten with 1.2i.

$\text{Num } f \star$  The variant  $\backslash\text{xintLDg}$  uses  $\backslash\text{xintNum}$ .

### 8.7 $\backslash\text{xintiAdd}$ , $\backslash\text{xintiiAdd}$

$\text{Num Num } f \star$   $\backslash\text{xintiAdd}\{N\}\{M\}$  returns the sum of the two numbers.  $\backslash\text{xintiiAdd}$  skips the  $\backslash\text{xintNum}$  overhead.

### 8.8 $\backslash\text{xintiSub}$ , $\backslash\text{xintiiSub}$

$\text{Num Num } f \star$   $\backslash\text{xintiSub}\{N\}\{M\}$  returns the difference  $N-M$ .  $\backslash\text{xintiiSub}$  skips the  $\backslash\text{xintNum}$  overhead.

### 8.9 $\backslash\text{xintiMul}$ , $\backslash\text{xintiiMul}$

$\text{Num Num } f \star$   $\backslash\text{xintiMul}\{N\}\{M\}$  returns the product of the two numbers.  $\backslash\text{xintiiMul}$  skips the  $\backslash\text{xintNum}$  overhead.

### 8.10 $\backslash\text{xintiSqr}$ , $\backslash\text{xintiiSqr}$

$\text{Num } ff \star$   $\backslash\text{xintiSqr}\{N\}$  returns the square.  $\backslash\text{xintiiSqr}$  skips the  $\backslash\text{xintNum}$  overhead.

### 8.11 \xintiPow, \xintiiPow

`\xintiPow{N}{x}` returns  $N^x$ . When  $x$  is zero, this is 1. If  $N=0$  and  $x<0$ , if  $|N|>1$  and  $x<0$ , an error is raised. There will also be an error naturally if  $x$  exceeds the maximal  $\varepsilon$ -TeX number 2147483647, but the real limit for huge exponents comes from either the computation time or the settings of some tex memory parameters.

Indeed, the maximal power of 2 which `xint` is able to compute explicitly is  $2^{2^{2^{17}}}=2^{131072}$  which has 39457 digits. This exceeds the maximal size on input for the `xintcore` multiplication, hence any  $2^N$  with a higher  $N$  will fail. On the other hand  $2^{2^{2^{16}}}$  has 19729 digits, thus it can be squared once to obtain  $2^{2^{2^{17}}}$  or multiplied by anything smaller, thus all exponents up to and including  $2^{17}$  are allowed (because the power operation works by squaring things and making products).

Side remark: after all it does pay to think! I almost melted my CPU trying by dichotomy to pinpoint the exact maximal allowable  $N$  for  $\text{xintiiPow } 2\{N\}$  before finally making the reasoning above. Indeed, each such computation with  $N > 130000$  activates the fan of my laptop and results in so warm a keyboard that I can hardly go on working on it! And it takes about 12 minutes for each  $\text{xintiiPow } 2\{N\}$  with such  $N$ 's of the order of 130000 (a.t.t.o.w.).

When `xintfrac` is loaded the type of the second argument to `\xintiPow` becomes  $f$ : fractional input is accepted but will be truncated to an integer; it still must be non-negative else the macro would produce fractions. For the version accepting negative (but still integer) exponents see `\xintPow`.

$f^{\text{num}}_x \star$  `\xintiiPow` is the variant which skips the `\xintNum` overhead for the first argument.

### 8.12 \xintiFac, \xintiiFac

$\text{num}_x \star \text{\texttt{\textbackslash xintiiFac}\{x\}}$  computes the factorial.

The (theoretically) allowable range is  $0 \leq x \leq 10000$ .

However the maximal possible computation depends on the values of some memory parameters of the `tex` executable: with the current default settings of TeXLive 2015, the maximal computable factorial (a.t.t.o.w. 2015/10/06) turns out to be 5971! which has 19956 digits.

`\xintiFac` is originally a synonym. With `xintfrac` loaded it applies `\xintNum` to its argument and thus accepts a fractional input but truncates it to an integer.

The `factorial` function, or equivalently `!` as post-fix operator is available in `\xintiiexpr`, `\xintexpr`:

```
\printnumber{\xinttheiiexpr 200!\relax}\par
788657867364790503552363213932185062295135977687173263294742533244359449963403342920304284012
198462390417721213891963883025764279024263710506192662495282993111346285727076331723739698892
439224456214516642402540332918641312274282948532775242424075739032403212574055795686602260312
904170324062351700858796178922227896237038973747200
0000000
```

See also `\xintFloatFac` from package `xintfrac` for the float variant, used in `\xintfloatexpr`.

### 8.13 \xintiDivision, \xintiiDivision

**ff**★ `\xintiiDivision{N}{M}` returns  $\{\text{quotient } Q\}\{\text{remainder } R\}$ . This is euclidean division:  $N = QM + R$ ,  $0 \leq R < |M|$ . So the remainder is always non-negative and the formula  $N = QM + R$  always holds independently of the signs of  $N$  or  $M$ . Division by zero is an error (even if  $N$  vanishes) and returns  $\{0\}\{0\}$ . It skips the overhead of parsing via `\xintNum`.

Num Num {0}{0}. It skips the overhead of parsing via `\xintNum`  
 $\frac{f}{f} \star$  `\xintiDivision` submits its arguments to `\xintNum`.

**8.14 `\xintiQuo`, `\xintiiQuo`**

*ff* ★ `\xintiiQuo{N}{M}` returns the quotient from the euclidean division. It skips the overhead of parsing via `\xintNum`.  
*Num Num*  
*f f* ★ `\xintiQuo` submits its arguments to `\xintNum`.

**8.15 `\xintiRem`, `\xintiiRem`**

*ff* ★ `\xintiiRem{N}{M}` returns the remainder from the euclidean division. It skips the overhead of parsing via `\xintNum`.  
*Num Num*  
*f f* ★ `\xintiRem` submits its arguments to `\xintNum`.

**8.16 `\xintiDivRound`, `\xintiiDivRound`**

*ff* ★ `\xintiiDivRound{N}{M}` returns the rounded value of the algebraic quotient  $N/M$  of two big integers. The rounding is ``away from zero.'' The macro skips the overhead of parsing via `\xintNum`.  
`\xintiiDivRound {100}{3}, \xintiiDivRound {101}{3}`  
*Num Num*  
*f f* ★ `\xintiDivRound` submits its arguments to `\xintNum`.  
**33, 34**

**8.17 `\xintiDivTrunc`, `\xintiiDivTrunc`**

*ff* ★ `\xintiiDivTrunc{N}{M}` computes the truncation towards zero of the algebraic quotient  $N/M$ . It skips the overhead of parsing the operands with `\xintNum`. For  $M > 0$  it is the same as `\xintiiQuo`.  
`$\xintiiQuo {1000}{-57}, \xintiiDivRound {1000}{-57}, \xintiiDivTrunc {1000}{-57}$`  
*Num Num*  
*f f* ★ `\xintiDivTrunc` submits its arguments to `\xintNum`.  
**-17, -18, -17**

**8.18 `\xintiMod`, `\xintiiMod`**

*ff* ★ `\xintiiMod{N}{M}` computes  $N - M * t(N/M)$ , where  $t(N/M)$  is the algebraic quotient truncated towards zero. The macro skips the overhead of parsing the operands with `\xintNum`. For  $M > 0$  it is the same as `\xintiiRem`.  
`$\xintiiRem {1000}{-57}, \xintiiMod {1000}{-57},`  
`\xintiiRem {-1000}{57}, \xintiiMod {-1000}{57}$`  
*Num Num*  
*f f* ★ `\xintiMod` submits its arguments to `\xintNum`.  
**31, 31, 26, -31**

For legacy reasons the macros next do not have *ii* in their names but they behave in the corresponding way, i.e. their argument must be a (long) integer in the strict format or a macro *f-expanding* to such digit tokens.

**8.19 `\xintDouble`, `\xintHalf`**

*f* ★ `\xintDouble{N}` computes  $2N$  and `\xintHalf{N}` computes  $N/2$  truncated towards zero. Rewritten for 1.2i.

**8.20 `\xintInc`, `\xintDec`**

*f* ★ `\xintInc{N}` evaluates to  $N+1$  and `\xintDec{N}` to  $N-1$ . Rewritten for 1.2i.

**8.21 `\xintDSL`**

*f* ★ `\xintDSL{N}` is decimal shift left, i.e. multiplication by ten. Rewritten with 1.2i and moved from *xint* to *xintcore*.

## 8.22 `\xintDSR`

*f* ★ `\xintDSR{N}` is truncated decimal shift right, *i.e.* it is the truncation of  $N/10$  towards zero. Rewritten with 1.2i and moved from *xint* to *xintcore*.

## 8.23 `\xintDSRr`

*f* ★ `\xintDSRr{N}` is rounded decimal shift right, *i.e.* it is the rounding of  $N/10$  away from zero. It is needed in *xintcore* for use by `\xintiDivRound`.  
New with 1.2i



## 9 Macros of the `xint` package

This package loads automatically `xintcore` (and `xintkernel`) hence all macros described in [section 8](#) are still available. Notice though that it does not load package `xinttools`.

|     |                                                                   |    |     |                                                                                                                                                          |     |
|-----|-------------------------------------------------------------------|----|-----|----------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| .1  | <code>\xintReverseDigits</code> .....                             | 93 | .26 | <code>\xintiiPrd</code> .....                                                                                                                            | 96  |
| .2  | <code>\xintLen</code> .....                                       | 94 | .27 | <code>\xintSgnFork</code> .....                                                                                                                          | 97  |
| .3  | <code>\xintCmp</code> , <code>\xintiiCmp</code> .....             | 94 | .28 | <code>\xintifSgn</code> , <code>\xintiiifSgn</code> .....                                                                                                | 97  |
| .4  | <code>\xintEq</code> , <code>\xintiiEq</code> .....               | 94 | .29 | <code>\xintifZero</code> , <code>\xintiiifZero</code> .....                                                                                              | 97  |
| .5  | <code>\xintNeq</code> , <code>\xintiiNeq</code> .....             | 94 | .30 | <code>\xintifNotZero</code> , <code>\xintiiifNotZero</code> .....                                                                                        | 97  |
| .6  | <code>\xintGt</code> , <code>\xintiiGt</code> .....               | 94 | .31 | <code>\xintifOne</code> , <code>\xintiiifOne</code> .....                                                                                                | 97  |
| .7  | <code>\xintLt</code> , <code>\xintiiLt</code> .....               | 94 | .32 | <code>\xintifTrueAelseB</code> , <code>\xintifFalseAelseB</code> .....                                                                                   | 97  |
| .8  | <code>\xintLtorEq</code> , <code>\xintiiLtorEq</code> .....       | 94 | .33 | <code>\xintifCmp</code> , <code>\xintiiifCmp</code> .....                                                                                                | 97  |
| .9  | <code>\xintGtorEq</code> , <code>\xintiiGtorEq</code> .....       | 95 | .34 | <code>\xintifEq</code> , <code>\xintiiifEq</code> .....                                                                                                  | 97  |
| .10 | <code>\xintIsZero</code> , <code>\xintiiIsZero</code> .....       | 95 | .35 | <code>\xintifGt</code> , <code>\xintiiifGt</code> .....                                                                                                  | 98  |
| .11 | <code>\xintNot</code> .....                                       | 95 | .36 | <code>\xintifLt</code> , <code>\xintiiifLt</code> .....                                                                                                  | 98  |
| .12 | <code>\xintIsNotZero</code> , <code>\xintiiIsNotZero</code> ..... | 95 | .37 | <code>\xintifOdd</code> , <code>\xintiiifOdd</code> .....                                                                                                | 98  |
| .13 | <code>\xintIsOne</code> , <code>\xintiiIsOne</code> .....         | 95 | .38 | <code>\xintiiMON</code> , <code>\xintiiMMON</code> .....                                                                                                 | 98  |
| .14 | <code>\xintAND</code> .....                                       | 95 | .39 | <code>\xintiiOdd</code> .....                                                                                                                            | 98  |
| .15 | <code>\xintOR</code> .....                                        | 95 | .40 | <code>\xintiiEven</code> .....                                                                                                                           | 98  |
| .16 | <code>\xintXOR</code> .....                                       | 95 | .41 | <code>\xintiSqrt</code> , <code>\xintiiSqrt</code> , <code>\xintiiSqrtR</code> ,<br><code>\xintiSquareRoot</code> , <code>\xintiiSquareRoot</code> ..... | 98  |
| .17 | <code>\xintANDof</code> .....                                     | 95 | .42 | <code>\xintiFac</code> , <code>\xintiiFac</code> .....                                                                                                   | 99  |
| .18 | <code>\xintORof</code> .....                                      | 95 | .43 | <code>\xintiBinomial</code> , <code>\xintiiBinomial</code> .....                                                                                         | 99  |
| .19 | <code>\xintXORof</code> .....                                     | 95 | .44 | <code>\xintiPFactorial</code> , <code>\xintiiPFactorial</code> .....                                                                                     | 100 |
| .20 | <code>\xintGeq</code> .....                                       | 95 | .45 | <code>\xintDSH</code> .....                                                                                                                              | 100 |
| .21 | <code>\xintiMax</code> , <code>\xintiiMax</code> .....            | 96 | .46 | <code>\xintDSHr</code> , <code>\xintDSx</code> .....                                                                                                     | 100 |
| .22 | <code>\xintiMin</code> , <code>\xintiiMin</code> .....            | 96 | .47 | <code>\xintDecSplit</code> , <code>\xintDecSplitL</code> ,<br><code>\xintDecSplitR</code> .....                                                          | 101 |
| .23 | <code>\xintiMaxof</code> , <code>\xintiiMaxof</code> .....        | 96 | .48 | <code>\xintiiE</code> .....                                                                                                                              | 101 |
| .24 | <code>\xintiMinof</code> , <code>\xintiiMinof</code> .....        | 96 |     |                                                                                                                                                          |     |
| .25 | <code>\xintiiSum</code> .....                                     | 96 |     |                                                                                                                                                          |     |

Version 1.0 was released 2013/03/28. This is 1.2j of 2016/12/22. The core arithmetic macros have been moved to separate package `xintcore`, which is automatically loaded by `xint`.

See the documentation of `xintcore` or [subsection 3.3.1](#) for the significance of the  $\overset{\text{Num}}{f}$ ,  $f$ ,  $\overset{\text{num}}{x}$  and  $\star$  margin annotations and some important background information.

### 9.1 `\xintReverseDigits`

$f \star$  `\xintReverseDigits{N}` will reverse the order of the digits of the number, preserving an optional upfront minus sign. `\xintRev` is the former denomination and is kept as an alias to it. Leading zeroes resulting from the operation are not removed. Contrarily to `\xintReverseOrder` this macro can only be used with digits and it first expands its argument (but beware that `-\x` will give an unexpected result as the minus sign immediately stops this expansion; one can use `\xintiiOpp{\x}` as argument.)

This macro has been rewritten for 1.2 and is faster for very long inputs. It is (almost) not used internally by the `xintcore` code, but the use of related routines explains to some extent the higher speed of release 1.2.

```
\fdef\x{\xintReverseDigits
 {-9876543210987654321098765432109876543210}}\meaning\x\par
\noindent\def\x{\xintReverseDigits {\xintReverseDigits
```

```
{-98765432109876543210987654321098765432109876543210}}\meaning\x\par
```

```
macro:->-0123456789012345678901234567890123456789
```

```
macro:->-9876543210987654321098765432109876543210
```

Notice that the output in this case with its leading zero is not in the strict integer format expected by the ``ii'` arithmetic macros.

## 9.2 `\xintLen`

Num  
f ★ `\xintLen{N}` returns the length of the number, not counting the sign.

```
\xintLen{-12345678901234567890123456789}=30
```

Extended by `xintfrac` to fractions: the length of `A/B[n]` is the length of `A` plus the length of `B` plus the absolute value of `n` and minus one (an integer input as `N` is internally represented in a form equivalent to `N/1[0]` so the minus one means that the extended `\xintLen` behaves the same as the original for integers).

```
\xintLen{-1e3/5.425}=11
```

The length is computed on the `A/B[n]` which would have been returned by `\xintRaw`: `\xintRaw {-1e3/5}.425}=-1/5425[6]`.

Let's point out that the whole thing should sum up to less than circa  $2^{31}$ , but this is a bit theoretical.

`\xintLen` is only for numbers or fractions. See also `\xintNthElt` from `xinttools`. See also `\xintLength` from `xintkernel` for counting tokens (or rather braced groups), more generally.

## 9.3 `\xintCmp`, `\xintiiCmp`

Num Num  
f f ★ `\xintCmp{N}{M}` returns 1 if  $N > M$ , 0 if  $N = M$ , and -1 if  $N < M$ . Extended by `xintfrac` to fractions (its output naturally still being either 1, 0, or -1).

ff ★ `\xintiiCmp` skips the `\xintNum` overhead.

## 9.4 `\xintEq`, `\xintiiEq`

Num Num  
f f ★ `\xintEq{N}{M}` returns 1 if  $N = M$ , 0 otherwise. Extended by `xintfrac` to fractions.

ff ★ `\xintiiEq` skips the `\xintNum` overhead.

## 9.5 `\xintNeq`, `\xintiiNeq`

Num Num  
f f ★ `\xintNeq{N}{M}` returns 0 if  $N = M$ , 1 otherwise. Extended by `xintfrac` to fractions.

ff ★ `\xintiiNeq` skips the `\xintNum` overhead.

## 9.6 `\xintGt`, `\xintiiGt`

Num Num  
f f ★ `\xintGt{N}{M}` returns 1 if  $N > M$ , 0 otherwise. Extended by `xintfrac` to fractions.

ff ★ `\xintiiGt` skips the `\xintNum` overhead.

## 9.7 `\xintLt`, `\xintiiLt`

Num Num  
f f ★ `\xintLt{N}{M}` returns 1 if  $N < M$ , 0 otherwise. Extended by `xintfrac` to fractions.

ff ★ `\xintiiLt` skips the `\xintNum` overhead.

## 9.8 `\xintLtorEq`, `\xintiiLtorEq`

Num Num  
f f ★ `\xintLtorEq{N}{M}` returns 1 if  $N \leq M$ , 0 otherwise. Extended by `xintfrac` to fractions.

ff ★ `\xintiiLtorEq` skips the `\xintNum` overhead.

9.9 `\xintGtorEq`, `\xintiiGtorEq`

$\text{Num}$   $\text{Num}$   
 $f$   $f$  ★ `\xintGtorEq{N}{M}` returns 1 if  $N \geq M$ , 0 otherwise. Extended by `xintfrac` to fractions.  
 $ff$  ★ `\xintiiGtorEq` skips the `\xintNum` overhead.

9.10 `\xintIsZero`, `\xintiiIsZero`

$\text{Num}$   
 $f$  ★ `\xintIsZero{N}` returns 1 if  $N=0$ , 0 otherwise. Extended by `xintfrac` to fractions.  
 $f$  ★ `\xintiiIsZero` skips the `\xintNum` overhead.

9.11 `\xintNot`

$\text{Num}$   
 $f$  ★ `\xintNot` is a synonym for `\xintIsZero`.

9.12 `\xintIsNotZero`, `\xintiiIsNotZero`

$\text{Num}$   
 $f$  ★ `\xintIsNotZero{N}` returns 1 if  $N \neq 0$ , 0 otherwise. Extended by `xintfrac` to fractions.  
 $f$  ★ `\xintiiIsNotZero` skips the `\xintNum` overhead.

9.13 `\xintIsOne`, `\xintiiIsOne`

$\text{Num}$   
 $f$  ★ `\xintIsOne{N}` returns 1 if  $N=1$ , 0 otherwise. Extended by `xintfrac` to fractions.  
 $f$  ★ `\xintiiIsOne` skips the `\xintNum` overhead.

9.14 `\xintAND`

$\text{Num}$   $\text{Num}$   
 $f$   $f$  ★ `\xintAND{N}{M}` returns 1 if  $N > 0$  and  $M > 0$  and zero otherwise. Extended by `xintfrac` to fractions.

9.15 `\xintOR`

$\text{Num}$   $\text{Num}$   
 $f$   $f$  ★ `\xintOR{N}{M}` returns 1 if  $N > 0$  or  $M > 0$  and zero otherwise. Extended by `xintfrac` to fractions.

9.16 `\xintXOR`

$\text{Num}$   $\text{Num}$   
 $f$   $f$  ★ `\xintXOR{N}{M}` returns 1 if exactly one of  $N$  or  $M$  is true (i.e. non-zero). Extended by `xintfrac` to fractions.

9.17 `\xintANDof`

$f \rightarrow *$   $\text{Num}$   
 $f$  ★ `\xintANDof{{a}{b}{c}...}` returns 1 if all are true (i.e. non zero) and zero otherwise. The list argument may be a macro, it (or rather its first token) is *f-expanded* first (each item also is *f-expanded*). Extended by `xintfrac` to fractions.

9.18 `\xintORof`

$f \rightarrow *$   $\text{Num}$   
 $f$  ★ `\xintORof{{a}{b}{c}...}` returns 1 if at least one is true (i.e. does not vanish). The list argument may be a macro, it is *f-expanded* first. Extended by `xintfrac` to fractions.

9.19 `\xintXORof`

$f \rightarrow *$   $\text{Num}$   
 $f$  ★ `\xintXORof{{a}{b}{c}...}` returns 1 if an odd number of them are true (i.e. does not vanish). The list argument may be a macro, it is *f-expanded* first. Extended by `xintfrac` to fractions.

9.20 `\xintGeq`

$\text{Num}$   $\text{Num}$   
 $f$   $f$  ★ `\xintGeq{N}{M}` returns 1 if the *absolute value* of the first number is at least equal to the absolute value of the second number. If  $|N| < |M|$  it returns 0. Extended by `xintfrac` to fractions. Important: the macro compares *absolute values*.

9.21 `\xintiMax`, `\xintiiMax`

$\text{Num Num}$   
 $f f \star$  `\xintiMax{N}{M}` returns the largest of the two in the sense of the order structure on the relative integers (i.e. the right-most number if they are put on a line with positive numbers on the right):  
`\xintiMax {-5}{-6}=-5`.

$f f \star$  The `\xintiiMax` macro skips the overhead of parsing the operands with `\xintNum`.

9.22 `\xintiMin`, `\xintiiMin`

$\text{Num Num}$   
 $f f \star$  `\xintiMin{N}{M}` returns the smallest of the two in the sense of the order structure on the relative integers (i.e. the left-most number if they are put on a line with positive numbers on the right):  
`\xintiMin {-5}{-6}=-6`.

$f f \star$  The `\xintiiMin` macro skips the overhead of parsing the operands with `\xintNum`.

9.23 `\xintiMaxof`, `\xintiiMaxof`

$f \rightarrow * f \star$  `\xintiMaxof{a}{b}{c}...` returns the maximum. The list argument may be a macro, it is *f-expanded* first. Each item is submitted to `\xintNum` normalization.

New with 1.2a `\xintiiMaxof` does the same, skips `\xintNum` normalization of items.

9.24 `\xintiMinof`, `\xintiiMinof`

$f \rightarrow * f \star$  `\xintiMinof{a}{b}{c}...` returns the minimum. The list argument may be a macro, it is *f-expanded* first. Each item is submitted to `\xintNum` normalization.

New with 1.2a `\xintiiMinof` does the same, skips `\xintNum` normalization of items.

9.25 `\xintiiSum`

$*f \star$  `\xintiiSum{braced things}` after expanding its argument expects to find a sequence of tokens (or braced material). Each is *f-expanded*, and the sum of all these numbers is returned. Note: the summands are not parsed by `\xintNum`.

`\xintiiSum{{123}{-98763450}}{\xintiiFac{7}}{\xintiMul{3347}{591}}=-96780210`  
`\xintiiSum{1234567890}=45`

An empty sum is no error and returns zero: `\xintiiSum {}=0`. A sum with only one term returns that number: `\xintiiSum {-1234}=-1234`. Attention that `\xintiiSum {-1234}` is not legal input and will make the  $\text{T}_{\text{E}}\text{X}$  run fail. On the other hand `\xintiiSum {1234}=10`.

9.26 `\xintiiPrd`

$*f \star$  `\xintiiPrd{braced things}` after expanding its argument expects to find a sequence of (of braced items or unbraced single tokens). Each is expanded (with the usual meaning), and the product of all these numbers is returned. Note: the operands are *not* parsed by `\xintNum`.

`\xintiiPrd{{-9876}}{\xintiiFac{7}}{\xintiMul{3347}{591}}=-98458861798080`  
`\xintiiPrd{123456789123456789}=131681894400`

An empty product is no error and returns 1: `\xintiiPrd {}=1`. A product reduced to a single term returns this number: `\xintiiPrd {-1234}=-1234`. Attention that `\xintiiPrd {-1234}` is not legal input and will make the  $\text{T}_{\text{E}}\text{X}$  compilation fail. On the other hand `\xintiiPrd {1234}=24`.

`$2^{200}3^{100}7^{100}=\printnumber`  
`{\xintiiPrd {{\xintiPow {2}{200}}{\xintiPow {3}{100}}{\xintiPow {7}{100}}}}$`

`220031007100 = 267872793166157757576627951700754840232474026637401534897445961481542641296549924900004440072407657271300001653120764065456211801435719940159033435392440282124389668222489227862988084382716133376`

With `xintexpr`, this would be easier:

`\xinttheiexpr 2^200*3^100*7^100\relax`

## 9.27 `\xintSgnFork`

`xnnn` ★ `\xintSgnFork{-1|0|1}{⟨A⟩}{⟨B⟩}{⟨C⟩}` expandably chooses to execute either the `⟨A⟩`, `⟨B⟩` or `⟨C⟩` code, depending on its first argument. This first argument should be anything expanding to either `-1`, `0` or `1` in a non self-delimiting way (i.e. a count register must be prefixed by `\the` and a `\numexp` r...`\relax` also must be prefixed by `\the`). This utility is provided to help construct expandable macros choosing depending on a condition which one of the package macros to use, or which values to confer to their arguments.

## 9.28 `\xintifSgn`, `\xintiiifSgn`

`Num f nnn` ★ Similar to `\xintSgnFork` except that the first argument may expand to a (big) integer (or a fraction if `xintfrac` is loaded), and it is its sign which decides which of the three branches is taken. Furthermore this first argument may be a count register, with no `\the` or `\number` prefix.

`f` ★ `\xintiiifSgn` skips the `\xintNum` overhead.

## 9.29 `\xintifZero`, `\xintiiifZero`

`Num f nn` ★ `\xintifZero{⟨N⟩}{⟨IsZero⟩}{⟨IsNotZero⟩}` expandably checks if the first mandatory argument `N` (a number, possibly a fraction if `xintfrac` is loaded, or a macro expanding to one such) is zero or not. It then either executes the first or the second branch. Beware that both branches must be present.

`f` ★ `\xintiiifZero` skips the `\xintNum` overhead.

## 9.30 `\xintifNotZero`, `\xintiiifNotZero`

`Num f nn` ★ `\xintifNotZero{⟨N⟩}{⟨IsNotZero⟩}{⟨IsZero⟩}` expandably checks if the first mandatory argument `N` (a number, possibly a fraction if `xintfrac` is loaded, or a macro expanding to one such) is not zero or is zero. It then either executes the first or the second branch. Beware that both branches must be present.

`f` ★ `\xintiiifNotZero` skips the `\xintNum` overhead.

## 9.31 `\xintifOne`, `\xintiiifOne`

`Num f nn` ★ `\xintifOne{⟨N⟩}{⟨IsOne⟩}{⟨IsNotOne⟩}` expandably checks if the first mandatory argument `N` (a number, possibly a fraction if `xintfrac` is loaded, or a macro expanding to one such) is one or not. It then either executes the first or the second branch. Beware that both branches must be present.

`f` ★ `\xintiiifOne` skips the `\xintNum` overhead.

## 9.32 `\xintifTrueAelseB`, `\xintifFalseAelseB`

`Num f nn` ★ `\xintifTrueAelseB{⟨N⟩}{⟨true branch⟩}{⟨false branch⟩}` is a synonym for `\xintifNotZero`. These macros have no lowercase versions, use `\xintifzero`, `\xintifnotzero`.

`Num f nn` ★ `\xintifFalseAelseB{⟨N⟩}{⟨false branch⟩}{⟨true branch⟩}` is a synonym for `\xintifZero`.

## 9.33 `\xintifCmp`, `\xintiiifCmp`

`Num Num f f nnn` ★ `\xintifCmp{⟨A⟩}{⟨B⟩}{⟨if A<B⟩}{⟨if A=B⟩}{⟨if A>B⟩}` compares its arguments and chooses accordingly the correct branch.

`ff` ★ `\xintiiifCmp` skips the `\xintNum` overhead.

## 9.34 `\xintifEq`, `\xintiiifEq`

`Num Num f f nn` ★ `\xintifEq{⟨A⟩}{⟨B⟩}{⟨YES⟩}{⟨NO⟩}` checks equality of its two first arguments (numbers, or fractions if `xintfrac` is loaded) and does the `YES` or the `NO` branch.

`ff` ★ `\xintiiifEq` skips the `\xintNum` overhead.

9.35 \xintifGt, \xintiiifGt

`Num Num f f nn` ★ `\xintifGt{⟨A⟩}{⟨B⟩}{⟨YES⟩}{⟨NO⟩}` checks if  $A > B$  and in that case executes the `YES` branch. Extended to fractions (in particular decimal numbers) by `xintfrac`.

`ff` ★ `\xintiifGt` skips the `\xintNum` overhead.

9.36 `\xintifLt`, `\xintiiiifLt`

Num Num  
f f nn ★ \xintifLt{A}{B}{{YES}}{{NO}} checks if  $A < B$  and in that case executes the YES branch. Extended to fractions (in particular decimal numbers) by xintfrac.  
ff ★ \xintiiifLt skips the \xintNum overhead.

9.37 \xintif0odd, \xintiiiif0odd

**f nn** ★ `\xintifOdd{⟨A⟩}{⟨YES⟩}{⟨NO⟩}` checks if A is and odd integer and in that case executes the YES branch.

**f** ★ `\xintiiifOdd` skips the `\xintNum` overhead.

The macros described next are all integer-only on input. Those with `ii` in their names skip the `\xintNum` parsing. The others, with `xintfrac` loaded, can have fractions as arguments, which will get truncated to integers via `\xintTTrunc`. On output, these macros always produce integers (with no `/B[N]`).

9.38 \xintiiMON. \xintiiMMON

**f★** `\xintiiMON{N}` returns  $(-1)^N$  and `\xintiiMMON{N}` returns  $(-1)^{N-1}$ . They skip the overhead of parsing via `\xintNum`.

**Num**  $\int$  ★ `\xintiiMON {-280914019374101929}=-1`, `\xintiiMMON {-280914019374101929}=1`  
 The variants `\xintMON` and `\xintMMON` use `\xintNum` and get extended to fractions by `xintfrac`.

9.39 \xintii0dd

$\text{xintNum}$   $\text{xintOdd}\{N\}$  is 1 if the number is odd and 0 otherwise. It skips the overhead of parsing via  $\text{xintNum}$ .  $\text{xintOdd}$  is the variant using  $\text{xintNum}$  and extended to fractions by  $\text{xintfrac}$ .

### 9.40 \xintiiEven

$\text{Num} \xrightarrow{f} \star$  `\xintiiEven{N}` is 1 if the number is even and 0 otherwise. It skips the overhead of parsing via `\xintNum`. `\xintEven` is the variant using `\xintNum` and extended to fractions by `\xintfrac`.

9.41 `\xintiSqrt`, `\xintiiSqrt`, `\xintiiSqrtR`, `\xintiSquareRoot`, `\xintiiSquareRoot`

**Num**  
**f** ★ `\xintiSqrt{N}` returns the largest integer whose square is at most equal to `N`. `\xintiiSqrt` is the variant skipping the `\xintNum` overhead. `\xintiiSqrtR` also skips the `\xintNum` overhead and it returns the rounded, not truncated, square root.

```
\begin{itemize}[nosep]
\item \xintiiSqrt {3000000000000000000000000000000000}
\item \xintiiSqrtR {3000000000000000000000000000000000}
\item \xintiiSqrt {\xintiie {3}{100}}
\end{itemize}
```

- 1732050807568877293
- 1732050807568877294
- 173205080756887729352744634150587236694280525381038

- 173205080756887729352744634150587236694280525381038

`\xintiSquareRoot{N}` returns  $\{M\}_d$  with  $d>0$ ,  $M^2-d=N$  and  $M$  smallest (hence  $=1+\text{\code\xintisqrt}\{N\}$ ).

`\xintiiSquareRoot` is the variant skipping the `\xintNum` overhead.

```
\xintAssign\xintiisquareRoot {170000000000000000000000000000}\to\A\B
\xintiisub{\xintiisqr\A}\B=\A\string^2-\B
```

```
170000000000000000000000000000=4123105625618^2-2799177881924
```

A rational approximation to  $\sqrt{N}$  is  $M - \frac{d}{2M}$  (this is a majorant and the error is at most  $1/2M$ ; if  $N$  is a perfect square  $k^2$  then  $M=k+1$  and this gives  $k+1/(2k+2)$ , not  $k$ ).

Package `xintfrac` has `\xintFloatSqrt` for square roots of floating point numbers.

## 9.42 `\xintiFac`, `\xintiFac`

Defined in `xintcore`, see subsection 8.12 for more info.

