

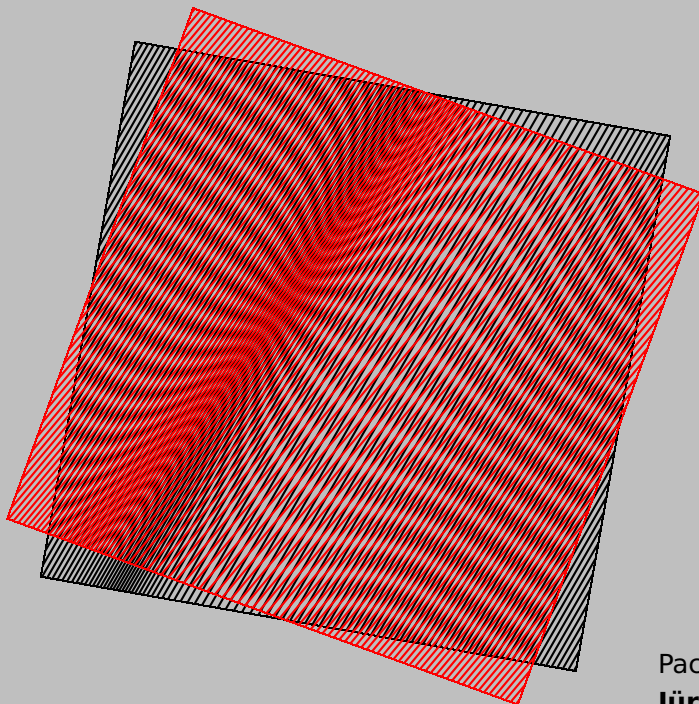
# PSTricks

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## **pst-moire v 1.0**

A PSTricks package to draw moiré patterns

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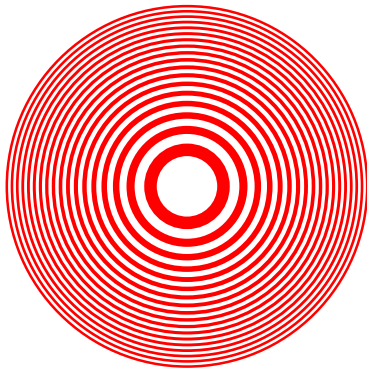
The `pst-moire` package makes it possible to very simply create a variety of patterns obtained either by dragging one pattern on another, or by rotating one on the other. Moiré effects sometimes look very interesting. This document provides the necessary commands and divers examples.

For the interested user, we present a section **Theory** (see pages 21-30) for the mathematical background of moiré patterns.

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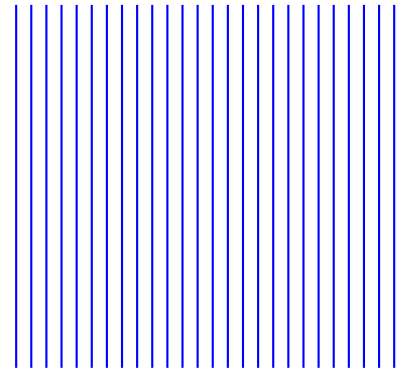
## 1 The provided patterns

Bands of Fresnel



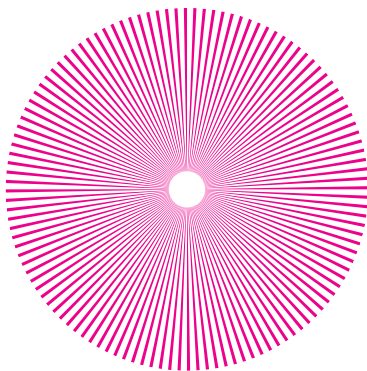
type=Fresnel

Equidistant lines



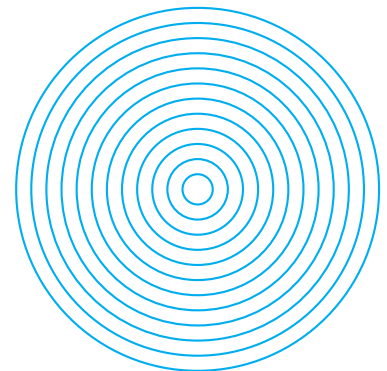
type=linear

Radii with  $3^\circ$



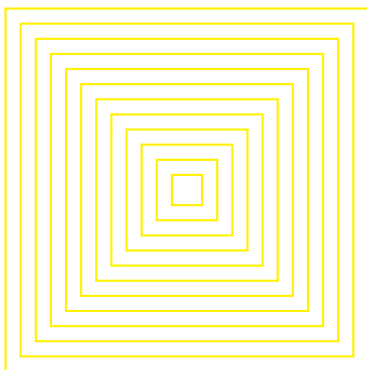
type=radial

Concentric circles



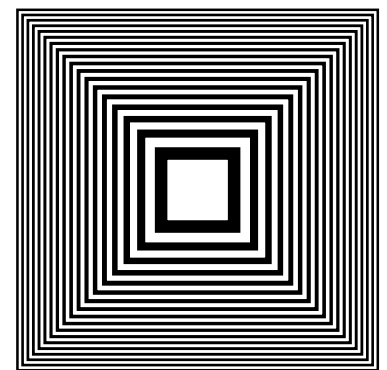
type=circle

Squares



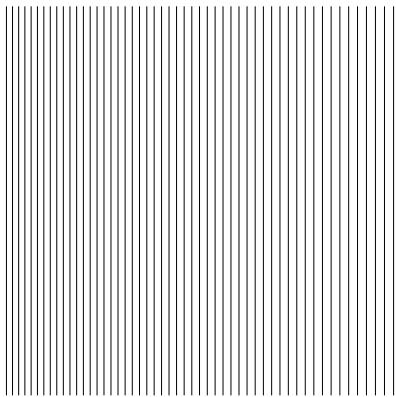
type=square

Squares of Newton



type=Newton

The pattern of H. Bouasse



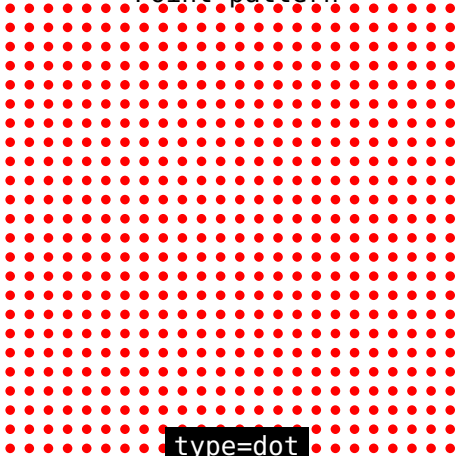
type=Bouasse

The pattern of Gauss



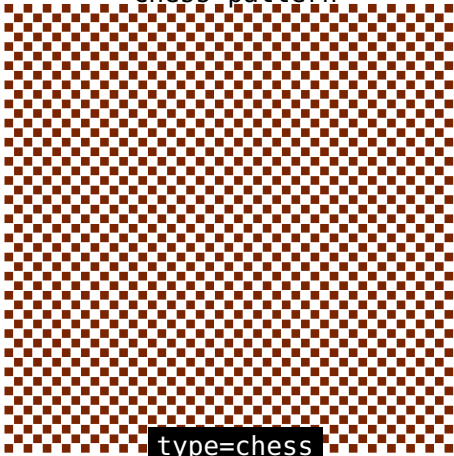
type=Gauss

Point pattern



type=dot

Chess pattern



type=chess

## 2 The command \psmoire

`\psmoire [Options] (x,y)`

The command `\psmoire` contains the options `type=`, `Rmax=`, `scale=`, `Alpha=`, `rotate=`, and `E=`. The optional argument `(x,y)` gives the x and y center of the image. If not chosen `(0,0)` is taken by default.

Name	Default	Meaning
<code>type</code>	Fresnel	The type of pattern
<code>Rmax</code>	6	The largest radius of the circles (in cm)
<code>scale</code>	1	Scaling factor for the image
<code>Alpha</code>	70	Slope of the lines for [ <code>type=Gauss</code> ]
<code>rotate</code>	0	Rotation of the figure in degrees.
<code>E</code>	0.25	Distance between two lines for [ <code>type=Gauss</code> ]

### Note:

- Thickness of the lines/circles: done with the usual PSTricks key `linewidth=1mm` (for example).
- Color of the lines/circles: done with the usual PSTricks key `linecolor=red` (for example).

We set the type of pattern like: `type=Gauss` (for example).

`\psmoire[options,type=Gauss](x,y)`

If no position coordinate is specified, the center of the image is placed at `(0,0)`. The thickness parameter does not effect the following types: Fresnel, Newton and radial

### 3 Opacity and Blendmodes

If we want to highlight the color of the intersecting area of the lines of two or more overlapping moiré patterns differently, we can either use *opacity* or *blendmodes*.

#### 3.1 Opacity

In case we want to add some opacity to the lines of the moiré patterns, we just set, i. e.

