# Cubic B-splines Using PSTricks 

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A cubic, uniform B-spline curve with control points $B_{0} \ldots B_{n}(n \geq 2)$ is a curve parametrized by the interval $[0, n]$, which is, except in degenerate cases, $C^{2}$-continuous (that is, has continuous curvature) and is on each interval $[k-1, k](0<k \leq n$ an integer $)$ given by a cubic Bézier curve whose control points are derived from the $\left(B_{k}\right)$. These curves are discussed in any reasonably modern text on Numerical Analysis. One easily accessible source is the UCLA lecture notes of Kirby Baker:
http://www.math.ucla.edu/~baker/149.1.02w/handouts/dd_splines.pdf
I'll focus on two special cases: (i) relaxed, uniform B-splines; (ii) periodic, uniform B-splines. 'Uniform' refers to the condition mentioned in the first paragraph: each Bézier sub-curve is parametrized by an interval of length 1. 'Relaxed' means that the curvature vanishes at the endpoints $t=0, t=n$. 'Periodic' means in effect that the $B_{i}$ repeat periodically, and the curve generated is a closed curve.

## 1 Quick summary of the macros

$\backslash$ psbspline $(1,1)(3,0)(5,2)(4,5)$ draws the relaxed, uniform B-spline interpolating the specified points.
$\backslash p s B s p l i n e(1,1)(3,0)(5,2)(4,5)$ draws the relaxed, uniform B-spline with specified control points.
$\backslash p s B s p l i n e\{B\}(1,1)(3,0)(5,2)(4,5)$ draws the relaxed, uniform B-spline with specified control points, using B as basename for the constructed points.
$\backslash$ psBsplineE $(\mathbf{1 , 1} \mathbf{1}(\mathbf{3 , 0})(\mathbf{5 , 2} \mathbf{( 4 , 5 )}$ is the same as $\backslash \operatorname{psBspline}(1,1)(3,0)(5,2)(4,5)$ except that it omits the first and last segments.
$\backslash$ psBsplineC $(\mathbf{1}, \mathbf{1})(\mathbf{3}, \mathbf{0})(5,2)(4,5)$ extends the specified points periodically, drawing a closed curve with the specified points as control points.
$\backslash$ psBsplineNodes $\{B\}\{4\}$ draws the relaxed, uniform B-spline with control points B0..B4.
$\backslash p s B s p l i n e N o d e s E\{B\}\{4\}$ is the same as $\backslash p s B s p l i n e N o d e s\{B\}\{4\}$ except that it omits the first and last segments.
$\backslash$ psBsplineNodesC $\{\mathrm{B}\}\{4\}$ extends the node sequence periodically, drawing a closed curve with them as control points.
\beztobsp(1,2)(-3,-4)(5,6)(-7,-8)\{B\} creates nodes B0..B3 for which the curve $\backslash p s B s p l i n e N o d e s E\{B\}\{3\}$ is identical to the Bézier curve determined by the specified points. (It does not draw the curve.)
\bspcurvepoints\{B\}\{5\}\{P\} creates PostScript arrays to describe a sequence of points along the curve that would be the result of the command $\backslash p s B s p l i n e N o d e s\{B\}\{5\}$, naming those arrays P.X, P.Y (for position), PNormal.X, PNormal.Y, PDelta.X and PDelta.Y. (Nothing is drawn.)
\bspcurvepointsE\{B\}\{5\}\{P\} does the same as $\backslash$ bspcurvepoints, but omits the first and last segments. (Nothing is drawn.)
\bspNode\{P\}\{5\}\{1.3\}\{Q\} requires that you first run $\backslash$ bspcurvepoints[E] to create PostScript arrays with basename P. It then sets a node $Q$ at position $t=1.3$ on the curve. (Nothing is drawn.)
\bspFnNode\{P\}\{5\}\{2.3\}\{Q\} requires that you first run $\backslash$ bspcurvepoints[E] to create PostScript arrays with basename $P$. It then sets a node Q at position $x=1.3$ on the curve. (Nothing is drawn.) The result is meaningful only for a B-spline curve that is the graph of a function of $x$ and where $x_{0}<x_{1}<\cdots$.
$\backslash$ psBsplineInterp\{S\}\{4\} will construct a sequence SB0..SB4 for which the associated B-spline curve interpolates S0..S4. (Nothing is drawn-you have to then issue the command $\backslash p s B s p l i n e N o d e s\{S B\}\{4\}$.)
$\backslash p s B s p l i n e I n t e r p C\{S\}\{4\}$ will construct a sequence SB0..SB5 for which the associated closed B-spline curve interpolates S0..S4. (Nothing is drawnyou have to then issue the command $\backslash \mathrm{psBsplineNodesC}\{\mathrm{SB}\}\{5\}$.)
\thickBspline\{B\}\{5\}\{12pt\}\{<graphic to clip>\} defines a clipping path 12pt wide around the B-spline curve with control points B0..B5, then draws the <graphic> clipped to that path.
$\backslash$ bspcurvenodes $\{\mathbf{P}\}\{\mathbf{Q}\}$ creates a node sequence $Q 0 \mathrm{Q} 1, \ldots$ from the position data in the arrays P.X, P.Y.