## 9.43 `\xintiBinomial`, `\xintiBinomial`

num num  
x x ★  
New with  
1.2f  
Changed  
(1.2h)

`\xintiBinomial{x}{y}` computes binomial coefficients.

`\xintiBinomial` is originally a synonym. With `xintfrac` loaded it applies `\xintNum` to its arguments and thus accepts fractional inputs but truncates them to an integer.

When  $x < 0$  an out-of-range error is raised. Else, if  $y < 0$  or if  $x < y$  the macro evaluates to 0 (it was a bit unfortunate that the 1.2f version deliberately raised an out-of-range error for the cases  $y < 0$  and  $y > x$ , with a positive  $x$ ).

The allowable range is  $0 \leq x \leq 99999999$ .

This theoretical range includes binomial coefficients with more than the roughly 19950 digits that the arithmetics of `xint` can handle. In such cases, the computation will end up in a low-level TeX error after a long time.

It turns out that  $\binom{65000}{32500}$  has 19565 digits and  $\binom{64000}{32000}$  has 19264 digits. The latter can be evaluated (this takes a long long time) but presumably not the former (I didn't try). Reasonable feasible evaluations are with binomial coefficients not exceeding about one thousand digits.

The `binomial` function is available in the `xintexpr` parsers.

```
\xinttheiexpr seq(binomial(100,i), i=47..53)\relax
84413487283064039501507937600, 93206558875049876949581681100, 98913082887808032681188722800,
100891344545564193334812497256, 98913082887808032681188722800, 93206558875049876949581681100,
84413487283064039501507937600
```

See `\xintFloatBinomial` from package `xintfrac` for the float variant, used in `\xintfloatexpr`.

In order to evaluate binomial coefficients  $\binom{x}{y}$  with  $x > 99999999$ , or even  $x \geq 2^{31}$ , but  $y$  is not too large, one may use an ad hoc function definition such as:

```
\xintdefunc mybigbinomial(x,y):='*(x-y+1..[1]..x)//y!;%
%
% without [1], x would have been limited to < 2^31
\printnumber{\xinttheexpr mybigbinomial(98765432109876543210,10)\relax}
24338098741940755592729533173058146177070669479669793038510211146784065843698581878582323710
27360575372715482389633359878460739973726786576925067784100587971261422326652270975592667517
4871960261
```

To get this functionality in macro form, one can do:

```
\xintNewIExpr\MyBigBinomial [2]{`*(#1-#2+1..[1]..#1)//#2!}
\printnumber{\MyBigBinomial {98765432109876543210}{10}}
24338098741940755592729533173058146177070669479669793038510211146784065843698581878582323710
27360575372715482389633359878460739973726786576925067784100587971261422326652270975592667517
4871960261
```

As we used `\xintNewIExpr`, this macro will only accept strict integers. Had we used `\xintNewExpr` the `\MyBigBinomial` would have accepted general fractions or decimal numbers, and computed the product at the numerator without truncating them to integers; but the factorial at the denominator would truncate its argument.



9.44 `\xintiPFactorial`, `\xintiiPFactorial`

`\xintiiPFactorial{a}{b}` computes the partial factorial  $(a+1)(a+2)\dots b$ . For  $a=b$  the product is considered empty hence returns 1.

`\xintiPFactorial` is originally a synonym. With `xintfrac` loaded it applies `\xintNum` to its arguments and thus accepts fractional inputs but truncates them to an integer.

Changed  
(1.2h)

The allowed range with 1.2f was  $0 \leq a \leq b \leq 99999999$ .

It was a bit unfortunate with 1.2f that the code deliberately raised an error if this condition was not obeyed by the arguments.

Starting with 1.2h,  $-100000000 \leq a, b \leq 99999999$  is accepted. The rule is to interpret the formula as the product of the  $j$ 's such that  $a < j \leq b$ , hence in particular if  $a \geq b$  the product is empty and the macro evaluates to 1.

Only for  $0 \leq a \leq b$  is the behaviour to be considered stable. For  $a > b$  or negative arguments, the definitive rules have not yet been fixed.

```
\xintiiPFactorial {100}{130}
```

```
69293021885203871012298422845822803287591970060789350400000000
```

This theoretical range allows computations whose result values would have more than the roughly 19950 digits that the arithmetics of `xint` can handle. In such cases, the computation will end up in a low-level TeX error after a long time.

The `pfactorial` function is available in the `xintexpr` parsers.

```
\xinttheiexpr pfactorial(100,130)\relax
```

```
69293021885203871012298422845822803287591970060789350400000000
```

See `\xintFloatPFactorial` from package `xintfrac` for the float variant, used in `\xintfloatexpr`.

In case values are needed with  $b > 99999999$ , or even  $b \geq 2^{31}$ , but  $b - a$  is not too large, one may use an ad hoc function definition such as:

```
\xintdefunc mybigpfac(a,b):='*(a+1..[1]..b);%
```

```
% without [1], b would have been limited to < 2^31
```

```
\printnumber{\xinttheexpr mybigpfac(98765432100,98765432120)\relax}
```

```
780008550175675280672981073130237784386530020290496474672081960281164994340505876568704893229
996306044822368535664039125614499125874046078441040781214726754618154427340986762834500699339
322948600573016997034009566576640000
```

The macros described next are strictly for integer-only arguments (which get only `f`-expanded, not filtered via `\xintNum`.)

9.45 `\xintDSH`

`\xintDSH{x}{N}` is parametrized decimal shift. When  $x$  is negative, it is like iterating `\xintDSL`  $|x|$  times (i.e. multiplication by  $10^{-x}$ ). When  $x$  positive, it is like iterating `\xintDSR`  $x$  times (and is more efficient), and for a non-negative  $N$  this is thus the same as the quotient from the euclidean division by  $10^x$ .

9.46 `\xintDSHr`, `\xintDSx`

`\xintDSHr{x}{N}` expects  $x$  to be zero or positive and it returns then a value  $R$  which is correlated to the value  $Q$  returned by `\xintDSH{x}{N}` in the following manner:

- if  $N$  is positive or zero,  $Q$  and  $R$  are the quotient and remainder in the euclidean division by  $10^x$  (obtained in a more efficient manner than using `\xintiDivision`),

- if  $N$  is negative let  $Q1$  and  $R1$  be the quotient and remainder in the euclidean division by  $10^{\wedge}x$  of the absolute value of  $N$ . If  $Q1$  does not vanish, then  $Q=-Q1$  and  $R=R1$ . If  $Q1$  vanishes, then  $Q=0$  and  $R=-R1$ .
- for  $x=0$ ,  $Q=N$  and  $R=0$ .

So one has  $N = 10^x Q + R$  if  $Q$  turns out to be zero or positive, and  $N = 10^x Q - R$  if  $Q$  turns out to be negative, which is exactly the case when  $N$  is at most  $-10^x$ .

$\text{num}_x f \star$  `\xintDSx{x}{N}` for  $x$  negative is exactly as `\xintDSH{x}{N}`, i.e. multiplication by  $10^{-x}$ . For  $x$  zero or positive it returns the two numbers `{Q}{R}` described above, each one within braces. So  $Q$  is `\xintDSH{x}{N}`, and  $R$  is `\xintDSHr{x}{N}`, but computed simultaneously.

### 9.47 `\xintDecSplit`, `\xintDecSplitL`, `\xintDecSplitR`

$\text{num}_x f \star \backslash \text{xintDecSplit}\{x\}\{N\}$  cuts the number into two pieces (each one within a pair of enclosing braces)  $\{L\}\{R\}$  where the decimal writing of  $N$  is the concatenation  $LR$ .

For `x` positive or null, `R` coincides with the `x` least significant digits and is *empty* if `x=0`. If `x` equals or exceeds the length of `N` the first piece `L` is *empty*.

When  $x$  is negative the first piece  $L$  contains the  $(-x)$  most significant digits and the second piece the remaining ones. Hence  $R$  is *empty* if  $x$  equals or exceeds the length of  $N$ .

Breaking change with 1.2i: formerly `N<0` was replaced by its absolute value. Now, a sign (positive or negative) will create an error. The `N` must consists only of digit tokens (after *f-expansion*). Leading zeroes are allowed.

`\xintDecSplitL{x}{N}` returns the first piece (unbraced) from the `\xintDecSplit` output.

$\text{num } x \star f$  `\xintDecSplit{x}{N}` returns the second piece (unbraced) from the `\xintDecSplit` output.

9.48 \xintiE

$f_x^{\text{num}} \star \backslash \text{xintiiE}\{N\}\{x\}$  serves to add zeros to the right of  $N$ .

$$\backslash \text{xintiiE} \{123\}\{89\}$$
[illegible]

## 10 Macros of the `xintfrac` package

This package loads automatically `xint` and `xintcore`, hence all macros described in [section 9](#) and [section 8](#) are available. Note that macros of those packages whose names contain `ii` are for integers only, not fractions. Those with a single `i` accept fractions but truncate them to integers.

|     |                                           |     |     |                                                       |     |
|-----|-------------------------------------------|-----|-----|-------------------------------------------------------|-----|
| .1  | <code>\xintNum</code>                     | 103 | .33 | <code>\xintiBinomial</code>                           | 111 |
| .2  | <code>\xintifInt</code>                   | 103 | .34 | <code>\xintiPFactorial</code>                         | 111 |
| .3  | <code>\xintLen</code>                     | 103 | .35 | <code>\xintPow</code>                                 | 111 |
| .4  | <code>\xintRaw</code>                     | 103 | .36 | <code>\xintSum</code>                                 | 111 |
| .5  | <code>\xintPRaw</code>                    | 103 | .37 | <code>\xintPrd</code>                                 | 111 |
| .6  | <code>\xintNumerator</code>               | 104 | .38 | <code>\xintCmp</code>                                 | 112 |
| .7  | <code>\xintDenominator</code>             | 104 | .39 | <code>\xintIsOne</code>                               | 112 |
| .8  | <code>\xintRawWithZeros</code>            | 104 | .40 | <code>\xintGeq</code>                                 | 112 |
| .9  | <code>\xintREZ</code>                     | 104 | .41 | <code>\xintMax</code>                                 | 112 |
| .10 | <code>\xintFrac</code>                    | 104 | .42 | <code>\xintMin</code>                                 | 112 |
| .11 | <code>\xintSignedFrac</code>              | 105 | .43 | <code>\xintMaxof</code>                               | 112 |
| .12 | <code>\xintFwOver</code>                  | 105 | .44 | <code>\xintMinof</code>                               | 112 |
| .13 | <code>\xintSignedFwOver</code>            | 105 | .45 | <code>\xintAbs</code>                                 | 112 |
| .14 | <code>\xintIrr</code>                     | 105 | .46 | <code>\xintSgn</code>                                 | 112 |
| .15 | <code>\xintJrr</code>                     | 105 | .47 | <code>\xintOpp</code>                                 | 113 |
| .16 | <code>\xintTrunc</code>                   | 105 | .48 | <code>\xintDigits, \xinttheDigits</code>              | 113 |
| .17 | <code>\xintiTrunc</code>                  | 106 | .49 | <code>\xintFloat</code>                               | 113 |
| .18 | <code>\xintTTrunc</code>                  | 106 | .50 | <code>\xintPFloat</code>                              | 113 |
| .19 | <code>\xintXTrunc</code>                  | 106 | .51 | <code>\xintFloatE</code>                              | 114 |
| .20 | <code>\xintRound</code>                   | 109 | .52 | <code>\xintFloatAdd</code>                            | 114 |
| .21 | <code>\xintiRound</code>                  | 109 | .53 | <code>\xintFloatSub</code>                            | 114 |
| .22 | <code>\xintFloor, \xintiFloor</code>      | 109 | .54 | <code>\xintFloatMul</code>                            | 114 |
| .23 | <code>\xintCeil, \xintiCeil</code>        | 109 | .55 | <code>\xintFloatDiv</code>                            | 115 |
| .24 | <code>\xintTFrac</code>                   | 109 | .56 | <code>\xintFloatFac</code>                            | 115 |
| .25 | <code>\xintE</code>                       | 110 | .57 | <code>\xintFloatBinomial</code>                       | 115 |
| .26 | <code>\xintAdd</code>                     | 110 | .58 | <code>\xintFloatPFactorial</code>                     | 115 |
| .27 | <code>\xintSub</code>                     | 110 | .59 | <code>\xintFloatPow</code>                            | 116 |
| .28 | <code>\xintMul</code>                     | 110 | .60 | <code>\xintFloatPower</code>                          | 116 |
| .29 | <code>\xintSqr</code>                     | 110 | .61 | <code>\xintFloatSqrt</code>                           | 117 |
| .30 | <code>\xintDiv</code>                     | 111 | .62 | <code>\xintiDivision, \xintiQuo, \xintiRem,</code>    |     |
| .31 | <code>\xintDivTrunc, \xintDivRound</code> | 111 |     | <code>\xintFDg, \xintLDg, \xintMON, \xintMMON,</code> |     |
| .32 | <code>\xintiFac</code>                    | 111 |     | <code>\xintOdd</code>                                 | 117 |

This package was first included in release 1.03 (2013/04/14) of the `xint` bundle. The general rule of the bundle that each macro first expands (what comes first, fully) each one of its arguments applies.

Frac  
*f*

*f* stands for an integer or a fraction (see [subsection 3.4](#) for the accepted input formats) or something which expands to an integer or fraction. It is possible to use in the numerator or the denominator of *f* count registers and even expressions with infix arithmetic operators, under some rules which are explained in the [subsection 3.6](#) section.

num  
*x*

As in the `xint.sty` documentation, *x* stands for something which will internally be embedded in a `\numexpr`. It may thus be a count register or something like `4*\count 255 + 17`, etc..., but must expand to an integer obeying the  $\text{T}_{\text{E}}\text{X}$  bound.

The fraction format on output is the scientific notation for the ``float'` macros, and the `A/B[n]` format for all other fraction macros, with the exception of `\xintTrunc`, `\xintRound` (which produce decimal numbers) and `\xintIrr`, `\xintJrr`, `\xintRawWithZeros` (which returns an `A/B` with no trailing `[n]`, and prints the `B` even if it is 1), and `\xintPraw` which does not print the `[n]` if `n=0` or the `B` if `B=1`.

To be certain to print an integer output without trailing `[n]` nor fraction slash, one should use either `\xintPraw {\xintIrr {f}}` or `\xintNum {f}` when it is already known that `f` evaluates to a (big) integer. For example `\xintPraw {\xintAdd {2/5}{3/5}}` gives a perhaps disappointing `5/5` whereas `\xintPraw {\xintIrr {\xintAdd {2/5}{3/5}}}` returns `1`. As we knew the result was an integer we could have used `\xintNum {\xintAdd {2/5}{3/5}}=1`.

Some macros (such as `\xintiTrunc`, `\xintiRound`, and `\xintiFac`) always produce integers on output.

Refer to [subsection 3.2](#) for general background information on how floating point numbers and evaluations are implemented.

## 10.1 `\xintNum`

*f* ★ The macro from *xint* is made a synonym to `\xintTrunc`.<sup>58</sup>

The original (which normalizes big integers to strict format) is still available as `\xintiNum`. It is imprudent to apply `\xintNum` to numbers with a large power of ten given either in scientific notation or with the `[n]` notation, as the macro will according to its definition add all the needed zeroes to produce an explicit integer in strict format.

## 10.2 `\xintifInt`

*Frac f nn* ★ `\xintifInt{f}{YES branch}{NO branch}` expandably chooses the `YES` branch if `f` reveals itself after expansion and simplification to be an integer. As with the other *xint* conditionals, both branches must be present although one of the two (or both, but why then?) may well be an empty brace pair `{}`. Spaces in-between the braced things do not matter, but a space after the closing brace of the `NO` branch is significant.

## 10.3 `\xintLen`

*Frac f* ★ The original macro is extended to accept a fraction on input.

```
\xintLen {201710/298219}=11, \xintLen {1234/1}=4, \xintLen {1234}=4
```

## 10.4 `\xintRaw`

*Frac f* ★ This macro 'prints' the fraction `f` as it is received by the package after its parsing and expansion, in a form `A/B[n]` equivalent to the internal representation: the denominator `B` is always strictly positive and is printed even if it has value 1.

```
\xintRaw{\the\numexpr 571*987\relax.123e-10/\the\numexpr -201+59\relax e-7}=
-563577123/142[-6]
```

## 10.5 `\xintPraw`

*Frac f* ★ `Praw` stands for 'pretty raw'. It does not show the `[n]` if `n=0` and does not show the `B` if `B=1`.

```
\xintPraw {123e10/321e10}=123/321, \xintPraw {123e9/321e10}=123/321[-1]
\xintPraw {\xintIrr{861/123}}=7 vz. \xintIrr{861/123}=7/1
```

See also `\xintFrac` (or `\xintFwOver`) for math mode. As is exemplified above the `\xintIrr` macro which puts the fraction into irreducible form does not remove the `/1` if the fraction is an integer.

<sup>58</sup> In earlier releases than 1.1, `\xintNum` did `\xintIrr` and then complained if the denominator was not 1, else, it silently removed the denominator.

One can use `\xintNum{f}` or `\xintPraw{\xintIrr{f}}` which produces the same output only if *f* is an integer (after simplification).

## 10.6 `\xintNumerator`

*Frac f* ★ This returns the numerator corresponding to the internal representation of a fraction, with positive powers of ten converted into zeroes of this numerator:

```
\xintNumerator {178000/25600000[17]}=1780000000000000000000
\xintNumerator {312.289001/20198.27}=312289001
\xintNumerator {178000e-3/256e5}=178000
\xintNumerator {178.000/256000000}=178000
```

As shown by the examples, no simplification of the input is done. For a result uniquely associated to the value of the fraction first apply `\xintIrr`.

## 10.7 `\xintDenominator`

*Frac f* ★ This returns the denominator corresponding to the internal representation of the fraction:<sup>59</sup>

```
\xintDenominator {178000/25600000[17]}=25600000
\xintDenominator {312.289001/20198.27}=20198270000
\xintDenominator {178000e-3/256e5}=25600000000
\xintDenominator {178.000/256000000}=25600000000
```

As shown by the examples, no simplification of the input is done. The denominator looks wrong in the last example, but the numerator was tacitly multiplied by 1000 through the removal of the decimal point. For a result uniquely associated to the value of the fraction first apply `\xintIrr`.

## 10.8 `\xintRawWithZeros`

*Frac f* ★ This macro 'prints' the fraction *f* (after its parsing and expansion) in A/B form, with A as returned by `\xintNumerator{f}` and B as returned by `\xintDenominator{f}`.

```
\xintRawWithZeros{\the\numexpr 571*987\relax.123e-10/\the\numexpr -201+59\relax e-7}=
-563577123/142000000
```

## 10.9 `\xintREZ`

*Frac f* ★ This macro normalizes a fraction by removing the powers of ten from its numerator and denominator:

```
\xintREZ {178000/25600000[17]}=178/256[15]
\xintREZ {17800000000000e30/2560000000000e15}=178/256[15]
```

As shown by the example, it does not otherwise simplify the fraction.

## 10.10 `\xintFrac`

*Frac f* ★ This is a  $\TeX$  only macro, to be used in math mode only. It will print a fraction, internally represented as something equivalent to  $A/B[n]$  as `\frac {A}{B}10^n`. The power of ten is omitted when  $n=0$ , the denominator is omitted when it has value one, the number being separated from the power of ten by a `\cdot`. `\xintFrac {178.000/256000000}` gives  $\frac{178000}{25600000}10^{-3}$ , `\xintFrac {178.000/1}` gives  $178000 \cdot 10^{-3}$ , `\xintFrac {3.5/5.7}` gives  $\frac{35}{57}$ , and `\xintFrac {\xintNum {\xintiFac{10}}/\xintiSqrr{\xintiFac {5}}}` gives 252. As shown by the examples, simplification of the input (apart from removing the decimal points and moving the minus sign to the numerator) is not done automatically and must be the result of macros such as `\xintIrr`, `\xintREZ`, or `\xintNum` (for fractions being in fact integers.)

<sup>59</sup> recall that the `[]` construct excludes presence of a decimal point.

### 10.11 `\xintSignedFrac`

Frac  
f ★ This is as `\xintFrac` except that a negative fraction has the sign put in front, not in the numerator.

```
\[\xintFrac {-355/113}=\xintSignedFrac {-355/113}\]
```

$$\frac{-355}{113} = -\frac{355}{113}$$

### 10.12 `\xintFwOver`

Frac  
f ★ This does the same as `\xintFrac` except that the `\over` primitive is used for the fraction (in case the denominator is not one; and a pair of braces contains the A`\over` B part). `\xintFwOver {178.0000/256000000}` gives  $\frac{178000}{25600000} \cdot 10^{-3}$ , `\xintFwOver {178.0000/1}` gives  $178000 \cdot 10^{-3}$ , `\xintFwOver {3.50/5.7}` gives  $\frac{35}{57}$ , and `\xintFwOver {\xintNum {\xintiiFac{10}}/\xintiSqr{\xintiiFac {5}}}` gives 252.

### 10.13 `\xintSignedFwOver`

Frac  
f ★ This is as `\xintFwOver` except that a negative fraction has the sign put in front, not in the numerator.

```
\[\xintFwOver {-355/113}=\xintSignedFwOver {-355/113}\]
```

$$\frac{-355}{113} = -\frac{355}{113}$$

### 10.14 `\xintIrr`

Frac  
f ★ This puts the fraction into its unique irreducible form:

```
\xintIrr {178.256/256.178}=6856/9853 = $\frac{6856}{9853}$
```

Note that the current implementation does not cleverly first factor powers of 2 and 5, so input such as `\xintIrr {2/3[100]}` will make *xintfrac* do the Euclidean division of  $2 \cdot 10^{100}$  by 3, which is a bit stupid.

Starting with release 1.08, `\xintIrr` does not remove the trailing /1 when the output is an integer. This was deemed better for various (stupid?) reasons and thus the output format is now always A/B with B>0. Use `\xintPRaw` on top of `\xintIrr` if it is needed to get rid of a possible trailing /1. For display in math mode, use rather `\xintFrac{\xintIrr {f}}` or `\xintFwOver{\xintIrr {f}}`.

### 10.15 `\xintJrr`

Frac  
f ★ This also puts the fraction into its unique irreducible form:

```
\xintJrr {178.256/256.178}=6856/9853
```

This is faster than `\xintIrr` for fractions having some big common factor in the numerator and the denominator.

```
\xintJrr {\xintiPow{\xintiiFac {15}}{3}/\xintiiPrdExpr {\xintiiFac{10}}{\xintiiFac{30}}{\xintiiFac{5}}\relax }=1001/51705840
```

But to notice the difference one would need computations with much bigger numbers than in this example. Starting with release 1.08, `\xintJrr` does not remove the trailing /1 when the output is an integer.

### 10.16 `\xintTrunc`

num  
x Frac  
f ★ `\xintTrunc{x}{f}` returns the integral part, a dot, and then the first x digits of the decimal expansion of the fraction f, except when the fraction is (or evaluates to) zero, then it simply prints 0 (with no dot).

The argument `x` must be non-negative, the behavior is currently undefined when `x<0` and will provoke errors.

Except when the input is (or evaluates to) exactly zero, the output contains exactly `x` digits after the decimal mark, thus the output may be `0.00...0` or `-0.00...0`, indicating that the original fraction was positive, respectively negative.

**Warning:** *it is not yet decided is this behavior is definitive.*

Currently *xintfrac* has no notion of a positive zero or a negative zero. Hence transitivity of `\xintTrunc` is broken for the case where the first truncation gives on output `0.00...0` or `-0.00...0`: a second truncation to less digits will then output `0`, whereas if it had been applied directly to the initial input it would have produced `0.00...0` or respectively `-0.00...0` (with less zeros).

If *xintfrac* distinguished zero, positive zero, and negative zero it would be possible to maintain transitivity.

The problem would also be fixed, even without distinguishing a negative zero on input, if `\xintTrunc` always produced `0.00...0` (with no sign) when the mathematical result is zero, discarding the information on original input being positive, zero, or negative.

I have multiple times hesitated about what to do and must postpone again final decision.

```
\xintTrunc {16}{-803.2028/20905.298}=-0.0384210165289200
\xintTrunc {20}{-803.2028/20905.298}=-0.03842101652892008523
\xintTrunc {10}{\xintPow {-11}{-11}}=-0.0000000000
\xintTrunc {12}{\xintPow {-11}{-11}}=-0.000000000003
\xintTrunc {12}{\xintAdd {-1/3}{3/9}}=0
```

The digits printed are exact up to and including the last one.

The macro is more efficient since 1.2i in the case where the `{f}` argument is already a decimal number, and not a general fraction, as it avoids doing then a division by a possibly big power of ten, replacing it by use of `\xintDecSplit`.

## 10.17 `\xintiTrunc`

$\frac{\text{num}}{x} \frac{\text{Frac}}{f}$  ★ `\xintiTrunc{x}{f}` returns the integer equal to  $10^x$  times what `\xintTrunc{x}{f}` would produce.

```
\xintiTrunc {16}{-803.2028/20905.298}=-384210165289200
\xintiTrunc {10}{\xintPow {-11}{-11}}=0
\xintiTrunc {12}{\xintPow {-11}{-11}}=-3
```

The difference between `\xintTrunc{0}{f}` and `\xintiTrunc{0}{f}` is that the latter never has the decimal mark always present in the former except for `f=0`. And `\xintTrunc{0}{-0.5}` returns ```-0.''` whereas `\xintiTrunc{0}{-0.5}` simply returns ```0''`.

## 10.18 `\xintTTrunc`

$\frac{\text{Frac}}{f}$  ★ `\xintTTrunc{f}` truncates to an integer (truncation towards zero). This is the same as `\xintiTrunc{c}{0}{f}` and as `\xintNum`.

## 10.19 `\xintXTrunc`

$\frac{\text{num}}{x} \frac{\text{Frac}}{f}$  ☆ `\xintXTrunc{x}{f}` is similar to `\xintTrunc` with the following important differences:

- it is completely expandable but not *f-expandable*, as is indicated by the hollow star in the margin,



- hence it can not be used as argument to the other package macros, but as it *f-expands* its `{f}` argument, it accepts arguments expressed with other *xintfrac* macros,
- it requires  $x > 0$ ,
- contrarily to `\xintTrunc` the number of digits on output is not limited to about 19950 and may go well beyond 100000 (this is mainly useful for outputting a decimal expansion to a file),
- when the mathematical result is zero, it always prints it as  $0.00\dots 0$  or  $-0.00\dots 0$  with  $x$  zeros after the decimal mark.

**Warning:** transitivity is broken too (see discussion of `\xintTrunc`), due to the sign in the last item. Hence *the definitive policy is yet to be fixed*.

Transitivity is here in the sense of using a first `\edef` and then a second one, because it is not possible to nest `\xintXTrunc` directly as argument to itself. Besides, although the number of digits on output isn't limited, nevertheless  $x$  should be less than about 19970 when the number of digits of the input (assuming it is expressed as a decimal number) is even bigger: `\xintXTrunc{230000}{Z}` after `\edef Z{\xintXTrunc{60000}{1/66049}}` raises an error in contrast with a direct `\xintXTrunc{30000}{1/66049}`. But `\xintXTrunc{30000}{123.456789}` works, because here the number of digits originally present is smaller than what is asked for, thus the routine only has to add trailing zeros, and this has no limitation (apart from  $\text{\TeX}$  main memory).

`\xintXTrunc` will expand fully in an `\edef` or a `\write` (`\message`, `\wlog`, ...) or in an `\xint-expression`, or as list argument to `\xintFor*`.

Here is an example session where the user checks that the decimal expansion of  $1/66049 = 1/257^2$  has the maximal period length  $257 * 256 = 65792$  (this period length must be a divisor of  $\phi(66049)$  and to check it is the maximal one it is enough to show that neither 32896 nor 256 are periods.)

```
$ rlwrap etex -jobname worksheet-66049
This is pdfTeX, Version 3.14159265-2.6-1.40.17 (TeX Live 2016) (preloaded format=etex)
restricted \write18 enabled.
**xintfrac.sty
entering extended mode
(/usr/local/texlive/2016/texmf-dist/tex/generic/xint/xintfrac.sty
(/usr/local/texlive/2016/texmf-dist/tex/generic/xint/xint.sty
(/usr/local/texlive/2016/texmf-dist/tex/generic/xint/xintcore.sty
(/usr/local/texlive/2016/texmf-dist/tex/generic/xint/xintkernel.sty)))
)% we load xinttools for \xintKeep, etc... \xintXTrunc itself has no more

)% any dependency on xinttools.sty since 1.2i

*\input xinttools.sty
(/usr/local/texlive/2016/texmf-dist/tex/generic/xint/xinttools.sty)
*\def\m#1;{\message{#1}}

*\m \the\numexpr 257*257\relax;
66049
*\m \the\numexpr 257*256\relax;
65792
)% Thus 1/66049 will have a period length dividing 65792.

)% Let us first check it is indeed periodical.

*\edef Z{\xintXTrunc{66000}{1/66049}}

)% Let's display the first decimal digits.

*\m \xintXTrunc{208}{Z};

0.00001514027464458205271843631243470756559523989765174340262532362337052794137
6856576178291874214598252812306015231116292449545034746930309315810988811337037
```

```

6538630410755651107511090251177156353616254598858423
*% let's now fetch the trailing digits

*\m \xintKeep{65792-66000}{\Z};% 208 trailing digits

0000151402746445820527184363124347075655952398976517434026253236233705279413768
5657617829187421459825281230601523111629244954503474693030931581098881133703765
38630410755651107511090251177156353616254598858423
*% yes they match! we now check that 65792/2 and 65792/257=256 aren't periods.

*\m \xintXTrunc{256}{\Z};

0.00001514027464458205271843631243470756559523989765174340262532362337052794137
6856576178291874214598252812306015231116292449545034746930309315810988811337037
6538630410755651107511090251177156353616254598858423291798513225029902042423049
554118911717058547442
*\m \xintXTrunc{256+256}{\Z};

0.00001514027464458205271843631243470756559523989765174340262532362337052794137
6856576178291874214598252812306015231116292449545034746930309315810988811337037
6538630410755651107511090251177156353616254598858423291798513225029902042423049
5541189117170585474420505987978621932201850141561567926842192917379521264515738
3154930430438008145467758785144362518736089872670290239064936637950612424109373
3440324607488379839210283274538600130206361943405653378552286938485064119063119
8049932625777831609865402958409665551333
*% now with 65792/2=32896. Problem: we can't do \xintXTrunc{32896+100}{\Z}

*% but only direct \xintXTrunc{32896+100}{1/66049}. Anyway we want to nest it

*% hence let's do it all with (slower) \xintKeep, \xintKeepUnbraced.

*\m \xintKeep {-100}{\xintKeepUnbraced{2+65792/2+100}{\Z}};

9999848597253554179472815636875652924344047601023482565973746763766294720586231
434238217081257854017
*% This confirms 32896 isn't a period length.

*% To conclude let's write the 66000 digits to the log.

*\wlog{\Z}

*% We want always more digits:

*\wlog{\xintXTrunc{150000}{1/66049}}

*\bye

```

The acute observer will have noticed that there is something funny when one compares the first digits with those after the middle-period:

```

0000151402746445820527184363124347075655952398976517434026253236233705279413768...
9999848597253554179472815636875652924344047601023482565973746763766294720586231...

```

Mathematical exercise: can you explain why the two indeed add to 9999...9999?

You can try your hands at this simpler one:

```

1/49=\xintTrunc{42+5}{1/49}...\newline
\xintTrim{2}{\xintTrunc{21}{1/49}}\newline
\xintKeep{-21}{\xintTrunc{42}{1/49}}

```

```

1/49=0.02040816326530612244897959183673469387755102040...
020408163265306122448

```

979591836734693877551

This was again an example of the type  $1/N$  with  $N$  the square of a prime. One can also find counter-examples within this class:  $1/31^2$  and  $1/37^2$  have an odd period length (465 and respectively 111) hence they can not exhibit the symmetry.

Mathematical challenge: prove generally that if the period length of the decimal expansion of  $1/p^r$  (with  $p$  a prime distinct from 2 and 5 and  $r$  a positive exponent) is even, then the above symmetry applies.

Releases earlier than 1.2i created a dependency of *xintfrac* on *xinttools* only for this macro, this dependency does not exist anymore.

## 10.20 `\xintRound`

$\text{num}_x^{\text{Frac}} f$  ★ `\xintRound{x}{f}` returns the start of the decimal expansion of the fraction  $f$ , rounded to  $x$  digits precision after the decimal point. The argument  $x$  should be non-negative. Only when  $f$  evaluates exactly to zero does `\xintRound` return 0 without decimal point. When  $f$  is not zero, its sign is given in the output, also when the digits printed are all zero.

```
\xintRound {16}{-803.2028/20905.298}=-0.0384210165289201
\xintRound {20}{-803.2028/20905.298}=-0.03842101652892008523
\xintRound {10}{\xintPow {-11}{-11}}=-0.0000000000
\xintRound {12}{\xintPow {-11}{-11}}=-0.000000000004
\xintRound {12}{\xintAdd {-1/3}{3/9}}=0
```

The identity `\xintRound {x}{-f}=-\xintRound {x}{f}` holds. And regarding  $(-11)^{-11}$  here is some more of its expansion:

```
-0.00000000000350493899481392497604003313162598556370...
```

## 10.21 `\xintiRound`

$\text{num}_x^{\text{Frac}} f$  ★ `\xintiRound{x}{f}` returns the integer equal to  $10^x$  times what `\xintRound{x}{f}` would return.

```
\xintiRound {16}{-803.2028/20905.298}=-384210165289201
\xintiRound {10}{\xintPow {-11}{-11}}=0
```

Differences between `\xintRound{0}{f}` and `\xintiRound{0}{f}`: the former cannot be used inside integer-only macros, and the latter removes the decimal point, and never returns -0 (and removes all superfluous leading zeroes.)