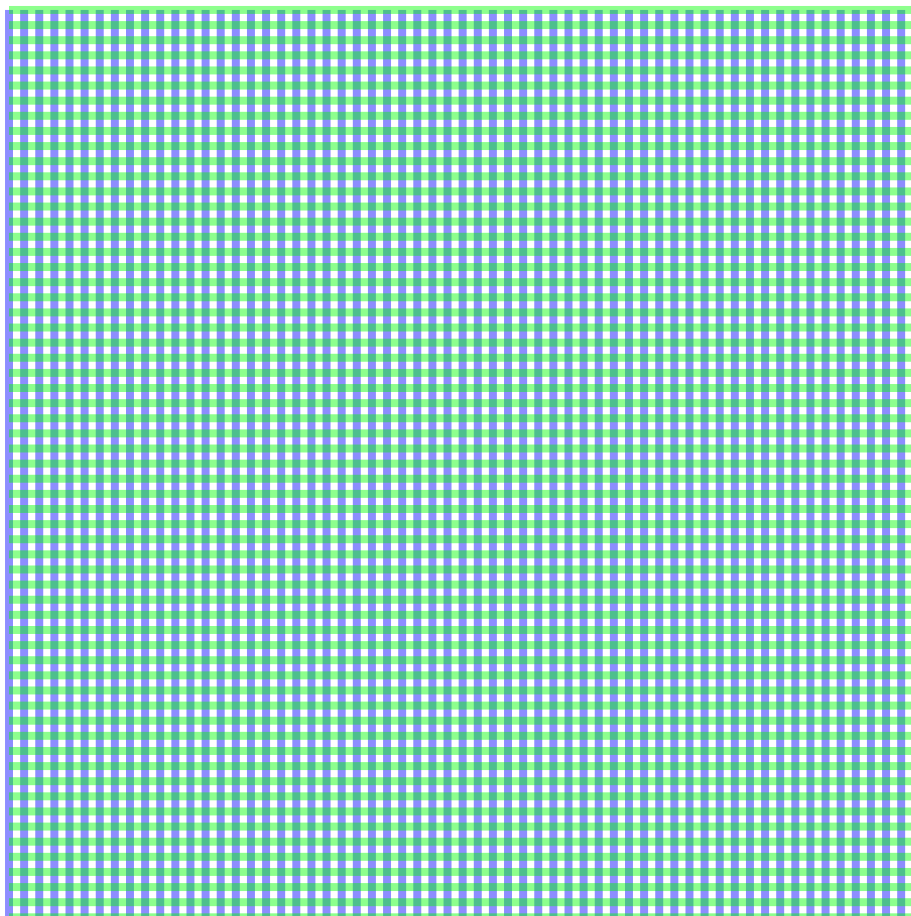
```
\pstVerb{%
0.45 .setopacityalpha
}
```

within the `\pspicture` environment.

Distiller users set:

```
\pstVerb{%
[ /ca 0.45 /CA 0.45 /SetTransparency pdfmark
}
```

**Note:** The value of the opacity needs to be between 0 and 1.



```
\begin{pspicture}(-6,-6)(6,6)
\pstVerb{%
0.45 .setopacityalpha
}
\psmoire[type=linear,linecolor=blue,linewidth=3pt]
\psmoire[type=linear,linecolor=green,linewidth=3pt,rotate=90]
\end{pspicture}
```

### 3.2 Blendmodes

In case we want to overlap various moiré patterns we can use the following blendmodes:

/Lighten, /Darken, /Normal, /Multiply, /Screen, /Overlay, /ColorDodge,  
/ColorBurn, /HardLight, /SoftLight, /Difference, /Exclusion, /Saturation,  
/Color, /Luminosity.

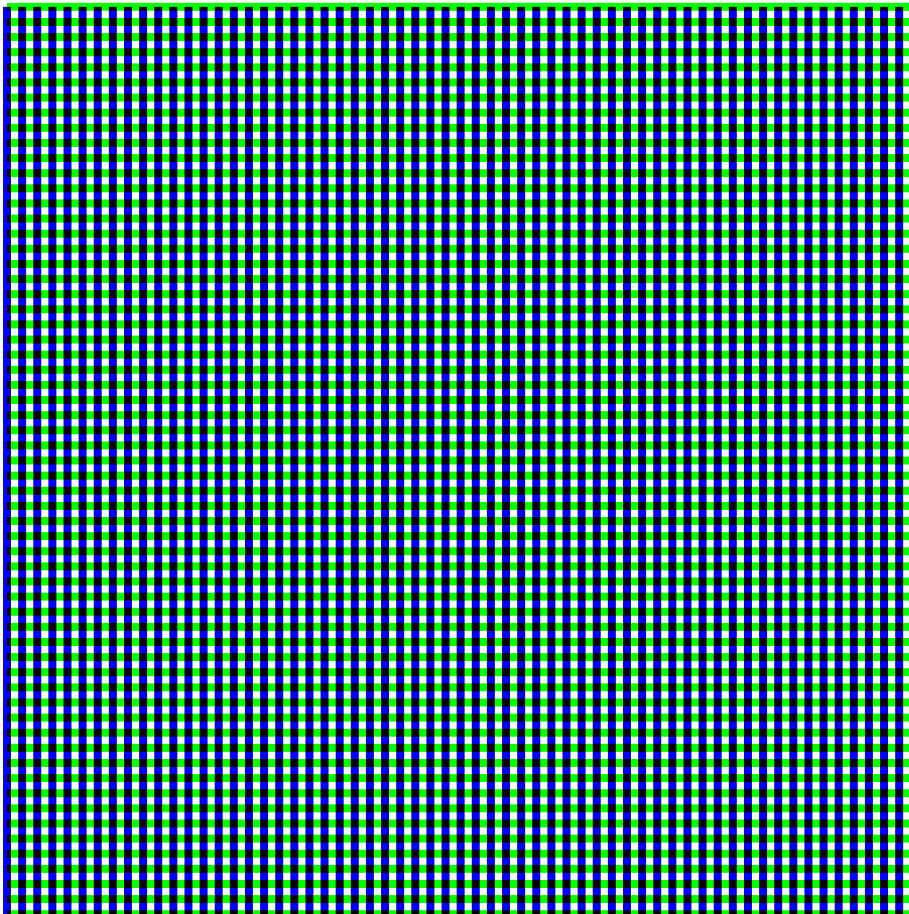
We just set, i. e.

```
\pstVerb{%  
/Darken .setblendmode  
}
```

within the \pspicture environment.

Distiller users set:

```
\pstVerb{%  
[ /BM /Darken /ca 1 /CA 1 /SetTransparency pdfmark  
}
```

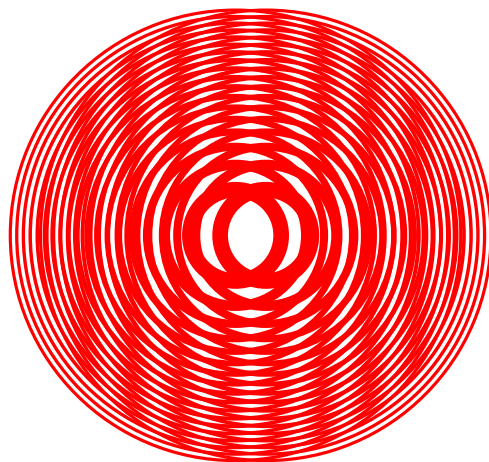


```
\begin{pspicture}(-6,-6)(6,6)  
\pstVerb{%  
/Darken .setblendmode  
}  
\psmoire[type=linear,linecolor=blue,linewidth=3pt]  
\psmoire[type=linear,linecolor=green,linewidth=3pt,rotate=90]  
\end{pspicture}
```



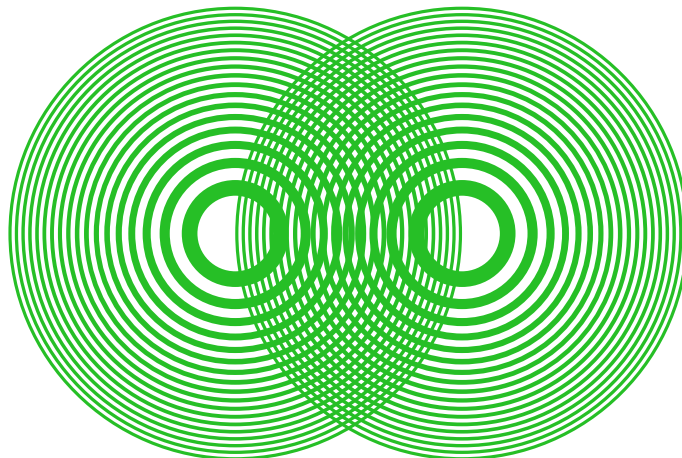
## 4 Examples

Bands of Fresnel 1



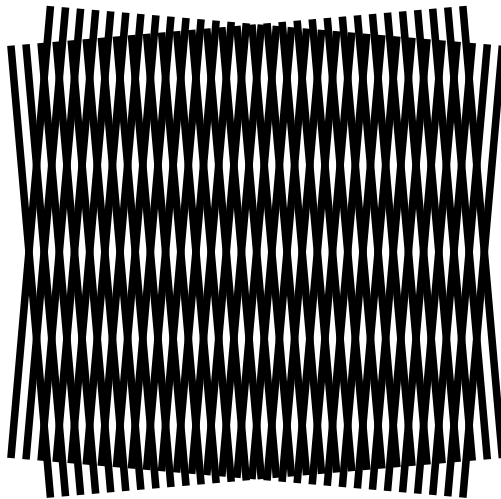
```
\begin{pspicture}(-4,-4)(4,4)
\psmoire[linecolor=red,scale=0.5](-0.2,0)
\psmoire[linecolor=red,scale=0.5](0.2,0)
\end{pspicture}
```