Details and examples are provided below.

## 2 Relaxed, Open B-spline

The algorithm to generate such a curve from a sequence of control points $B_{0}, \cdots, B_{n}$ is as follows:

- The curve starts at $B_{0}$ and ends at $B_{n}$. (Important: $n \geq 2$.)
- Divide each line segment $B_{k-1} B_{k}$ into equal thirds, with subdivision points labeled $R_{k-1}, L_{k}$ respectively, so that $B_{k}$ has $L_{k}$ as its immediate neighbor to the left, and $R_{k}$ as its immediate neighbor to the right.
- For $0<k<n$, divide the line segment $L_{k} R_{k}$ in half, letting $S_{k}$ denote the midpoint. In effect, for $0<k<n, S_{k}=\left(B_{k-1}+4 B_{k}+B_{k+1}\right) / 6$.
- Let $S_{0}=B_{0}$ and $S_{n}=B_{n}$.
- For $0<k \leq n$, construct the cubic Bézier curve with control points $S_{k-1}, R_{k-1}, L_{k}, S_{k}$, parametrized by $k-1 \leq t \leq k$.

The pst-Bspline package implements this algorithm as \psBspline, whose simplest form is, for example
$\backslash$ psBspline $(.5, .5)(2,0)(5,2)(6,4)(4,5)(2,4)$
The coordinates are the B-spline control points. Aside from the usual keywords, like linestyle, linecolor and arrows, there is a Boolean keyword showframe. The effect of showframe=true is to show the intermediate points and lines in the algorithm described above.

There is another optional argument that can be applied if you wish to be able to refer to any of the points constructed in the algorithm. By example,
$\backslash p s B s p l i n e\{B\}(.5, .5)(2,0)(5,2)(6,4)(4,5)(2,4)$
sets the root of the naming scheme to B , the effect of which is that the Bspline control points will be nodes of type \pnode with names B0, B1 and so on, the other points being similarly named BLO, BL1, ... , BRO, BR1, ... , BSO, BS1, ... . For example, to draw a line between BL1 and BS4, just use \ncline(BL1) (BS4).

The algorithm depends for the most part on the flexibility of nodes, and above all the \multido macro, which allows one to construct with relative ease items that look and feel like arrays. Use of \SpecialCoor is essential.

There is a closely related macro $\backslash p s B s p l i n e E$ which removes the first and last Bézier segments, much as \psecurve acts in relation to \pscurve, allowing one to draw B-splines with non-zero curvature at the endpoints.

```
\documentclass{article}
\usepackage{pstricks}
\usepackage{multido,pst-node,pst-bspline}
\pagestyle{empty}
\begin{document}
\SpecialCoor % essential for pst-bspline package
\psset{unit=.6in}
\begin{pspicture}[showgrid=true](-.5,-.5)(6,5)
\psBspline[showframe=true]{B}(.5,.5)(2,0)(5,2) (6,4) (4,5) (2,4)
\multido{\i=0+1}{5}{\uput[20](B\i){B\i}}
\uput[90](B5){B5}\uput[90](BS1){S1}\uput[90](BS2){S2}
\uput[180](BS3){S3}\uput[270](BS4){S4}\uput[-45](BR1){R1}
\uput[-45](BL2){L2}
\end{pspicture}
\end{document}
```