## 10.22 `\xintFloor`, `\xintiFloor`

$\text{Frac}_f$  ★ `\xintFloor {f}` returns the largest relative integer  $N$  with  $N \leq f$ .

```
\xintFloor {-2.13}=-3/1[0], \xintFloor {-2}=-2/1[0], \xintFloor {2.13}=2/1[0]
```

$\text{Frac}_f$  ★ `\xintiFloor {f}` does the same but without adding the `/1[0]`.

```
\xintiFloor {-2.13}=-3, \xintiFloor {-2}=-2, \xintiFloor {2.13}=2
```

## 10.23 `\xintCeil`, `\xintiCeil`

$\text{Frac}_f$  ★ `\xintCeil {f}` returns the smallest relative integer  $N$  with  $N > f$ .

```
\xintCeil {-2.13}=-2/1[0], \xintCeil {-2}=-2/1[0], \xintCeil {2.13}=3/1[0]
```

$\text{Frac}_f$  ★ `\xintiCeil {f}` does the same but without adding the `/1[0]`.

## 10.24 `\xintTFrac`

$\text{Frac}_f$  ★ `\xintTFrac{f}` returns the fractional part,  $f=\text{trunc}(f)+\text{frac}(f)$ . Thus if  $f < 0$ , then  $-1 < \text{frac}(f) \leq 0$

and if  $f > 0$  one has  $0 \leq \text{frac}(f) < 1$ . The *T* stands for 'Trunc', and there should exist also similar macros associated respectively with 'Round', 'Floor', and 'Ceil', each type of rounding to an integer deserving arguably to be associated with a fractional 'modulo'. By sheer laziness, the package currently implements only the 'modulo' associated with 'Truncation'. Other types of modulo may be obtained more clumsily via a combination of the rounding with a subsequent subtraction from *f*.

Notice that the result is filtered through `\xintREZ`, and will thus be of the form  $A/B[N]$ , where neither *A* nor *B* has trailing zeros. But the output fraction is not reduced to smallest terms.

The function call in expressions (`\xintexpr`, `\xintfloatexpr`) is `frac`. Inside `\xintexpr..relax` *x*, the function `frac` is mapped to `\xintTFrac`. Inside `\xintfloatexpr..relax`, `frac` first applies `\xintTFrac` to its argument (which may be an exact fraction with more digits than the floating point precision) and only in a second stage makes the conversion to a floating point number with the precision as set by `\xintDigits` (default is 16).

```
\xintTFrac {1235/97}=71/97[0] \xintTFrac {-1235/97}=-71/97[0]
\xintTFrac {1235.973}=973/1[-3] \xintTFrac {-1235.973}=-973/1[-3]
\xintTFrac {1.122435727e5}=5727/1[-4]
```

## 10.25 `\xintE`

$\frac{\text{Frac}}{f} \frac{\text{num}}{x}$  ★ `\xintE {f}{x}` multiplies the fraction *f* by  $10^x$ . The second argument *x* must obey the  $\text{T}_{\text{E}}\text{X}$  bounds. Example:

```
\count 255 123456789 \xintE {10}{\count 255}->10/1[123456789]
```

Be careful that for obvious reasons such gigantic numbers should not be given to `\xintNum`, or added to something with a widely different order of magnitude, as the package always works to get the exact result. There is *no problem* using them for *float* operations:

```
\xintFloatAdd {1e1234567890}{1}=1.000000000000000e1234567890
```

## 10.26 `\xintAdd`

$\frac{\text{Frac}}{f} \frac{\text{Frac}}{f}$  ★ Computes the addition of two fractions. To keep for integers the integer format on output use `\xintiAdd`.

Checks if one denominator is a multiple of the other. Else multiplies the denominators.

## 10.27 `\xintSub`

$\frac{\text{Frac}}{f} \frac{\text{Frac}}{f}$  ★ Computes the difference of two fractions (`\xintSub{F}{G}` computes  $F-G$ ). To keep for integers the integer format on output use `\xintiSub`.

Checks if one denominator is a multiple of the other. Else multiplies the denominators.

## 10.28 `\xintMul`

$\frac{\text{Frac}}{f} \frac{\text{Frac}}{f}$  ★ Computes the product of two fractions. To keep for integers the integer format on output use `\xintiMul`.

No reduction attempted.

## 10.29 `\xintSqr`

$\frac{\text{Frac}}{f}$  ★ Computes the square of one fraction. To maintain for integer input an integer format on output use `\xintiSqr`.

10.30 `\xintDiv`

$\frac{F}{f} \frac{F}{f} \star$  Computes the quotient of two fractions. (`\xintDiv{F}{G}` computes  $F/G$ ). To keep for integers the integer format on output use `\xintiMul`.  
No reduction attempted.

10.31 `\xintDivTrunc`, `\xintDivRound`

$\frac{F}{f} \frac{F}{f} \star$  Computes the quotient of the two arguments then either truncates or rounds to an integer.

10.32 `\xintiFac`

$\frac{Num}{f} \star$  With *xintfrac* loaded `\xintiFac` is extended to allow a fraction  $f$  as input, it will be truncated first to an integer  $n$  before the evaluation of the factorial. The output is an integer in strict format, without a trailing `/1[0]`. See the `\xintiiFac` doc for more info.

10.33 `\xintiBinomial`

$\frac{Num}{f} \frac{Num}{f} \star$  With *xintfrac* loaded `\xintiBinomial` is extended to allow fractional inputs which will be truncated to integers before the evaluation of the binomial. The output is an integer in strict format, without a trailing `/1[0]`. See the `\xintiiBinomial` doc for the current allowable range.

10.34 `\xintiPFactorial`

$\frac{Num}{f} \frac{Num}{f} \star$  With *xintfrac* loaded `\xintiPFactorial` is extended to allow fractional inputs which will be truncated to integers before the evaluation of the partial factorial. The output is an integer in strict format, without a trailing `/1[0]`. See the `\xintiiPFactorial` doc for more info.

10.35 `\xintPow`

$\frac{Frac}{f} \frac{Num}{f} \star$  `\xintPow{f}{x}`: computes  $f^x$  with  $f$  a fraction and  $x$  possibly also, but  $x$  will first get truncated to a (positive or negative) integer.

The output will now always be in the form  $A/B[n]$  (even when the exponent vanishes: `\xintPow {2/2}{3}{0}=1/1[0]`).

The macro handling only integers is available as `\xintiPow`. Only `\xintPow` accepts negative exponent, as this produces fractions.

Within an `\xintiexpr..relax` the infix operator `^` is mapped to `\xintiiPow`; within an `\xintexpr`-ession it is mapped to `\xintPow`.

10.36 `\xintSum`

$f \rightarrow \star \frac{Frac}{f} \star$  This computes the sum of fractions. The output will now always be in the form  $A/B[n]$ . The original, for big integers only (in strict format), is available as `\xintiiSum`.

```
\xintSum {{1282/2196921}{-281710/291927}{4028/28612}}
-15113784906302076/18350036010217404[0]
```

No simplification attempted.

10.37 `\xintPrd`

$f \rightarrow \star \frac{Frac}{f} \star$  TThis computes the product of fractions. The output will now always be in the form  $A/B[n]$ . The original, for big integers only (in strict format), is available as `\xintiiPrd`.

```
\xintPrd {{1282/2196921}{-281710/291927}{4028/28612}}
-1454721142160/18350036010217404[0]
```

No simplification attempted.

10.38 `\xintCmp`

$\frac{f}{f}$   $\frac{f}{f}$  ★ This compares two fractions `F` and `G` and produces `-1`, `0`, or `1` according to `F<G`, `F=G`, `F>G`. For choosing branches according to the result of comparing `f` and `g`, see `\xintifCmp`.

10.39 `\xintIsOne`

$\frac{f}{f}$  ★ This returns `1` if the fraction is `1` and `0` if not.  
`\xintIsOne {21921379213/21921379213}` but `\xintIsOne {1.00000000000000000000000000000001}`  
`1` but `0`

10.40 `\xintGeq`

$\frac{f}{f}$   $\frac{f}{f}$  ★ This compares the *absolute values* of two fractions. `\xintGeq{f}{g}` returns `1` if  $|f| \geq |g|$  and `0` if not.  
 May be used for expandably branching as: `\xintSgnFork{\xintGeq{f}{g}}{\code for |f|<|g|}{code for |f|≥|g|}`

10.41 `\xintMax`

$\frac{f}{f}$   $\frac{f}{f}$  ★ The maximum of two fractions. But now `\xintMax {2}{3}` returns `3/1[0]`. The original, for use with (possibly big) integers only with no need of normalization, is available as `\xintiiMax: \xintiiMax {2}{3}=3`.  
 $\frac{ff}{f}$   $\frac{ff}{f}$  ★ There is also `\xintiMax` which works with fractions but first truncates them to integers.  
`\xintMax {2.5}{7.2}` but `\xintiMax {2.5}{7.2}`  
`72/1[-1]` but `7`

10.42 `\xintMin`

$\frac{f}{f}$   $\frac{f}{f}$  ★ The maximum of two fractions. The original, for use with (possibly big) integers only with no need of normalization, is available as `\xintiiMin: \xintiiMin {2}{3}=2`.  
 $\frac{ff}{f}$   $\frac{ff}{f}$  ★ There is also `\xintiMin` which works with fractions but first truncates them to integers.  
`\xintMin {2.5}{7.2}` but `\xintiMin {2.5}{7.2}`  
`25/1[-1]` but `2`

10.43 `\xintMaxof`

$f \rightarrow * \frac{f}{f}$  ★ The maximum of any number of fractions, each within braces, and the whole thing within braces.  
`\xintMaxof {{1.23}{1.2299}{1.2301}}` and `\xintMaxof {{-1.23}{-1.2299}{-1.2301}}`  
`12301/1[-4]` and `-12299/1[-4]`

10.44 `\xintMinof`

$f \rightarrow * \frac{f}{f}$  ★ The minimum of any number of fractions, each within braces, and the whole thing within braces.  
`\xintMinof {{1.23}{1.2299}{1.2301}}` and `\xintMinof {{-1.23}{-1.2299}{-1.2301}}`  
`12299/1[-4]` and `-12301/1[-4]`

10.45 `\xintAbs`

$\frac{f}{f}$  ★ The absolute value. Note that `\xintAbs {-2}=2/1[0]` whereas `\xintiAbs {-2}=2`.

10.46 `\xintSgn`

$\frac{f}{f}$  ★ The sign of a fraction.

10.47 `\xintOpp`

$\frac{\text{num}}{\text{den}}$   $\frac{\text{Frac}}{f}$  ★ The opposite of a fraction. Note that `\xintOpp {3}` now outputs `-3/1[0]` whereas `\xintiOpp {3}` returns `-3`.

10.48 `\xintDigits`, `\xinttheDigits`

The syntax `\xintDigits := D;` (where spaces do not matter) assigns the value of `D` to the number of digits to be used by floating point operations. The default is `16`. The maximal value is `32767`. The macro `\xinttheDigits` serves to print the current value.

10.49 `\xintFloat`

$\frac{\text{num}}{\text{den}}$   $\frac{\text{Frac}}{f}$  ★ The macro `\xintFloat [P]{f}` has an optional argument `P` which replaces the current value of `\xinttheDigits`. The (rounded truncation of the) fraction `f` is then printed in scientific form, with `P` digits, a lowercase `e` and an exponent `N`. The first digit is from `1` to `9`, it is preceded by an optional minus sign and is followed by a dot and `P-1` digits, the trailing zeroes are not trimmed. In the exceptional case where the rounding went to the next power of ten, the output is `10.0...0eN` (with a sign, perhaps). The sole exception is for a zero value, which then gets output as `0.e0` (in an `\xintCmp` test it is the only possible output of `\xintFloat` or one of the 'Float' macros which will test positive for equality with zero).

```
\xintFloat[32]{1234567/7654321}=1.6129020457856418616360615134902e-1
\xintFloat[32]{1/\xintiiFac{100}}=1.0715102881254669231835467595192e-158
```

The argument to `\xintFloat` may be an `\xinttheexpr`-ession, like the other macros; only its final evaluation is submitted to `\xintFloat`: the inner evaluations of chained arguments are not at all done in 'floating' mode. For this one must use `\xintthefloatexpr`.

10.50 `\xintPFloat`

$\frac{\text{num}}{\text{den}}$   $\frac{\text{Frac}}{f}$  ★ { The macro `\xintPFloat [P]{f}` is like `\xintFloat` but 'pretty-prints' the output. Its behaviour has changed with release 1.2f: there is only one simplification rule now which is that decimal notation (with possibly needed extra zeros) is used in place of scientific notation when the exponent would end up being between `-5` and `5` inclusive.<sup>60</sup>

Changed (1.2f) If the input vanishes the output will be `0.` with a decimal mark (the original version printed `0` with no decimal mark).<sup>61</sup>

`\xintthefloatexpr` applies this macro to its output (or each of its outputs, if comma separated). Currently trailing zeros are not trimmed.

```
\begingroup\def\test #1{#1$\to\}$\xintPFloat{#1}}%
\string\xintDigits\ at \xinttheDigits
\begin{itemize}[nosep]
\item \test {0}
\item \test {1.23456789e-7}
\item \test {1.23456789e-6}
\item \test {1.23456789e-5}
\item \test {1.23456789e-4}
\item \test {1.23456789e-3}
\item \test {1.23456789e-2}
\item \test {1.23456789e-1}
\item \test {1.23456789e0}
\item \test {1.23456789e1}
\item \test {1.23456789e2}
\item \test {1.23456789e3}
```

<sup>60</sup> In the exceptional case of an input rounded up towards next power of ten, the exponent referred to here is the integer `N` in `10.0...0eN` with a total number of zeroes equal to the precision. <sup>61</sup> Currently there are no subnormal numbers, and no underflow because the exponent is only limited by the maximal T<sub>E</sub>X number; thus underflow situations would manifest themselves via low-level arithmetic overflow errors.



```

\item \test {1.23456789e4}
\item \test {1.23456789e5}
\item \test {1.23456789e6}
\item \test {1.23456789e7}
\end{itemize}
\endgroup

```

`\xintDigits at 16`

- $0 \rightarrow 0.$
- $1.23456789e-7 \rightarrow 1.2345678900000000e-7$
- $1.23456789e-6 \rightarrow 1.2345678900000000e-6$
- $1.23456789e-5 \rightarrow 0.00001234567890000000$
- $1.23456789e-4 \rightarrow 0.00012345678900000000$
- $1.23456789e-3 \rightarrow 0.00123456789000000000$
- $1.23456789e-2 \rightarrow 0.01234567890000000000$
- $1.23456789e-1 \rightarrow 0.12345678900000000000$
- $1.23456789e0 \rightarrow 1.23456789000000000000$
- $1.23456789e1 \rightarrow 12.34567890000000000000$
- $1.23456789e2 \rightarrow 123.45678900000000000000$
- $1.23456789e3 \rightarrow 1234.56789000000000000000$
- $1.23456789e4 \rightarrow 12345.67890000000000000000$
- $1.23456789e5 \rightarrow 123456.78900000000000000000$
- $1.23456789e6 \rightarrow 1.234567890000000000e6$
- $1.23456789e7 \rightarrow 1.234567890000000000e7$

### 10.51 `\xintFloatE`

$\frac{\text{num}}{X}$   $\frac{\text{Frac num}}{f X}$  ★

`\xintFloatE [P]{f}{x}` multiplies the input `f` by  $10^x$ , and converts it to float format according to the optional first argument or current value of `\xinttheDigits`.

```
\xintFloatE {1.23e37}{53}=1.2300000000000000e90
```

### 10.52 `\xintFloatAdd`

$\frac{\text{num}}{X}$   $\frac{\text{Frac Frac}}{f f}$  ★  
Changed  
(1.2f)

`\xintFloatAdd [P]{f}{g}` first replaces `f` and `g` with their float approximations `f'` and `g'` to `P` significant places or to the precision from `\xintDigits`. It then produces the sum `f'+g'`, correctly rounded to nearest with the same number of significant places.

As for `\xintFloat`, in case of rounding up to next power of ten, the value may exceptionally come out as  $10.0\dots0eN$  with a total of `P+1` digits.

See [subsection 3.2](#) for more.

### 10.53 `\xintFloatSub`

$\frac{\text{num}}{X}$   $\frac{\text{Frac Frac}}{f f}$  ★  
Changed  
(1.2f)

`\xintFloatSub [P]{f}{g}` first replaces `f` and `g` with their float approximations `f'` and `g'` to `P` significant places or to the precision from `\xintDigits`. It then produces the difference `f'-g'` correctly rounded to nearest `P`-float.

As for `\xintFloat`, in case of rounding up to next power of ten, the value may exceptionally come out as  $10.0\dots0eN$  with a total of `P+1` digits.

See [subsection 3.2](#) for more.

### 10.54 `\xintFloatMul`

$\frac{\text{num}}{X}$   $\frac{\text{Frac Frac}}{f f}$  ★  
Changed  
(1.2f)

`\xintFloatMul [P]{f}{g}` first replaces `f` and `g` with their float approximations `f'` and `g'` to `P` (or `\xinttheDigits`) significant places. It then correctly rounds the product `f'*g'` to nearest `P`-float.

See [subsection 3.2](#) for more.

It is obviously much needed that the author improves its algorithms to avoid going through the exact  $2P$  or  $2P-1$  digits before throwing to the waste-bin half of those digits !

### 10.55 `\xintFloatDiv`

num.  $\frac{f}{x}$   $\frac{f}{f}$  ★  
Changed (1.2f) `\xintFloatDiv [P]{f}{g}` first replaces  $f$  and  $g$  with their float approximations  $f'$  and  $g'$  to  $P$  (or `\xinttheDigits`) significant places. It then correctly rounds the fraction  $f'/g'$  to nearest  $P$ -float.  
See [subsection 3.2](#) for more.

### 10.56 `\xintFloatFac`

num.  $\frac{f}{x}$   $\frac{f}{f}$  ★  
New with 1.2 `\xintFloatFac[P]{f}` returns the factorial with either `\xinttheDigits` or  $P$  digits of precision.  
The exact theoretical value differs from the calculated one  $Y$  by an absolute error strictly less than  $0.6 \text{ ulp}(Y)$ .  
`\$1000!\approx{\$}\xintFloatFac [30]{1000}`  
`1000! ≈ 4.02387260077093773543702433923e2567` The computation proceeds via doing explicitly the product, as the Stirling formula cannot be used for lack so far of `exp/log`.  
The maximal allowed argument is 99999999, but already 100000! currently takes, for 16 digits of precision, a few seconds on my laptop (it returns `2.824229407960348e456573`).  
The `factorial` function is available in `\xintfloatexpr`:  
`\xintthefloatexpr factorial(1000)\relax % same as 1000!`  
`4.023872600770938e2567`

### 10.57 `\xintFloatBinomial`

num.  $\frac{f}{x}$   $\frac{f}{f}$  ★  
New with 1.2f `\xintFloatBinomial[P]{x}{y}` computes binomial coefficients with either `\xinttheDigits` or  $P$  digits of precision.  
When  $x < 0$  an out-of-range error is raised. Else (this was changed in 1.2h, see [subsection 9.43](#)), if  $y < 0$  or if  $x < y$  the macro evaluates to `0.e0`.  
The exact theoretical value differs from the calculated one  $Y$  by an absolute error strictly less than  $0.6 \text{ ulp}(Y)$ .  
`\${3000}\choose 1500\approx{\$}\xintFloatBinomial [24]{3000}{1500}`  
 `$\binom{3000}{1500} \approx 1.79196793754756005073269e901$`   
The `binomial` function is available in `\xintfloatexpr`:  
`\xintthefloatexpr binomial(3000,1500)\relax`  
`1.791967937547560e901`  
The computation is based on the formula  $(x-y+1) \dots x/y!$  (here one arranges  $y \leq x-y$  naturally).

### 10.58 `\xintFloatPFactorial`

num.  $\frac{f}{x}$   $\frac{f}{f}$  ★  
New with 1.2f  
Changed (1.2h) `\xintFloatPFactorial[P]{x}{y}` computes the product  $(x+1) \dots y$ .  
The inputs  $x$  and  $y$  must evaluate to non-negative integers less in absolute value than  $10^8$ . For  $x=y$  the product is considered empty hence the returned value is 1.  
It was a bit unfortunate with 1.2f that the code deliberately raised an error if the condition  $0 \leq x \leq y < 10^8$  was violated. See [subsection 9.44](#) for the now prevailing rules.  
But only for the range  $0 \leq x \leq y < 10^8$  is it to be considered that the behaviour is fixed and will not change in the future.  
The exact theoretical value differs from the calculated one  $Y$  by an absolute error strictly less than  $0.6 \text{ ulp}(Y)$ .  
The `pfactorial` function is available in `\xintfloatexpr`:  
`\xintthefloatexpr pfactorial(2500,5000)\relax`

2.595989917947957e8914

## 10.59 `\xintFloatPow`

$\frac{\text{num}}{X} \cdot \frac{\text{Frac num}}{f} \cdot \frac{\text{num}}{X} \star$  `\xintFloatPow [P]{f}{x}` uses either the optional argument `P` or the value of `\xinttheDigits`. It computes a floating approximation to  $f^x$ . The precision `P` must be at least 1, naturally.

The exponent `x` will be fed to a `\numexpr`, hence count registers are accepted on input for this `x`. And the absolute value  $|x|$  must obey the  $\text{T}_{\text{EX}}$  bound. For larger exponents use the slightly slower routine `\xintFloatPower` which allows the exponent to be a fraction simplifying to an integer and does not limit its size. This slightly slower routine is the one to which `^` is mapped inside `\xintthefloatexpr... \relax`.

Changed (1.2f) The argument `f` is first rounded to `P` significant places to give `f'`. The output `Z` is such that the exact  $f'^x$  differs from `Z` by an absolute error less than  $0.52 \text{ ulp}(Z)$ .

```
\xintFloatPow [8]{3.1415}{1234567890}=1.6122066e613749456
```

## 10.60 `\xintFloatPower`

$\frac{\text{num}}{X} \cdot \frac{\text{Frac Num}}{f} \cdot \frac{\text{Num}}{f} \star$  `\xintFloatPower[P]{f}{g}` computes a floating point value  $f^g$  where the exponent `g` is not constrained to be at most the  $\text{T}_{\text{EX}}$  bound 2147483647. It may even be a fraction `A/B` but must simplify to a (possibly big) integer. The exponent of the output however *must* at any rate obey the  $\text{T}_{\text{EX}}$  2147483647 bound.

Changed (1.2f) The argument `f` is first rounded to `P` significant places to give `f'`. The output `Z` is such that the exact  $f'^g$  differs from `Z` by an absolute error less than  $0.6 \text{ ulp}(Z)$  (actually  $0.52 \text{ ulp}(Z)$ ).<sup>62</sup>

```
\xintFloatPower [8]{3.1415}{3e9}=1.4317729e1491411192
```

Notice that  $3e9 > 2^{31}$ .

Here is another example:

```
\xintFloatPower [48]{1.1547}{\xintiipow {2}{35}}
```

computes  $(1.1547)^{2^{35}} = (1.1547)^{34359738368}$  to be approximately

2.785,837,382,571,371,438,495,789,880,733,698,213,205,183,990,48  $\times 10^{2,146,424,193}$

Again,  $2^{35}$  exceeds  $\text{T}_{\text{EX}}$ 's bound, but `\xintFloatPower` allows it, what counts is the exponent of the result which, while dangerously close to  $2^{31}$  is not quite there yet.<sup>63</sup>

Inside an `\xintfloatexpr`-ession, `\xintFloatPower` is the function to which `^` is mapped.<sup>64,65</sup> Thus the same computation as above can be done via the non-expandable assignment `\xintDigits:=48`; and

```
\xintthefloatexpr 1.1547^(2^35)\relax
```

There is a subtlety here that the  $2^{35}$  will be evaluated as a floating point number but fortunately it only has 11 digits, hence the final evaluation is done with a correct exponent. It is safer, and also more efficient to code the above rather as:

```
\xintthefloatexpr 1.1547^{\xintiipow 2^35}\relax\relax
```

to guarantee no loss of digits in the exponent.

There is an important difference between `\xintFloatPower [Q]{X}{Y}` and `\xintthefloatexpr [Q] X^Y \relax`: in the former case the computation is done with `Q` digits or precision (but if `X` and `Y` themselves stand for some floating point macros with arguments, their respective evaluations obey the precision `\xinttheDigits` or as set optionally in the macro calls themselves), whereas with `\xintthefloatexpr [Q]` the evaluation of the expression proceeds with `\xinttheDigits` digits of precision, and the final result is then rounded to `Q` digits: thus this makes real sense only if used with `Q < \xinttheDigits`.

<sup>62</sup> For the  $0.6 \text{ ulp}(Z)$  bound, earlier releases had potentially a problem for negative exponents; the final reverse was done with only two guard digits (due to the implementation of `\xintFloat`), and as a result the final error could conceivably exceed  $0.6 \text{ ulp}(Z)$ , although remaining smaller than  $0.7 \text{ ulp}(Z)$ .<sup>63</sup> The printing of the result was done via the `\numprint` macro from the <http://ctan.org/pkg/numprint> package.

<sup>64</sup> I am constantly hesitating about this. In my testing, switching to `\xintFloatPow` would only bring from 5% to perhaps 20% gain for computations with 16 digits of precision and moderately sized exponent.

<sup>65</sup> Since 1.2f, the `^` operator handles also half-integer exponents.

### 10.61 \xintFloatSqrt

`[ $\frac{\text{num}}{x}$ ]  $f^{\text{frac}}$  ★ \xintFloatSqrt[P]{f}` computes a floating point approximation of  $\sqrt{f}$ , either using the optional precision `P` or the value of `\xinttheDigits`.

More precisely the macro achieves so-called *correct rounding*: the produced value is the rounding to  $P$  significant places of the abstract exact value, *if the input has itself at most  $P$  digits* (and an arbitrary exponent).

New with  
1.2f

```
\xintFloatSqrt [89]{10}\newline
\xintFloatSqrt [89]{100}\newline
\xintFloatSqrt [89]{123456789}\newline
And now some tests to check that correct rounding applies correctly (sic):\newline
(we observe in passing illustrations that rounding to nearest is not transitive)\newline
\xintFloatSqrt {5625000075000001}\newline
\xintFloatSqrt [24]{5625000075000001}\newline
\xintFloatSqrt [32]{5625000075000001}\newline
\xintFloatSqrt [48]{56250000000000000000000000750000000000000000000000001}\newline
\xintFloatSqrt [64]{56250000000000000000000000750000000000000000000000001}\newline
\xintFloatSqrt [80]{56250000000000000000000000750000000000000000000000001}\par
```

[illegible]

(we observe in passing illustrations that rounding to nearest is not transitive)

7.500000050000000e7

7.5000000500000005000000e7

7.500000050000000499999966666667e7

7.50000000000000000000000050000000000000000000000e23

```
7.5000000000000000000000005000000000000000000500000000000000e23
```

7.50000000000000000000000050000000000000000000000004999999999999999999966666667e23

10.62 `\xintiDivision`, `\xintiQuo`, `\xintiRem`, `\xintFDg`, `\xintLDg`, `\xintMON`,  
`\xintMMON`, `\xintOdd`

$\frac{f}{f}$   $\star$  These macros accept a fraction (or two) on input but will truncate it (them) to an integer using `\xintNum` (which is the same as `\xintTTrunc`). On output they produce integers without `/` nor `[N]`.

All have variants from package `xint` whose names start with `xintii` rather than `xint`; these variants accept on input only integers in the strict format (they do not use `\xintNum`). They thus have less overhead, and may be used when one is dealing exclusively with (big) integers.

[illegible]

## 11 Macros of the `xintexpr` package

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The `xintexpr` package was first released with version 1.07 (2013/05/25) of the `xint` bundle. It was substantially enhanced with release 1.1 from 2014/10/28.

Release 1.2 removed a limitation to numbers of at most 5000 digits, and there is now a float variant of the factorial. Also the ``pseudo-functions'' `qint`, `qfrac`, `qfloat` ('q' for quick), were added to handle very big inputs and avoid scanning it digit per digit.

The package loads automatically `xintfrac` and `xinttools` (it is now the only arithmetic package from the `xint` bundle which loads `xinttools`).

- for using the `gcd` and `lcm` functions, it is necessary to load package `xintgcd`.

```
\xinttheexpr lcm (2^5*7*13^10*17^5, 2^3*13^15*19^3, 7^3*13*23^2)\relax
2894379441338000036761046087608864
```

- for allowing hexadecimal numbers (uppercase letters) on input, it is necessary to load package `xintbinhex`.

```
\xinttheexpr "A*"B*"C*"D*"D*"F, "FF.FF, reduce("FF.FFF + 16^-3)\relax
3346200, 25599609375[-8], 256
```

Please refer to [section 2](#) for a more detailed description of the syntax elements for expressions.

### 11.1 The `\xintexpr` expressions

- ✱ An `xintexpression` is a construct `\xintexpr<expandable_expression>\relax` where the expandable expression is read and completely expanded from left to right.

An `\xintexpr... \relax` must end in a `\relax` (which will be absorbed). Like a `\numexpr` expression, it is not printable as is, nor can it be directly employed as argument to the other package macros. For this one must use one of the three equivalent forms:

- ✱ • `\thexintexpr<expandable_expression>\relax`, or
- ✱ • `\xinttheexpr<expandable_expression>\relax`, or
- ✱ • `\xintthe\xintexpr<expandable_expression>\relax`.

The computations are done *exactly*, and with no simplification of the result. See `\xintfloatexpr` for a similar parser which rounds each operation inside the expression to `\xinttheDigits` digits of precision.

As an alternative and equivalent syntax to

```
\xintexpr round(<expression>, D)\relax
```

there is<sup>66</sup>

```
\xintiexpr [D] <expression> \relax
```

The parameter *D* must be zero or positive.<sup>67</sup> Perhaps some future version will give a meaning to using a negative *D*.<sup>68</sup>

- the expression may contain arbitrarily many levels of nested parenthesized sub-expressions,
- the expression may contain explicitly or from a macro expansion a sub-expression `\xintexpr... \relax`, which itself may contain a sub-expressions etc. . .
- to let sub-contents evaluate as a sub-unit it should thus be either
  1. parenthesized,
  2. or a sub-expression `\xintexpr... \relax`.
- to use an expression as argument to the other package macros, or more generally to macros which expand their arguments, one must use the `\xinttheexpr... \relax` or `\xintthe\xintexpr... \relax` forms.
- similarly, printing the result itself must be done with these forms.
- one should not use `\xinttheexpr... \relax` as a sub-constituent of an `\xintexpr... \relax` but only the `\xintexpr... \relax` form which is more efficient in this context.
- each *xintexpression*, whether prefixed or not with `\xintthe`, is completely expandable and obtains its result in two expansion steps.

See [section 2](#) for the primary information on built-in operators and functions. This section now adds some complementary information.

- An expression is built the standard way with opening and closing parentheses, infix operators, and (big) numbers, with possibly a fractional part, and/or scientific notation (except for `\xintiexpr` which only admits big integers). All variants work with comma separated expressions. On output each comma will be followed by a space. A decimal number must have digits either before or after the decimal mark.
- As everything gets expanded, the characters `.`, `+`, `-`, `*`, `/`, `^`, `!`, `&`, `|`, `?`, `:`, `<`, `>`, `=`, `(`, `)`, `"`, `]`, `[`, `@` and the comma `,` should not (if used in the expression) be active. For example, the French language in *Babel* system, for pdf<sub>TeX</sub>, activates `!`, `?`, `;` and `:`. Turn off the activity before the expressions.

Alternatively the macro `\xintexprSafeCatcodes` resets all characters potentially needed by `\xintexpr` to their standard catcodes and `\xintexprRestoreCatcodes` restores the status prevailing at the time of the previous `\xintexprSafeCatcodes`.

- Count registers and `\numexpr`-essions are accepted (LaTeX's counters can be inserted using `\val` `blue`) natively without `\the` or `\number` as prefix. Also dimen registers and control sequences, skip registers and control sequences (TeX's lengths), `\dimexpr`-essions, `\glueexpr`-essions are automatically unpacked using `\number`, discarding the stretch and shrink components and giving the dimension value in *sp* units (1/65536th of a TeX point). Furthermore, tacit multiplication is implied, when the (count or dimen or glue) register or variable, or the (`\numexpr` or `\dimexpr` or `\glueexpr`) expression is immediately prefixed by a (decimal) number. See [subsection 2.3](#) for the complete rules of tacit multiplication.



<sup>66</sup> For truncation rather than rounding, one uses `\xintexpr trunc(<expression>, D)\relax`. <sup>67</sup> *D*=0 corresponds to using `round(<expression>)` not `round(<expression>,0)` which would leave a trailing dot. Same for `trunc`. There is also function `float` for floating point rounding to `\xinttheDigits` or the given number of significant digits as second argument. <sup>68</sup> Thanks to KT for this suggestion. Sorry for the delay in implementing it... matter of formatting the output and corresponding choice of user interface are still in need of some additional thinking.