Bands of Fresnel 2



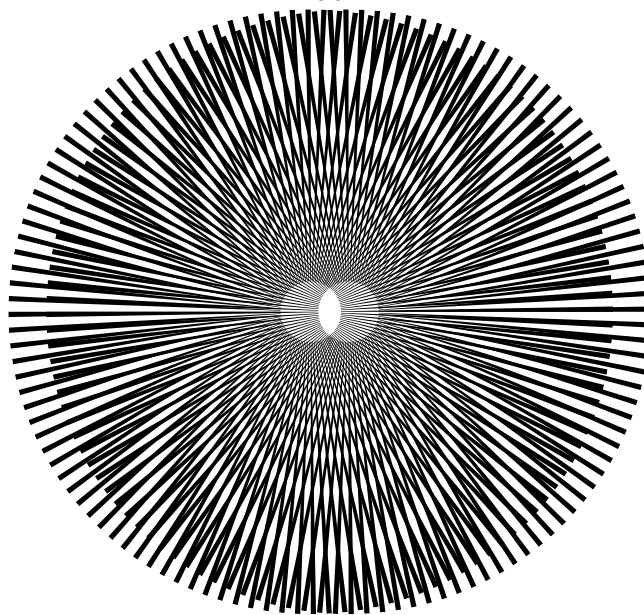
```
\begin{pspicture}(-4,-4)(4,4)
\psmoire[linecolor=green,scale=0.5](-1.5,0)
\psmoire[linecolor=green,scale=0.5](1.5,0)
\end{pspicture}
```

Lines



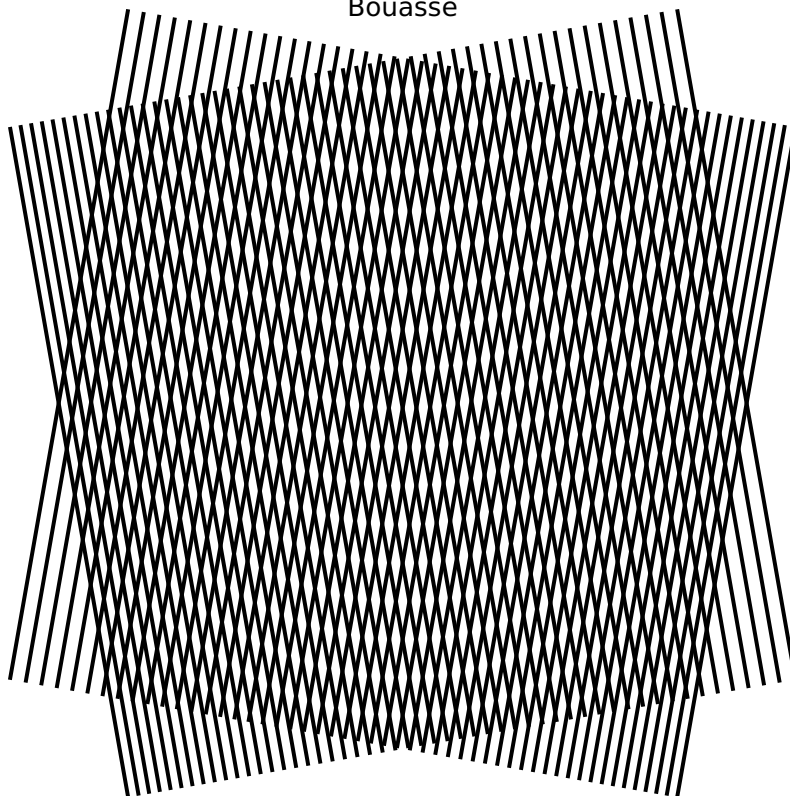
```
\begin{pspicture}(-4,-4)(4,4)
\psmoire[scale=0.5,type=linear,rotate=5,linewidth=0.1]
\psmoire[scale=0.5,type=linear,rotate=-5,linewidth=0.1]
\end{pspicture}
```

Radii



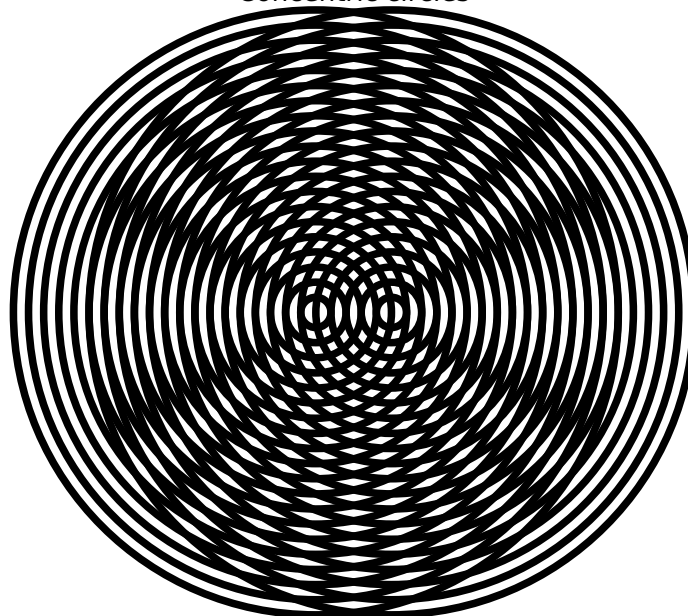
```
\begin{pspicture}(-4,-4)(4,4)
\psmoire[Rmax=4,type=radial](-0.25,0)
\psmoire[Rmax=4,type=radial](0.25,0)
\end{pspicture}
```

Bouasse



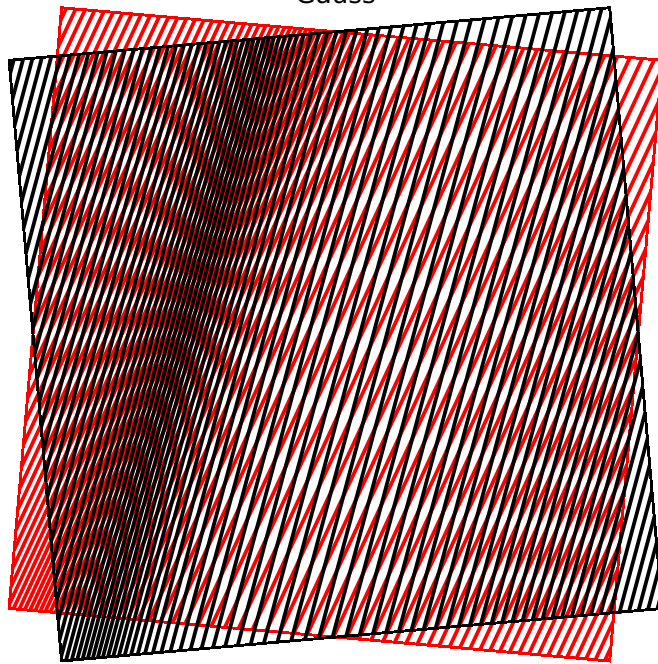
```
\begin{pspicture}(-6,-6)(6,6)
\psmoire[type=Bouasse,rotate=10]
\psmoire[type=Bouasse,rotate=170]
\end{pspicture}
```

Concentric circles



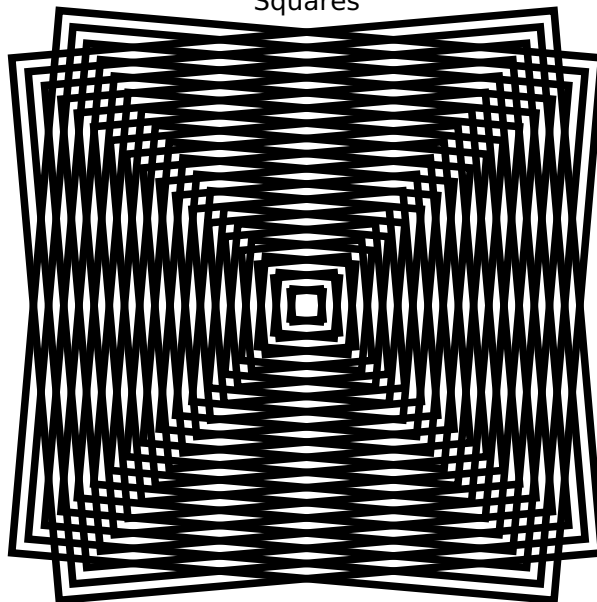
```
\begin{pspicture}(-5,-5)(5,5)
\psmoire[Rmax=5,type=circle](-0.2,0)
\psmoire[Rmax=5,type=circle](0.2,0)
\end{pspicture}
```