5

4

3

2

1


```
\documentclass{article}
\usepackage{pstricks}
\usepackage{multido,pst-node,pst-bspline}
\pagestyle{empty}
\begin{document}
\SpecialCoor % essential for pst-bspline package
\psset{unit=.6in}
\begin{pspicture}[showgrid=true](-.5,-.5)(6,5)
\psBsplineE[showframe=true]{B}(.5,.5)(2,0)(5,2)(6,4)(4,5)(2,4)
\multido{\i=0+1}{5}{\uput[20](B\i){B\i}}
\uput[90](B5){B5}
\uput[90](BS1){S1}
\uput[90](BS2){S2}
\uput[180](BS3){S3}
\uput[270](BS4){S4}
\uput[-45](BR1){R1}
\uput[-45](BL2){L2}
\end{pspicture}
\end{document}
```

5

4

3

2

1

0


### 2.1 Bézier curves as B-spline curves

Consider a Bézier curve $\mathscr{C}$ with control points $S_{1}, C_{1}, C_{2}, S_{2}$. To identify $\mathscr{C}$ as a B-spline curve of the type discussed above, consider the problem of finding the B-spline control points $B_{0}, B_{1}, B_{2}, B_{3}$ for which $\backslash$ psBsplineE yields $\mathscr{C}$. This is a simple problem in linear algebra whose solution is:
$\mathrm{B} 0=6 \mathrm{~S} 1-7 \mathrm{C} 1+2 \mathrm{C} 2$
B1 $=2 \mathrm{C} 1-\mathrm{C} 2$
$\mathrm{B} 2=-\mathrm{C} 1+2 \mathrm{C} 2$
B3=6S2+2C1-7C2
In other words, any cubic Bézier curve may be considered to be the output of $\backslash \mathrm{psBspl}$ lineE for the $B_{i}$ described above. In this way, all macros described for B-spline curves may be applied to an arbitrary Bézier curve as a special case. The macro
\beztobsp(S1)(C1)(C2)(S2) \{B\}
results in defining $B_{0}, \ldots, B_{3}$ exactly as above.

## 3 Periodic B-spline

The result here is a closed curve. The algorithm is essentially the same as in the preceding case, except:

- Extend $B_{i}$ periodically with period $n+1$, so that $B_{n+1}=B_{0}$ and $B_{n+2}=B_{1}$.
- Construct $R_{i}, L_{i}$ for $0<i<n+2$, as above.
- Construct $S_{k}$ as above (midpoint of $L_{k} R_{k}$ ), for $0<k<n+2$.
- Set $S_{0}=S_{n+1}$.
- For $0<k \leq n+1$, construct the cubic Bézier curve with control points $S_{k-1}, R_{k-1}, L_{k}, S_{k}$, parametrized by $k-1 \leq t \leq k$.
The macro in this case is $\backslash$ psBsplineC, where the C stands for Closed. The code, being implemented as a $\backslash$ pscustom object, does not accept the doubleline keyword, but does accept, for example,
fillstyle=solid,fillcolor=gray

```
\documentclass{article}
\usepackage{pstricks}
\usepackage{multido,pst-node,pst-bspline}
\pagestyle{empty}
\begin{document}
\SpecialCoor % essential for pst-bspline package
\psset{unit=.6in}
\begin{pspicture}[showgrid=true](-.5,-.5)(6,5)
\psBsplineC[showframe=true]{B}(.5,.5)(2,0)(5,2)(6,4)(4,5)(2,4)
\multido{\i=0+1}{5}{\uput[20](B\i){B\i}}
\uput[90](B5){B5}\uput[90](BS1){S1}
\uput[90](BS2){S2}\uput[180](BS3){S3}
\uput[270](BS4){S4}\uput[-45](BR1){R1}
\uput[-45](BL2){L2}
\end{pspicture}
\end{document}
```

5

4

3

2

1

0
0


## 4 Related constructions

There are in addition three additional macros that draw similar curves, but organized in a slightly different way. They are particularly useful when there is a sequence of points already defined as \pnodes. Here is a simple way to define such a sequence.