- With a macro `\x` defined like this:

```
\def\x {\xintexpr \a + \b \relax} or \edef\x {\xintexpr \a+\b\relax}
```

one may then do `\xintthe\x`, either for printing the result on the page or to use it in some other macros expanding their arguments. The `\edef` does the computation immediately but keeps it in an internal private format. Naturally, the `\edef` is only possible if `\a` and `\b` are already defined. With both approaches the `\x` can be inserted in other expressions, as for example (assuming naturally as we use an `\edef` that in the 'yet-to-be computed' case the `\a` and `\b` now have some suitable meaning):

```
\edef\y {\xintexpr \x^3\relax}
```

- There is also `\xintboolexpr ... \relax` and `\xinttheboolexpr ... \relax`. Same as `\xintexpr` with the final result converted to 1 if it is not zero. See also `\xintifboolexpr` (subsection 11.14) and the `bool` and `togl` functions in section 11. Here is an example:

```
\xintNewBoolExpr \AssertionA[3]{ #1 && (#2|#3) }
\xintNewBoolExpr \AssertionB[3]{ #1 || (#2) }
\xintNewBoolExpr \AssertionC[3]{ xor(#1,#2,#3) }
{\centering\normalcolor\xintFor #1 in {0,1} \do {%
 \xintFor #2 in {0,1} \do {%
 \xintFor #3 in {0,1} \do {%
 #1 AND (#2 OR #3) is \textcolor[named]{OrangeRed}{\AssertionA {#1}{#2}{#3}}\hfil
 #1 OR (#2 AND #3) is \textcolor[named]{OrangeRed}{\AssertionB {#1}{#2}{#3}}\hfil
 #1 XOR #2 XOR #3 is \textcolor[named]{OrangeRed}{\AssertionC {#1}{#2}{#3}}\}}}

0 AND (0 OR 0) is 0 0 OR (0 AND 0) is 0 0 XOR 0 XOR 0 is 0
0 AND (0 OR 1) is 0 0 OR (0 AND 1) is 0 0 XOR 0 XOR 1 is 1
0 AND (1 OR 0) is 0 0 OR (1 AND 0) is 0 0 XOR 1 XOR 0 is 1
0 AND (1 OR 1) is 0 0 OR (1 AND 1) is 1 0 XOR 1 XOR 1 is 0
1 AND (0 OR 0) is 0 1 OR (0 AND 0) is 1 1 XOR 0 XOR 0 is 1
1 AND (0 OR 1) is 1 1 OR (0 AND 1) is 1 1 XOR 0 XOR 1 is 0
1 AND (1 OR 0) is 1 1 OR (1 AND 0) is 1 1 XOR 1 XOR 0 is 0
1 AND (1 OR 1) is 1 1 OR (1 AND 1) is 1 1 XOR 1 XOR 1 is 1
```

This example used for efficiency `\xintNewBoolExpr`. See also the subsection 11.6.

- There is `\xintfloatexpr ... \relax` where the algebra is done in floating point approximation (also for each intermediate result). Use the syntax `\xintDigits:=N;` to set the precision. Default: 16 digits.

```
\xintthefloatexpr 2^1000000\relax: 9.990020930143845e30102
```

The square-root operation can be used in `\xintexpr`, it is computed as a float with the precision set by `\xintDigits` or by the optional second argument:

```
\xinttheexpr sqrt(2,60)\relax\newline
Here the [60] is to avoid truncation to |\xinttheDigits| of precision on output.\newline
\printnumber{\xintthefloatexpr [60] sqrt(2,60)\relax}
```

```
141421356237309504880168872420969807856967187537694807317668[-59]
```

Here the [60] is to avoid truncation to `\xinttheDigits` of precision on output.

```
1.41421356237309504880168872420969807856967187537694807317668
```

Floats are quickly indispensable when using the power function, as exact results will easily have hundreds, if not thousands, of digits.

```
\xintDigits:=48;\xintthefloatexpr 2^1000000\relax
```

```
9.99002093014384507944032764330033590980429139054e30102
```

Only integer and (in `\xintfloatexpr... \relax`) half-integer exponents are allowed.

- if one uses macros within `\xintexpr... \relax` one should obviously take into account that the parser will not see the macro arguments, hence one cannot use the syntax there, except if the arguments are themselves wrapped as `\xinttheexpr... \relax` and assuming the macro *f-expands* these arguments.



## 11.2 `\numexpr` or `\dimexpr` expressions, count and dimension registers and variables

Count registers, count control sequences, dimen registers, dimen control sequences (like `\parindent`), skips and skip control sequences, `\numexpr`, `\dimexpr`, `\glueexpr`, `\fontdimen` can be inserted directly, they will be unpacked using `\number` which gives the internal value in terms of scaled points for the dimensional variables: `1pt = 65536sp` (stretch and shrink components are thus discarded).

Tacit multiplication (see [subsection 2.3](#)) is implied, when a number or decimal number prefixes such a register or control sequence.  $\TeX$  lengths are skip control sequences and  $\TeX$  counters should be inserted using `\value`.

Release 1.2 of the `\xintexpr` parser also recognizes and prefixes with `\number` the `\ht`, `\dp`, and `\wd`  $\TeX$  primitives as well as the `\fontcharht`, `\fontcharwd`, `\fontchardp` and `\fontcharic`  $\varepsilon$ - $\TeX$  primitives.

In the case of numbered registers like `\count255` or `\dimen0` (or `\ht0`), the resulting digits will be re-parsed, so for example `\count255 0` is like `100` if `\the\count255` would give `10`. The same happens with inputs such as `\fontdimen6\font`. And `\numexpr 35+52\relax` will be exactly as if `87` as been encountered by the parser, thus more digits may follow: `\numexpr 35+52\relax 000` is like `87000`. If a new `\numexpr` follows, it is treated as what would happen when `\xintexpr` scans a number and finds a non-digit: it does a tacit multiplication.

```
\xinttheexpr \numexpr 351+877\relax\numexpr 1000-125\relax\relax{} is the same
as \xinttheexpr 1228*875\relax.
```

*1074500 is the same as 1074500.*

Control sequences however (such as `\parindent`) are picked up as a whole by `\xintexpr`, and the numbers they define cannot be extended extra digits, a syntax error is raised if the parser finds digits rather than a legal operation after such a control sequence.

A token list variable must be prefixed by `\the`, it will not be unpacked automatically (the parser will actually try `\number`, and thus fail). Do not use `\the` but only `\number` with a dimen or skip, as the `\xintexpr` parser doesn't understand `pt` and its presence is a syntax error. To use a dimension expressed in terms of points or other  $\TeX$  recognized units, incorporate it in `\dimexpr...\relax`.

Regarding how dimensional expressions are converted by  $\TeX$  into scaled points see also [subsection 3.7](#).

## 11.3 Catcodes and spaces

Active characters may (and will) break the functioning of `\xintexpr`. Inside an expression one may prefix, for example a `:` with `\string`. Or, for a more radical way, there is `\xintexprSafeCatcodes`. This is a non-expandable step as it changes catcodes.

### 11.3.1 `\xintexprSafeCatcodes`

This macro sets the catcodes of the relevant characters to safe values. This is used internally by `\xintNewExpr` (restoring the catcodes on exit), hence `\xintNewExpr` does not have to be protected against active characters.

Attention however that if the whole

```
\xintNewExpr \foo [N] {<expression with #1,...>}
```

has been fetched as a macro argument, it will be too late then for `\xintNewExpr` to sanitize the catcodes of the (active) characters within the expression.

### 11.3.2 `\xintexprRestoreCatcodes`

Restores the catcodes to the earlier state.

Spaces inside an `\xinttheexpr...\relax` should mostly be innocuous (except inside macro arguments).

`\xintexpr` and `\xinttheexpr` are for the most part agnostic regarding catcodes: (unbraced) digits, binary operators, minus and plus signs as prefixes, dot as decimal mark, parentheses, may be indifferently of catcode letter or other or subscript or superscript, ..., it doesn't matter.<sup>69</sup>

The characters `+`, `-`, `*`, `/`, `^`, `!`, `&`, `|`, `?`, `:`, `<`, `>`, `=`, `(`, `)`, `"`, `[`, `]`, `;`, the dot and the comma should not be active if in the expression, as everything is expanded along the way. If one of them is active, it should be prefixed with `\string`.

The exclamation mark `!` should have its standard catcode: with catcode letter it is used internally and hence will confuse the parsers if it comes from the expression.

Digits, slash, square brackets, minus sign, in the output from an `\xinttheexpr` are all of catcode 12. For `\xintthefloatexpr` the ``e'` in the output has its standard catcode ``letter'`.

A macro with arguments will expand and grab its arguments before the parser may get a chance to see them, so the situation with catcodes and spaces is not the same within such macro arguments.

## 11.4 Expandability, `\xinteval`

As is the case with all other package macros `\xintexpr` *f-expands* (in two steps) to its final (non-printable) result; and `\xinttheexpr` *f-expands* (in two steps) to the chain of digits (and possibly minus sign `-`, decimal mark `.`, fraction slash `/`, scientific `e`, square brackets `[`, `]`) representing the result.

Starting with 1.09j, an `\xintexpr...\relax` can be inserted without `\xintthe` prefix inside an `\edef`, `f`, or a `\write`. It expands to a private more compact representation (five tokens) than `\xinttheexpr` or `\xintthe\xintexpr`.

The material between `\xintexpr` and `\relax` should contain only expandable material.

The once expanded `\xintexpr` is `\romannumeral0\xinteval`. And there is similarly `\xintieval`, `\xintntieval`, and `\xintfloateval`. For the other cases one can use `\romannumeral-`0` as prefix. For an example of expandable algorithms making use of chains of `\xinteval`-uations connected via `\expand\after` see subsection 2.9.

An expression can only be legally finished by a `\relax` token, which will be absorbed.

It is quite possible to nest expressions among themselves; for example, if one needs inside an `\xintiexpr...\relax` to do some computations with fractions, rounding the final result to an integer, one just has to insert `\xintiexpr...\relax`. The functioning of the infix operators will not be in the least affected from the fact that the surrounding ``environment'` is the `\xintiexpr` one.

## 11.5 Memory considerations

The parser creates an undefined control sequence for each intermediate computation evaluation: addition, subtraction, etc. ... Thus, a moderately sized expression might create 10, or 20 such control sequences. On my T<sub>E</sub>X installation, the memory available for such things is of circa 200, 000 multi-letter control words. So this means that a document containing hundreds, perhaps even thousands of expressions will compile with no problem.

Besides the hash table, also T<sub>E</sub>X main memory is impacted. Thus, if *xintexpr* is used for computing plots<sup>70</sup>, this may cause a problem. In my testing and with current TL2015 memory settings, I ran into problems after doing about *ten thousand* evaluations (for example  $(\#1+\#2)*\#3-\#1*\#3-\#2*\#3$ ) each with number having *hundreds* of digits. Typical error message can be:

```
./testaleatoires.tex:243: TeX capacity exceeded, sorry [pool size=6134970].
<argument> ...19140037877484848545931233090884903
```

There is a (partial) solution.<sup>71</sup>

A document can possibly do tens of thousands of evaluations only if some identical formulae are being used repeatedly, with varying arguments (from previous computations possibly) or coming

<sup>69</sup> Furthermore, although `\xintexpr` uses `\string`, it is escape-char agnostic. It should work with any `\escapechar` setting including -1. <sup>70</sup> this is not very probable as so far *xint* does not include a mathematical library with floating point calculations, but provides only the basic operations of algebra. <sup>71</sup> which convinced me that I could stick with the parser implementation despite its potential impact on the hash-table and other parts of T<sub>E</sub>X's memory.

from data being fetched from a file. Most certainly, there will be a few dozens formulae at most, but they will be used again and again with varying inputs.

With the `\xintNewExpr` macro, it is possible to convert once and for all an expression containing parameters into an expandable macro with parameters. Only this initial definition of this macro actually activates the `\xintexpr` parser and will (very moderately) impact the hash-table: once this unique parsing is done, a macro with parameters is produced which is built-up recursively from the `\xintAdd`, `\xintMul`, etc... macros, exactly as it would be necessary to do without the facilities of the *xintexpr* package.

Notice that since 1.2c the `\xintdefunc` construct allows an alternative to `\xintNewExpr` whose syntax uses arbitrary letters rather than macro parameters `#1`, `#2`, ..., `#9`. The declared function must still be used inside an expression, but its use will need only as many `\csname`'s as were needed for the function arguments plus one more for encapsulating the function result.

## 11.6 The `\xintNewExpr` macro

The macro is used as:

`\xintNewExpr{\myformula}[n]{<stuff>}`, where

- *<stuff>* will be inserted inside `\xinttheexpr . . . \relax`,
- *n* is an integer between zero and nine, inclusive, which is the number of parameters of `\myformula`,
- the placeholders `#1`, `#2`, ..., `#n` are used inside *<stuff>* in their usual rôle,<sup>72 73</sup>
- the [*n*] is mandatory, even for *n*=0.<sup>74</sup>
- the macro `\myformula` is defined without checking if it already exists,  $\TeX$  users might prefer to do first `\newcommand*\myformula {}` to get a reasonable error message in case `\myformula` already exists,
- the protection against active characters is done automatically (as long as the whole thing has not already been fetched as a macro argument and the catcodes correspondingly already frozen).

It will be a completely expandable macro entirely built-up using `\xintAdd`, `\xintSub`, `\xintMul`, `\xintDiv`, `\xintPow`, etc... as corresponds to the expression written with the infix operators. Macros created by `\xintNewExpr` can thus be nested.

```
\xintNewFloatExpr \FA [2]{(#1+#2)^10}
\xintNewFloatExpr \FB [2]{sqrt(#1*#2)}
\begin{enumerate}[nosep]
 \item \FA {5}{5}
 \item \FB {30}{10}
 \item \FA {\FB {30}{10}}{\FB {40}{20}}
\end{enumerate}
```

1. 1.0000000000000000e10
2. 17.32050807568877
3. 3.891379490446502e16

The use of `\xintNewExpr` circumvents the impact of the `\xintexpr` parsers on  $\TeX$ 's memory: it is useful if one has a formula which has to be re-evaluated thousands of times with distinct inputs each with dozens, or hundreds of digits.

A `''formula''` created by `\xintNewExpr` is thus a macro whose parameters are given to a possibly very complicated combination of the various macros of *xint* and *xintfrac*. Consequently, one can not use at all any infix notation in the inputs, but only the formats which are recognized by the *xintfrac* macros.

<sup>72</sup> if `\xintNewExpr` is used inside a macro, the `#`'s must be doubled as usual. <sup>73</sup> the `#`'s will in practice have their usual catcode, but category code other `#`'s are accepted too. <sup>74</sup> there is some use for `\xintNewExpr[0]` compared to an `\edef` as `\xintNewExpr` has some built-in catcode protection.

This is thus quite different from a macro with parameters which one would have defined via a simple `\def` or `\newcommand` as for example:

```
\newcommand\myformula [1]{\xinttheexpr (#1)^3\relax}
```

Such a macro `\myformula`, if it was used tens of thousands of times with various big inputs would end up populating large parts of TeX's memory. It would thus be better for such use cases to go for:

```
\xintNewExpr\myformula [1]{#1^3\relax}
```

Here naturally the situation is over-simplified and it would be even simpler to go directly for the use of the macro `\xintPow` or `\xintPower`.

`\xintNewExpr` tries to do as many evaluations as are possible at the time the macro parameters are still parameters. Let's see a few examples. For this I will use `\meaning` which reveals the contents of a macro.

1. the examples use a mysterious `\fixmeaning` macro, which is there to get in the display `\romannumeral`^^@` rather than the frankly cabalistic `\romannumeral`` which made the admiration of the readers of the documentation dated 2015/10/19 (the second ``` stood for an ascii code zero token as per T1 encoded `newtxtt` font). Thus the true meaning is `fixed'` to display something different which is how the macro could be defined in a standard `tex` source file (modulo, as one can see in example, the use of characters such as `:` as letters in control sequence names). Prior to 1.2a, the meaning would have started with a more mundane `\romannumeral`0`, but I decided at the time of releasing 1.2a to imitate the serious guys and switch for the more hacky yet `\romannumeral`^^@` everywhere in the source code (not only in the macros produced by `\xintNewExpr`), or to be more precise for an equivalent as the caret has catcode letter in `xint`'s source code, and I had to use another character.
2. the meaning reveals the use of some private macros from the `xint` bundle, which should not be directly used. If the things look a bit complicated, it is because they have to cater for many possibilities.
3. the point of showing the meaning is also to see what has already been evaluated in the construction of the macros.

```
\xintNewIIExpr\FA [1]{13*25*78*#1+2826*292}\fixmeaning\FA
macro:#1->\romannumeral`^^@\xintCSV::csv {\xintiiAdd {\xintiiMul {25350}{#1}}{825192}}

\xintNewIExpr\FA [2]{(3/5*9/7*13/11*#1-#2)*3^7}
\printnumber{\fixmeaning\FA}
macro:#1#2->\romannumeral`^^@\xintSPRaw::csv {\xintRound::csv {0}{\xintMul {\xintSub {\xintMu
1 {351/385[0]}{#1}}{#2}}{2187/1[0]}}}

% an example with optional parameter
\xintNewIExpr\FA [3]{[24] (#1+#2)/(#1-#2)^#3}
\printnumber{\fixmeaning\FA}
macro:#1#2#3->\romannumeral`^^@\xintSPRaw::csv {\xintRound::csv {24}{\xintDiv {\xintAdd {#1}{2
#2}}{\xintPow {\xintSub {#1}{#2}}{#3}}}}

\xintNewFloatExpr\FA [2]{[12] 3.1415^3*#1-#2^5}
\printnumber{\fixmeaning\FA}
macro:#1#2->\romannumeral`^^@\xintPFloat::csv {12}{\XINTinFloatSub {\XINTinFloatMul {310035332
39837500[-14]}{#1}}{\XINTinFloatPowerH {#2}{5}}}

\xintNewExpr\DET[9]{ #1*#5*#9+#2*#6*#7+#3*#4*#8-#1*#6*#8-#2*#4*#9-#3*#5*#7 }
\printnumber{\fixmeaning\DET}
macro:#1#2#3#4#5#6#7#8#9->\romannumeral`^^@\xintSPRaw::csv {\xintSub {\xintSub {\xintSub {\xi
ntAdd {\xintAdd {\xintMul {\xintMul {#1}{#5}}{#9}}{\xintMul {\xintMul {#2}{#6}}{#7}}}{\xintMul
{\xintMul {#3}{#4}}{#8}}}{\xintMul {\xintMul {#1}{#6}}{#8}}}{\xintMul {\xintMul {#2}{#4}}{#9}}
}{\xintMul {\xintMul {#3}{#5}}{#7}}}}

Notice that since 1.2c it is perhaps more natural to do:
% attention that «ad» would try to use non-existent variable "ad"
```

```

\xintdeffunc det2(a, b, c, d) := a*d - b*c ;
% This is impossible because we must use single letters :
% \xintdeffunc det3(x_11, x_12, x_13, x_21, x_22, x_23, x_31, x_32, x_33) :=
% x_11 * det2 (x_22, x_23, x_32, x_33) + x_21 * det2 (x_32, x_33, x_12, x_13)
% + x_31 * det2 (x_12, x_13, x_22, x_23);
\xintdeffunc det3 (a, b, c, u, v, w, x, y, z) := a*v*z + b*w*x + c*u*y - b*u*z - c*v*x - a*w*y ;
\xinttheexpr det3 (1,1,1,1,2,4,1,3,9), det3 (1,10,100,1,100,10000,1,1000,1000000),
90*900*990, reduce(det3 (1,1/2,1/3,1/2,1/3,1/4,1/3,1/4,1/5))\relax\newline
\xintdeffunc det3bis (a, b, c, u, v, w, x, y, z) :=
a*det2(v,w,y,z)-b*det2(u,w,x,z)+c*det2(u,v,x,y);
\pdfsetrandomseed 123456789 % xint.pdf should be predictable from xint.dtx !
\xinttheexpr subs(subs(subs(subs(subs(subs(subs(subs(
% we use one extra pair of parentheses to hide the commas from the subs
(a, b, c, u, v, w, x, y, z, det3 (a, b, c, u, v, w, x, y, z),
det3bis (a, b, c, u, v, w, x, y, z)),
z=\pdfuniformdeviate 1000), y=\pdfuniformdeviate 1000), x=\pdfuniformdeviate 1000),
w=\pdfuniformdeviate 1000), v=\pdfuniformdeviate 1000), u=\pdfuniformdeviate 1000),
c=\pdfuniformdeviate 1000), b=\pdfuniformdeviate 1000), a=\pdfuniformdeviate 1000)\relax

```

2, 80190000, 80190000, 1/2160

339, 694, 33, 664, 31, 921, 891, 763, 353, 188129929, 188129929

The last computation with its nine nested `subs` can be coded more economically (and efficiently), exploiting the fact that a single dummy variable can expand to a whole list:

```

\pdfsetrandomseed 123456789 % xint.pdf should be predictable from xint.dtx !
\xinttheexpr subs(L, det3(L), det3bis(L)), % parentheses used to hide the inner commas
L=\pdfuniformdeviate 1000, \pdfuniformdeviate 1000, \pdfuniformdeviate 1000,
\pdfuniformdeviate 1000, \pdfuniformdeviate 1000, \pdfuniformdeviate 1000,
\pdfuniformdeviate 1000, \pdfuniformdeviate 1000, \pdfuniformdeviate 1000)\relax

```

339, 694, 33, 664, 31, 921, 891, 763, 353, 188129929, 188129929

With `\xintverbosetrue` we will find in the log:

```

Function det3 for \xintexpr parser associated to \XINT_expr_userfunc_det3 w
ith meaning macro:#1,#2,#3,#4,#5,#6,#7,#8,#9,->\xintSub {\xintSub {\xintSub {\x
intAdd {\xintAdd {\xintMul {\xintMul {\#1}{\#5}}{\#9}}{\xintMul {\xintMul {\#2}{\#6}
}{\#7}}{\xintMul {\xintMul {\#3}{\#4}}{\#8}}{\xintMul {\xintMul {\#2}{\#4}}{\#9}}}{\
xintMul {\xintMul {\#3}{\#5}}{\#7}}{\xintMul {\xintMul {\#1}{\#6}}{\#8}}
Package xintexpr Info: (on line 11)

```

```

Function det3bis for \xintexpr parser associated to \XINT_expr_userfunc_det
3bis with meaning macro:#1,#2,#3,#4,#5,#6,#7,#8,#9,->\xintAdd {\xintSub {\xintM
ul {\#1}{\xintSub {\xintMul {\#5}{\#9}}{\xintMul {\#6}{\#8}}}}{\xintMul {\#2}{\xintSu
b {\xintMul {\#4}{\#9}}{\xintMul {\#6}{\#7}}}}{\xintMul {\#3}{\xintSub {\xintMul {\#
4}{\#8}}{\xintMul {\#5}{\#7}}}}

```

Lists, including Python-like selectors, are compatible with `\xintNewExpr`:<sup>75</sup>

```

\xintNewExpr\Foo[5]{\empty[#1..[#2]..[#3]..[#4]:[#5]}
\begin{itemize}[nosep]
\item |\Foo{1}{3}{90}{20}{30}|->\Foo{1}{3}{90}{20}{30}
\item |\Foo{1}{3}{90}{-40}{-15}|->\Foo{1}{3}{90}{-40}{-15}
\item |\Foo{1.234}{-0.123}{-10}{3}{7}|->\Foo{1.234}{-0.123}{-10}{3}{7}
\end{itemize}
\def\test {\Foo {0}{10}{100}{3}{6}}\meaning\test +++

```

- `\Foo{1}{3}{90}{20}{30}`->61, 64, 67, 70, 73, 76, 79, 82, 85, 88
- `\Foo{1}{3}{90}{-40}{-15}`->1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, 40, 43, 46, 49, 52, 55, 58, 61, 64, 67, 70, 73
- `\Foo{1.234}{-0.123}{-10}{3}{7}`->865[-3], 742[-3], 619[-3], 496[-3]

macro:->30, 40, 50+++

In this last example the macro `\Foo` will not be able to handle an empty `#4` or `#5`: this is only

<sup>75</sup> The `\empty` token is optional here, but it would be needed in case of `\xintNewFloatExpr` or `\xintNewIExpr`.

possible in an expression, because the parser identifies `[:` or `:]` and handles them appropriately. During the construction of `\Foo` the parser will find `][#4:` and not `][:`.

The `\xintdeffunc`, `\xintdefiifunc`, `\xintdeffloatfunc` declarators added to `xintexpr` since release 1.2c are based on the same underlying mechanism as `\xintNewExpr`, `\xintNewIIExpr`, ... The discussion that follows applies to them too.

### 11.6.1 Conditional operators and `\NewExpr`

The `?` and `??` conditional operators cannot be parsed by `\xintNewExpr` when they contain macro parameters `#1`, ..., `#9` within their scope. However replacing them with the functions `if` and, respectively `ifsgn`, the parsing should succeed. And the created macro will *not evaluate the branches to be skipped*, thus behaving exactly like `?` and `??` would have in the `\xintexpr`.

```
\xintNewExpr\Formula [3]{ if((#1>#2) && (#2>#3), sqrt(#1-#2)*sqrt(#2-#3), #1^2+#3/#2) }%
\printnumber{\fixmeaning\Formula }
macro:#1#2#3->\romannumeral`^^@\xintSPRaw::csv {\xintiifNotZero {\xintAND {\xintGt {#1}{#2}}
{\xintGt {#2}{#3}}}{\xintMul {\XINTinFloatSqrtdigits {\xintSub {#1}{#2}}{\XINTinFloatSqrtdig
its {\xintSub {#2}{#3}}}}{\xintAdd {\xintPow {#1}{2}}{\xintDiv {#3}{#2}}}}
```

This formula (with its `\xintiifNotZero`) will gobble the false branch without evaluating it when used with given arguments.

Remark: the meaning above reveals some of the private macros used by the package. They are not for direct use.

Another example


```
\xintNewExpr\myformula[3]{ ifsgn(#1,#2/#3,#2-#3,#2*#3) }%
\fixmeaning\myformula
macro:#1#2#3->\romannumeral`^^@\xintSPRaw::csv {\xintiifSgn {#1}{\xintDiv {#2}{#3}}{\xintSub
{#2}{#3}}{\xintMul {#2}{#3}}}
```

Again, this macro gobbles the false branches, as would have the operator `??` inside an `\xintexpr`-session.

### 11.6.2 External macros and `\NewExpr`; the `protect` function

For macros within such a created `xint`-formula macro, there are two cases:

- the macro does not involve the numbered parameters in its arguments: it may then be left as is, and will be evaluated once during the construction of the formula,
- it does involve at least one of the macro parameters as argument. Then:

 { the whole thing (macro + argument) should be `protect`-ed, not in the `TeX` sense (!), but in the following way: `protect(\macro {#1})`.

Here is a silly example illustrating the general principle: the macros here have equivalent functional forms which are more convenient; but some of the more obscure package macros of `xint` dealing with integers do not have functions pre-defined to be in correspondance with them, use this mechanism could be applied to them.

```
\xintNewExpr\formulaA[2]{protect(\xintRound{#1}{#2}) - protect(\xintTrunc{#1}{#2})}%
\printnumber{\fixmeaning\formulaA}

\xintNewIIExpr\formulaB [3]{rem(#1,quo(protect(\the\numexpr #2\relax),#3))}%
\noindent\printnumber{\fixmeaning\formulaB }
```



```
macro:#1#2->\romannumeral`^^@\xintSPRaw::csv {\xintSub {\xintRound {#1}{#2}}{\xintTrunc {#1}{#2}}
#2}}}
macro:#1#2#3->\romannumeral`^^@\xintCSV::csv {\xintiiRem {#1}{\xintiiQuo {\the \numexpr #2\relax}{#3}}}
```

Only macros involving the #1, #2, etc. . . should be protected in this way; the +, \*, etc. . . symbols, the functions from the *\xintexpr* syntax, none should ever be included in a protected string.

### 11.6.3 Limitations of *\xintNewExpr* and *\xintdeffunc*

*\xintNewExpr* will pre-evaluate everything as long as it does not contain the macro parameters #1, #2, . . . and the special measures to take when these are inside branches to ? and ?? (replace these operators by *if* and *ifsgn*) or as arguments to macros external to *xintexpr* (use *protect*) were discussed in [subsubsection 11.6.1](#) and [subsubsection 11.6.2](#).

The main remaining limitation is that expressions with dummy variables are compatible with *\xintNewExpr* only to the extent that the iterated-over list of values does not depend on the macro parameters #1, #2, . . . For example, this works:

```
\xintNewExpr \FA [2] {reduce(add((t+#1)/(t+#2), t=0..5))}
\FA {1}{1}, \FA {1}{2}, \FA {2}{3}
```

6, 617/140, 1339/280 but the 5 can not be abstracted into a third argument #3.

There are no restriction on using macro parameters #1, #2, . . . with list constructs. For example, this works:

```
\xintNewExpr \FB [3] {[4] `+`([1/3..[#1/3]..#2]*#3)}
\begin{itemize}[nosep]
\item \FB {1}{10/3}{100} % (1/3+2/3+...+10/3)*100
\item \FB {5}{5}{20} % (1/3+6/3+11/3)*20
\item \FB {3}{4}{1} % (1/3+4/3+7/3+10/3)*1
\end{itemize}
```

- 1833.3333
- 120.0000
- 7.3333

Some simple expressions with *add* or *mul* can be also expressed with *`+`* and *`\*`* and list operations. But there is no hope for *seq*, *iter*, etc. . . if the #1, #2, . . . are used inside the list argument: *seq(x(x+#1)(x+#2),x=1..#3)* is currently not compatible with *\xintNewExpr*. But *seq(x(x+#1)(x+#2), x=1..10)* has no problem.

All the preceeding applies identically for *\xintdeffunc*, *\xintdefiifunc*, *\xintdeffloatfunc* which share the same routines as *\xintNewExpr*, *\xintNewIExpr*, . . ., replacing the #1, #2, . . . in the discussion by the letters used as function arguments.

There is a final syntax restriction which however applies only to *\xintNewExpr* et. al., and not to *\xintdeffunc*, *\xintdefiifunc*, *\xintdeffloatfunc* : it is possible to use sub-expressions only if they use *\xintexpr*, those with *\xinttheexpr* are illegal.

```
\xintNewExpr \FC [4] {#1+\xintexpr #2*#3\relax + #4}
\printnumber{\fixmeaning\FC}
```

```
macro:#1#2#3#4->\romannumeral`^^@\xintSPRaw::csv {\xintAdd {\xintAdd {#1}{\xintMul {#2}{#3}}}{#4}}
```

works, but already

```
\xintNewExpr \FD [1] {#1+\xinttheexpr 1\relax}
```

doesn't. On the other hand

```
\xintdeffunc FD(t) := t + \xinttheexpr 1\relax ;
```

and even

```
\xintdeffunc FE(t,u) := t + \xinttheexpr u\relax ;
```

have no issue. Anyway, one should never use *\xinttheexpr* for sub-expressions but only *\xintexpr*, so this restriction on the *\xintNewExpr* syntax isn't really one.



## 11.7 The `\xintNewFunction` macro

New with 1.2h See [subsection 2.6.3](#) for its documentation.

## 11.8 `\xintiexpr`, `\xinttheiexpr`

- ✱ ★ Equivalent to doing `\xintexpr round(...)\relax` (more precisely, `round` is applied to each one of the evaluated values, if the expression was comma separated). Thus, only the *final result* value is rounded to an integer. Half integers are rounded towards  $+\infty$  for positive numbers and towards  $-\infty$  for negative ones.

An optional parameter `d>0` within brackets, immediately after `\xintiexpr` is allowed: it instructs the expression to do its final rounding to the nearest value with that many digits after the decimal mark, i.e., `\xintiexpr [d] <expression>\relax` is equivalent (in case of a single expression) to `\xintexpr round(<expression>, d)\relax`.

`\xintiexpr [0] ...` is the same as `\xintiexpr ...`.<sup>76</sup>

If truncation rather than rounding is needed use (in case of a single expression, naturally) `\xintexpr trunc(...)\relax` for truncation to an integer or `\xintexpr trunc(...,d)\relax` for truncation to a decimal number with `d>0` digits after the decimal mark.

Perhaps in the future some meaning will be given to using negative value for the optional parameter `d`.<sup>77</sup>

New with 1.2h `\thexintiexpr` is synonym to `\xinttheiexpr`.

## 11.9 `\xintiexpr`, `\xinttheiexpr`

- ✱ ★ This variant does not know fractions. It deals almost only with long integers. Comma separated lists of expressions are allowed.

It maps `/` to the *rounded* quotient. The operator `//` is, like in `\xintexpr...\relax`, mapped to *truncated* division. The euclidean quotient (which for positive operands is like the truncated quotient) was, prior to release 1.1, associated to `/.` The function `quo(a,b)` can still be employed.

The `\xintiexpr`-essions use the `'ii'` macros for addition, subtraction, multiplication, power, square, sums, products, euclidean quotient and remainder.

The `round`, `trunc`, `floor`, `ceil` functions are still available, and are about the only places where fractions can be used, but `/` within, if not somehow hidden will be executed as integer rounded division. To avoid this one can wrap the input in `qfrac`: this means however that none of the normal expression parsing will be executed on the argument.