Gauss



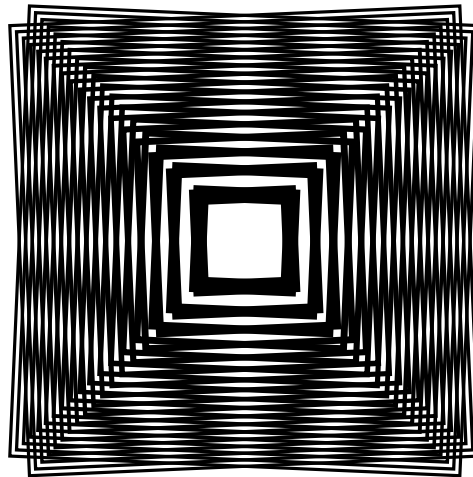
```
\begin{pspicture}(-5,-5)(5,5)
\psmoire[type=Gauss,rotate=-10]
\psmoire[type=Gauss]
\end{pspicture}
```

Squares



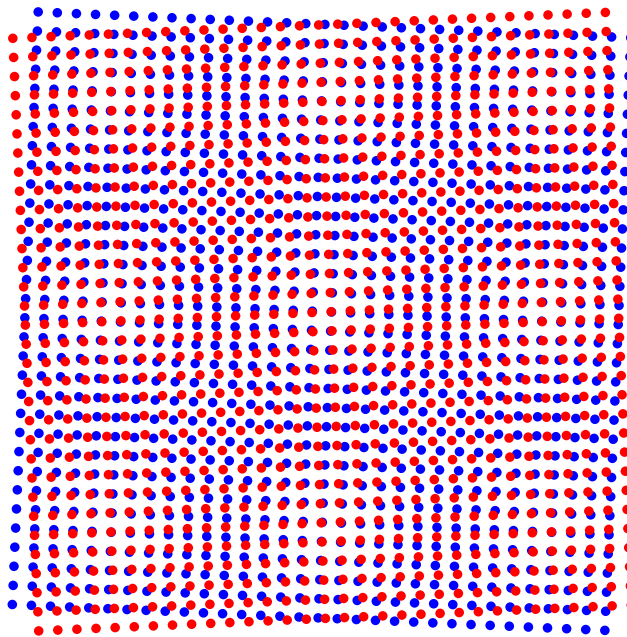
```
\psmoire[type=square,rotate=-5]
\psmoire[type=square,rotate=5]
```

Squares of Newton



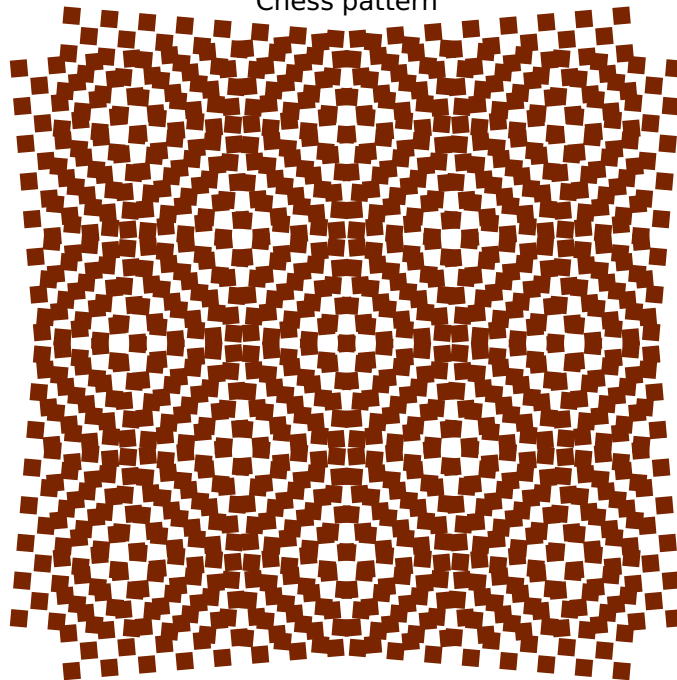
```
\begin{pspicture}(-4,-4)(4,4)
\psmoire[type=Newton,rotate=-2.5]
\psmoire[type=Newton,rotate=2.5]
\end{pspicture}
```

Point pattern



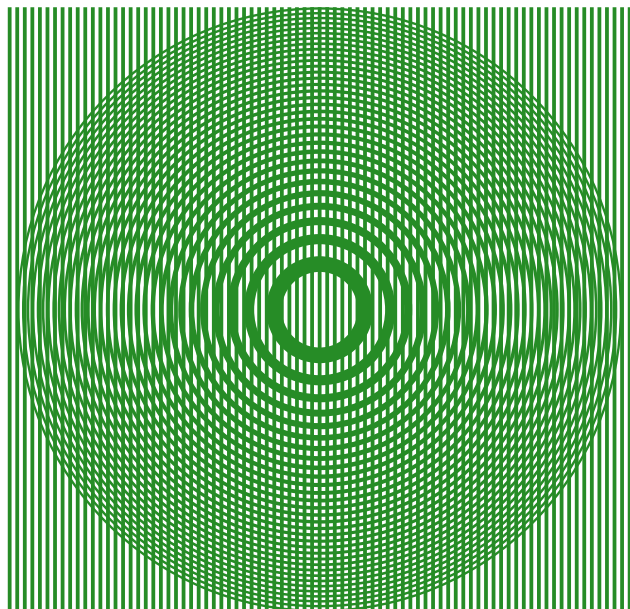
```
\begin{pspicture}(-5,-5)(5,5)
\psmoire[type=dot,linecolor=blue,rotate=-2.5]
\psmoire[type=dot,rotate=2.5,linecolor=red]
\end{pspicture}
```

Chess pattern



```
\psset{Rmax=4,dotstyle=square*,dotsize=0.25cm}
\begin{pspicture}(-4,-4)(4,5)
\psmoire[type=chess,rotate=-5]
\psmoire[type=chess,rotate=5]
\end{pspicture}
```

Bands of Fresnel + lines



```
\begin{pspicture}(-4,-4)(4,4)
\psmoire[type=Fresnel]
\psmoire[type=linear]
\psmoire[type=linear](-0.1,0)
\end{pspicture}
```

These rotating moirés were obtained with the use of the `pst-lens` package. It is the reproduction, with the tools of PSTricks, of the photograph 6, page 137 of the book “*Les phénomènes naturels*” of the Library **Pour la Science**, Berlin (1978). This photograph is accompanied by the following comment:

*« Ces moirés tournants apparaissent quand les lentilles placées sur une trame et observées avec une autre trame identique à la première. La grosse lentille (convergente) réduit la trame inférieure, tandis que la petite lentille (divergente) la grossit. En conséquence, les moirés obtenus ont des sens de rotation opposés. Une figure de moiré ondulée traduit la présence d’aberrations optiques dans la lentille. »*

**Animation:** Big lens: magnification of 1.2, small lens: magnification of 0.8

```
\psset{unit=0.5}
\begin{animateinline}[controls,palindrome,
    begin={\begin{pspicture}(-8.5,-8.5)(8.5,8.5)},
    end={\end{pspicture}}]{10}% 10 image/s
\multiframe{20}{i=-10+1}{%
\psset{LensHandle=false,LensShadow=false}
\psset{linecolor=red,linewidth=0.1,type=linear}
\psmoire%
\PstLens[LensMagnification=1.2,LensSize=2](1,1.5){\psmoire}
\PstLens[LensMagnification=0.8,LensSize=1.5](-2,-2){\psmoire}
\psmoire[linecolor=black,rotate=i]}
\end{animateinline}
```

## 5 Animations

Some interactive moiré JavaScript based applications can be found on:

<https://melusine.eu.org/syracuse/G/pstricks/pst-moire/moirej/>

### Animation 1:

```
\begin{animateinline}[%
  controls,palindrome,
  begin={\begin{pspicture}(-6,-6)(6,6)},
  end={\end{pspicture}}
]{5}% 5 image/s
\multiframe{36}{i=0+1}{%
\psmoire[type=Newton,rotate=-\i]%
\psmoire[type=Newton,rotate=\i]%
}
\end{animateinline}
```



**Animation 2:**

```

\begin{animateinline}[%
  controls,palindrome,
  begin={\begin{pspicture}(-6,-6)(6,6)},
  end={\end{pspicture}}
]{10}% 10 image/s
\multiframe{36}{r=0+0.1}{%
\psmoire[scale=0.85,type=linear,rotate=-\r,linewidth=0.1,linecolor=red](0,0)%
\psmoire[scale=0.85,type=linear,rotate=\r,linewidth=0.1,linecolor=red](0,0)%
}
\end{animateinline}

```

**Animation 3:**

```
\begin{animateinline}[%
  controls,palindrome,
  begin={\begin{pspicture}(-6,-6)(6,6)},
  end={\end{pspicture}}
]{5}% 5 image/s
\multiframe{20}{r=0+0.025}{%
\psset{linewidth=2.5pt}
\psmoire[type=circle,linecolor=moire1](0,\r)
\psmoire[type=circle,linecolor=moire2](0,-\r)
\psmoire[type=circle,linecolor=moire3](\r,0)
\psmoire[type=circle,linecolor=moire4]( -\r,0)
}
\end{animateinline}
```