### 4.1 The pnodes macro

The line
$\backslash$ pnodes $\{P\}(2,1.5)(3,4)(5,1)$
defines a sequence of $\backslash$ pnodes with the node root P : $\mathrm{P} 0=(2,1.5), \mathrm{P} 1=(3,4)$ and $\mathrm{P} O=(5,1)$. The sequence may be any (reasonable) length. The macro leaves an entry in the console saying that it has defined nodes P0 .. P2. The three new macros are:

```
\psBsplineNodes{<node root>}{<top index>}
\psBsplineNodesC{<node root>}{<top index>}
\psBsplineNodesE{<node root>}{<top index>}
```

corresponding to the macros $\backslash$ psBspline, $\backslash p s B s p l i n e C$ and $\backslash p s B s p l i n e E$. The difference is that the macros with Nodes in the name have as arguments the root node name and the last index, rather than the list of points. For example, with the above definition of P in force, $\backslash \mathrm{psBsplineNodes}\{\mathrm{P}\}\{2\}$ has exactly the same effect as $\backslash$ psBspline $(2,1.5)(3,4)(5,1)$.

### 4.2 The \bspcurvepoints macros

There are two macros that provide for B-spline curves essentially the same functionality as the $\backslash p s c u r v e p o i n t s ~ m a c r o ~ f r o m ~ p s t r i c k s-a d d . ~(T h a t ~ m a c r o ~$ takes as input a parametric curve and constructs as output (at the PostScript level) arrays of data associated with the curve: the positions of points along the curve, the increment from the previous point and a normal vector to the curve. The principal uses for such data are (i) the \pspolylineticks macro from pstricks-add, which allows placement of ticks and other marks along a curve that has been approximated by a polyline; (ii) the \polyIntersections macro from pst-node, which allows you to find the points of intersection
of the curve (approximated by a polyline) and an arbitrary line.) The macros

```
\bspcurvepoints{<source name>}{<source max index>}{<dest. name>}
\bspcurvepointsE{<source name>}{<source max index>}{<dest. name>}
```

work, in the first case, for a relaxed, uniform B-spline curve, and in the second, for such a curve with its initial and final segments removed, corresponding to the output from $\backslash p s B s p l i n e E$ rather than $\backslash p s B s p l i n e$. In both cases, you may set the keyword plotpoints (default value: 50) to change the number of sample points on each Bézier component. This will result in the construction of PostScript arrays with indices from 0 to $n=$ num of segments $\times$ (plotpoints-1). After running
$\backslash$ pnodes $\{\mathrm{B}\}(1,2)(3,-1)(4,1)(6,2) \%$ define B0.. B 3
$\backslash$ bspcurvepoints[plotpoint $=11]\{B\}\{3\}\{P\}$
the following PostScript arrays are created, each indexed from 0 to 30:
P.X, P.Y (position)

PNormal.X, PNormal.Y (normal vector)
PDelta.X, PDelta.Y (increment from previous position)
and these may be used in the usual way to create nodes. For example,
\pnode(! P.X 8 get P.Y 8 get) $\{Q\}$
\pnode(! PNormal.X 8 get PNormal.Y get)\{Dir\}
\psrline(Q)(1cm; $\{($ Dir) $\}$ )
places $Q$ at the position on the curve with index 8 , defines Dir to be a normal vector at that point, then draws a line from $Q$ of length 1 cm in the direction of that normal.

### 4.3 Setting nodes on a B-spline curve

To set a node at parameter value $t$ on a B-spline curve after running $\backslash$ bspcurvepoints [E], call the macro
\bspNode\{<control point root>\}\{<top index>\}\{<t>\}\{<node name>\}
For example, if I have constructed a B-spline curve using control points $B_{0}, \ldots, B_{5}$, then $\backslash \mathrm{bspNode}\{\mathrm{B}\}\{5\}\{2.1\}\{\mathrm{Q}\}$ defines a node named $Q$ at $t=$ 2.1.