To understand the illustrative examples, recall that `round` and `trunc` have a second (non negative) optional argument. In a normal `\xintexpr`-essions, `round` and `trunc` are mapped to `\xintRound` and `\xintTrunc`, in `\xintiexpr`-essions, they are mapped to `\xintiRound` and `\xintiTrunc`.

```
\xinttheiexpr 5/3, round(5/3,3), trunc(5/3,3), trunc(\xintDiv {5}{3},3),
trunc(\xintRaw {5/3},3)\relax{} are problematic, but
%
\xinttheiexpr 5/3, round(qfrac(5/3),3), trunc(qfrac(5/3),3), floor(qfrac(5/3)),
ceil(qfrac(5/3))\relax{} work!
```

2, 2000, 2000, 2000, 2000 are problematic, but 2, 1667, 1666, 1, 2 work!

On the other hand decimal numbers and scientific numbers can be used directly as arguments to the `num`, `round`, or any function producing an integer.

<sup>76</sup> Incidentally using `round(...,0)` in place of `round(...)` in `\xintexpr` would leave a trailing dot in the produced value. <sup>77</sup> Thanks to KT for this suggestion.

Scientific numbers will be represented with as many zeroes as necessary, thus one does not want to insert `num(1e100000)` for example in an `\xintiexpression` !

```
\xinttheiexpr num(13.4567e3)+num(10000123e-3)\relax % should (num truncates) compute 13456+10000
23456
```

The `reduce` function is not available and will raise an error. The `frac` function also. The `sqr` function is mapped to `\xintiiSqrt` which gives a truncated square root. The `sqrtr` function is mapped to `\xintiiSqrtR` which gives a rounded square root.

One can use the Float macros if one is careful to use `num`, or `round` etc. . . on their output.

```
\xinttheiexpr \xintFloatSqrt [20]{2}, \xintFloatSqrt [20]{3}\relax % no operations
```

```
\noindent The next example requires the |round|, and one could not put the |+| inside it:
```

```
\xinttheiexpr round(\xintFloatSqrt [20]{2},19)+round(\xintFloatSqrt [20]{3},19)\relax
```

(the second argument of `|round|` and `|trunc|` tells how many digits from after the decimal mark one should keep.)

```
14142135623730950488[-19], 17320508075688772935[-19]
```

The next example requires the `round`, and one could not put the `+` inside it:

```
31462643699419723423
```

(the second argument of `round` and `trunc` tells how many digits from after the decimal mark one should keep.)

The whole point of `\xintiexpr` is to gain some speed in *integer-only* algorithms, and the above explanations related to how to nevertheless use fractions therein are a bit peripheral. We observed (2013/12/18) of the order of 30% speed gain when dealing with numbers with circa one hundred digits (1.2: this info may be obsolete).

New with 1.2h `\thexintiexpr` is synonym to `\xinttheiexpr`.

### 11.10 `\xintboolexpr`, `\xinttheboolexpr`

- x ★ Equivalent to doing `\xintexpr ... \relax` and returning 1 if the result does not vanish, and 0 if the result is zero. As `\xintexpr`, this can be used on comma separated lists of expressions, and will return a comma separated list of 0's and 1's.

New with 1.2h `\thexintboolexpr` is synonym to `\xinttheboolexpr`.

### 11.11 `\xintfloatexpr`, `\xintthefloatexpr`

- x ★ `\xintfloatexpr... \relax` is exactly like `\xintexpr... \relax` but with the four binary operations and the power function are mapped to `\xintFloatAdd`, `\xintFloatSub`, `\xintFloatMul`, `\xintFloatDiv` and `\xintFloatPower`, respectively.<sup>78</sup>

The target precision for the computation is from the current setting of `\xintDigits`. Comma separated lists of expressions are allowed.

An optional (positive) parameter within brackets is allowed: the final float will have that many digits of precision. This is provided to get rid of possibly irrelevant last digits, thus makes sense only if this parameter is less than the `\xinttheDigits` precision.

Since 1.2f all float operations first round their arguments; a parsed number is not rounded prior to its use as operand to such a float operation.

New with 1.2h `\thexintfloatexpr` is synonym to `\xintthefloatexpr`.

```
\xintDigits:=36;
```

```
\xintthefloatexpr (1/13+1/121)*(1/179-1/173)/(1/19-1/18)\relax
0.00564487459334466559166166079096852897
```

<sup>78</sup> Since 1.2f the `^` handles half-integer exponents, contrarily to `\xintFloatPower`.

```
\xintthefloatexpr\xintexpr (1/13+1/121)*(1/179-1/173)/(1/19-1/18)\relax\relax
0.00564487459334466559166166079096852912
```

The latter is the rounding of the exact result. The former one has its last three digits wrong due to the cumulative effect of rounding errors in the intermediate computations, as compared to exact evaluations.

Changed  
(1.2f)

I recall here from [subsection 3.2](#) that with release 1.2f the float macros for addition, subtraction, multiplication and division round their arguments first to *P* significant places with *P* the asked-for precision of the output; and similarly the power macros and the square root macro. This does not modify anything for computations with arguments having at most *P* significant places already.

## 11.12 Using an expression parser within another one

This was already illustrated before. In the following:

```
\xintthefloatexpr \xintexpr add(1/i, i=1234..1243)\relax ^100\relax
5.136088460396579e-210, the inner sum is computed exactly. Then it will be rounded to \xinttheD\
igits significant digits, and then its power will be evaluated as a float operation. One should
avoid the "\xintthe" parsers in inner positions as this induces digit by digit parsing of the inner
computation result by the outer parser. Here is the same computation done with floats all the way:
\xintthefloatexpr add(1/i, i=1234..1243)^100\relax
5.136088460396643e-210
```

Not surprisingly this differs from the previous one which was exact until raising to the 100th power.

The fact that the inner expression occurs inside a bigger one has nil influence on its behaviour. There is the limitation though that the outputs from `\xintexpr` and `\xintfloatexpr` can not be used directly in `\xinttheiexpr` integer-only parser. But one can do:

```
\xinttheiexpr round(\xintfloatexpr 3.14^10\relax)\relax % or trunc
93174
```

## 11.13 The `\xintthecoords` macro

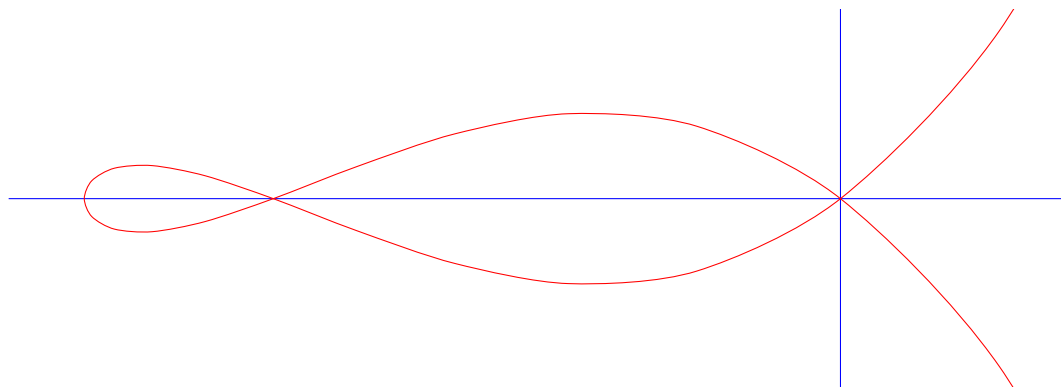
It converts a comma separated list into the format for list of coordinates as expected by the `TikZ coordinates` syntax. The code had to work around the problem that `TikZ` seemingly allows only a maximal number of about one hundred expansion steps for the list to be entirely produced. Presumably to catch an infinite loop.

```
\begin{figure}[htbp]
\centering\begin{tikzpicture}[scale=10]\xintDigits:=8;
\clip (-1.1,-.25) rectangle (.3,.25);
\draw [blue] (-1.1,0)--(1,0);
\draw [blue] (0,-1)--(0,+1);
\draw [red] plot[smooth] coordinates {%
\xintthecoords % (converts what is next into (x1, y1) (x2, y2)... format)
\xintfloatexpr seq((x^2-1,mul(x-t,t=-1+[0..4]/2)),x=-1.2..[0.1]..+1.2) \relax };
\end{tikzpicture}
\caption{Coordinates with \cs{xintthecoords}.}
\end{figure}
```

`\xintthecoords` should be followed immediately by `\xintfloatexpr` or `\xintiexpr` or `\xintiiexpr`, but not `\xintthefloatexpr`, etc. . .

Besides, as `TikZ` will not understand the `A/B[N]` format which is used on output by `\xintexpr`, `\xi\ntexpr` is not really usable with `\xintthecoords` for a `TikZ` picture, but one may use it on its own, and the reason for the spaces in and between coordinate pairs is to allow if necessary to print on the page for examination with about correct line-breaks.

```
\edef\x{\xintthecoords \xintexpr rrseq(1/2,1/3; @1+@2, x=1..20)\relax }
\meaning\x +++
```

Figure 1: Coordinates with `\xintthecoords`.

```
macro:->(1/2, 1/3) (5/6, 7/6) (12/6, 19/6) (31/6, 50/6) (81/6, 131/6) (212/6, 343/6) (555/6,
898/6) (1453/6, 2351/6) (3804/6, 6155/6) (9959/6, 16114/6) (26073/6, 42187/6)+++
```

### 11.14 `\xintifboolexpr`

**xnn ★** `\xintifboolexpr{<expr>}{YES}{NO}` does `\xinttheexpr <expr>\relax` and then executes the YES or the NO branch depending on whether the outcome was non-zero or zero. `<expr>` can involve various `&` and `|`, parentheses, `all`, `any`, `xor`, the `bool` or `togl` operators, but is not limited to them: the most general computation can be done, the test is on whether the outcome of the computation vanishes or not.

Will not work on an expression composed of comma separated sub-expressions.

### 11.15 `\xintifboolfloatexpr`

**xnn ★** `\xintifboolfloatexpr{<expr>}{YES}{NO}` does `\xintthefloatexpr <expr>\relax` and then executes the YES or the NO branch depending on whether the outcome was non zero or zero.

### 11.16 `\xintifbooliiexpr`

**xnn ★** `\xintifbooliiexpr{<expr>}{YES}{NO}` does `\xinttheiiexpr <expr>\relax` and then executes the YES or the NO branch depending on whether the outcome was non zero or zero.

### 11.17 `\xintNewFloatExpr`

This is exactly like `\xintNewExpr` except that the created formulas are set-up to use `\xintthefloatexpr`. The precision used for the computation will be the one given by `\xinttheDigits` at the time of use of the created formulas. However, the numbers hard-wired in the original expression will have been evaluated with the then current setting for `\xintDigits`.

```
\xintNewFloatExpr \f [1] {sqrt(#1)}
\ f {2} (with \xinttheDigits{} of precision).

{\xintDigits := 32;\f {2} (with \xinttheDigits{} of precision).}

\xintNewFloatExpr \f [1] {sqrt(#1)*sqrt(2)}
\ f {2} (with \xinttheDigits {} of precision).

{\xintDigits := 32;\f {2} (?? we thought we had a higher precision. Explanation next)}
```

The `sqrt(2)` in the second formula was computed with only `\xinttheDigits{}` of precision. Setting `|\xinttheDigits|` to a higher value at the time of definition will confirm that the result above is from a mismatch of the precision for `|sqrt(2)|` at the time of its evaluation and the precision for the new `|sqrt(2)|` with `|#1=2|` at the time of use.

```
{\xintDigits := 32;\xintNewFloatExpr \f [1] {sqrt(#1)*sqrt(2)}
\f {2} (with \xinttheDigits {} of precision)}
```

1.414213562373095 (with 16 of precision).

1.4142135623730950488016887242097 (with 32 of precision).

2.0000000000000000 (with 16 of precision).

1.9999999999999999309839899395125 (?? we thought we had a higher precision. Explanation next)

The `sqrt(2)` in the second formula was computed with only 16 of precision. Setting `\xinttheDigits` to a higher value at the time of definition will confirm that the result above is from a mismatch of the precision for `sqrt(2)` at the time of its evaluation and the precision for the new `sqrt(2)` with `#1=2` at the time of use.

2.0000000000000000000000000000000000 (with 32 of precision)

### 11.18 `\xintNewIExpr`

Like `\xintNewExpr` but using `\xinttheiexpr`.

### 11.19 `\xintNewIIExpr`

Like `\xintNewExpr` but using `\xinttheiiexpr`.

### 11.20 `\xintNewBoolExpr`

Like `\xintNewExpr` but using `\xinttheboolexpr`.

### 11.21 Technicalities

As already mentioned `\xintNewExpr\myformula[n]` does not check the prior existence of a macro `\myformula`. And the number of parameters `n` given as mandatory argument within square brackets should be (at least) equal to the number of parameters in the expression.

Obviously I should mention that `\xintNewExpr` itself can not be used in an expansion-only context, as it creates a macro.

The `\escapechar` setting may be arbitrary when using `\xintexpr`.

The format of the output of `\xintexpr<stuff>\relax` is a `!` (with catcode 11) followed by various things:

```
\edef\f {\xintexpr 1.23^10\relax }\meaning\f
```

```
macro:->!\XINT_expr_usethe \XINT_protectii \XINT_expr_print \.=792594609605189126649/1[-20]
```

Note that `\xintexpr` expands in an `\edef`, contrarily to `\numexpr` which is non-expandable, if not prefixed by `\the`, `\number`, or `\romannumeral` or in some other context where  $\TeX$  is building a number. See [subsection 2.9](#) for some illustration.


I decided to put all intermediate results (from each evaluation of an infix operators, or of a parenthesized subpart of the expression, or from application of the minus as prefix, or of the exclamation sign as postfix, or any encountered braced material) inside `\csname...\endcsname`, as this can be done expandably and encapsulates an arbitrarily long fraction in a single token (left with undefined meaning), thus providing tremendous relief to the programmer in his/her expansion control.

As the `\xintexpr` computations corresponding to functions and infix or postfix operators are done inside `\csname...\endcsname`, the *f-expandability* could possibly be dropped and one could imagine implementing the basic operations with expandable but not *f-expandable* macros (as `\xintXTrunc`.) I have not investigated that possibility.

Syntax errors in the input such as using a one-argument function with two arguments will generate low-level  $\TeX$  processing unrecoverable errors, with cryptic accompanying message.

Some other problems will give rise to 'error messages' macros giving some indication on the location and nature of the problem. Mainly, an attempt has been made to handle gracefully missing or extraneous parentheses.

However, this mechanism is completely inoperant for parentheses involved in the syntax of the `seq`, `add`, `mul`, `subs`, `rseq` and `rrseq` functions, and missing parentheses may cause the parser to fetch tokens beyond the ending `\relax` necessarily ending up in cryptic low-level  $\TeX$ -errors.

 { Note that the `,<letter>=` part must be visible, it can not arise from expansion (the equal sign does not have to be an equal sign, it can be any token and will be gobbled). However for `iter`, `iter2`, `r`, `rseq`, `rrseq`, the initial values delimited by a `;` are parsed in the normal way, and in particular may be braced or arise from expansion. This is useful as the `;` may be hidden from `\xintdeffunc` as `{;}` for example. Again, this remark does not apply to the comma `,` which precedes the `<letter>=` part. The comma will be fetched by delimited macros and must be there. Nesting is handled by checking (again using suitable delimited macros) that parentheses are suitably balanced.

Note that `\relax` is mandatory (contrarily to the situation for `\numexpr`).

## 11.22 Acknowledgements (2013/05/25)

I was greatly helped in my preparatory thinking, prior to producing such an expandable parser, by the commented source of the `l3fp` package, specifically the `l3fp-parse.dtx` file (in the version of April-May 2013; I think there was in particular a text called ``roadmap'' which was helpful). Also the source of the `calc` package was instructive, despite the fact that here for `\xintexpr` the principles are necessarily different due to the aim of achieving expandability.

## 12 Macros of the `xintbinhex` package

|    |                            |     |    |                             |     |
|----|----------------------------|-----|----|-----------------------------|-----|
| .1 | <code>\xintDecToHex</code> | 134 | .5 | <code>\xintBinToHex</code>  | 135 |
| .2 | <code>\xintDecToBin</code> | 134 | .6 | <code>\xintHexToBin</code>  | 135 |
| .3 | <code>\xintHexToDec</code> | 134 | .7 | <code>\xintCHexToBin</code> | 135 |
| .4 | <code>\xintBinToDec</code> | 134 |    |                             |     |

This package was first included in the 1.08 (2013/06/07) release of `xint`. It provides expandable conversions of arbitrarily big integers to and from binary and hexadecimal.

The argument is first *f-expanded*. It then may start with an optional minus sign (unique, of category code other), followed with optional leading zeroes (arbitrarily many, category code other) and then ```digits``` (hexadecimal letters may be of category code letter or other, and must be up-percased). The optional (unique) minus sign (plus sign is not allowed) is kept in the output. Leading zeroes are allowed, and stripped. The hexadecimal letters on output are of category code letter, and up-percased.

### 12.1 `\xintDecToHex`

*f* ★ Converts from decimal to hexadecimal.

```
\xintDecToHex{2718281828459045235360287471352662497757247093699959574966967627724076630353}
547594571382178525166427427466391932003}
->11A9397C66949A97051F7D0A817914E3E0B17C41B11C48BAEF2B5760BB38D272F46DCE46C6032936BF37DAC918}
814C63
```

### 12.2 `\xintDecToBin`

*f* ★ Converts from decimal to binary.

```
\xintDecToBin{2718281828459045235360287471352662497757247093699959574966967627724076630353}
547594571382178525166427427466391932003}
->100011010100100111001011111000110011010010100100110101001011100000101000111110111110100001}
01010000001011110010001010011100011111000001011000101111100010000011011000100011100010010001}
01110101110111100101011010101110110000010111011001110001101001001110010111101000110110111001}
11001000110110001100000001100101001001101101011111001101111011011100100100011000100000010}
100110001100011
```

### 12.3 `\xintHexToDec`

*f* ★ Converts from hexadecimal to decimal.

```
\xintHexToDec{11A9397C66949A97051F7D0A817914E3E0B17C41B11C48BAEF2B5760BB38D272F46DCE46C603}
2936BF37DAC918814C63}
->271828182845904523536028747135266249775724709369995957496696762772407663035354759457138217}
8525166427427466391932003
```

### 12.4 `\xintBinToDec`

*f* ★ Converts from binary to decimal.

```
\xintBinToDec{1000110101001001110010111110001100110100101001001101010010111000001010001111}
10111110100001010100000010111100100010100111000111110000010110001011111000100000110110001000}
11100010010001011101011101111001010110101011101100000101110110011100011010010011100101111010}
0011011011100111001000110110001100000001100101001001101101011111001101111011011001001000}
11000100000010100110001100011}
->271828182845904523536028747135266249775724709369995957496696762772407663035354759457138217}
8525166427427466391932003
```



## 12.5 `\xintBinToHex`

*f* ★ Converts from binary to hexadecimal.

```
\xintBinToHex{1000110101001001110010111110001100110100101001001101010010111000001010001111101111010000101010000001011110010001010011100011111000001011000101111100010000011011000100011100010010011101011101110010101101110010101101010111011000001011101100111000110100100111001011110101001100110010001100110001100110001100000011001010010011011010111111001101111101101100100100011000100000010100110001100011}
->11A9397C66949A97051F7D0A817914E3E0B17C41B11C48BAEF2B5760BB38D272F46DCE46C6032936BF37DAC918814C63
```

## 12.6 `\xintHexToBin`

*f* ★ Converts from hexadecimal to binary.

```
\xintHexToBin{11A9397C66949A97051F7D0A817914E3E0B17C41B11C48BAEF2B5760BB38D272F46DCE46C6032936BF37DAC918814C63}
->10001101010010011100101111100011001101001010010011010100101110000010100011111011111010000101010000001011110010001010011100011111000001011000101111100010000110110001000111000100100010111010111011110010110101011101100000101110110011100011010010011100101111010001101101110011100100011001100000011001010010011011010111111001101111101101011001001000110001000000100110001100011
```

## 12.7 `\xintCHexToBin`

*f* ★ Also converts from hexadecimal to binary. Faster on inputs with at least one hundred hexadecimal digits.

```
\xintCHexToBin{11A9397C66949A97051F7D0A817914E3E0B17C41B11C48BAEF2B5760BB38D272F46DCE46C6032936BF37DAC918814C63}
->1000110101001001110010111110001100110100101001001101010010111000001010001111101111101000010101000000101111001000101001110001111100000101100010111110001000001101100010001110001001000101110101110111100101101010111011000001011101100111000110100100111001011110100011011011100111001000110110001100000011001010010011011010111111001101111101101011001001000110001000000100110001100011
```

## 13 Macros of the *xintgcd* package

|    |                                   |     |    |                                             |     |
|----|-----------------------------------|-----|----|---------------------------------------------|-----|
| .1 | <code>\xintGCD, \xintiiGCD</code> | 136 | .6 | <code>\xintEuclideanAlgorithm</code>        | 136 |
| .2 | <code>\xintGCDof</code>           | 136 | .7 | <code>\xintBezoutAlgorithm</code>           | 137 |
| .3 | <code>\xintLCM, \xintiiLCM</code> | 136 | .8 | <code>\xintTypesetEuclideanAlgorithm</code> | 137 |
| .4 | <code>\xintLCMof</code>           | 136 | .9 | <code>\xintTypesetBezoutAlgorithm</code>    | 137 |
| .5 | <code>\xintBezout</code>          | 136 |    |                                             |     |

This package was included in the original release 1.0 (2013/03/28) of the *xint* bundle.

Since release 1.09a the macros filter their inputs through the `\xintNum` macro, so one can use count registers, or fractions as long as they reduce to integers.

Since release 1.1, the two ```typeset``` macros require the explicit loading by the user of package *xinttools*.

### 13.1 `\xintGCD, \xintiiGCD`

*Num Num*  
*f f* ★ `\xintGCD{N}{M}` computes the greatest common divisor. It is positive, except when both *N* and *M* vanish, in which case the macro returns zero.

```
\xintGCD{10000}{1113}=1
\xintiiGCD{123456789012345}{9876543210321}=3
```

*ff* ★ `\xintiiGCD` skips the `\xintNum` overhead.

### 13.2 `\xintGCDof`

*Num*  
*f → \* f* ★ `\xintGCDof{{a}{b}{c}...}` computes the greatest common divisor of all integers *a*, *b*, ... The list argument may be a macro, it is *f-expanded* first and must contain at least one item.

### 13.3 `\xintLCM, \xintiiLCM`

*Num Num*  
*f f* ★ `\xintGCD{N}{M}` computes the least common multiple. It is 0 if one of the two integers vanishes.  
*ff* ★ `\xintiiLCM` skips the `\xintNum` overhead.

### 13.4 `\xintLCMof`

*Num*  
*f → \* f* ★ `\xintLCMof{{a}{b}{c}...}` computes the least common multiple of all integers *a*, *b*, ... The list argument may be a macro, it is *f-expanded* first and must contain at least one item.

### 13.5 `\xintBezout`

*Num Num*  
*f f* ★ `\xintBezout{N}{M}` returns five numbers *A*, *B*, *U*, *V*, *D* within braces. *A* is the first (expanded, as usual) input number, *B* the second, *D* is the GCD, and *UA - VB = D*.

```
\xintAssign {\xintBezout {10000}{1113}}\to\X
\meaning\X:macro:->\xintBezout {10000}{1113}.
\xintAssign {\xintBezout {10000}{1113}}\to\A\B\U\V\D
\A:10000, \B:1113, \U:-131, \V:-1177, \D:1.
\xintAssign {\xintBezout {123456789012345}{9876543210321}}\to\A\B\U\V\D
\A:123456789012345, \B:9876543210321, \U:256654313730, \V:3208178892607, \D:3.
```

### 13.6 `\xintEuclideanAlgorithm`

*Num Num*  
*f f* ★ `\xintEuclideanAlgorithm{N}{M}` applies the Euclidean algorithm and keeps a copy of all quotients and remainders.

```
\xintAssign {\xintEuclideanAlgorithm {10000}{1113}}\to\X
```

`\meaning\X:macro:->\xintEuclideanAlgorithm {10000}{1113}.`

The first token is the number of steps, the second is *N*, the third is the GCD, the fourth is *M* then the first quotient and remainder, the second quotient and remainder, ... until the final quotient and last (zero) remainder.

### 13.7 `\xintBezoutAlgorithm`

*Num Num*  
*f f* ★ `\xintBezoutAlgorithm{N}{M}` applies the Euclidean algorithm and keeps a copy of all quotients and remainders. Furthermore it computes the entries of the successive products of the 2 by 2 matrices  $\begin{pmatrix} q & 1 \\ 1 & 0 \end{pmatrix}$  formed from the quotients arising in the algorithm.

`\xintAssign {\xintBezoutAlgorithm {10000}{1113}}\to\X`

`\meaning\X:macro:->\xintBezoutAlgorithm {10000}{1113}.`

The first token is the number of steps, the second is *N*, then 0, 1, the GCD, *M*, 1, 0, the first quotient, the first remainder, the top left entry of the first matrix, the bottom left entry, and then these four things at each step until the end.

### 13.8 `\xintTypesetEuclideanAlgorithm`

*Num Num*  
*f f* Requires explicit loading by the user of package *xinttools*.

This macro is just an example of how to organize the data returned by `\xintEuclideanAlgorithm`. Copy the source code to a new macro and modify it to what is needed.

`\xintTypesetEuclideanAlgorithm {123456789012345}{9876543210321}`

123456789012345 = 12 × 9876543210321 + 4938270488493

9876543210321 = 2 × 4938270488493 + 2233335

4938270488493 = 2211164 × 2233335 + 536553

2233335 = 4 × 536553 + 87123

536553 = 6 × 87123 + 13815

87123 = 6 × 13815 + 4233

13815 = 3 × 4233 + 1116

4233 = 3 × 1116 + 885

1116 = 1 × 885 + 231

885 = 3 × 231 + 192

231 = 1 × 192 + 39

192 = 4 × 39 + 36

39 = 1 × 36 + 3

36 = 12 × 3 + 0

### 13.9 `\xintTypesetBezoutAlgorithm`

*Num Num*  
*f f* Requires explicit loading by the user of package *xinttools*.

This macro is just an example of how to organize the data returned by `\xintBezoutAlgorithm`. Copy the source code to a new macro and modify it to what is needed.

`\xintTypesetBezoutAlgorithm {10000}{1113}`

10000 = 8 × 1113 + 1096

8 = 8 × 1 + 0

1 = 8 × 0 + 1

1113 = 1 × 1096 + 17

9 = 1 × 8 + 1

1 = 1 × 1 + 0

1096 = 64 × 17 + 8

584 = 64 × 9 + 8

65 = 64 × 1 + 1

### 13 Macros of the *xintgcd* package

```
17 = 2 × 8 + 1
1177 = 2 × 584 + 9
131 = 2 × 65 + 1
8 = 8 × 1 + 0
10000 = 8 × 1177 + 584
1113 = 8 × 131 + 65
131 × 10000 - 1177 × 1113 = -1
```

## 14 Macros of the *xintseries* package

|    |                                         |     |     |                                           |     |
|----|-----------------------------------------|-----|-----|-------------------------------------------|-----|
| .1 | <code>\xintSeries</code> .....          | 139 | .7  | <code>\xintFxFtPowerSeries</code> .....   | 147 |
| .2 | <code>\xintiSeries</code> .....         | 140 | .8  | <code>\xintFxFtPowerSeriesX</code> .....  | 148 |
| .3 | <code>\xintRationalSeries</code> .....  | 141 | .9  | <code>\xintFloatPowerSeries</code> .....  | 149 |
| .4 | <code>\xintRationalSeriesX</code> ..... | 144 | .10 | <code>\xintFloatPowerSeriesX</code> ..... | 149 |
| .5 | <code>\xintPowerSeries</code> .....     | 145 | .11 | Computing $\log 2$ and $\pi$ .....        | 150 |
| .6 | <code>\xintPowerSeriesX</code> .....    | 147 |     |                                           |     |

This package was first released with version 1.03 (2013/04/14) of the *xint* bundle.

The  $\overset{\text{Frac}}{f}$  expansion type of various macro arguments is only a  $\overset{\text{Num}}{f}$  if only *xint* but not *xintfrac* is loaded. The macro `\xintiSeries` is special and expects summing big integers obeying the strict format, even if *xintfrac* is loaded.

The arguments serving as indices are of the  $\overset{\text{num}}{x}$  expansion type.

In some cases one or two of the macro arguments are only expanded at a later stage not immediately.

### 14.1 `\xintSeries`

$\overset{\text{num}}{x} \overset{\text{num}}{x} \overset{\text{Frac}}{f} \star$  `\xintSeries{A}{B}{\coeff}` computes  $\sum_{n=A}^{n=B} \text{"coeff-n"}$ . The initial and final indices must obey the `\numexpr` constraint of expanding to numbers at most  $2^{31}-1$ . The `\coeff` macro must be a one-parameter *f-expandable* macro, taking on input an explicit number *n* and producing some number or fraction `\coeff{n}`; it is expanded at the time it is needed.<sup>79</sup>

```
\def\coeff #1{\xintiiMON{#1}/#1.5} % (-1)^n/(n+1/2)
\def\w {\xintSeries {0}{50}{\coeff}} % we want to re-use it
\def\z {\xintJrr {\w}[0]} % the [0] for a microsecond gain.
% \xintJrr preferred to \xintIrr: a big common factor is suspected.
% But numbers much bigger would be needed to show the greater efficiency.
\[\sum_{n=0}^{n=50} \frac{(-1)^n}{n+\frac{1}{2}} = \xintFrac{\z}{\w}
```

$$\sum_{n=0}^{n=50} \frac{(-1)^n}{n + \frac{1}{2}} = \frac{173909338287370940432112792101626602278714}{110027467159390003025279917226039729050575}$$

The definition of `\coeff` as `\xintiiMON{#1}/#1.5` is quite suboptimal. It allows *#1* to be a big integer, but anyhow only small integers are accepted as initial and final indices (they are of the  $\overset{\text{num}}{x}$  type). Second, when the *xintfrac* parser sees the *#1.5* it will remove the dot hence create a denominator with one digit more. For example  $1/3.5$  turns internally into  $10/35$  whereas it would be more efficient to have  $2/7$ . For info here is the non-reduced `\w`:

$$\frac{24489212733740439818553118189578822128979076445102691650390625}{154936248757874299375548246172975814272155426442623138427734375} 10^1$$

It would have been bigger still in releases earlier than 1.1: now, the *xintfrac* `\xintAdd` routine does not multiply blindly denominators anymore, it checks if one is a multiple of the other. However it does not practice systematic reduction to lowest terms.