**Animation 4:**

```
\begin{animateinline}[%
  controls,palindrome,
  begin={\begin{pspicture}(-6,-6)(6,6)},
  end={\end{pspicture}}
]{5}% 5 image/s
\multiframe{30}{r=0+0.025}{%
\psset{linewidth=1pt}
\psmoire[type=radial,linecolor=red](\r,0)
\psmoire[type=radial,linecolor=green]( -\r,0)
\psmoire[type=radial,linecolor=blue](0,-\r)
}
\end{animateinline}
```

**Animation 5:**

This idea came from a post card “**turn the top part**”, bought in a boutique of the centre Beaubourg in Paris, showing the phenomenon of the moiré effect and redesigned with PSTricks.

```

\def\myMoiré{%
\psset{dimen=inner,linewidth=0pt}
\def\carre{%
\nodes{AL}{0,0}(-1.5,1.5)(-1.5,1.2)(-1.5,0.9)(-1.5,0.7)(-1.5,0.4)(-1.5,0.2)(-1.5,0)%
(-1.5,-0.2)(-1.5,-0.4)(-1.5,-0.6)(-1.5,-0.75)(-1.5,-0.9)(-1.5,-1.05)(-1.5,-1.15)%
(-1.5,-1.25)(-1.5,-1.3)(-1.5,-1.4)(-1.5,-1.45)
\nodes{AR}{0,0}(1.5,-1.5)(1.5,-1.2)(1.5,-0.9)(1.5,-0.7)(1.5,-0.4)(1.5,-0.2)(1.5,0)%
(1.5,0.2)(1.5,0.4)(1.5,0.6)(1.5,0.75)(1.5,0.9)(1.5,1.05)(1.5,1.15)(1.5,1.25)%
(1.5,1.3)(1.5,1.4)(1.5,1.45)
\multido{\iA=1+2,\iB=2+2}{9}{\pspolygon*(AL\iA)(AR\iA)(AR\iB)(AL\iB)}%
\nodes{BL}{0,0}(-1.2,1.5)(-0.9,1.5)(-0.7,1.5)(-0.4,1.5)(-0.2,1.5)(0,1.5)(0.2,1.5)%
(0.4,1.5)(0.6,1.5)(0.75,1.5)(0.9,1.5)(1.05,1.5)(1.15,1.5)(1.25,1.5)(1.3,1.5)%
(1.4,1.5)(1.45,1.5)(1.5,1.5)
\nodes{BR}{0,0}(1.2,-1.5)(0.9,-1.5)(0.7,-1.5)(0.4,-1.5)(0.2,-1.5)(0,-1.5)(-0.2,-1.5)%
(-0.4,-1.5)(-0.6,-1.5)(-0.75,-1.5)(-0.9,-1.5)(-1.05,-1.5)(-1.15,-1.5)(-1.25,-1.5)%
(-1.3,-1.5)(-1.4,-1.5)(-1.45,-1.5)(-1.5,-1.5)
\multido{\iA=1+2,\iB=2+2}{9}{\pspolygon*(BL\iA)(BR\iA)(BR\iB)(BL\iB)}%
\def\half{%
\rput(0,0){\carre}
\rput(-3,0){\psscalebox{-1 1}{\carre}}
}
\def\pattern{%
\rput(0,0){\half}
\rput(0,-3){\psscalebox{1 -1}{\half}}
}
\multido{\iA=0+6}{2}{\multido{\iB=0+6}{3}{\rput(\iA,\iB){\pattern}}}}

\psset{unit=0.75}
\begin{animateinline}[controls,loop,
  begin={\begin{pspicture}(-7,-19)(10,5)},
  end={\end{pspicture}}]{10}% 10 frames/s (velocity of the animation)
\multiframe{11}{i=0+2}{% number of frames
\rput(0,0){\myMoiré}
\psrotate(1.5,-7){\i}{\myMoiré}
}
\multiframe{21}{i=20+2}{%
\rput(0,0){\myMoiré}
\psrotate(1.5,-7){\i}{\myMoiré}
}
\multiframe{10}{i=-20+2}{%
\rput(0,0){\myMoiré}
\psrotate(1.5,-7){\i}{\myMoiré}
}
\end{animateinline}

```

## 6 Theory—for the interested user

### 6.1 The contribution of “éditions Kangourou”

“Le Kangourou des mathématiques”: <http://www.mathkang.org/> published a revue in 2002, “*Les malices du Kangourou*” that contains a magnificent article from pages 18 to 26 titled “*Mirifiques et mirobolants moirés*” and on the back cover “*La règle à moirer*” (“The ruler”). The article and the ruler are available at the following addresses:

<http://www.mathkang.org/cite/moires9p.pdf>  
<http://www.mathkang.org/cite/moirer.html>

The ruler can be purchased at the following address:

<http://www.mathkang.org/catalogue/prodmoir.html>

In the article the sketches are very beautiful and the part “*Mathématisation du phénomène*” is remarkable! It contains the following moirés:

- a network of parallel straight lines superposed each with a rotation in different direction;
- shifted superposition of two networks consisting of radial rays (or rather sectors);
- shifted superposition of two frames of Fresnel rings, well known as Newton’s rings observed in optics.

## 6.2 The contribution of Henri Bouasse

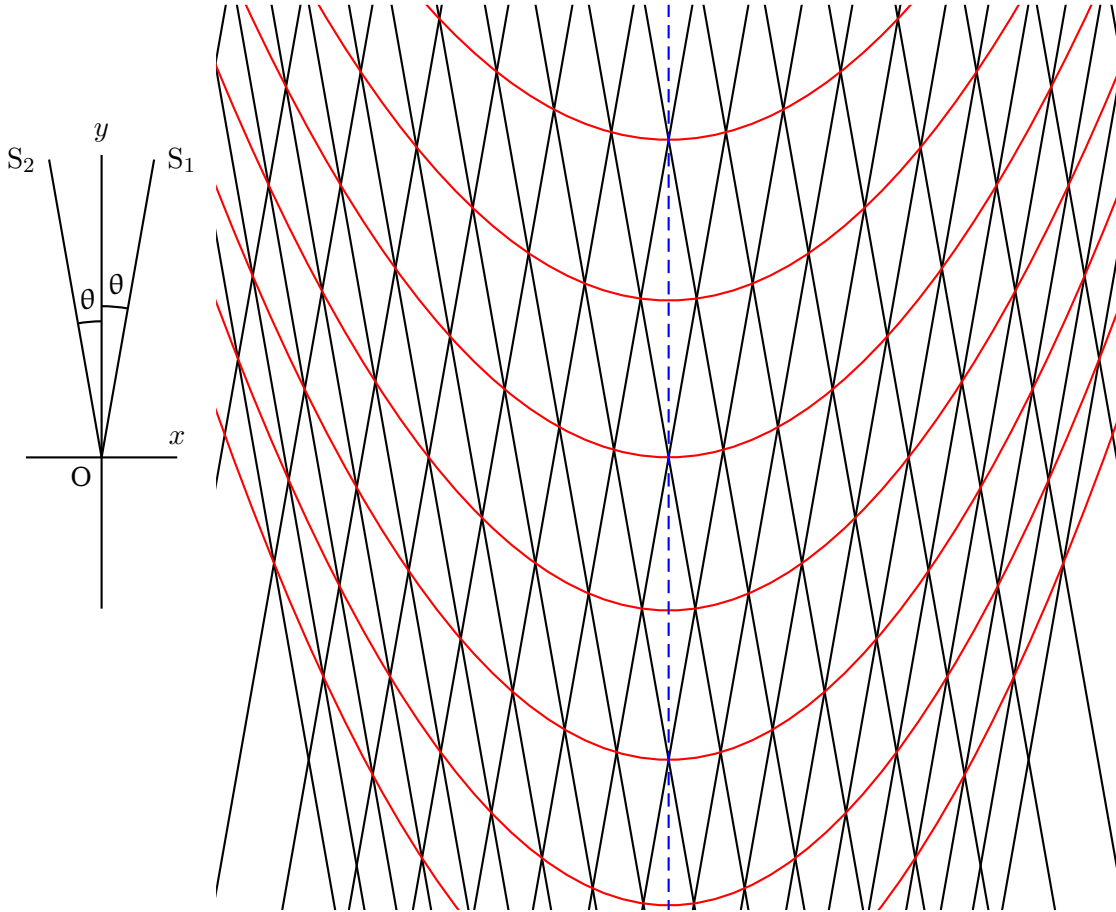
This is the chapter of his book *Vision et reproduction des formes et des couleurs* published at Librairie Delagrave in Paris in 1917. His demonstration and the diagram within his book have been reproduced here:

### Parallel straight lines

1° — Consider two straight lines respectively parallel:

$$x \cos \theta + y \sin \theta = bt + ct^2 \quad x \cos \theta - y \sin \theta = b\tau - c\tau^2 \quad (1)$$

For  $t = \tau = 0$ , we get the two lines  $OS_2$  and  $OS_1$ ; they obviously have the same angle  $\theta$  with the axis  $Oy$ .