The macro $\backslash$ bspcurvenodes $\{P\}\{R\}$ creates a node sequence R0..Rn at the locations specified by the arrays P.X, P.Y. (Those arrays must first have been created with one of the \bspcurvepoints macros.)

### 4.4 B-spline function curves

By this we mean an open B-spline curve which is the graph of a function $y=f(x)$ and whose orientation is toward the right. It is not analytically simple to specify a formula for $f$ in most cases, and to compute $y$ from $x$ involves (a) finding the index of the Bézier segment containing $x$; (b) solving the cubic $x(t)=x$ for $t$; (c) substituting in $y(t)$. The package provides a macro to perform these calculations after generating the data using \bspcurvepoints[E]:
\bspfnNode\{<control point root>\}\{<top index>\}\{<x0>\}\{<node name>\}

## 5 B-spline Interpolation

This is the inverse problem. Being given points $\left(S_{k}\right)_{0 \leq k \leq n}$, the goal is to produce the B-spline control points $B_{k}$ leading to the points $S_{k}$, so that the associated B-spline curve interpolates the $S_{k}$.

### 5.1 Open curve

We discuss first the case of an open, uniform B-spline curve with relaxed endpoints. According to the discussion above, we have to solve the equations

$$
\begin{aligned}
& B_{0}=S_{0} \\
& B_{0}+4 B_{1}+B_{2}=6 S_{1} \\
& B_{1}+4 B_{2}+B_{3}=6 S_{2} \\
& \cdots \\
& B_{n-2}+4 B_{n-1}+B_{n}=6 S_{n-1} \\
& B_{n}=S_{n}
\end{aligned}
$$

for the $B_{k}$. In matrix form, this becomes the tridiagonal system

$$
\left(\begin{array}{ccccc}
4 & 1 & & & \\
1 & 4 & 1 & & \\
& 1 & 4 & 1 & \\
& & \cdots & & 1 \\
& & & 1 & 4
\end{array}\right)\left(\begin{array}{c}
B_{1} \\
B_{2} \\
B_{3} \\
\cdots \\
B_{n-1}
\end{array}\right)=\left(\begin{array}{c}
6 S_{1}-S_{0} \\
6 S_{2} \\
6 S_{3} \\
\cdots \\
6 S_{n-1}-S_{n}
\end{array}\right)
$$

The LU decomposition of the tridiagonal matrix may be seen to take the form

$$
\left(\begin{array}{ccccc}
1 & & & & \\
m_{1} & 1 & & & \\
& m_{2} & 1 & & \\
& & \cdots & & \\
& & & m_{n-2} & 1
\end{array}\right)\left(\begin{array}{ccccc}
m_{1}^{-1} & 1 & & & \\
& m_{2}^{-1} & 1 & & \\
& & m_{3}^{-1} & 1 & \\
& & & \cdots & 1 \\
& & & & m_{n-1}^{-1}
\end{array}\right)
$$

where $m_{1}=1 / 4, m_{k+1}=1 /\left(4-m_{k}\right)$ for $k=1, \cdots, n-2$. The solution of the original system is therefore accomplished in two steps, introducing intermediate points ( $R_{k}$ ), by (in pseudo-code)

```
R_1=6*S_1-S_0
for i=2 to n-2
    R_i=6*S_i-m_{i-1}* R_{i-1}
R_{n-1}=(6*S_{n-1}-S_n)-m_{n-2}*R_{n-2}
B_{n-1}=m_{n-1}*R_{n-1}
for i=n-2 downto 1
    B_i=m_i*(R_i-B_{i+1})
```

The code for the $\backslash \mathrm{psBsplineInterp}$ command uses this algorithm to solve for the $B_{k}$ as nodes, except that in order to save node memory, the $B$ nodes are substituted in place for the $R$ nodes, so that, for example, the first step becomes B_1=6*S_1-S_0.