A more efficient way to code `\coeff` is illustrated next.

```
\def\coeff #1{\the\numexpr\ifodd #1 -2\else2\fi\relax/\the\numexpr 2*#1+1\relax [0]}%
% The [0] in \coeff is a tiny optimization: in its presence the \xintfracname parser
% sees something which is already in internal format.
\def\w {\xintSeries {0}{50}{\coeff}}
\[\sum_{n=0}^{n=50} \frac{(-1)^n}{n+\frac{1}{2}} = \xintFrac{\w}{\w}
```

<sup>79</sup> `\xintiiMON` is like `\xintMON` but does not parse its argument through `\xintNum`, for efficiency; other macros of this type are `\xintiiAdd`, `\xintiiMul`, `\xintiiSum`, `\xintiiPrd`, `\xintiiMMON`, `\xintiiLDg`, `\xintiiFDg`, `\xintiiOdd`, ...

$$\sum_{n=0}^{n=50} \frac{(-1)^n}{n + \frac{1}{2}} = \frac{164344324681565538708346588536037139153384730}{103975956465623552858889521778607543952793375}$$

The reduced form `\z` as displayed above only differs from this one by a factor of 945.

```
\def\coeffleibnitz #1{\the\numexpr\ifodd #1 1\else-1\fi\relax/#1[0]}
\cnta 1
\loop % in this loop we recompute from scratch each partial sum!
% we can afford that, as \xintSeries is fast enough.
\noindent\hbox to 2em{\hfil\texttt{\the\cnta.} }%
\xintTrunc {12}{\xintSeries {1}{\cnta}{\coeffleibnitz}}\dots
\endgraf
\ifnum\cnta < 30 \advance\cnta 1 \repeat
```

|                       |                       |                       |
|-----------------------|-----------------------|-----------------------|
| 1. 1.000000000000...  | 11. 0.736544011544... | 21. 0.716390450794... |
| 2. 0.500000000000...  | 12. 0.653210678210... | 22. 0.670935905339... |
| 3. 0.833333333333...  | 13. 0.730133755133... | 23. 0.714414166209... |
| 4. 0.583333333333...  | 14. 0.658705183705... | 24. 0.672747499542... |
| 5. 0.783333333333...  | 15. 0.725371850371... | 25. 0.712747499542... |
| 6. 0.616666666666...  | 16. 0.662871850371... | 26. 0.674285961081... |
| 7. 0.759523809523...  | 17. 0.721695379783... | 27. 0.711322998118... |
| 8. 0.634523809523...  | 18. 0.666139824228... | 28. 0.675608712404... |
| 9. 0.745634920634...  | 19. 0.718771403175... | 29. 0.710091471024... |
| 10. 0.645634920634... | 20. 0.668771403175... | 30. 0.676758137691... |

## 14.2 \xintiSeries

num num  
X X f ★

`\xintiSeries{A}{B}{\coeff}` computes  $\sum_{n=A}^{n=B}$  “coeff-*n*” where `\coeff{n}` must *f-expand* to a (possibly long) integer in the strict format.

```
\def\coeff #1{\xintiTrunc {40}{\xintMON{#1}/#1.5}}%
% better:
\def\coeff #1{\xintiTrunc {40}
{\the\numexpr 2*\xintiiMON{#1}\relax/\the\numexpr 2*#1+1\relax [0]}}%
% better still:
\def\coeff #1{\xintiTrunc {40}
{\the\numexpr\ifodd #1 -2\else2\fi\relax/\the\numexpr 2*#1+1\relax [0]}}%
% (-1)^n/(n+1/2) times 10^40, truncated to an integer.
\[\sum_{n=0}^{n=50} \frac{(-1)^n}{n+\frac{1}{2}} \approx
\xintTrunc {40}{\xintiSeries {0}{50}{\coeff}[-40]}\dots\]
```

$$\sum_{n=0}^{n=50} \frac{(-1)^n}{n + \frac{1}{2}} \approx 1.5805993064935250412367895069567264144810$$

We should have cut out at least the last two digits: truncating errors originating with the first coefficients of the sum will never go away, and each truncation introduces an uncertainty in the last digit, so as we have 40 terms, we should trash the last two digits, or at least round at 38 digits. It is interesting to compare with the computation where rounding rather than truncation is used, and with the decimal expansion of the exactly computed partial sum of the series:

```
\def\coeff #1{\xintiRound {40} % rounding at 40
{\the\numexpr\ifodd #1 -2\else2\fi\relax/\the\numexpr 2*#1+1\relax [0]}}%
% (-1)^n/(n+1/2) times 10^40, rounded to an integer.
\[\sum_{n=0}^{n=50} \frac{(-1)^n}{n+\frac{1}{2}} \approx
\xintTrunc {40}{\xintiSeries {0}{50}{\coeff}[-40]}\dots\]
\def\exactcoeff #1%
{\the\numexpr\ifodd #1 -2\else2\fi\relax/\the\numexpr 2*#1+1\relax [0]}}%
\[\sum_{n=0}^{n=50} \frac{(-1)^n}{n+\frac{1}{2}}
= \xintTrunc {50}{\xintSeries {0}{50}{\exactcoeff}}\dots\]
```

$$\sum_{n=0}^{n=50} \frac{(-1)^n}{n + \frac{1}{2}} \approx 1.5805993064935250412367895069567264144804$$

$$\sum_{n=0}^{n=50} \frac{(-1)^n}{n + \frac{1}{2}} = 1.58059930649352504123678950695672641448068680288367 \dots$$

This shows indeed that our sum of truncated terms estimated wrongly the 39th and 40th digits of the exact result<sup>80</sup> and that the sum of rounded terms fared a bit better.

### 14.3 `\xintRationalSeries`

`\xintRationalSeries{A}{B}{f}{ratio}` evaluates  $\sum_{n=A}^{n=B} F(n)$ , where  $F(n)$  is specified indirectly via the data of  $f=F(A)$  and the one-parameter macro `\ratio` which must be such that `\macro{n}` expands to  $F(n)/F(n-1)$ . The name indicates that `\xintRationalSeries` was designed to be useful in the cases where  $F(n)/F(n-1)$  is a rational function of  $n$  but it may be anything expanding to a fraction. The macro `\ratio` must be an expandable-only compatible macro and expand to its value after iterated full expansion of its first token.  $A$  and  $B$  are fed to a `\numexpr` hence may be count registers or arithmetic expressions built with such; they must obey the TeX bound. The initial term  $f$  may be a macro `\f`, it will be expanded to its value representing  $F(A)$ .

```
\def\ratio #1{2/#1[0]}% 2/n, to compute exp(2)
\cnta 0 % previously declared count
\begin{quote}
\loop \fdef\z {\xintRationalSeries {0}{\cnta}{1}{ratio }}%
\noindent$\sum_{n=0}^{\the\cnta} \frac{2^n}{n!}=
\xintTrunc{12}\z\dots=
\xintFrac\z=\xintFrac{\xintIrr\z}\vtop to 5pt{}\par
\ifnum\cnta<20 \advance\cnta 1 \repeat
\end{quote}
```

$$\sum_{n=0}^0 \frac{2^n}{n!} = 1.000000000000 \dots = 1 = 1$$

$$\sum_{n=0}^1 \frac{2^n}{n!} = 3.000000000000 \dots = 3 = 3$$

$$\sum_{n=0}^2 \frac{2^n}{n!} = 5.000000000000 \dots = \frac{10}{2} = 5$$

$$\sum_{n=0}^3 \frac{2^n}{n!} = 6.333333333333 \dots = \frac{38}{6} = \frac{19}{3}$$

$$\sum_{n=0}^4 \frac{2^n}{n!} = 7.000000000000 \dots = \frac{168}{24} = 7$$

$$\sum_{n=0}^5 \frac{2^n}{n!} = 7.266666666666 \dots = \frac{872}{120} = \frac{109}{15}$$

$$\sum_{n=0}^6 \frac{2^n}{n!} = 7.355555555555 \dots = \frac{5296}{720} = \frac{331}{45}$$

$$\sum_{n=0}^7 \frac{2^n}{n!} = 7.380952380952 \dots = \frac{37200}{5040} = \frac{155}{21}$$

$$\sum_{n=0}^8 \frac{2^n}{n!} = 7.387301587301 \dots = \frac{297856}{40320} = \frac{2327}{315}$$

$$\sum_{n=0}^9 \frac{2^n}{n!} = 7.388712522045 \dots = \frac{2681216}{362880} = \frac{20947}{2835}$$

$$\sum_{n=0}^{10} \frac{2^n}{n!} = 7.388994708994 \dots = \frac{26813184}{3628800} = \frac{34913}{4725}$$

$$\sum_{n=0}^{11} \frac{2^n}{n!} = 7.389046015712 \dots = \frac{294947072}{39916800} = \frac{164591}{22275}$$

$$\sum_{n=0}^{12} \frac{2^n}{n!} = 7.389054566832 \dots = \frac{3539368960}{479001600} = \frac{691283}{93555}$$

$$\sum_{n=0}^{13} \frac{2^n}{n!} = 7.389055882389 \dots = \frac{46011804672}{6227020800} = \frac{14977801}{2027025}$$

<sup>80</sup> as the series is alternating, we can roughly expect an error of  $\sqrt{40}$  and the last two digits are off by 4 units, which is not contradictory to our expectations.



$$\begin{aligned}\sum_{n=0}^{14} \frac{2^n}{n!} &= 7.389056070325 \dots = \frac{644165281792}{87178291200} = \frac{314533829}{42567525} \\ \sum_{n=0}^{15} \frac{2^n}{n!} &= 7.389056095384 \dots = \frac{9662479259648}{1307674368000} = \frac{4718007451}{638512875} \\ \sum_{n=0}^{16} \frac{2^n}{n!} &= 7.389056098516 \dots = \frac{154599668219904}{20922789888000} = \frac{1572669151}{212837625} \\ \sum_{n=0}^{17} \frac{2^n}{n!} &= 7.389056098884 \dots = \frac{2628194359869440}{355687428096000} = \frac{16041225341}{2170943775} \\ \sum_{n=0}^{18} \frac{2^n}{n!} &= 7.389056098925 \dots = \frac{47307498477912064}{6402373705728000} = \frac{103122162907}{13956067125} \\ \sum_{n=0}^{19} \frac{2^n}{n!} &= 7.389056098930 \dots = \frac{898842471080853504}{121645100408832000} = \frac{4571749222213}{618718975875} \\ \sum_{n=0}^{20} \frac{2^n}{n!} &= 7.389056098930 \dots = \frac{17976849421618118656}{2432902008176640000} = \frac{68576238333199}{9280784638125}\end{aligned}$$

```
\def\ratio #1{-1/#1[0]}% -1/n, comes from the series of exp(-1)
\cnta 0 % previously declared count
\begin{quote}
\loop
\fddefz {\xintRationalSeries {0}{\cnta}{1}{\ratio }}%
\noindent$\sum_{n=0}^{\the\cnta} \frac{(-1)^n}{n!}=
\xintTrunc{20}\z\dotso=\xintFrac{\z}=\xintFrac{\xintIrr\z}$%
\vtop to 5pt{}\par
\ifnum\cnta<20 \advance\cnta 1 \repeat
\end{quote}
```

$$\begin{aligned}\sum_{n=0}^0 \frac{(-1)^n}{n!} &= 1.00000000000000000000 \dots = 1 = 1 \\ \sum_{n=0}^1 \frac{(-1)^n}{n!} &= 0 \dots = 0 = 0 \\ \sum_{n=0}^2 \frac{(-1)^n}{n!} &= 0.50000000000000000000 \dots = \frac{1}{2} = \frac{1}{2} \\ \sum_{n=0}^3 \frac{(-1)^n}{n!} &= 0.33333333333333333333 \dots = \frac{2}{6} = \frac{1}{3} \\ \sum_{n=0}^4 \frac{(-1)^n}{n!} &= 0.37500000000000000000 \dots = \frac{9}{24} = \frac{3}{8} \\ \sum_{n=0}^5 \frac{(-1)^n}{n!} &= 0.36666666666666666666 \dots = \frac{44}{120} = \frac{11}{30} \\ \sum_{n=0}^6 \frac{(-1)^n}{n!} &= 0.36805555555555555555 \dots = \frac{265}{720} = \frac{53}{144} \\ \sum_{n=0}^7 \frac{(-1)^n}{n!} &= 0.36785714285714285714 \dots = \frac{1854}{5040} = \frac{103}{280} \\ \sum_{n=0}^8 \frac{(-1)^n}{n!} &= 0.36788194444444444444 \dots = \frac{14833}{40320} = \frac{2119}{5760} \\ \sum_{n=0}^9 \frac{(-1)^n}{n!} &= 0.36787918871252204585 \dots = \frac{133496}{362880} = \frac{16687}{45360} \\ \sum_{n=0}^{10} \frac{(-1)^n}{n!} &= 0.36787946428571428571 \dots = \frac{1334961}{3628800} = \frac{16481}{44800} \\ \sum_{n=0}^{11} \frac{(-1)^n}{n!} &= 0.36787943923360590027 \dots = \frac{14684570}{39916800} = \frac{1468457}{3991680} \\ \sum_{n=0}^{12} \frac{(-1)^n}{n!} &= 0.36787944132128159905 \dots = \frac{176214841}{479001600} = \frac{16019531}{43545600} \\ \sum_{n=0}^{13} \frac{(-1)^n}{n!} &= 0.36787944116069116069 \dots = \frac{2290792932}{6227020800} = \frac{63633137}{172972800} \\ \sum_{n=0}^{14} \frac{(-1)^n}{n!} &= 0.36787944117216190628 \dots = \frac{32071101049}{87178291200} = \frac{2467007773}{6706022400} \\ \sum_{n=0}^{15} \frac{(-1)^n}{n!} &= 0.36787944117139718991 \dots = \frac{481066515734}{1307674368000} = \frac{34361893981}{93405312000} \\ \sum_{n=0}^{16} \frac{(-1)^n}{n!} &= 0.36787944117144498468 \dots = \frac{7697064251745}{20922789888000} = \frac{15549624751}{42268262400} \\ \sum_{n=0}^{17} \frac{(-1)^n}{n!} &= 0.36787944117144217323 \dots = \frac{130850092279664}{355687428096000} = \frac{8178130767479}{22230464256000} \\ \sum_{n=0}^{18} \frac{(-1)^n}{n!} &= 0.36787944117144232942 \dots = \frac{2355301661033953}{6402373705728000} = \frac{138547156531409}{376610217984000}\end{aligned}$$

$$\sum_{n=0}^{19} \frac{(-1)^n}{n!} = 0.36787944117144232120\dots = \frac{44750731559645106}{121645100408832000} = \frac{92079694567171}{250298560512000}$$

$$\sum_{n=0}^{20} \frac{(-1)^n}{n!} = 0.36787944117144232161\dots = \frac{895014631192902121}{2432902008176640000} = \frac{4282366656425369}{11640679464960000}$$

We can incorporate an indeterminate if we define `\ratio` to be a macro with two parameters: `\def\ratioexp #1#2{\xintDiv{#1}{#2}}%` `x/n: x=#1, n=#2`. Then, if `\x` expands to some fraction `x`, the macro

```
\xintRationalSeries {0}{b}{1}{\ratioexp{\x}}
```

will compute  $\sum_{n=0}^{b} x^n/n!$ :

```
\cnta 0
\def\ratioexp #1#2{\xintDiv{#1}{#2}}% #1/#2
\loop
\noindent
$\sum_{n=0}^{\the\cnta} (.57)^n/n! = \xintTrunc {50}
{\xintRationalSeries {0}{\cnta}{1}{\ratioexp{.57}}}\dots$
\vtop to 5pt {} \endgraf
\ifnum\cnta<50 \advance\cnta 10 \repeat
```

$$\sum_{n=0}^0 (.57)^n/n! = 1.0000000000000000000000000000000000000000000000000000000\dots$$

$$\sum_{n=0}^{10} (.57)^n/n! = 1.76826705137947002480668058035714285714285714285714\dots$$

$$\sum_{n=0}^{20} (.57)^n/n! = 1.76826705143373515162089324271187082272833005529082\dots$$

$$\sum_{n=0}^{30} (.57)^n/n! = 1.76826705143373515162089339282382144915484884979430\dots$$

$$\sum_{n=0}^{40} (.57)^n/n! = 1.76826705143373515162089339282382144915485219867776\dots$$

$$\sum_{n=0}^{50} (.57)^n/n! = 1.76826705143373515162089339282382144915485219867776\dots$$

Observe that in this last example the `x` was directly inserted; if it had been a more complicated explicit fraction it would have been worthwhile to use `\ratioexp\x` with `\x` defined to expand to its value. In the further situation where this fraction `x` is not explicit but itself defined via a complicated, and time-costly, formula, it should be noted that `\xintRationalSeries` will do again the evaluation of `\x` for each term of the partial sum. The easiest is thus when `x` can be defined as an `\edef`. If however, you are in an expandable-only context and cannot store in a macro like `\x` the value to be used, a variant of `\xintRationalSeries` is needed which will first evaluate this `\x` and then use this result without recomputing it. This is `\xintRationalSeriesX`, documented next.

Here is a slightly more complicated evaluation:

```
\cnta 1
\begin{multicols}{2}
\loop \fdef\z {\xintRationalSeries
{\cnta}
{2*\cnta-1}
{\xintiPow {\the\cnta}{\cnta}/\xintiiFac{\cnta}}
{\ratioexp{\the\cnta}}}%
\fdef\w {\xintRationalSeries {0}{2*\cnta-1}{1}{\ratioexp{\the\cnta}}}%
\noindent
$\sum_{n=\the\cnta}^{\the\cnta} {\the\cnta}^{2*\cnta-1\relax} \frac{\the\cnta^n}{n!} /
\sum_{n=0}^{\the\cnta} {\the\cnta}^{2*\cnta-1\relax} \frac{\the\cnta^n}{n!} =
\xintTrunc{8}{\xintDiv\z\w}\dots$ \vtop to 5pt {} \endgraf
\ifnum\cnta<20 \advance\cnta 1 \repeat
\end{multicols}
```

$$\sum_{n=1}^1 \frac{1^n}{n!} / \sum_{n=0}^1 \frac{1^n}{n!} = 0.50000000\dots$$

$$\sum_{n=2}^3 \frac{2^n}{n!} / \sum_{n=0}^3 \frac{2^n}{n!} = 0.52631578\dots$$

$$\sum_{n=3}^5 \frac{3^n}{n!} / \sum_{n=0}^5 \frac{3^n}{n!} = 0.53804347\dots$$

$$\sum_{n=4}^7 \frac{4^n}{n!} / \sum_{n=0}^7 \frac{4^n}{n!} = 0.54317053\dots$$

$$\sum_{n=5}^9 \frac{5^n}{n!} / \sum_{n=0}^9 \frac{5^n}{n!} = 0.54502576\dots$$

$$\sum_{n=6}^{11} \frac{6^n}{n!} / \sum_{n=0}^{11} \frac{6^n}{n!} = 0.54518217\dots$$

$$\sum_{n=7}^{13} \frac{7^n}{n!} / \sum_{n=0}^{13} \frac{7^n}{n!} = 0.54445274\dots$$

$$\sum_{n=8}^{15} \frac{8^n}{n!} / \sum_{n=0}^{15} \frac{8^n}{n!} = 0.54327992\dots$$

$$\sum_{n=9}^{17} \frac{9^n}{n!} / \sum_{n=0}^{17} \frac{9^n}{n!} = 0.54191055\dots$$

$$\sum_{n=10}^{19} \frac{10^n}{n!} / \sum_{n=0}^{19} \frac{10^n}{n!} = 0.54048295\dots$$

$$\begin{aligned}\sum_{n=11}^{21} \frac{11^n}{n!} / \sum_{n=0}^{21} \frac{11^n}{n!} &= 0.53907332\dots \\ \sum_{n=12}^{23} \frac{12^n}{n!} / \sum_{n=0}^{23} \frac{12^n}{n!} &= 0.53772178\dots \\ \sum_{n=13}^{25} \frac{13^n}{n!} / \sum_{n=0}^{25} \frac{13^n}{n!} &= 0.53644744\dots \\ \sum_{n=14}^{27} \frac{14^n}{n!} / \sum_{n=0}^{27} \frac{14^n}{n!} &= 0.53525726\dots \\ \sum_{n=15}^{29} \frac{15^n}{n!} / \sum_{n=0}^{29} \frac{15^n}{n!} &= 0.53415135\dots\end{aligned}$$

$$\begin{aligned}\sum_{n=16}^{31} \frac{16^n}{n!} / \sum_{n=0}^{31} \frac{16^n}{n!} &= 0.53312615\dots \\ \sum_{n=17}^{33} \frac{17^n}{n!} / \sum_{n=0}^{33} \frac{17^n}{n!} &= 0.53217628\dots \\ \sum_{n=18}^{35} \frac{18^n}{n!} / \sum_{n=0}^{35} \frac{18^n}{n!} &= 0.53129566\dots \\ \sum_{n=19}^{37} \frac{19^n}{n!} / \sum_{n=0}^{37} \frac{19^n}{n!} &= 0.53047810\dots \\ \sum_{n=20}^{39} \frac{20^n}{n!} / \sum_{n=0}^{39} \frac{20^n}{n!} &= 0.52971771\dots\end{aligned}$$

## 14.4 `\xintRationalSeriesX`

um  
X f f f ★

`\xintRationalSeriesX{A}{B}{\first}{\ratio}{\g}` is a parametrized version of `\xintRationalSeries` where `\first` is now a one-parameter macro such that `\first{\g}` gives the initial term and `\ratio` is a two-parameter macro such that `\ratio{n}{\g}` represents the ratio of one term to the previous one. The parameter `\g` is evaluated only once at the beginning of the computation, and can thus itself be the yet unevaluated result of a previous computation.

Let `\ratio` be such a two-parameter macro; note the subtle differences between

```
\xintRationalSeries {A}{B}{\first}{\ratio{\g}}
and \xintRationalSeriesX {A}{B}{\first}{\ratio}{\g}.
```

First the location of braces differ... then, in the former case `\first` is a *no-parameter* macro expanding to a fractional number, and in the latter, it is a *one-parameter* macro which will use `\g`. Furthermore the *X* variant will expand `\g` at the very beginning whereas the former non-*X* former variant will evaluate it each time it needs it (which is bad if this evaluation is time-costly, but good if `\g` is a big explicit fraction encapsulated in a macro).

The example will use the macro `\xintPowerSeries` which computes efficiently exact partial sums of power series, and is discussed in the next section.

```
\def\firstterm #1{1[0]}% first term of the exponential series
% although it is the constant 1, here it must be defined as a
% one-parameter macro. Next comes the ratio function for exp:
\def\ratioexp #1#2{\xintDiv {#1}{#2}}% x/n
% These are the (-1)^{n-1}/n of the log(1+h) series:
\def\coefflog #1{\the\numexpr\ifodd #1 1\else-1\fi\relax/#1[0]}%
% Let L(h) be the first 10 terms of the log(1+h) series and
% let E(t) be the first 10 terms of the exp(t) series.
% The following computes E(L(a/10)) for a=1,...,12.
\begin{multicols}{3}\raggedcolumns
\cnta 0
\loop
\noindent\xintTrunc {18}{%
\xintRationalSeriesX {0}{9}{\firstterm}{\ratioexp}
{\xintPowerSeries{1}{10}{\coefflog}{\the\cnta[-1]}}}\dots
\endgraf
\ifnum\cnta < 12 \advance \cnta 1 \repeat
\end{multicols}
```

|                           |                         |                         |
|---------------------------|-------------------------|-------------------------|
| 1.00000000000000000000... | 1.499954310225476533... | 1.907197560339468199... |
| 1.099999999999083906...   | 1.599659266069210466... | 1.845117565491393752... |
| 1.199999998111624029...   | 1.698137473697423757... | 1.593831932293536053... |
| 1.299999835744121464...   | 1.791898112718884531... |                         |
| 1.399996091955359088...   | 1.870485649686617459... |                         |

These completely exact operations rapidly create numbers with many digits. Let us print in full the raw fractions created by the operation illustrated above:

```
E(L(1[-1]))=837588191708763593233567174687168686839043767250040308005177927752494629866184)
007635859046126459421356378683685505198743151968256/761443810644964678986073374720000000000[
-90] (length of numerator: 129)
```

```
E(L(12[-2]))=852817067917170404743367455694175958595958834849647782275488039683039028140142
627504107752450752216393977420945142612214186817956418552109426998619906107437170238454024542
66764149586292947053094306437899694978421745516544/76144381064496467898607337472000000000[-2
180] (length of numerator: 219)
```

```
E(L(123[-3]))=85510139934748417382331557840332916647230257508795878817033811296662463224972
19142331115922733076372551695958545295492787962370297931258555334838173016993380276766882172
119374626466595481228434620385193134459556100336250033905803253996685925486649200488748729352
7193753147231854444914269381901852177272347295744/76144381064496467898607337472000000000[-22
70] (length of numerator: 309)
```

We see that the denominators here remain the same, as our input only had various powers of ten as denominators, and *xintfrac* efficiently assemble (some only, as we can see) powers of ten. Notice that 1 more digit in an input denominator seems to mean 90 more in the raw output. We can check that with some other test cases:

```
E(L(1/7))=246928403777284535542005925363994435551841707513610888309477702059412077893312772
8456047080539072743831005696/2160623533139083201661118734213279886660128668842038173763451722
79206429151601320351772754579554304000000000[0] (length of numerator: 108; length of denomina-
tor: 108)
```

```
E(L(1/71))=31693003114420993773601541095216519886197494696368120129960391285745860214771192
416176332877917435184310890684584136895442264511996499581982071012712604174613973484964435392
24405185605375986452422252984559104/312528225156095910823136583628983100903663468032171880552
179095599208106488310703560230149547919271401955564834180272077013758540888195207115933497192
437205503937460406242219012273841825343258755072000000000[0] (length of numerator: 206; length
of denominator: 206)
```

```
E(L(1/712))=3003564353778406020559670408411885925389093114199308387996560136260710297841742
496819290884958041362038132421744055614153154268292413172870530372734533290558141538915173252
756941123200263645694953665349180314390511046104875297961920582057259996416578066159049290482
98946463533146662233869249/299935178105220909766968489591763105361775507557039697364359215352
224604108923285325397380419112021214124247158817340492547166400824709873409851519325042814942
240645967888744414705331478482078635497788470006171032646666387826770190191301139308374215312
81047806202596610291401752576000000000[0] (length of numerator: 288; length of denominator:
288)
```

Thus decimal numbers such as `0.123` (equivalently `123[-3]`) give less computing intensive tasks than fractions such as `1/712`: in the case of decimal numbers the (raw) denominators originate in the coefficients of the series themselves, powers of ten of the input within brackets being treated separately. And even then the numerators will grow with the size of the input in a sort of linear way, the coefficient being given by the order of series: here 10 from the log and 9 from the exp, so 90. One more digit in the input means 90 more digits in the numerator of the output: obviously we can not go on composing such partial sums of series and hope that *xint* will joyfully do all at the speed of light!

Hence, truncating the output (or better, rounding) is the only way to go if one needs a general calculus of special functions. This is why the package *xintseries* provides, besides `\xintSeries`, `\xintRationalSeries`, or `\xintPowerSeries` which compute exact sums, `\xintFxFtPowerSeries` for fixed-point computations and a (tentative naive) `\xintFloatPowerSeries`.

## 14.5 `\xintPowerSeries`

$\sum_{n=A}^B \frac{\text{num}}{X} \frac{\text{Frac}}{f} \frac{\text{Frac}}{f} \star$  `\xintPowerSeries{A}{B}{\coeff}{f}` evaluates the sum  $\sum_{n=A}^{n=B} \text{"coeff-n"} \cdot f^n$ . The initial and final indices are given to a `\numexpr` expression. The `\coeff` macro (which, as argument to `\xintPowerSeries` is expanded only at the time `\coeff{n}` is needed) should be defined as a one-parameter expandable macro, its input will be an explicit number.

The `f` can be either a fraction directly input or a macro `\f` expanding to such a fraction. It is actually more efficient to encapsulate an explicit fraction `f` in such a macro, if it has big

numerators and denominators ('big' means hundreds of digits) as it will then take less space in the processing until being (repeatedly) used.

This macro computes the exact result (one can use it also for polynomial evaluation), using a Horner scheme which helps avoiding a denominator build-up (this problem however, even if using a naive additive approach, is much less acute since release 1.1 and its new policy regarding `\xint-Add`).

```
\def\geom #1{1[0]} % the geometric series
\def\ f {5/17[0]}
\[\sum_{n=0}^{n=20} \Bigl(\frac{5}{17}\Bigr)^n
=\xintFrac{\xintIrr{\xintPowerSeries {0}{20}{\geom}{\f}}}
=\xintFrac{\xinttheexpr (17^21-5^21)/12/17^20\relax}\]
```

$$\sum_{n=0}^{n=20} \left(\frac{5}{17}\right)^n = \frac{5757661159377657976885341}{4064231406647572522401601} = \frac{69091933912531895722624092}{48770776879770870268819212}$$

```
\def\coefflog #1{1/#1[0]}% 1/n
\def\ f {1/2[0]}%
\[\log 2 \approx \sum_{n=1}^{n=20} \frac{1}{n \cdot 2^n}
=\xintFrac {\xintIrr {\xintPowerSeries {1}{20}{\coefflog}{\f}}}\]
\[\log 2 \approx \sum_{n=1}^{n=50} \frac{1}{n \cdot 2^n}
=\xintFrac {\xintIrr {\xintPowerSeries {1}{50}{\coefflog}{\f}}}\]
```

$$\log 2 \approx \sum_{n=1}^{20} \frac{1}{n \cdot 2^n} = \frac{42299423848079}{61025172848640}$$

$$\log 2 \approx \sum_{n=1}^{50} \frac{1}{n \cdot 2^n} = \frac{60463469751752265663579884559739219}{87230347965792839223946208178339840}$$

```
\setlength{\columnsep}{0pt}
\begin{multicols}{3}
\cnta 1 % previously declared count
\loop % in this loop we recompute from scratch each partial sum!
% we can afford that, as \xintPowerSeries is fast enough.
\noindent\hbox to 2em{\hfil\texttt{\the\cnta.} }%
\xintTrunc {12}
{\xintPowerSeries {1}{\cnta}{\coefflog}{\f}}\dots
\endgraf
\ifnum \cnta < 30 \advance\cnta 1 \repeat
\end{multicols}
```

|                       |                       |                       |
|-----------------------|-----------------------|-----------------------|
| 1. 0.500000000000...  | 11. 0.693109245355... | 21. 0.693147159757... |
| 2. 0.625000000000...  | 12. 0.693129590407... | 22. 0.693147170594... |
| 3. 0.666666666666...  | 13. 0.693138980431... | 23. 0.693147175777... |
| 4. 0.682291666666...  | 14. 0.693143340085... | 24. 0.693147178261... |
| 5. 0.688541666666...  | 15. 0.693145374590... | 25. 0.693147179453... |
| 6. 0.691145833333...  | 16. 0.693146328265... | 26. 0.693147180026... |
| 7. 0.692261904761...  | 17. 0.693146777052... | 27. 0.693147180302... |
| 8. 0.692750186011...  | 18. 0.693146988980... | 28. 0.693147180435... |
| 9. 0.692967199900...  | 19. 0.693147089367... | 29. 0.693147180499... |
| 10. 0.693064856150... | 20. 0.693147137051... | 30. 0.693147180530... |

```
\def\coeffarctg #1{1/\the\numexpr\ifodd #1 -2*#1-1\else2*#1+1\fi\relax}%
% the above gives (-1)^n/(2n+1). The sign being in the denominator,
% **** no [0] should be added ****,
% else nothing is guaranteed to work (even if it could by sheer luck)
% Notice in passing this aspect of \numexpr:
```

```
% **** \numexpr -(1)\relax is illegal !!! ****
\def\frac{1/25[0]}{1/5^2}
\[\mathrm{Arctg}(\frac{1}{5})\approx \frac{1}{5}\sum_{n=0}^{15} \frac{(-1)^n}{(2n+1)25^n} = \frac{165918726519122955895391793269168}{840539304153062403202056884765625}
```

$$\mathrm{Arctg}\left(\frac{1}{5}\right) \approx \frac{1}{5} \sum_{n=0}^{15} \frac{(-1)^n}{(2n+1)25^n} = \frac{165918726519122955895391793269168}{840539304153062403202056884765625}$$

## 14.6 \xintPowerSeriesX

This is the same as `\xintPowerSeries` apart from the fact that the last parameter `f` is expanded once and for all before being then used repeatedly. If the `f` parameter is to be an explicit big fraction with many (dozens) digits, rather than using it directly it is slightly better to have some macro `\g` defined to expand to the explicit fraction and then use `\xintPowerSeries` with `\g`; but if `f` has not yet been evaluated and will be the output of a complicated expansion of some `\f`, and if, due to an expanding only context, doing `\edef\g{\f}` is no option, then `\xintPowerSeriesX` should be used with `\f` as last parameter.

```
\def\ratioexp #1#2{\xintDiv {#1}{#2}}% x/n
% These are the (-1)^{n-1}/n of the log(1+h) series:
\def\coefflog #1{\the\numexpr\ifodd #1 1\else-1\fi\relax/#1[0]}%
% Let L(h) be the first 10 terms of the log(1+h) series and
% let E(t) be the first 10 terms of the exp(t) series.
% The following computes L(E(a/10)-1) for a=1,..., 12.
\begin{multicols}{3}\raggedcolumns
\cnta 1
\loop
\noindent\xintTrunc {18}{%
 \xintPowerSeriesX {1}{10}{\coefflog}
 {\xintSub
 {\xintRationalSeries {0}{9}{1[0]}\ratioexp{\the\cnta[-1]}}
 {1}}}\dots
\endgraf
\ifnum\cnta < 12 \advance \cnta 1 \repeat
\end{multicols}
```

|                         |                         |                            |
|-------------------------|-------------------------|----------------------------|
| 0.099999999998556159... | 0.499511320760604148... | -1.597091692317639401...   |
| 0.199999995263443554... | 0.593980619762352217... | -12.648937932093322763...  |
| 0.299999338075041781... | 0.645144282733914916... | -66.259639046914679687...  |
| 0.399974460740121112... | 0.398118280111436442... | -304.768437445462801227... |

## 14.7 \xintFxFtPowerSeries

`\xintFxFtPowerSeries{A}{B}{\coeff}{f}{D}` computes  $\sum_{n=A}^{n=B} \text{"coeff-n"} \cdot f^n$  with each term of the series truncated to `D` digits after the decimal point. As usual, `A` and `B` are completely expanded through their inclusion in a `\numexpr` expression. Regarding `D` it will be similarly be expanded each time it is used inside an `\xintTrunc`. The one-parameter macro `\coeff` is similarly expanded at the time it is used inside the computations. Idem for `f`. If `f` itself is some complicated macro it is thus better to use the variant `\xintFxFtPowerSeriesX` which expands it first and then uses the result of that expansion.

The current (1.04) implementation is: the first power `f^A` is computed exactly, then truncated. Then each successive power is obtained from the previous one by multiplication by the exact value of `f`, and truncated. And `\coeff{n} \cdot f^n` is obtained from that by multiplying by `\coeff{n}` (untruncated) and then truncating. Finally the sum is computed exactly. Apart from that `\xintFxFtPowerSeries` (where `FxFt` means 'fixed-point') is like `\xintPowerSeries`.