**Figure 1:** Moiré: parallel lines

The curves of the intersection points, which correspond to the small diagonals of the parallelograms, satisfy the following condition:

$$t - \tau = \mu = \text{constant}$$

Adding and reordering the equations (1):

$$\begin{aligned} 2x \cos \theta &= (b + c\mu)(t + \tau) \\ 2y \sin \theta &= b\mu + c(t^2 + \tau^2) = b\mu + c(\mu^2 + 2t\tau) \end{aligned}$$

To complete the elimination, we will use the following relation:

$$(t + \tau)^2 - 4t\tau = \mu^2$$

Thus:

$$\frac{4x^2 \cos^2 \theta}{(b + c\mu)^2} - \frac{4y \sin \theta}{c} + \frac{2b\mu}{c} + \mu^2 = 0 \quad (2)$$

The wanted curves are parabolas which have the  $Oy$  axis in common.

Its vertices are given by:

$$y = \frac{\mu(2b + c\mu)}{4 \sin \theta} \quad (3)$$

2° — The parameter  $c$  is small compared to the parameter  $b$ , so the equations simplify.

The equation (2) becomes:

$$\frac{4x^2 \cos^2 \theta}{b^2} - 4y \sin \theta + 2b\mu = 0$$

This is the same parabola for all the values of  $\mu$  sliding parallelly to  $Oy$ . The vertices are given by:

$$y = \mu \frac{b}{2 \sin \theta} \quad (4)$$

The radius of curvature at the vertex of the parabola is:

$$R = \frac{b^2 \sin \theta}{2c \cos^2 \theta}$$

If the parallel straight lines are equidistant ( $c = 0$ ), the parabolas degenerate to straight lines (4); in other words, the radius of curvature becomes infinite.

3° — To make an experiment, we trace with “China ink” on a paper 51 parallel lines with a length of i. e. 20 cm, where the distance between two adjacent lines increases from 2 mm (between the first two lines) to 3 mm (between the last two lines), following the formula:

$$s = 2t + 0.01t^2$$

We take a photo by reducing to the half or a quarter. We generate two diapositives<sup>1</sup>. We realize the phenomenon when placing one over the other by rotating one of them.

We think that if you had followed the given instructions, you might be as well convinced—as we are—it would have been a pity to have left this beautiful demonstration “of that time” in oblivion!

### 6.3 The humble contributions of our group

These demonstrations contain:

1. the moirés of Newton. In fact it is a construction similar to that of the Fresnel rings. Here the progression of the squares is such that the areas between two consecutive squares are equal to the area of the central square. One out of every two intervals is made opaque. The resulting moiré figures are equilateral hyperbolas.
2. the moirés obtained by the superposition of Fresnel rings and a network of parallel lines result as well in Fresnel rings.

The source files (L<sup>A</sup>T<sub>E</sub>X) and pdf are found within the repository:

<http://melusine.eu.org/syracuse/G/pstricks/pst-moire/moiredoc/>

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<sup>1</sup> spelling of the time.

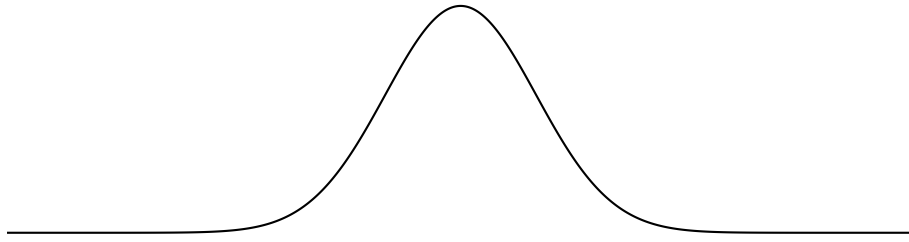
## 6.4 The construction of a Gauss network

This method is discussed on page 136 of the book “*Les phénomènes naturels*” edited in 1978 by the revue “*Pour la Science*” and distributed by the Berlin editions.

« The Gauss network is obtained by drawing a series of equidistant vertical lines on a Gaussian curve, then by drawing parallel oblique lines passing through the points of intersection between the vertical lines and the Gaussian curve. »

### Gaussian curve

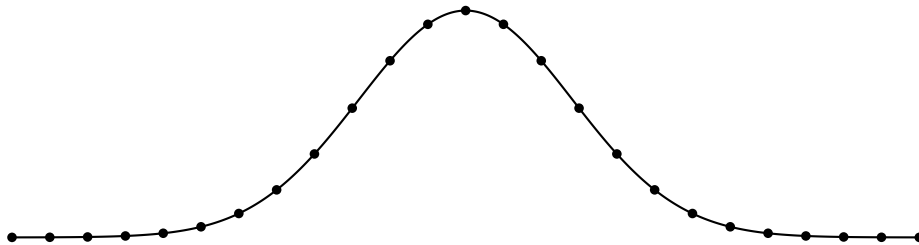
$$y = ae^{-(kx)^2}$$



### Determination of the points of intersection

The equidistant vertical line network has for equation:  $x = ne$ ,  $e$  is the spacing et  $n$  an integer.

The ordinates of the intersection points are:  $y_n = ae^{-(kne)^2}$ . Within the following figure, we set the spacing to 0.5.



### Drawing the network of the stright lines

We determine the equations of the straight lines passing through these points and which are inclined by an angle  $\alpha$  with respect to the horizontal.

The general equation of such a line is given by:  $y = x \tan(\alpha) + b$ , we determine  $b$  to go through one of the previous points.

$$ne \tan(\alpha) + b = ae^{-(kne)^2}$$

so we get  $b$ .

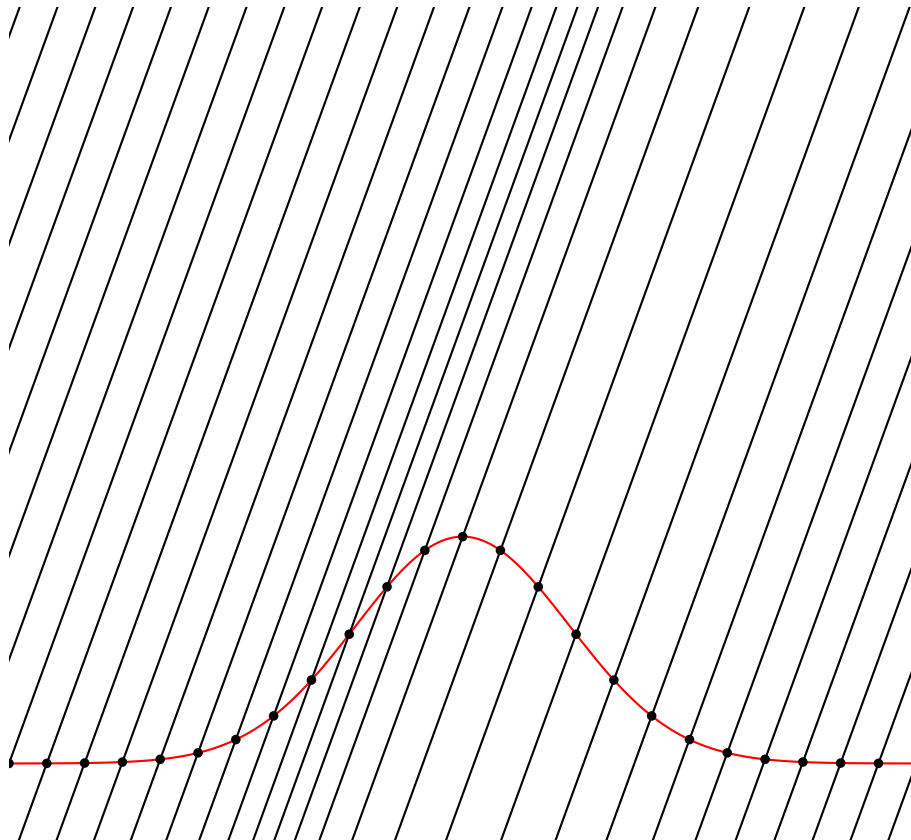
$$b = ae^{-(kne)^2} - ne \tan(\alpha)$$



For every value of  $n$  we get a straight line.

$$y = x \tan(\alpha) + ae^{-(kne)^2} - ne \tan(\alpha)$$

Let's draw some of these straight lines. Setting  $a = 3$ ,  $k = 0.5$ ,  $-20 < n < +20$ ,  $e = 0.5$  and  $\alpha = 70^\circ$