Assuming you have previously defined nodes s0 $\cdots$ S4,
$\backslash$ psBsplineInterp\{S\}\{4\}
will construct a sequence SB0 $\cdots$ SB4 of nodes at the B-spline control points for the relaxed, uniform cubic B-spline interpolating the Sk , and this curve may then be rendered with the command
$\backslash p s B s p l i n e N o d e s\{S B\}\{4\}$

If you don't care about keeping track of the internal operations and names for nodes, you may generate the curve directly with, for example,

```
\psbspline(0,0)(.5,.1)(1.5,.6)(2.5,1.4)(3.5,1.8)(4.5,1.7)%
(5.8,1.0)(7.5,.25)(10,0)
```


### 5.2 Closed (periodic) case

We turn now to the periodic uniform B-spline curve interpolating $n$ points $S_{0}, \ldots, S_{n-1}$. Extend the sequence periodically with period $n$, so that $S_{n}=S_{0}$, $S_{n+1}=S_{1}, S_{-1}=S_{n-1}$, and so on. In order to find the periodic control points $B_{k}$, we have to solve the $n$ equations

$$
\begin{gathered}
B_{n}+4 B_{1}+B_{2}=6 S_{1} \\
B_{1}+4 B_{2}+B_{3}=6 S_{2} \\
\cdots \\
B_{n-2}+4 B_{n-1}+B_{n}=6 S_{n-1} \\
B_{n-1}+4 B_{n}+B_{1}=6 S_{n}
\end{gathered}
$$

for the $B_{k}, 1 \leq k \leq n$. In matrix form, this becomes the system

$$
\left(\begin{array}{ccccc}
4 & 1 & & & 1 \\
1 & 4 & 1 & & \\
& 1 & 4 & 1 & \\
& & \cdots & & 1 \\
1 & & & 1 & 4
\end{array}\right)\left(\begin{array}{c}
B_{1} \\
B_{2} \\
B_{3} \\
\cdots \\
B_{n}
\end{array}\right)=\left(\begin{array}{c}
6 S_{1} \\
6 S_{2} \\
6 S_{3} \\
\cdots \\
6 S_{n}
\end{array}\right)
$$

Let $\left(x_{k}, y_{k}\right)=6 S_{k}$. We perform Gaussian elimination on the matrix

$$
\left(\begin{array}{ccccccc}
4 & 1 & & & 1 & x_{1} & y_{1} \\
1 & 4 & 1 & & & x_{2} & y_{2} \\
& 1 & 4 & 1 & & x_{3} & y_{3} \\
& & \cdots & & 1 & & \\
1 & & & 1 & 4 & x_{n} & y_{n}
\end{array}\right)
$$

As in the previous case, let $m_{1}=0.25, m_{k}=1 /\left(4-m_{k-1}\right)$ for $k \geq 2$. The factor $m_{k}$ will be the multiplier of row $k$ after the previous row operation, in order to normalize the row. These are the steps in the procedure.

- Initialize: multiply row 1 by $m_{1}$ so that its first entry $(1,1)$ is 1 . Replace $x_{1}$ by $m_{1} x_{1}$ and $y_{1}$ by $m_{1} y_{1}$. Entry $(1, n)$ is $m_{1}$.
- Subtract new row 1 from row 2 and multiply the resulting row by $m_{2}$. The leading entry $(2,1)$ becomes 1 . Entry $(2, n)$ becomes $-m_{1} m_{2}$, and $x_{2}, y_{2}$ are updated to $m_{2}\left(x_{2}-x_{1}\right), m_{2}\left(y_{2}-y_{1}\right)$. The superdiagonal entry $(2,3)$ is the only other non-zero entry, and its new value is $m_{2}$.
- Subtract new row 1 from row $n$, so that its leading entry $(n, 2)$ is $-m_{1}$.
- Subtract new row 2 from row 3 and multiply the result by $m_{3}$. The leading entry $(3,3)$ becomes 1 and the entry $(3, n)$ becomes $m_{1} m_{2}$, with $x_{3}, y_{3}$ updating to $m_{3}\left(x_{3}-x_{2}\right), m_{3}\left(y_{3}-y_{2}\right)$. The superdiagonal entry $(3,4)$ is now $m_{3}$.
- Subtract new row 2 times $-m_{1}$ from row $n$, whose leading entry $(n, 3)$ is now $m_{1} m_{2}$.
- Continue in this way until row $n-2$ has been subtracted as above from row $n-1$, multiplying the result by $m_{n-1}$, and a suitable multiple has been subtracted from row $n$. The leading entry of row $n-1$ (column $n-1)$ is 1 and its $n^{\text {th }}$ entry is $1-(-1)^{n} m_{1} \cdots m_{n-2}$. Row $n$ has leading entry in column $n-1$, equal to 1 .
- Finally, subtract an appropriate multiple of row $n-1$ from row $n$ so that row $n$ has leading entry in column $n$. The resulting matrix is upper triangular, and we may now substitute back starting from the last row to give a complete reduction.