There should be a variant for things of the type  $\sum c_n \frac{f^n}{n!}$  to avoid having to compute the factorial from scratch at each coefficient, the same way `\xintFxFtPowerSeries` does not compute  $f^n$  from scratch at each  $n$ . Perhaps in the next package release.

$$e^{-\frac{1}{2}} \approx$$

|                        |                        |                        |
|------------------------|------------------------|------------------------|
| 1.00000000000000000000 | 0.60653056795634920635 | 0.60653065971263344622 |
| 0.50000000000000000000 | 0.60653066483754960317 | 0.60653065971263342289 |
| 0.62500000000000000000 | 0.60653065945526069224 | 0.60653065971263342361 |
| 0.60416666666666666667 | 0.60653065972437513778 | 0.60653065971263342359 |
| 0.60677083333333333333 | 0.60653065971214266299 | 0.60653065971263342359 |
| 0.60651041666666666667 | 0.60653065971265234943 | 0.60653065971263342359 |
| 0.60653211805555555555 | 0.60653065971263274611 |                        |

```
\def\coeffexp #1{\xintiiFac {#1}[0]}% 1/n!
\def\ f {-1/2[0]}% [0] for faster input parsing
\cnta 0 % previously declared \count register
\noindent\loop
$\xintFxFtPowerSeries {0}{\cnta}{\coeffexp}{\f}{20}$\\
\ifnum\cnta<19 \advance\cnta 1 \repeat\par
\xintFxFtPowerSeries {0}{19}{\coeffexp}{\f}{25}= 0.6065306597126334236037992
```

It is no difficulty for *xintfrac* to compute exactly, with the help of `\xintPowerSeries`, the nineteenth partial sum, and to then give (the start of) its exact decimal expansion:

$$\begin{aligned} \text{\xintPowerSeries {0}{19}{\coeffexp}{\f}} &= \frac{38682746160036397317757}{63777066403145711616000} \\ &= 0.606530659712633423603799152126\dots \end{aligned}$$

Thus, one should always estimate a priori how many ending digits are not reliable: if there are  $N$  terms and  $N$  has  $k$  digits, then digits up to but excluding the last  $k$  may usually be trusted. If we are optimistic and the series is alternating we may even replace  $N$  with  $\sqrt{N}$  to get the number  $k$  of digits possibly of dubious significance.

## 14.8 `\xintFxFtPowerSeriesX`

`\xintFxFtPowerSeriesX{A}{B}{\coeff}{\f}{D}` computes, exactly as `\xintFxFtPowerSeries`, the sum of  $\text{\coeff{n}} \cdot \text{\f}^n$  from  $n=A$  to  $n=B$  with each term of the series being truncated to  $D$  digits after the decimal point. The sole difference is that `\f` is first expanded and it is the result of this which is used in the computations.

Let us illustrate this on the numerical exploration of the identity

$$\log(1+x) = -\log(1/(1+x))$$

Let  $L(h)=\log(1+h)$ , and  $D(h)=L(h)+L(-h/(1+h))$ . Theoretically thus,  $D(h)=0$  but we shall evaluate  $L(h)$  and  $-h/(1+h)$  keeping only 10 terms of their respective series. We will assume  $h < 0.5$ . With only ten terms kept in the power series we do not have quite 3 digits precision as  $2^{10} = 1024$ . So it wouldn't make sense to evaluate things more precisely than, say circa 5 digits after the decimal points.

```
\cnta 0
\def\coefflog #1{\the\numexpr\ifodd#1 1\else-1\fi\relax/#1[0]}% (-1)^{n-1}/n
\def\coeffalt #1{\the\numexpr\ifodd#1 -1\else1\fi\relax [0]}% (-1)^n
\begin{multicols}2
\loop
\noindent \hbox to 2.5cm {\hss\texttt{D(\the\cnta/100): }}%
\xintAdd {\xintFxFtPowerSeriesX {1}{10}{\coefflog}{\the\cnta [-2]}}{5}}
{\xintFxFtPowerSeriesX {1}{10}{\coefflog}
{\xintFxFtPowerSeriesX {1}{10}{\coeffalt}{\the\cnta [-2]}}{5}}
{5}}\endgraf
\ifnum\cnta < 49 \advance\cnta 7 \repeat
\end{multicols}
```



```

D(0/100): 0/1[0]
D(7/100): 2/1[-5]
D(14/100): 2/1[-5]
D(21/100): 3/1[-5]
D(28/100): 4/1[-5]
D(35/100): 4/1[-5]
D(42/100): 9/1[-5]
D(49/100): 42/1[-5]

```

Let's say we evaluate functions on  $[-1/2, +1/2]$  with values more or less also in  $[-1/2, +1/2]$  and we want to keep 4 digits of precision. So, roughly we need at least 14 terms in series like the geometric or log series. Let's make this 15. Then it doesn't make sense to compute intermediate summands with more than 6 digits precision. So we compute with 6 digits precision but return only 4 digits (rounded) after the decimal point. This result with 4 post-decimal points precision is then used as input to the next evaluation.

```

\begin{multicols}2
\loop
\noindent \hbox to 2.5cm {\hss\texttt{D(\the\cnta/100): }}%
\dt{\xintRound{4}
{\xintAdd {\xintFxFtPowerSeriesX {1}{15}{\coefflog}{\the\cnta [-2]}}{6}}
{\xintFxFtPowerSeriesX {1}{15}{\coefflog}
{\xintRound {4}{\xintFxFtPowerSeriesX {1}{15}{\coeffalt}
{\the\cnta [-2]}}{6}}}}
{6}}}%
}}\endgraf
\ifnum\cnta < 49 \advance\cnta 7 \repeat
\end{multicols}
D(0/100): 0
D(7/100): 0.0000
D(14/100): 0.0000
D(21/100): -0.0001
D(28/100): -0.0001
D(35/100): -0.0001
D(42/100): -0.0000
D(49/100): -0.0001

```

Not bad... I have cheated a bit: the 'four-digits precise' numeric evaluations were left unrounded in the final addition. However the inner rounding to four digits worked fine and made the next step faster than it would have been with longer inputs. The morale is that one should not use the raw results of `\xintFxFtPowerSeriesX` with the `D` digits with which it was computed, as the last are to be considered garbage. Rather, one should keep from the output only some smaller number of digits. This will make further computations faster and not less precise. I guess there should be some macro to do this final truncating, or better, rounding, at a given number  $D' < D$  of digits. Maybe for the next release.

## 14.9 `\xintFloatPowerSeries`

`\xintFloatPowerSeries[P]{A}{B}{\coeff}{f}` computes  $\sum_{n=A}^{n=B} \text{"coeff-n"} \cdot f^n$  with a floating point precision given by the optional parameter `P` or by the current setting of `\xintDigits`.

In the current, preliminary, version, no attempt has been made to try to guarantee to the final result the precision `P`. Rather, `P` is used for all intermediate floating point evaluations. So rounding errors will make some of the last printed digits invalid. The operations done are first the evaluation of  $f^A$  using `\xintFloatPow`, then each successive power is obtained from this first one by multiplication by `f` using `\xintFloatMul`, then again with `\xintFloatMul` this is multiplied with `\coeff{n}`, and the sum is done adding one term at a time with `\xintFloatAdd`. To sum up, this is just the naive transformation of `\xintFxFtPowerSeries` from fixed point to floating point.

```

\def\coefflog #1{\the\numexpr\ifodd#1 1\else-1\fi\relax/#1[0]}%
\xintFloatPowerSeries [8]{1}{30}{\coefflog}{-1/2[0]}
-6.9314718e-1

```

## 14.10 `\xintFloatPowerSeriesX`

`\xintFloatPowerSeriesX[P]{A}{B}{\coeff}{f}` is like `\xintFloatPowerSeries` with the difference that `f` is expanded once and for all at the start of the computation, thus allowing efficient chain-

$\frac{f}{f}$  ★

ing of such series evaluations.

```
\def\coeffexp #1{\xintiiFac {#1}[0]}% 1/n! (exact, not float)
\def\coefflog #1{\the\numexpr\ifodd#1 1\else-1\fi\relax/#1[0]}%
\xintFloatPowerSeriesX [8]{0}{30}{\coeffexp}
{\xintFloatPowerSeries [8]{1}{30}{\coefflog}{-1/2[0]}}
5.0000001e-1
```

## 14.11 Computing $\log 2$ and $\pi$

In this final section, the use of `\xintFxpTPowerSeries` (and `\xintPowerSeries`) will be illustrated on the (expandable... why make things simple when it is so easy to make them difficult!) computations of the first digits of the decimal expansion of the familiar constants  $\log 2$  and  $\pi$ .

Let us start with  $\log 2$ . We will get it from this formula (which is left as an exercise):

$$\log(2) = -2 \log(1-13/256) - 5 \log(1-1/9)$$

The number of terms to be kept in the log series, for a desired precision of  $10^{-D}$  was roughly estimated without much theoretical analysis. Computing exactly the partial sums with `\xintPowerSeries` and then printing the truncated values, from  $D=0$  up to  $D=100$  showed that it worked in terms of quality of the approximation. Because of possible strings of zeroes or nines in the exact decimal expansion (in the present case of  $\log 2$ , strings of zeroes around the fortieth and the sixtieth decimals), this does not mean though that all digits printed were always exact. In the end one always end up having to compute at some higher level of desired precision to validate the earlier result.

Then we tried with `\xintFxpTPowerSeries`: this is worthwhile only for  $D$ 's at least 50, as the exact evaluations are faster (with these short-length  $f$ 's) for a lower number of digits. And as expected the degradation in the quality of approximation was in this range of the order of two or three digits. This meant roughly that the 3+1=4 ending digits were wrong. Again, we ended up having to compute with five more digits and compare with the earlier value to validate it. We use truncation rather than rounding because our goal is not to obtain the correct rounded decimal expansion but the correct exact truncated one.

```
\def\coefflog #1{1/#1[0]}% 1/n
\def\xa {13/256[0]}% we will compute log(1-13/256)
\def\xb {1/9[0]}% we will compute log(1-1/9)
\def\LogTwo #1%
% get log(2)=-2log(1-13/256)- 5log(1-1/9)
{% we want to use \printnumber, hence need something expanding in two steps
% only, so we use here the \romannumeral0 method
\romannumeral0\expandafter\LogTwoDoIt \expandafter
% Nb Terms for 1/9:
{\the\numexpr #1*150/143\expandafter}\expandafter
% Nb Terms for 13/256:
{\the\numexpr #1*100/129\expandafter}\expandafter
% We print #1 digits, but we know the ending ones are garbage
{\the\numexpr #1\relax}% allows #1 to be a count register
}%
\def\LogTwoDoIt #1#2#3%
% #1=nb of terms for 1/9, #2=nb of terms for 13/256,
{% #3=nb of digits for computations, also used for printing
\xinttrunc {#3} % lowercase form to stop the \romannumeral0 expansion!
{\xintAdd
{\xintMul {2}{\xintFxpTPowerSeries {1}{#2}{\coefflog}{\xa}{#3}}}
{\xintMul {5}{\xintFxpTPowerSeries {1}{#1}{\coefflog}{\xb}{#3}}}%
}%
}%
\noindent $\log 2 \approx \LogTwo {60}\dots$\endgraf
```

```

\noindent\approx{}\printnumber{\LogTwo {65}}\dots\endgraf
\noindent\approx{}\printnumber{\LogTwo {70}}\dots\endgraf
log 2 \approx 0.693147180559945309417232121458176568075500134360255254120484...
 \approx 0.69314718055994530941723212145817656807550013436025525412068000711...
 \approx 0.6931471805599453094172321214581765680755001343602552541206800094933723...

```

Here is the code doing an exact evaluation of the partial sums. We have added a +1 to the number of digits for estimating the number of terms to keep from the log series: we experimented that this gets exactly the first  $D$  digits, for all values from  $D=0$  to  $D=100$ , except in one case ( $D=40$ ) where the last digit is wrong. For values of  $D$  higher than 100 it is more efficient to use the code using `\xintFxFtPowerSeries`.

```

\def\LogTwo #1% get log(2)=-2log(1-13/256)- 5log(1-1/9)
{%
 \romannumeral0\expandafter\LogTwoDoIt \expandafter
 {\the\numexpr (#1+1)*150/143\expandafter}\expandafter
 {\the\numexpr (#1+1)*100/129\expandafter}\expandafter
 {\the\numexpr #1\relax}%
}%
\def\LogTwoDoIt #1#2#3%
{% #3=nb of digits for truncating an EXACT partial sum
 \xinttrunc {#3}
 {\xintAdd
 {\xintMul {2}{\xintPowerSeries {1}{#2}{\coefflog}{\xa}}}
 {\xintMul {5}{\xintPowerSeries {1}{#1}{\coefflog}{\xb}}}%
 }%
}%

```

Let us turn now to  $\pi$ , computed with the Machin formula (but see also the approach via the [Brent-Salamin algorithm](#) with `\xintfloatexpr`) Again the numbers of terms to keep in the two `arctg` series were roughly estimated, and some experimentations showed that removing the last three digits was enough (at least for  $D=0-100$  range). And the algorithm does print the correct digits when used with  $D=1000$  (to be convinced of that one needs to run it for  $D=1000$  and again, say for  $D=1010$ .) A theoretical analysis could help confirm that this algorithm always gets better than  $10^{-D}$  precision, but again, strings of zeroes or nines encountered in the decimal expansion may falsify the ending digits, nines may be zeroes (and the last non-nine one should be increased) and zeroes may be nine (and the last non-zero one should be decreased).

```

\def\coeffarctg #1{\the\numexpr ifodd#1 -1\else1\fi\relax/%
 \the\numexpr 2*#1+1\relax [0]}%
%
\def\coeffarctg #1{\romannumeral0\xintmon{#1}/\the\numexpr 2*#1+1\relax }%
\def\xa {1/25[0]}% 1/5^2, the [0] for faster parsing
\def\xb {1/57121[0]}% 1/239^2, the [0] for faster parsing
\def\Machin #1{% #1 may be a count register, \Machin {\mycount} is allowed
 \romannumeral0\expandafter\MachinA \expandafter
 % number of terms for arctg(1/5):
 {\the\numexpr (#1+3)*5/7\expandafter}\expandafter
 % number of terms for arctg(1/239):
 {\the\numexpr (#1+3)*10/45\expandafter}\expandafter
 % do the computations with 3 additional digits:
 {\the\numexpr #1+3\expandafter}\expandafter
 % allow #1 to be a count register:
 {\the\numexpr #1\relax }}%
\def\MachinA #1#2#3#4%
{\xinttrunc {#4}
 {\xintSub
 {\xintMul {16/5}{\xintFxFtPowerSeries {0}{#1}{\coeffarctg}{\xa}{#3}}}
 {\xintMul {4/239}{\xintFxFtPowerSeries {0}{#2}{\coeffarctg}{\xb}{#3}}}%
 }%
}
\begin{framed}

```

```
\[\pi = \Machin {60}\dots \]
\end{framed}
```

$$\pi = 3.141592653589793238462643383279502884197169399375105820974944\dots$$

Here is a variant `\MachinBis`, which evaluates the partial sums exactly using `\xintPowerSeries`, before their final truncation. No need for a ```+3''` then.

```
\def\MachinBis #1% #1 may be a count register,
% the final result will be truncated to #1 digits post decimal point
 \romannumeral0\expandafter\MachinBisA \expandafter
 % number of terms for arctg(1/5):
 {\the\numexpr #1*5/7\expandafter}\expandafter
 % number of terms for arctg(1/239):
 {\the\numexpr #1*10/45\expandafter}\expandafter
 % allow #1 to be a count register:
 {\the\numexpr #1\relax }}%
\def\MachinBisA #1#2#3%
{\xinttrunc {#3} %
 {\xintSub
 {\xintMul {16/5}{\xintPowerSeries {0}{#1}{\coeffarctg}{\xa}}}%
 {\xintMul {4/239}{\xintPowerSeries {0}{#2}{\coeffarctg}{\xb}}}%
 }}%
```

Let us use this variant for a loop showing the build-up of digits:

```
\begin{multicols}{2}
 \cnta 0 % previously declared \count register
 \loop \noindent
 \centeredline{\dtl{\MachinBis{\cnta}}}%
 \ifnum\cnta < 30
 \advance\cnta 1 \repeat
\end{multicols}
```

|                  |                                  |
|------------------|----------------------------------|
|                  | 3.141592653589793                |
| 3.               | 3.1415926535897932               |
| 3.1              | 3.14159265358979323              |
| 3.14             | 3.141592653589793238             |
| 3.141            | 3.1415926535897932384            |
| 3.1415           | 3.14159265358979323846           |
| 3.14159          | 3.141592653589793238462          |
| 3.141592         | 3.1415926535897932384626         |
| 3.1415926        | 3.14159265358979323846264        |
| 3.14159265       | 3.141592653589793238462643       |
| 3.141592653      | 3.1415926535897932384626433      |
| 3.1415926535     | 3.14159265358979323846264338     |
| 3.14159265358    | 3.141592653589793238462643383    |
| 3.141592653589   | 3.1415926535897932384626433832   |
| 3.1415926535897  | 3.14159265358979323846264338327  |
| 3.14159265358979 | 3.141592653589793238462643383279 |

You want more digits and have some time? compile this copy of the `\Machin` with `etex` (or `pdftex`):

```
% Compile with e-TeX extensions enabled (etex, pdftex, ...)
\input xintfrac.sty
\input xintseries.sty
% pi = 16 Arctg(1/5) - 4 Arctg(1/239) (John Machin's formula)
\def\coeffarctg #1{\the\numexpr\ifodd#1 -1\else1\fi\relax/%
 \the\numexpr 2*#1+1\relax [0]}%
\def\xa {1/25[0]}%
```

```

\def\xb {1/57121[0]}%
\def\Machin #1{%
 \romannumeral0\expandafter\MachinA \expandafter
 {\the\numexpr (#1+3)*5/7\expandafter}\expandafter
 {\the\numexpr (#1+3)*10/45\expandafter}\expandafter
 {\the\numexpr #1+3\expandafter}\expandafter
 {\the\numexpr #1\relax }}%
\def\MachinA #1#2#3#4%
{\xinttrunc {#4}
 {\xintSub
 {\xintMul {16/5}{\xintFxFtPowerSeries {0}{#1}{\coeffarctg}{\xa}{#3}}}
 {\xintMul {4/239}{\xintFxFtPowerSeries {0}{#2}{\coeffarctg}{\xb}{#3}}}%
 }}%
\pdfresettimer
\edef\Z {\Machin {1000}}
\odef\W {\the\pdfelapsedtime}
\message{\Z}
\message{computed in \xintRound {2}{\W/65536} seconds.}
\bye

```

This will log the first 1000 digits of  $\pi$  after the decimal point. On my laptop (a 2012 model) this took about 5.05 seconds last time I tried.<sup>81 82</sup>

As mentioned in the introduction, the file `pi.tex` by D. ROEGEL shows that orders of magnitude faster computations are possible within  $\text{T}_{\text{E}}\text{X}$ , but recall our constraints of complete expandability and be merciful, please.

**Why truncating rather than rounding?** One of our main competitors on the market of scientific computing, a canadian product (not encumbered with expandability constraints, and having barely ever heard of  $\text{T}_{\text{E}}\text{X}$  ;-), prints numbers rounded in the last digit. Why didn't we follow suit in the macros `\xintFxFtPowerSeries` and `\xintFxFtPowerSeriesX`? To round at  $D$  digits, and excluding a rewrite or cloning of the division algorithm which anyhow would add to it some overhead in its final steps, `xintfrac` needs to truncate at  $D+1$ , then round. And rounding loses information! So, with more time spent, we obtain a worst result than the one truncated at  $D+1$  (one could imagine that additions and so on, done with only  $D$  digits, cost less; true, but this is a negligible effect per summand compared to the additional cost for this term of having been truncated at  $D+1$  then rounded). Rounding is the way to go when setting up algorithms to evaluate functions destined to be composed one after the other: exact algebraic operations with many summands and an  $f$  variable which is a fraction are costly and create an even bigger fraction; replacing  $f$  with a reasonable rounding, and rounding the result, is necessary to allow arbitrary chaining.

But, for the computation of a single constant, we are really interested in the exact decimal expansion, so we truncate and compute more terms until the earlier result gets validated. Finally if we do want the rounding we can always do it on a value computed with  $D+1$  truncation.

<sup>81</sup> With 1.09i and earlier *xint*, this used to be 42 seconds; starting with 1.09j, and prior to 1.2, it was 16 seconds (this was probably due to a more efficient division with denominators at most 9999). The 1.2 *xintcore* achieves a further gain at 5.6 seconds. <sup>82</sup> With `\xintDigits :=1001`, the non-optimized implementation with the *iter* of *xintexpr* fame using the *Brent-Salamin algorithm*, took, last time I tried (1.2i), about 7 seconds on my laptop (the last two digits were wrong, which is ok as they serve as guard digits), and for obtaining about 500 digits, it was about 1.7s. This is not bad, taking into account that the syntax is almost free rolling speech, contrarily to the code above for the Machin formula computation; we would like to use the quadratically convergent Brent-Salamin algorithm for more digits, but with such computations with numbers of one thousand digits we are beyond the border of the reasonable range for *xint*. Innocent people not knowing what it means to compute with  $\text{T}_{\text{E}}\text{X}$ , and with the extra constraint of expandability will wonder why this is at least thousands of times slower than with any other language (with a little Python program using the *Decimal* library, I timed the Brent-Salamin algorithm to 4.4ms for about 1000 digits and 1.14ms for 500 digits.) I will just say that for example digits are represented and manipulated via their ascii-code ! all computations must convert from ascii-code to cpu words; furthermore nothing can be stored away. And there is no memory storage with  $O(1)$  time access... if expandability is to be verified.

## 15 Macros of the *xintcfrac* package

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This package was first included in release 1.04 (2013/04/25) of the *xint* bundle. It was kept almost unchanged until 1.09m of 2014/02/26 which brings some new macros: `\xintFtoC`, `\xintCtoF`, `\xintCtoCv`, dealing with sequences of braced partial quotients rather than comma separated ones, `\xintFGtoC` which is to produce ``guaranteed'' coefficients of some real number known approximately, and `\xintGGCFrac` for displaying arbitrary material as a continued fraction; also, some changes to existing macros: `\xintFtoCs` and `\xintCntoCs` insert spaces after the commas, `\xintCstoF` and `\xintCstoCv` authorize spaces in the input also before the commas.

This section contains:

1. an [overview](#) of the package functionalities,
2. a description of each one of the package macros,
3. further illustration of their use via the study of the [convergents of  \$e\$](#) .

### 15.1 Package overview

The package computes partial quotients and convergents of a fraction, or conversely start from coefficients and obtain the corresponding fraction; three macros `\xintCFrac`, `\xintGCFrac` and `\xintGGCFrac` are for typesetting (the first two assume that the coefficients are numeric quantities acceptable by the *xintfrac* `\xintFrac` macro, the last one will display arbitrary material), the others can be nested (if applicable) or see their outputs further processed by other macros from the *xint* bundle, particularly the macros of *xinttools* dealing with sequences of braced items or comma separated lists.

A *simple* continued fraction has coefficients  $[c_0, c_1, \dots, c_N]$  (usually called partial quotients, but I dislike this entrenched terminology), where  $c_0$  is a positive or negative integer and the others are positive integers.

Typesetting is usually done via the *amsmath* macro `\cfrac`:

```
\[c_0 + \cfrac{1}{c_1 + \cfrac{1}{c_2 + \cfrac{1}{c_3 + \cfrac{1}{\ddots}}}} \]
```

$$c_0 + \frac{1}{c_1 + \frac{1}{c_2 + \frac{1}{c_3 + \frac{1}{\ddots}}}}$$

Here is a concrete example:

```
\[\xintFrac {208341/66317}=\xintCFrac {208341/66317}\]
```

$$\frac{208341}{66317} = 3 + \frac{1}{7 + \frac{1}{15 + \frac{1}{1 + \frac{1}{292 + \frac{1}{2}}}}}$$

But it is the macro `\xintCFrac` which did all the work of *computing* the continued fraction and using `\cfrac` from *amsmath* to typeset it.

A *generalized* continued fraction has the same structure but the numerators are not restricted to be 1, and numbers used in the continued fraction may be arbitrary, also fractions, irrationals, complex, indeterminates.<sup>83</sup> The *centered* continued fraction is an example:

```
\[\xintFrac {915286/188421}=\xintGCFrac {5+-1/7+1/39+-1/53+-1/13}
=\xintCFrac {915286/188421}\]
```

$$\frac{915286}{188421} = 5 - \frac{1}{7 + \frac{1}{39 - \frac{1}{53 - \frac{1}{13}}}} = 4 + \frac{1}{1 + \frac{1}{6 + \frac{1}{38 + \frac{1}{1 + \frac{1}{51 + \frac{1}{1 + \frac{1}{12}}}}}}}$$

The macro `\xintGCFrac`, contrarily to `\xintCFrac`, does not compute anything, it just typesets starting from a generalized continued fraction in inline format, which in this example was input literally. We also used `\xintCFrac` for comparison of the two types of continued fractions.

To let  $\TeX$  compute the centered continued fraction of  $f$  there is `\xintFtoCC`:

```
\[\xintFrac {915286/188421}\to\xintFtoCC {915286/188421}\]
```

$$\frac{915286}{188421} \rightarrow 5 + -1/7 + 1/39 + -1/53 + -1/13$$

The package macros are expandable and may be nested (naturally `\xintCFrac` and `\xintGCFrac` must be at the top level, as they deal with typesetting).

```
\[\xintGCFrac {\xintFtoCC{915286/188421}}\]
```

<sup>83</sup> *xintcfrac* may be used with indeterminates, for basic conversions from one inline format to another, but not for actual computations. See `\xintGGCFrac`.



$$5 - \frac{1}{7 + \frac{1}{39 - \frac{1}{53 - \frac{1}{13}}}}$$

The `\inline` format expected on input by `\xintGCFrac` is

$$a_0 + b_0/a_1 + b_1/a_2 + b_2/a_3 + \cdots + b_{n-2}/a_{n-1} + b_{n-1}/a_n$$

Fractions among the coefficients are allowed but they must be enclosed within braces. Signed integers may be left without braces (but the `+` signs are mandatory). No spaces are allowed around the plus and fraction symbols. The coefficients may themselves be macros, as long as these macros are *f-expandable*.

```
\[\xintFrac{\xintGctoF {1+-1/57+\xintPow {-3}{7}/\xintiQuo {132}{25}}}{\xintGCFrac {1+-1/57+\xintPow {-3}{7}/\xintiQuo {132}{25}}}\]
```

$$\frac{1907}{1902} = 1 - \frac{1}{57 - \frac{2187}{5}}$$

To compute the actual fraction one has `\xintGctoF`:

```
\[\xintFrac{\xintGctoF {1+-1/57+\xintPow {-3}{7}/\xintiQuo {132}{25}}}{\xintGctoF {1+-1/57+\xintPow {-3}{7}/\xintiQuo {132}{25}}}\]
```

For non-numeric input there is `\xintGGCFrac`.

```
\[\xintGGCFrac {a_0+b_0/a_1+b_1/a_2+b_2/\ddots+\ddots/a_{n-1}+b_{n-1}/a_n}\]
```

$$a_0 + \frac{b_0}{a_1 + \frac{b_1}{a_2 + \frac{b_2}{\ddots + \frac{b_{n-1}}{a_{n-1} + \frac{b_{n-1}}{a_n}}}}}$$

For regular continued fractions, there is a simpler comma separated format:

```
\[-7,6,19,1,33\to\xintFrac{\xintCstoF{-7,6,19,1,33}}{\xintCstoF{-7,6,19,1,33}}\]
```

$$-7, 6, 19, 1, 33 \rightarrow \frac{-28077}{4108} = -7 + \frac{1}{6 + \frac{1}{19 + \frac{1}{1 + \frac{1}{33}}}}$$

The macro `\xintFtoCs` produces from a fraction *f* the comma separated list of its coefficients.

```
\[\xintFrac{1084483/398959}=[\xintFtoCs{1084483/398959}]\]
```

$$\frac{1084483}{398959} = [2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 2]$$

If one prefers other separators, one can use the two arguments macros `\xintFtoCx` whose first argument is the separator (which may consist of more than one token) which is to be used.

```
\[\xintFrac{2721/1001}=\xintFtoCx {+1/}{2721/1001}\cdots\]
```

$$\frac{2721}{1001} = 2 + 1/(1 + 1/(2 + 1/(1 + 1/(1 + 1/(4 + 1/(1 + 1/(1 + 1/(6 + 1/(2) \cdots)))$$

This allows under Plain  $\text{\TeX}$  with `amstex` to obtain the same effect as with  $\text{\LaTeX}$  + `\amsmath` + `\xintCFrac`:

```
$$\xintFwOver{2721/1001}=\xintFtoCx {+\cfrac1{\ }}{2721/1001}\endcfrac$$
```

As a shortcut to `\xintFtoCx` with separator `1+/,` there is `\xintFtoGC`:

```
2721/1001=\xintFtoGC {2721/1001}
```

`2721/1001=2+1/1+1/2+1/1+1/1+1/4+1/1+1/1+1/6+1/2` Let us compare in that case with the output of `\xintFtoCC`:

```
2721/1001=\xintFtoCC {2721/1001}
```

`2721/1001=3+-1/4+-1/2+1/5+-1/2+1/7+-1/2` To obtain the coefficients as a sequence of braced numbers, there is `\xintFtoC` (this is a shortcut for `\xintFtoCx {}`). This list (sequence) may then be manipulated using the various macros of *xinttools* such as the non-expandable macro `\xintAssignArray` or the expandable `\xintApply` and `\xintListWithSep`.

Conversely to go from such a sequence of braced coefficients to the corresponding fraction there is `\xintCtoF`.

The `\printnumber` (subsection 1.3) macro which we use in this document to print long numbers can also be useful on long continued fractions.

```
\printnumber{\xintFtoCC {35037018906350720204351049/244241737886197404558180}}
```

`143+1/2+1/5+-1/4+-1/4+-1/4+-1/3+1/2+1/2+1/6+-1/22+1/2+1/10+-1/5+-1/11+-1/3+1/4+-1/2+1/2+1/4+-1/2+1/23+1/3+1/8+-1/6+-1/9` If we apply `\xintGctoF` to this generalized continued fraction, we discover that the original fraction was reducible:

```
\xintGctoF {143+1/2+...+-1/9}=2897319801297630107/20197107104701740
```

When a generalized continued fraction is built with integers, and numerators are only 1's or -1's, the produced fraction is irreducible. And if we compute it again with the last sub-fraction omitted we get another irreducible fraction related to the bigger one by a Bezout identity. Doing this here we get:

```
\xintGctoF {143+1/2+...+-1/6}=328124887710626729/2287346221788023
```

and indeed:

$$\begin{vmatrix} 2897319801297630107 & 328124887710626729 \\ 20197107104701740 & 2287346221788023 \end{vmatrix} = 1$$

The various fractions obtained from the truncation of a continued fraction to its initial terms are called the convergents. The macros of *xintcfrac* such as `\xintFtoCv`, `\xintFtoCCv`, and others which compute such convergents, return them as a list of braced items, with no separator (as does `\xintFtoC` for the partial quotients). Here is an example:

```
\[\xintFrac{915286/188421}\to
```

```
\xintListWithSep{,}{\xintApply\xintFrac{\xintFtoCv{915286/188421}}}\]
```

$$\frac{915286}{188421} \rightarrow 4, 5, \frac{34}{7}, \frac{1297}{267}, \frac{1331}{274}, \frac{69178}{14241}, \frac{70509}{14515}, \frac{915286}{188421}$$

```
\[\xintFrac{915286/188421}\to
```

```
\xintListWithSep{,}{\xintApply\xintFrac{\xintFtoCCv{915286/188421}}}\]
```

$$\frac{915286}{188421} \rightarrow 5, \frac{34}{7}, \frac{1331}{274}, \frac{70509}{14515}, \frac{915286}{188421}$$

We thus see that the 'centered convergents' obtained with `\xintFtoCCv` are among the fuller list of convergents as returned by `\xintFtoCv`.

Here is a more complicated use of `\xintApply` and `\xintListWithSep`. We first define a macro which will be applied to each convergent:

```
\newcommand{\mymacro}[1]{\xintFrac{#1}=\xintFtoCs{#1}}\vtop to 6pt{}}
```

Next, we use the following code:

```
$$\xintFrac{49171/18089}\to{}
```

```
\xintListWithSep {, }{\xintApply{\mymacro}{\xintFtoCv{49171/18089}}}
```

It produces:

$\frac{49171}{18089} \rightarrow 2 = [2], 3 = [3], \frac{8}{3} = [2, 1, 2], \frac{11}{4} = [2, 1, 3], \frac{19}{7} = [2, 1, 2, 2], \frac{87}{32} = [2, 1, 2, 1, 1, 4], \frac{106}{39} = [2, 1, 2, 1, 1, 5], \frac{193}{71} = [2, 1, 2, 1, 1, 4, 2], \frac{1264}{465} = [2, 1, 2, 1, 1, 4, 1, 1, 6], \frac{1457}{536} = [2, 1, 2, 1, 1, 4, 1, 1, 7], \frac{2721}{1001} = [2, 1, 2, 1, 1, 4, 1, 1, 6, 2], \frac{23225}{8544} = [2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8], \frac{49171}{18089} = [2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 2]$

The macro `\xintCntoF` allows to specify the coefficients as a function given by a one-parameter macro. The produced values do not have to be integers.

```
\def\cn #1{\xintiiPow {2}{#1}}% 2^n
\[\xintFrac{\xintCntoF {6}{\cn}}=\xintCFrac [1]{\xintCntoF {6}{\cn}}\]
```

$$\frac{3541373}{2449193} = 1 + \frac{1}{2 + \frac{1}{4 + \frac{1}{8 + \frac{1}{16 + \frac{1}{32 + \frac{1}{64}}}}}}$$

Notice the use of the optional argument `[1]` to `\xintCFrac`. Other possibilities are `[r]` and (default) `[c]`.

```
\def\cn #1{\xintPow {2}{-#1}}%
\[\xintFrac{\xintCntoF {6}{\cn}}=\xintGCFrac [r]{\xintCntoGC {6}{\cn}}=
[\xintFtoCs {\xintCntoF {6}{\cn}}]\]
```

$$\frac{3159019}{2465449} = 1 + \frac{1}{\frac{1}{2} + \frac{1}{\frac{1}{4} + \frac{1}{\frac{1}{8} + \frac{1}{\frac{1}{16} + \frac{1}{\frac{1}{32} + \frac{1}{64}}}}}} = [1, 3, 1, 1, 4, 14, 1, 1, 1, 1, 79, 2, 1, 1, 2]$$

We used `\xintCntoGC` as we wanted to display also the continued fraction and not only the fraction returned by `\xintCntoF`.