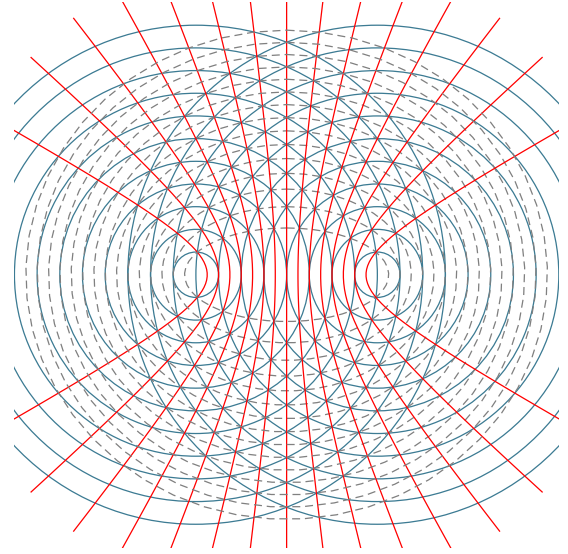
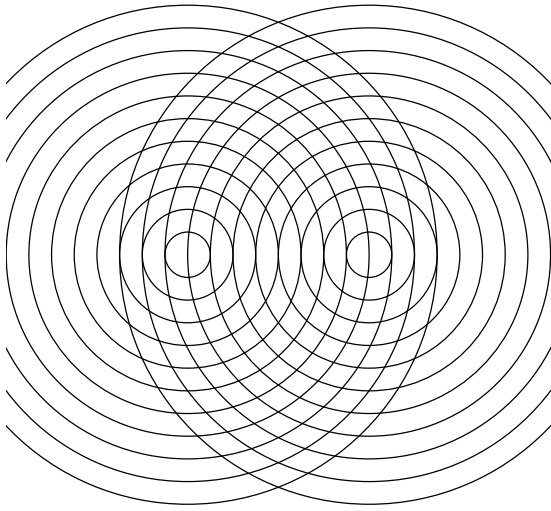


```
\begin{pspicture*}(-6,-1)(6,10)
\parametricplot[plotpoints=1000,linecolor=red]{-6}{6}{%
    t
    3 2.71828 0.5 t mul dup mul neg exp mul}
\pstVerb{/A 3 def
    /K 0.5 def
    /E 0.5 def
    /Alpha 70 def
    /m {Alpha dup sin exch cos div} bind def % pente de la droite
}%
\multido{\n=-20+1}{41}{%
\pnode(! \n\space E mul % x
    A 2.71828 K \n\space E mul mul dup mul neg exp mul){A}
\psdot(A)
\rput(A){\psline(! -4 -4 m mul)(! 4 4 m mul)}
}
\end{pspicture*}
```

The last step is to translate these lines into PostScript code.

## 6.5 Some moiré figures

### Circles + Circles



#### Mathematization

Equidistant radii that increase like:  $r_n = n \cdot a$ , with  $a > 0$ .

We have

$$(x - c)^2 + y^2 = k^2 p^2$$

$$(x + c)^2 + y^2 = k^2 q^2$$

It is necessarily:  $p - q = m \in \mathbb{Z}$ , thus

$$p = \frac{1}{k} \sqrt{(x - c)^2 + y^2}$$

$$q = \frac{1}{k} \sqrt{(x + c)^2 + y^2}$$

and  $p - q = m$

$$\sqrt{(x - c)^2 + y^2} - \sqrt{(x + c)^2 + y^2} = km$$

$$r_p = p \cdot k \quad r_q = q \cdot k$$

$$r_p - r_q = (p - q) \cdot k = m \cdot k$$

The points of the moiré curves are such that the difference in distances to the two centers is constant. The moiré curves are hyperbolas focussing the centers of circles.

We pose:  $a = \frac{km}{2}$  and  $b^2 = c^2 - a^2$ . The equations of this family of hyperbolas are written like:

$$x = a \cosh(t)$$

$$y = b \sinh(t)$$

or:

$$x = \frac{a}{\cos(t)}$$

$$y = b \tan(t)$$

If we go from a point of intersection  $(p, q)$  to a point  $(p + 1, q - 1)$ , the sum of the distances remains constant.

As a result, we say:

$$r_p + r_q = (p + q) \cdot k = n \cdot k$$

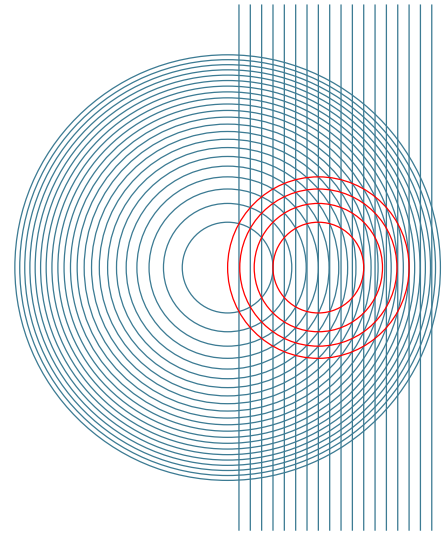
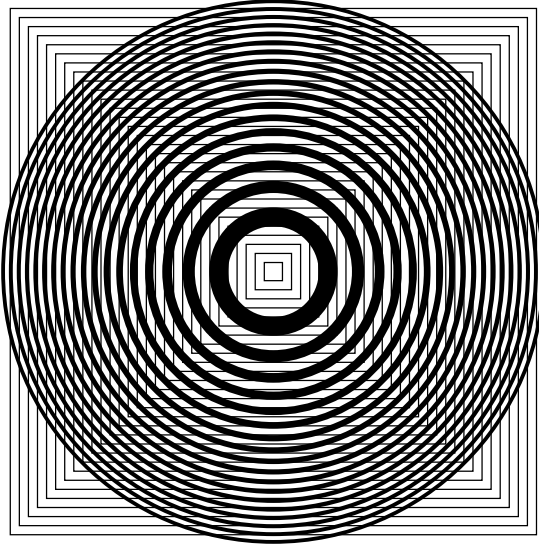
This family of moiré curves are ellipses with equations like:

$$x = a \cos(t) \quad y = b \sin(t)$$

with:

$$b^2 = a^2 - c^2$$

### Squares + Fresnel rings



#### Mathematization

The abscissa of the edges of the square with  $x > 0$  increase with:  $x_n = a \cdot n$ , with  $a > 0$

The radii increase with:  $r_n = \sqrt{n}$ .

We have

$$x = ap$$

$$x^2 + y^2 = q$$

$$p = \frac{x}{a}$$

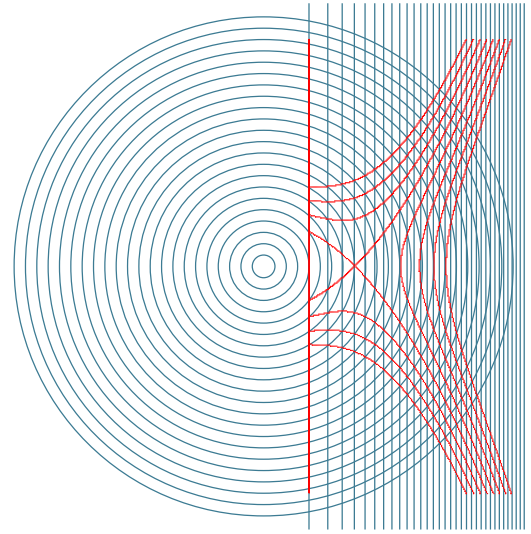
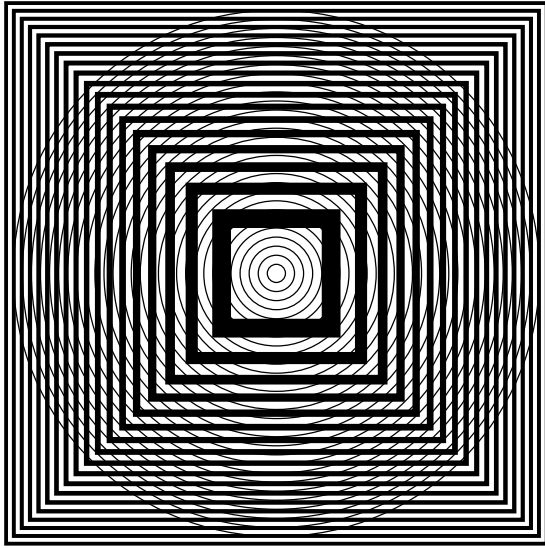
$$q = x^2 + y^2$$

On a curve of moiré, we verify:  $p - q = m \in \mathbb{Z}$ :

$$\left(x - \frac{1}{2a}\right)^2 + y^2 = m + \frac{1}{4a^2}$$

This family of moiré curves are circles with the center at  $(\frac{1}{2a}, 0)$  and with a radius of  $r_m = \sqrt{m + \frac{1}{4a^2}}$

## Circle + Squares of Newton



### Mathematization

The abscissa of the edges of the square with  $x > 0$  increase with:  $x_n = \sqrt{n}$ .

The radii increase with:  $r_n = n \cdot a$ , with  $a > 0$ .