Here are the steps in pseudocode. We keep track of row $n$ with the array $b_{k}$, column $n$ with the array $c_{k}$. The indices for both run from 1 to $n$.

```
m(1)=0.25
for k=2 to n-1
    m(k)=1/(4-m(k-1))
b(1)=1
b(n-1)=1
b(n)=4
c(n-1)=1% don't need c(n), =b(n)
%multiply first row by m1
c(1)=m(1)
x(1)=m(1)*x(1)
y(1)=m(1)*y(1)
for k=2 to n-1
    %subtract normalized row k-1 from row k, renormalize row k
    c(k)=m(k)*(c(k)-c(k-1))%note that initially, c(k)=0 for 1<k<n-1
```

```
    x(k)=m(k)*(x(k)-x(k-1))
    y(k)=m(k)*(y(k)-y(k-1))
    %subtract normalized row k-1 times b(k-1) from row n
    b(k)=b(k)-b(k-1)*m(k-1)
    b}(n)=b(n)-c(k-1)*b(k-1
    x(n)=x(n)-x(k-1)*b(k-1)
    y(n)=y(n)-y(k-1)*b(k-1)
% subtract row n-1 times b(n-1) from row n, renormalize by 1/b(n)
b}(n)=b(n)-b(n-1)*c(n-1
x(n)=(x(n)-x(n-1)*b(n-1))/b(n)
y(n)=(y(n)-y(n-1)*b(n-1))/b(n)
%work back
x(n-1)=x(n-1)-c(n-1)*x(n)
y(n-1)=y(n-1)-c(n-1)*y(n)
for k=n-2 downto 1
    x(k)=x(k)-m(k)* x (k+1)-c(k)*x(n)
    y(k)=y(k)-m(k)* y(k+1)-c(k)*y(n)
```

This algorithm is implemented in $\mathrm{T}_{\mathrm{E}} \mathrm{X} /$ PostScript code in pst-bspline.tex and may be invoked using the macro
$\backslash p s B s p l i n e$ InterpC $\{<$ node root>\} $<$ index>\}
You must previously have defined a sequence, say S0 $\cdots$ S100 of $\backslash$ pnodes that you plan to interpolate with a closed curve. If you used $\backslash$ pnodes to do this, it would have constructed a macro \Snodecount to store the value 100 . Then
$\backslash p s B s p l i n e I n t e r p C\{S\}\{100\}$
constructs the sequence SBO ... SB100 of B-spline control points (appending B to the root name) for a closed curve interpolating S0 ... S100, which may then be rendered with the command
\psBsplineNodesC\{SB\}\{101\}
with any keywords options you wish.
 S, setting S101=S0, and changing \Snodecount to take the value 101. This is convenient when you use the construction:

```
\pnodes{S}()()()()% sets \Snodecount to 3
\psBsplineInterpC{S}{\Snodecount}% constructs B-spline control pts SBO..SB4
```

```
\psBsplineNodesC{SB}{\Snodecount}
```

The following example illustrates that there is a difference between $\backslash$ psccurve and B-spline interpolation, the former having a rounder appearance. Generally speaking, B-spline interpolation comes closer to minimizing the average curvature.

```
\documentclass{article}
\usepackage{pstricks}
\usepackage{pst-bspline,pstricks-add}
\begin{document}
\begin{pspicture}[showgrid=true](-.5,-.5)(6,5)
\pnodes{P}(0,1)(2,0)(5,2)(6,4)(4,5)(2,4)%
\psBsplineInterpC{P}{5}%
\psBsplineNodesC*[linecolor=gray!40]{PB}{5}%
\psccurve[linecolor=red,showpoints=true](0,1)(2,0)(5,2)(6,4)(4,5)(2,4)
\end{pspicture}
\end{document}
```