There are also `\xintGCntoF` and `\xintGCntoGC` which allow the same for generalized fractions. An initial portion of a generalized continued fraction for  $\pi$  is obtained like this

```
\def\an #1{\the\numexpr 2*#1+1\relax }%
\def\bn #1{\the\numexpr (#1+1)*(#1+1)\relax }%
\[\xintFrac{\xintDiv {4}{\xintGCntoF {5}{\an}{\bn}}} =
\cfrac{4}{\xintGCFrac{\xintGCntoGC {5}{\an}{\bn}}} =
\xintTrunc {10}{\xintDiv {4}{\xintGCntoF {5}{\an}{\bn}}}\dots\]
```

$$\frac{92736}{29520} = \frac{4}{1 + \frac{1}{3 + \frac{4}{5 + \frac{9}{7 + \frac{16}{9 + \frac{25}{11}}}}}} = 3.1414634146\dots$$

We see that the quality of approximation is not fantastic compared to the simple continued fraction of  $\pi$  with about as many terms:

```
\[\xintFrac{\xintCstoF{3,7,15,1,292,1,1}}=
\xintGCFrac{3+1/7+1/15+1/1+1/292+1/1+1/1}=
\xintTrunc{10}{\xintCstoF{3,7,15,1,292,1,1}}\dots\]
```

$$\frac{208341}{66317} = 3 + \frac{1}{7 + \frac{1}{15 + \frac{1}{1 + \frac{1}{292 + \frac{1}{1 + \frac{1}{1}}}}}} = 3.1415926534\dots$$

When studying the continued fraction of some real number, there is always some doubt about how many terms are valid, when computed starting from some approximation. If  $f \leq x \leq g$  and  $f, g$  both have the same first  $K$  partial quotients, then  $x$  also has the same first  $K$  quotients and convergents. The macro `\xintFGtoC` outputs as a sequence of braced items the common partial quotients of its two arguments. We can thus use it to produce a sure list of valid convergents of  $\pi$  for example, starting from some proven lower and upper bound:

```
$$\pi\to [\xintListWithSep{,}
{\xintFGtoC {3.14159265358979323}{3.14159265358979324}}, \dots]$$
\noindent$\pi\to\xintListWithSep{,\allowbreak;}
{\xintApply{\xintFrac}
{\xintCtoCv{\xintFGtoC {3.14159265358979323}{3.14159265358979324}}}}, \dots$
\pi \to [3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, \dots]
```

$$\pi \rightarrow 3, \frac{22}{7}, \frac{333}{106}, \frac{355}{113}, \frac{103993}{33102}, \frac{104348}{33215}, \frac{208341}{66317}, \frac{312689}{99532}, \frac{833719}{265381}, \frac{1146408}{364913}, \frac{4272943}{1360120}, \frac{5419351}{1725033}, \frac{80143857}{25510582}, \frac{165707065}{52746197}, \frac{245850922}{78256779}, \frac{411557987}{131002976}, \dots$$

## 15.2 `\xintCFrac`

*Frac*  
*f*

`\xintCFrac{f}` is a math-mode only,  $\TeX$  with `amsmath` only, macro which first computes then displays with the help of `\cfrac` the simple continued fraction corresponding to the given fraction. It admits an optional argument which may be `[l]`, `[r]` or (the default) `[c]` to specify the location of the one's in the numerators of the sub-fractions. Each coefficient is typeset using the `\xintFrac` macro from the *xintfrac* package. This macro is *f-expandable* in the sense that it prepares expandably the whole expression with the multiple `\cfrac`'s, but it is not completely expandable naturally as `\cfrac` isn't.

### 15.3 `\xintGCfrac`

*f* `\xintGCfrac{a+b/c+d/e+f/g+h/...+x/y}` uses similarly `\cfrac` to prepare the typesetting with the `\msmath` `\cfrac` ( $\TeX$ ) of a generalized continued fraction given in inline format (or as macro which will *f-expand* to it). It admits the same optional argument as `\xintCFrac`. Plain  $\TeX$  with `amstex` users, see `\xintGctoGCx`.

```
\[\xintGCfrac {1+\xintPow{1.5}{3}/{1/7}+{-3/5}/\xintiiFrac {6}}\]
```

$$1 + \frac{3375 \cdot 10^{-3}}{\frac{1}{7} - \frac{3}{5}}$$

This is mostly a typesetting macro, although it does provoke the expansion of the coefficients. See `\xintGctoF` if you are impatient to see this specific fraction computed.

It admits an optional argument within square brackets which may be either `[l]`, `[c]` or `[r]`. Default is `[c]` (numerators are centered).

Numerators and denominators are made arguments to the `\xintFrac` macro. This allows them to be themselves fractions or anything *f-expandable* giving numbers or fractions, but also means however that they can not be arbitrary material, they can not contain color changing macros for example. One of the reasons is that `\xintGCfrac` tries to determine the signs of the numerators and chooses accordingly to use + or -.

### 15.4 `\xintGGCFrac`

*f* `\xintGGCFrac{a+b/c+d/e+f/g+h/...+x/y}` is a clone of `\xintGCfrac`, hence again  $\TeX$  specific with package `amsmath`. It does not assume the coefficients to be numbers as understood by *xintcffrac*. The macro can be used for displaying arbitrary content as a continued fraction with `\cfrac`, using only plus signs though. Note though that it will first *f-expand* its argument, which may be thus be one of the *xintcffrac* macros producing a (general) continued fraction in inline format, see `\xintFtoCx` for an example. If this expansion is not wished, it is enough to start the argument with a space.

```
\[\xintGGCFrac {1+q/1+q^2/1+q^3/1+q^4/1+q^5/\ddots}\]
```

$$1 + \frac{q}{1 + \frac{q^2}{1 + \frac{q^3}{1 + \frac{q^4}{1 + \frac{q^5}{\ddots}}}}}$$

### 15.5 `\xintGctoGCx`

*nnf* ★ `\xintGctoGCx{sepa}{sepb}{a+b/c+d/e+f/...+x/y}` returns the list of the coefficients of the generalized continued fraction of *f*, each one within a pair of braces, and separated with the help of *sepa* and *sepb*. Thus

```
\xintGctoGCx ::{1+2/3+4/5+6/7} gives 1:2;3:4;5:6;7
```

The following can be used by Plain  $\TeX$ +`amstex` users to obtain an output similar as the ones produced by `\xintGCfrac` and `\xintGGCFrac`:

```
$$\xintGctoGCx {+\cfrac{\{\}\{\}\{a+b/\dots\}\endcfrac}{}
$\xintGctoGCx {+\cfrac{\xintFwOver{\{\}\{\}\xintFwOver}\{a+b/\dots\}\endcfrac}{}$
```

## 15.6 \xintFtoC

**Frac**  $\star$  `\xintFtoC{f}` computes the coefficients of the simple continued fraction of `f` and returns them as a list (sequence) of braced items.

```
\fdef\test{\xintFtoC{-5262046/89233}}\texttt{\meaning\test}
```

macro:  $\rightarrow \{-59\}\{33\}\{27\}\{100\}$

## 15.7 \xintFtoCs

**Frac**  $\star$  `\xintFtoCs{f}` returns the comma separated list of the coefficients of the simple continued fraction of `f`. Notice that starting with `1.09m` a space follows each comma (mainly for usage in text mode, as in math mode spaces are produced in the typeset output by  $\text{\TeX}$  itself).

```
\[\xintSignedFrac{-5262046/89233} \to [\xintFtoCs{-5262046/89233}]\]
```

$$-\frac{5262046}{89233} \rightarrow [-59, 33, 27, 100]$$

## 15.8 \xintFtoCx

$\frac{f}{n}$  ★ `\xintFtoCx{sep}{f}` returns the list of the coefficients of the simple continued fraction of `f` separated with the help of `sep`, which may be anything (and is kept unexpanded). For example, with Plain TeX and `amstex`,

$$x_{\text{intFtoCx}} = \frac{-5262046}{89233}$$

will display the continued fraction using `\cfrac`. Each coefficient is inside a brace pair `{ }`, allowing a macro to end the separator and fetch it as argument, for example, again with Plain  $\text{\TeX}$  and `amstex`:

```
\def\highlight #1{\ifnum #1>200 \textcolor{red}{#1}\else #1\fi}
```

$$\frac{104348}{33215}$$

Due to the different and extremely cumbersome syntax of `\cfrac` under  $\text{\LaTeX}$  it proves a bit tortuous to obtain there the same effect. Actually, it is partly for this purpose that 1.09m added `\xintGGCFrac`. We thus use `\xintFtoCx` with a suitable separator, and then the whole thing as argument to `\xintGGCFrac`:

```
\def\highlight #1{\ifnum #1>200 \fcolorbox{blue}{white}{\boldmath\color{red}$#1$}%
```

```
\else #1\fi}
```

$$\frac{208341}{66317}$$

$$\begin{array}{r}
 3 + \frac{1}{7} \\
 7 + \frac{1}{15} \\
 15 + \frac{1}{1} \\
 1 + \frac{1}{292} + \frac{1}{2}
 \end{array}$$

## 15.9 \xintFtoGC

`\xintFtoGC{f}` does the same as `\xintFtoCx{+1/}{f}`. Its output may thus be used in the package macros expecting such an 'inline format'.

```
566827/208524=\xintFtoGC {566827/208524}
```

$$566827/208524=2+1/1+1/2+1/1+1/1+1/4+1/1+1/1+1/6+1/1+1/1+1/8+1/1+1/1+1/11$$

## 15.10 \xintFGtoC

$\frac{f}{f}$  ★ `\xintFGtoC{f}{g}` computes the common initial coefficients to two given fractions `f` and `g`. Notice that any real number `f < x < g` or `f > x > g` will then necessarily share with `f` and `g` these common initial coefficients for its regular continued fraction. The coefficients are output as a sequence of braced numbers. This list can then be manipulated via macros from `xinttools`, or other macros of `xintfrac`.

```

\fdef\test{\xintFGtoC{-5262046/89233}{-5314647/90125}}\texttt{\meaning\test}
macro:->{-59}{33}{27}
\fdef\test{\xintFGtoC{3.141592653}{3.141592654}}\texttt{\meaning\test}
macro:->{3}{7}{15}{1}
\fdef\test{\xintFGtoC{3.1415926535897932384}{3.1415926535897932385}}\meaning\test
macro:->{3}{7}{15}{1}{292}{1}{1}{1}{2}{1}{3}{1}{14}{2}{1}{1}{2}{2}{2}
\xintRound {30}{\xintCstoF{\xintListWithSep{,}{\test}}}{
3.141592653589793238386377506390
\xintRound {30}{\xintCtoF{\test}}{
3.141592653589793238386377506390
\fdef\test{\xintFGtoC{1.41421356237309}{1.4142135623731}}\meaning\test
macro:->{1}{2}

```

### 15.11 \xintFtoCC

`\xintFtoCC{f}` returns the 'centered' continued fraction of `f`, in 'inline format'.

$$\frac{566827}{208524} = \frac{3 + \frac{-1}{4} + \frac{-1}{2} + \frac{1}{5} + \frac{-1}{2} + \frac{1}{7} + \frac{-1}{2} + \frac{1}{9} + \frac{-1}{2} + \frac{1}{11}}{\frac{566827}{208524}} = \frac{\frac{566827}{208524}}{\frac{566827}{208524}}$$

$$\frac{566827}{208524} = 3 - \frac{1}{4 - \frac{1}{2 + \frac{1}{5 - \frac{1}{2 + \frac{1}{7 - \frac{1}{2 + \frac{1}{9 - \frac{1}{2 + \frac{1}{11}}}}}}}}}$$

## 15.12 \xintCstoF

**f** ★ `\xintCstoF{a,b,c,d,...,z}` computes the fraction corresponding to the coefficients, which may be fractions or even macros expanding to such fractions. The final fraction may then be highly reducible. Starting with release 1.09m spaces before commas are allowed and trimmed automatically (spaces after commas were already silently handled in earlier releases).

$$\frac{-1+1/3+1/-5+1/7+1/-9+1/11+1/-13}{-1, 3, -5, 7, -9, 11, -13} = \frac{-1+1/3+1/-5+1/7+1/-9+1/11+1/-13}{-1, 3, -5, 7, -9, 11, -13}$$



$$-1 + \frac{1}{3 + \frac{1}{-5 + \frac{1}{7 + \frac{1}{-9 + \frac{1}{11 + \frac{1}{-13}}}}}} = -\frac{75887}{118187} = -\frac{75887}{118187}$$

```
\[\xintGCFrac{{1/2}+1/{1/3}+1/{1/4}+1/{1/5}}=\xintFrac{\xintCstoF {1/2,1/3,1/4,1/5}}\]
```

$$\frac{1}{2} + \frac{1}{\frac{\frac{1}{3} + \frac{1}{\frac{\frac{1}{4} + \frac{1}{\frac{1}{5}}}}}} = \frac{159}{66}$$

A generalized continued fraction may produce a reducible fraction (`\xintCstoF` tries its best not to accumulate in a silly way superfluous factors but will not do simplifications which would be obvious to a human, like simplification by 3 in the result above).

### 15.13 `\xintCtoF`

*f* ★ `\xintCtoF{a}{b}{c}...{z}` computes the fraction corresponding to the coefficients, which may be fractions or even macros.

```
\xintCtoF {\xintApply {\xintiiPow 3}{\xintSeq {1}{5}}}
```

```
14946960/4805083
```

```
\[\xintFrac{14946960/4805083}=\xintCFrac {14946960/4805083}\]
```

$$\frac{14946960}{4805083} = 3 + \frac{1}{9 + \frac{1}{27 + \frac{1}{81 + \frac{1}{243}}}}$$

In the example above the power of 3 was already pre-computed via the expansion done by `\xintApply`, but if we try with `\xintApply { \xintiiPow 3}` where the space will stop this expansion, we can check that `\xintCtoF` will itself provoke the needed coefficient expansion.

### 15.14 `\xintGctoF`

*f* ★ `\xintGctoF{a+b/c+d/e+f/g+.....+v/w+x/y}` computes the fraction defined by the inline generalized continued fraction. Coefficients may be fractions but must then be put within braces. They can be macros. The plus signs are mandatory.

```
\[\xintGCFrac {1+\xintPow{1.5}{3}/{1/7}+{-3/5}/\xintiiFac {6}} =
\xintFrac{\xintGctoF {1+\xintPow{1.5}{3}/{1/7}+{-3/5}/\xintiiFac {6}}} =
\xintFrac{\xintIrr{\xintGctoF
{1+\xintPow{1.5}{3}/{1/7}+{-3/5}/\xintiiFac {6}}}}\]
```

$$1 + \frac{3375 \cdot 10^{-3}}{\frac{1}{7} - \frac{3}{720}} = \frac{88629000}{3579000} = \frac{29543}{1193}$$

```
\[\xintGCFrac{{1/2}+{2/3}/{4/5}+{1/2}/{1/5}+{3/2}/{5/3}} =
\xintFrac{\xintGctoF {{1/2}+{2/3}/{4/5}+{1/2}/{1/5}+{3/2}/{5/3}}} \]
```

$$\frac{1}{2} + \frac{\frac{2}{3}}{\frac{4}{5} + \frac{\frac{1}{2}}{\frac{1}{5} + \frac{3}{\frac{2}{5}}}} = \frac{4270}{4140}$$

The macro tries its best not to accumulate superfluous factor in the denominators, but doesn't reduce the fraction to irreducible form before returning it and does not do simplifications which would be obvious to a human.

### 15.15 `\xintCstoCv`

*f* ★ `\xintCstoCv{a,b,c,d,...,z}` returns the sequence of the corresponding convergents, each one within braces.

It is allowed to use fractions as coefficients (the computed convergents have then no reason to be the real convergents of the final fraction). When the coefficients are integers, the convergents are irreducible fractions, but otherwise it is not necessarily the case.

```
\xintListWithSep{\xintCstoCv{1,2,3,4,5,6}}
1/1:3/2:10/7:43/30:225/157:1393/972
\xintListWithSep{\xintCstoCv{1,1/2,1/3,1/4,1/5,1/6}}
1/1:3/1:9/7:45/19:225/159:1575/729
\[\xintListWithSep{\to}{\xintApply\xintFrac{\xintCstoCv {\xintPow
{- .3}{-5},7.3/4.57,\xintCstoF{3/4,9,-1/3}}}}\]
-100000/243 → -72888949/177390 → -2700356878/6567804
```

### 15.16 `\xintCtoCv`

*f* ★ `\xintCtoCv{{a}{b}{c}...{z}}` returns the sequence of the corresponding convergents, each one within braces.

```
\fdef\test{\xintCtoCv {1111111111}}\texttt{\meaning\test}
macro:->{1/1}{2/1}{3/2}{5/3}{8/5}{13/8}{21/13}{34/21}{55/34}{89/55}{144/89}
```

### 15.17 `\xintGctoCv`

*f* ★ `\xintGctoCv{a+b/c+d/e+f/g+.....+v/w+x/y}` returns the list of the corresponding convergents. The coefficients may be fractions, but must then be inside braces. Or they may be macros, too.

The convergents will in the general case be reducible. To put them into irreducible form, one needs one more step, for example it can be done with `\xintApply\xintIrr`.

```
\[\xintListWithSep{,}{\xintApply\xintFrac
{\xintGctoCv{3+{-2}/{7/2}+{3/4}/12+{-56}/3}}}\]
\[\xintListWithSep{,}{\xintApply\xintFrac{\xintApply\xintIrr
{\xintGctoCv{3+{-2}/{7/2}+{3/4}/12+{-56}/3}}}}\]
```

$$3, \frac{17}{7}, \frac{834}{342}, \frac{1306}{542}$$

$$3, \frac{17}{7}, \frac{139}{57}, \frac{653}{271}$$

### 15.18 `\xintFtoCv`

$\frac{f}{f}$  ★ `\xintFtoCv{f}` returns the list of the (braced) convergents of  $f$ , with no separator. To be treated with `\xintAssignArray` or `\xintListWithSep`.

```
\[\xintListWithSep{\to}{\xintApply\xintFrac{\xintFtoCv{5211/3748}}}\]
```

$$1 \rightarrow \frac{3}{2} \rightarrow \frac{4}{3} \rightarrow \frac{7}{5} \rightarrow \frac{25}{18} \rightarrow \frac{32}{23} \rightarrow \frac{57}{41} \rightarrow \frac{317}{228} \rightarrow \frac{374}{269} \rightarrow \frac{691}{497} \rightarrow \frac{5211}{3748}$$

### 15.19 `\xintFtoCCv`

$\frac{f}{f}$  ★ `\xintFtoCCv{f}` returns the list of the (braced) centered convergents of  $f$ , with no separator. To be treated with `\xintAssignArray` or `\xintListWithSep`.

```
\[\xintListWithSep{\to}{\xintApply\xintFrac{\xintFtoCCv{5211/3748}}}\]
```

$$1 \rightarrow \frac{4}{3} \rightarrow \frac{7}{5} \rightarrow \frac{32}{23} \rightarrow \frac{57}{41} \rightarrow \frac{374}{269} \rightarrow \frac{691}{497} \rightarrow \frac{5211}{3748}$$

### 15.20 `\xintCntoF`

$\frac{num}{x} f$  ★ `\xintCntoF{N}{\macro}` computes the fraction  $f$  having coefficients  $c(j)=\macro{j}$  for  $j=0,1,\dots,N$ . The  $N$  parameter is given to a `\numexpr`. The values of the coefficients, as returned by `\macro` do not have to be positive, nor integers, and it is thus not necessarily the case that the original  $c(j)$  are the true coefficients of the final  $f$ .

```
\def\macro #1{\the\numexpr 1+#1*#1\relax} \xintCntoF {5}{\macro}
```

72625/49902[0]

This example shows that the fraction is output with a trailing number in square brackets (representing a power of ten), this is for consistency with what do most macros of *xintcfrac*, and does not have to be always this annoying [0] as the coefficients may for example be numbers in scientific notation. To avoid these trailing square brackets, for example if the coefficients are known to be integers, there is always the possibility to filter the output via `\xintPRaw`, or `\xintIrr` (the latter is overkill in the case of integer coefficients, as the fraction is guaranteed to be irreducible then).

### 15.21 `\xintGCntoF`

$\frac{num}{x} ff$  ★ `\xintGCntoF{N}{\macroA}{\macroB}` returns the fraction  $f$  corresponding to the inline generalized continued fraction  $a_0+b_0/a_1+b_1/a_2+\dots+b_{(N-1)}/a_N$ , with  $a(j)=\macroA{j}$  and  $b(j)=\macroB{j}$ . The  $N$  parameter is given to a `\numexpr`.

```
\def\coeffA #1{\the\numexpr #1+4-3*((#1+2)/3)\relax}%
\def\coeffB #1{\the\numexpr \ifodd #1 -\fi 1\relax}% (-1)^n
\[\xintGCfrac{\xintGCntoGC {6}{\coeffA}{\coeffB}}{=}
\xintFrac{\xintGCntoF {6}{\coeffA}{\coeffB}}{\]
```

$$1 + \frac{1}{2 - \frac{1}{3 + \frac{1}{1 - \frac{1}{2 + \frac{1}{3 - \frac{1}{1}}}}}} = \frac{39}{25}$$

There is also `\xintGCtoGC` to get the 'inline format' continued fraction.

## 15.22 `\xintCtoCs`

`\xintCtoCs{N}{\macro}` produces the comma separated list of the corresponding coefficients, from  $n=0$  to  $n=N$ . The  $N$  is given to a `\numexpr`.

```
\xintCtoCs {5}{\macro}
```

1, 2, 5, 10, 17, 26

```
\[\xintFrac{\xintCtoF{5}{\macro}}=\xintCFrac{\xintCtoF {5}{\macro}}\]
```

$$\frac{72625}{49902} = 1 + \frac{1}{2 + \frac{1}{5 + \frac{1}{10 + \frac{1}{17 + \frac{1}{26}}}}}$$

## 15.23 `\xintCtoGC`

`\xintCtoGC{N}{\macro}` evaluates the  $c(j)=\macro{j}$  from  $j=0$  to  $j=N$  and returns a continued fraction written in inline format:  $\{c(0)\}+1/\{c(1)\}+1/\dots+1/\{c(N)\}$ . The parameter  $N$  is given to a `\numexpr`. The coefficients, after expansion, are, as shown, being enclosed in an added pair of braces, they may thus be fractions.

```
\def\macro #1{\the\numexpr\ifodd#1 -1-#1\else1+#1\fi\relax/\the\numexpr 1+#1*#1\relax}
```

```
\fdef\x{\xintCtoGC {5}{\macro}}\meaning\x
```

```
\[\xintGCfrac{\xintCtoGC {5}{\macro}}\]
```

macro:->{1/\the \numexpr 1+0\*0\relax }+1/{-2/\the \numexpr 1+1\*1\relax }+1/{3/\the \numexpr 1+2\*2\relax }+1/{-4/\the \numexpr 1+3\*3\relax }+1/{5/\the \numexpr 1+4\*4\relax }+1/{-6/\the \numexpr 1+5\*5\relax }

$$1 + \frac{1}{-\frac{2}{2} + \frac{1}{\frac{3}{5} + \frac{1}{-\frac{4}{10} + \frac{1}{\frac{5}{17} + \frac{1}{-\frac{6}{26}}}}}}$$

15.24 `\xintGCntoGC`

$\text{num}$   
 $\text{x}$  *ff* ★ `\xintGCntoGC{N}{\macroA}{\macroB}` evaluates the coefficients and then returns the corresponding `{a0}+{b0}/{a1}+{b1}/{a2}+...+{b(N-1)}/{aN}` inline generalized fraction. `N` is given to a `\numexpr`. The coefficients are enclosed into pairs of braces, and may thus be fractions, the fraction slash will not be confused in further processing by the continued fraction slashes.

```
\def\an #1{\the\numexpr #1*#1*#1+1\relax}%
\def\bn #1{\the\numexpr \ifodd#1 -\fi 1*(#1+1)\relax}%
\xintGCntoGC {5}{\an}{\bn}=\xintGCFrac {\xintGCntoGC {5}{\an}{\bn}} =
\displaystyle\xintFrac {\xintGCntoF {5}{\an}{\bn}}{\par
```

$$1 + \frac{1}{2 - \frac{1}{9 + \frac{1}{28 - \frac{1}{65 + \frac{1}{126}}}}} = \frac{5797655}{3712466}$$

15.25 `\xintCstoGC`

*f* ★ `\xintCstoGC{a,b,...,z}` transforms a comma separated list (or something expanding to such a list) into an 'inline format' continued fraction `{a}+1/{b}+1/...+1/{z}`. The coefficients are just copied and put within braces, without expansion. The output can then be used in `\xintGCFrac` for example.

```
\[\xintGCFrac {\xintCstoGC {-1,1/2,-1/3,1/4,-1/5}}=\xintSignedFrac{\xintCstoF {-1,1/2,-1/3,1/4,-1/5}}\]
```

$$-1 + \frac{1}{\frac{1}{2} + \frac{1}{-\frac{1}{3} + \frac{1}{\frac{1}{4} + \frac{1}{-\frac{1}{5}}}}} = -\frac{145}{83}$$

15.26 `\xintiCstoF`, `\xintiGctoF`, `\xintiCstoCv`, `\xintiGctoCv`

*f* ★ Essentially the same as the corresponding macros without the 'i', but for integer-only input. Infinitesimally faster, mainly for internal use by the package.

15.27 `\xintGctoGC`

*f* ★ `\xintGctoGC{a+b/c+d/e+f/g+.....+v/w+x/y}` expands (with the usual meaning) each one of the coefficients and returns an inline continued fraction of the same type, each expanded coefficient being enclosed within braces.

```
\fdef\x {\xintGctoGC {1+\xintPow{1.5}{3}/{1/7}+{-3/5}/%
\xintiiFac {6}+\xintCstoF {2,-7,-5}/16}} \meaning\x
```

**macro:** `->{1}+{3375/1[-3]}/{1/7}+{-3/5}/{720}+{67/36}/{16}`

To be honest I have forgotten for which purpose I wrote this macro in the first place.

## 15.28 Euler's number e

Let us explore the convergents of Euler's number e. The volume of computation is kept minimal by the following steps:

- a comma separated list of the first 36 coefficients is produced by `\xintCnCs`,
- this is then given to `\xintiCstoCv` which produces the list of the convergents (there is also `\xintCstoCv`, but our coefficients being integers we used the infinitesimally faster `\xintiCstoCv`),
- then the whole list was converted into a sequence of one-line paragraphs, each convergent becomes the argument to a macro printing it together with its decimal expansion with 30 digits after the decimal point.
- A count register `\cnta` was used to give a line count serving as a visual aid: we could also have done that in an expandable way, but well, let's relax from time to time...

```
\def\cn #1{\the\numexpr\ifcase \numexpr #1+3-3*((#1+2)/3)\relax
1\or1\or2*(#1/3)\fi\relax }
% produces the pattern 1,1,2,1,1,4,1,1,6,1,1,8,... which are the
% coefficients of the simple continued fraction of e-1.
\cnta 0
\def\mymacro #1{\advance\cnta by 1
\noindent
\hbox to 3em {\hfil\small\dt{\the\cnta.} }%
$\xintTrunc {30}{\xintAdd {1[0]}{#1}}\dots=
\xintFrac{\xintAdd {1[0]}{#1}}{#1}}$}%
\xintListWithSep{\vtop to 6pt}{\vbox to 12pt}{\par}
{\xintApply\mymacro{\xintiCstoCv{\xintCnCs {35}{\cn}}}}
```

1.  $2.00000000000000000000000000000000 \dots = 2$
2.  $3.00000000000000000000000000000000 \dots = 3$
3.  $2.66666666666666666666666666666666 \dots = \frac{8}{3}$
4.  $2.75000000000000000000000000000000 \dots = \frac{11}{4}$
5.  $2.714285714285714285714285714285 \dots = \frac{19}{7}$
6.  $2.71875000000000000000000000000000 \dots = \frac{87}{32}$
7.  $2.717948717948717948717948717948 \dots = \frac{106}{39}$
8.  $2.718309859154929577464788732394 \dots = \frac{193}{71}$
9.  $2.718279569892473118279569892473 \dots = \frac{1264}{465}$
10.  $2.718283582089552238805970149253 \dots = \frac{1457}{536}$
11.  $2.718281718281718281718281718281 \dots = \frac{2721}{1001}$
12.  $2.718281835205992509363295880149 \dots = \frac{23225}{8544}$
13.  $2.718281822943949711891042430591 \dots = \frac{25946}{9545}$
14.  $2.718281828735695726684725523798 \dots = \frac{49171}{18089}$
15.  $2.718281828445401318035025074172 \dots = \frac{517656}{190435}$
16.  $2.718281828470583721777828930962 \dots = \frac{566827}{208524}$

17.  $2.718281828458563411277850606202\dots = \frac{1084483}{398959}$
18.  $2.718281828459065114074529546648\dots = \frac{13580623}{4996032}$
19.  $2.718281828459028013207065591026\dots = \frac{14665106}{5394991}$
20.  $2.718281828459045851404621084949\dots = \frac{28245729}{10391023}$
21.  $2.718281828459045213521983758221\dots = \frac{410105312}{150869313}$
22.  $2.718281828459045254624795027092\dots = \frac{438351041}{161260336}$
23.  $2.718281828459045234757560631479\dots = \frac{848456353}{312129649}$
24.  $2.718281828459045235379013372772\dots = \frac{14013652689}{5155334720}$
25.  $2.718281828459045235343535532787\dots = \frac{14862109042}{5467464369}$
26.  $2.718281828459045235360753230188\dots = \frac{28875761731}{10622799089}$
27.  $2.718281828459045235360274593941\dots = \frac{534625820200}{196677847971}$
28.  $2.718281828459045235360299120911\dots = \frac{563501581931}{207300647060}$
29.  $2.718281828459045235360287179900\dots = \frac{1098127402131}{403978495031}$
30.  $2.718281828459045235360287478611\dots = \frac{22526049624551}{8286870547680}$
31.  $2.718281828459045235360287464726\dots = \frac{23624177026682}{8690849042711}$
32.  $2.718281828459045235360287471503\dots = \frac{46150226651233}{16977719590391}$
33.  $2.718281828459045235360287471349\dots = \frac{1038929163353808}{382200680031313}$
34.  $2.718281828459045235360287471355\dots = \frac{1085079390005041}{399178399621704}$
35.  $2.718281828459045235360287471352\dots = \frac{2124008553358849}{781379079653017}$
36.  $2.718281828459045235360287471352\dots = \frac{52061284670617417}{19152276311294112}$

One can with no problem compute much bigger convergents. Let's get the 200th convergent. It turns out to have the same first 268 digits after the decimal point as  $e-1$ . Higher convergents get more and more digits in proportion to their index: the 500th convergent already gets 799 digits correct! To allow speedy compilation of the source of this document when the need arises, I limit here to the 200th convergent.

```
\fdef\z {\xintCnToF {199}{\cn}}%
\begingroup\parindent 0pt \leftskip 2.5cm
\indent\llap {Numerator = }\printnumber{\xintNumerator\z}\par
\indent\llap {Denominator = }\printnumber{\xintDenominator\z}\par
\indent\llap {Expansion = }\printnumber{\xintTrunc{268}\z}\dots\par\endgroup
```

Numerator = 5689640388718962675975238923158078752938890176679174460572320245471922969611182  
23017524386017499531081773136701241708609749634329382906

Denominator = 3311238176697376193062563608163567533654688237293144381562056154632466597285812  
86546133769206314891601955061457059255337661142645217223

Expansion = 1.71828182845904523536028747135266249775724709369995957496696762772407663035352  
4759457138217852516642742746639193200305992181741359662904357290033429526059562  
3073813232862794349076323382988075319525101901157383418793070215408914993488412  
675092447614606680822648001684774118...

One can also use a centered continued fraction: we get more digits but there are also more computations as the numerators may be either 1 or -1.



This documentation has been compiled without the source code, which is available in the separate file:  
`sourcexint.pdf`,  
which will open in a PDF viewer via `texdoc sourcexint.pdf`. To produce a single file including both the user documentation and the source code, run `tex xint.dtx` to generate `xint.tex` (if not already available), then edit `xint.tex` to set the `\NoSourceCode` toggle to `0`, then run thrice `latex` on `xint.tex` and finally `dvipdfmx` on `xint.dvi`. Alternatively, run `pdflatex` either directly on `xint.dtx`, or on `xint.tex` with `\NoSourceCode` set to `0`.