We have:

$$x = \sqrt{p}$$

$$x^2 + y^2 = a^2 q^2$$

It is necessarily:  $p - q = m \in \mathbb{Z}$ ,  
thus

$$p = x^2$$

$$q = \frac{1}{a} \sqrt{x^2 + y^2}$$

and  $p - q = m$

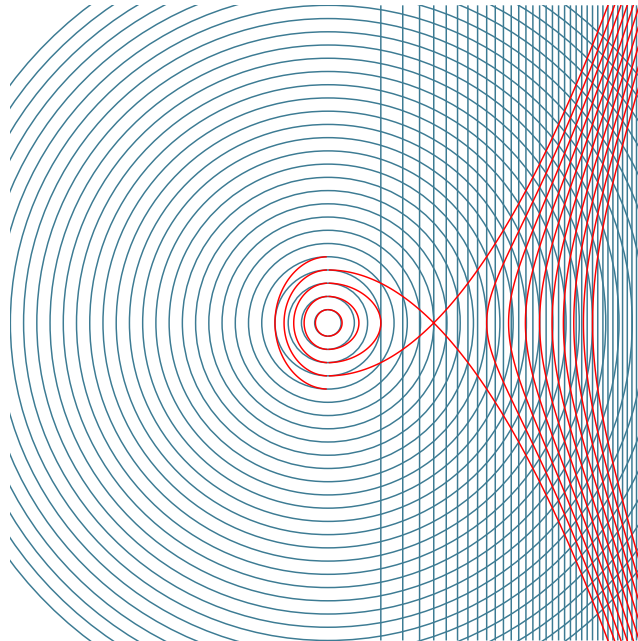
$$x^2 - \frac{1}{a} \sqrt{x^2 + y^2} = m$$

In polar coordinates:

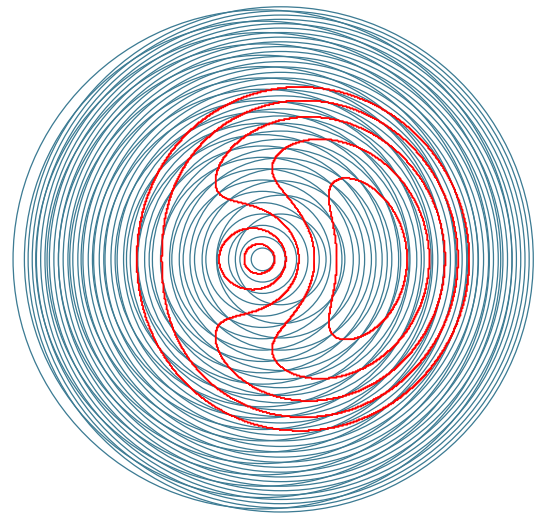
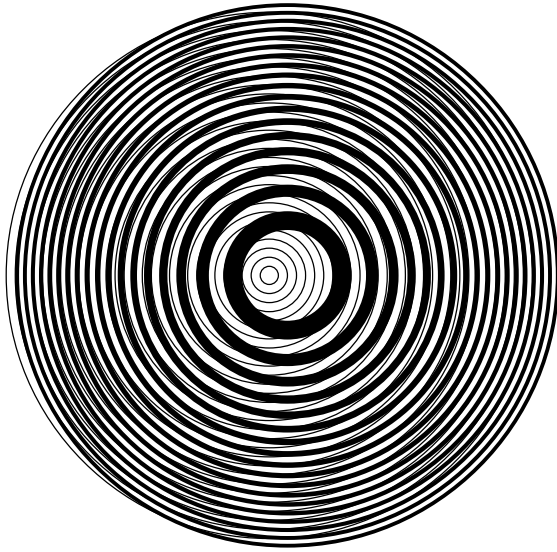
$$\rho^2 (\cos \theta)^2 - \frac{\rho}{a} - m = 0$$

$$\Delta = \left( \frac{-1}{a} \right)^2 + 4m (\cos \theta)^2$$

$$\rho = \frac{\frac{1}{a} \pm \sqrt{\Delta}}{2(\cos \theta)^2}$$



## Circles + Fresnel rings



### Mathematization

The radii of the Fresnel circles increase:  $r_n = \sqrt{n}$ .

The radii of the equidistant circles increase:  $r_n = n \cdot a$ , with  $a > 0$ .

The centers of the circles are placed at  $(x_M, 0)$  and  $(-x_M, 0)$ , we have  $p$  and  $q$  as integers:

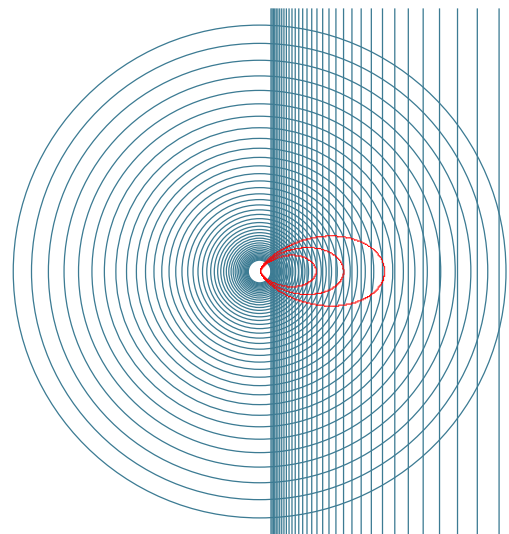
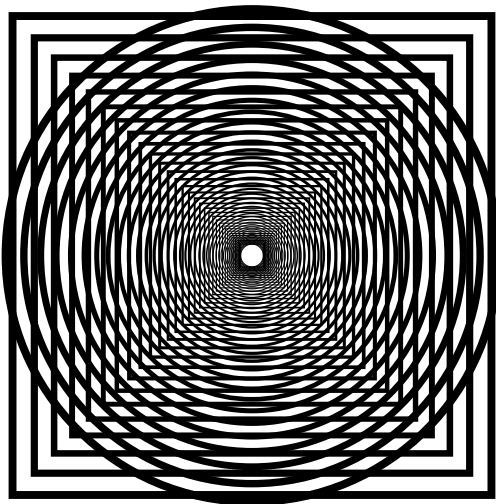
$$\begin{aligned}(x - x_M)^2 + y^2 &= p \\ (x + x_M)^2 + y^2 &= a^2 q^2 \\ p &= (x - x_M)^2 + y^2 \\ q &= \frac{1}{a} \sqrt{(x + x_M)^2 + y^2}\end{aligned}$$

One moiré curve line is determined by:  $p - q = m \in \mathbb{Z}$ , thus:

$$(x - x_M)^2 + y^2 - \frac{1}{a} \sqrt{(x + x_M)^2 + y^2} = m$$

which is the implicit equation of a moiré curve line with  $m$ .

## Circles + Squares (both of increasing thickness)



**Mathematization**

The abscissa of the edges of the square with  $x > 0$  increase with:  $x_n = \frac{1}{4}a^n$ , with  $a > 1$ .

The radii increase with:  $r_n = \frac{1}{4}b^n$ , with  $b > 1$ .

We have

$$\begin{aligned} x_p &= \frac{1}{4}a^p \\ r_q^2 &= x^2 + y^2 \\ x^2 + y^2 &= \frac{1}{16}b^{2q} \end{aligned}$$

If we consider the point determined by the intersection  $p \cap q$ , the next point will be  $(p+1) \cap (q+1)$ , the next one at  $(p+2) \cap (q+2)$ , etc., so that the difference between the indices remains constant. As a result, the moiré lines are characterized by the relation  $p - q = m \in \mathbb{Z}$ ,  $m$  determines a moiré curve.

$$\begin{aligned} p &= \frac{\ln(4x)}{\ln a} \\ q &= \frac{\ln[16(x^2 + y^2)]}{2 \ln b} \end{aligned}$$

and  $p - q = m$

$$\ln(4x)^{\frac{1}{\ln a}} - \ln[16(x^2 + y^2)]^{\frac{1}{2 \ln b}} = m$$

finally gives

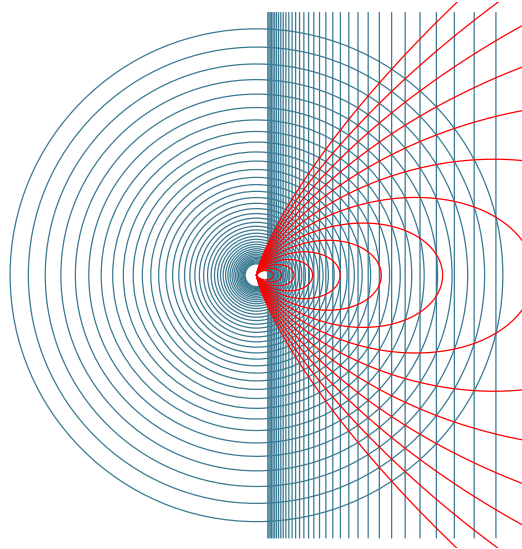
$$\frac{(4x)^{\frac{1}{\ln a}}}{[16(x^2 + y^2)]^{\frac{1}{2 \ln b}}} = e^m$$

We transform this implicit equation into a polar equation by setting  $\rho^2 = x^2 + y^2$  and  $x = \rho \cos \theta$ .

Setting  $\alpha = \frac{1}{\ln a}$  and  $\beta = \frac{1}{2 \ln b}$ . The equation becomes:

$$\begin{aligned} \frac{\rho^\alpha (\cos \theta)^\alpha \cdot 4^\alpha}{\rho^{2\beta} \cdot 4^{2\beta}} &= e^m \\ \rho^{\alpha-2\beta} (\cos \theta)^\alpha &= e^m \cdot 4^{2\beta-\alpha} \\ \rho &= \frac{1}{4} \left( \frac{e^m}{(\cos \theta)^\alpha} \right)^{\frac{1}{\alpha-2\beta}} \end{aligned}$$

We can trace some elements of this family of curves:



## 7 List of all optional arguments for *pst-moire*

Key	Type	Default
Rmax	ordinary	[none]
scale	ordinary	[none]
Alpha	ordinary	[none]
rotate	ordinary	[none]
E	ordinary	[none]
type	ordinary	[none]