Slight difference between psccurve and B-spline interpolation


A B-spline curve can in many cases provide a good function interpolation mechanism, but the result is not guaranteed to be the graph of a function.
\begin\{pspicture\}(-.5,-.5) } ( 6 , 4 )
$\backslash$ psdots $(0,3.5)(1, .5)(3,2.5)(4,0)(5,2)(6, .5) \%$
\pnodes\{S\}(0,3.5)(1,.5)(3,2.5)(4,0)(5,2)(6,.5)\% S0..S5
\psBsplineInterp\{S\}\{5\}\%SB0..SB5
\psBsplineNodes\{SB\}\{5\}\% draw B-spline with control pts SBO..SB5
\bspcurvepoints[plotpoints=10]\{SB\}\{5\}\{P\}
\% construct the PS arrays
$\backslash$ bspFnNode\{SB\}\{5\}\{4.5\}\{QQ\}\% node QQ on curve at $x=4.5$
$\backslash$ psdot[linecolor=red](QQ)\%
$\backslash$ psaxes $(0,0)(-.5,-.5)(6,4)$
\end\{pspicture\} }


```
\documentclass{article}
\usepackage{pstricks}
\usepackage{pst-bspline,pstricks-add}
\begin{document}
\psset{unit=.25in}
\begin{pspicture}[showgrid=true](-.5,-.5)(6,5)
\pnodes{P}(0,1)(2,0) (5,2) (6,4) (4,5) (2,4)
\pnode(3,3){C}
\multido{\ra=0+.05,\rb=1+.05,\i=30+1}{40}{%
\psBsplineC*[linecolor=blue!\i!brown]{B}%
([nodesep=\ra]{C}P0)([nodesep=\ra]{C}P1)%
([nodesep=\ra]{C}P2)([nodesep=\ra]{C}P3)%
([nodesep=\ra]{C}P4)([nodesep=\ra]{C}P5)}
\end{pspicture}
\end{document}
```



## 6 Thick B-spline curves

Inspired by the package pst-thick, we provide a similar option for curves generated as B -spline interpolations. The new macro that accomplishes this is
\thickBspline\#1\#2\#3\#4
\%\#1=root | \#2=nsegments | \#3=thickness | \#4=items to clip
which expects the following data.

- A node sequence. This can be constructed with a command like
$\backslash$ pnodes $\{S\}(0,0)(5,1)(4,4)(1,3) \%$
which declares nodes S0..S3, and sets the macro \Snodecount to 3 .
- An interpolation command, such as
$\backslash p s B s p l i n e I n t e r p\{S\}\{\backslash$ Snodecount $\} \%$
which creates a framework of B-spline control points SBO..SB3.
- Create the interpolating curve and the Bézier control points for its components, with names like SBRO..SBR2,SBL1..SBL3 etc, using
\psBsplineNodes[linestyle=none]\{SB\}\{\Snodecount\}\%
(The [linestyle=none] may be omitted if you want the curve to show.)
- Create a clipping path of specified thickness around the interpolating curve and place graphics to be clipped:
\thickBspline[plotpoints=50, linestyle=none]\{S\}\{3\}\{20pt \}\%
$\{\backslash p s l i n e[l i n e c o l o r=r e d, l i n e s t y l e=s o l i d](0,0)(6,6)\} \%$
(The [linestyle=none] controls whether the clipping path is rendered, and plotpoints controls the number of subdivisions of each Bézier component. Its default value is 50 .)

The clipping path is drawn by default positively oriented so that objects are clipped to its interior. By specifying the keyword reverseclip, the clipping path will be reversed so that objects are clipped to the exterior.

```
\documentclass{article}
\usepackage{pstricks}
\usepackage{pst-bspline,pstricks-add}
\begin{document}
\begin{pspicture}[showgrid=true](-.5,-.5)(6,5)
\pnodes{S}(1,0)(5,1)(4,4)(1,3)%
\psdots(1,0)(5,1)(4,4)(1,3)%
\psBsplineInterp{S}{\Snodecount}%
\psBsplineNodes[linestyle=none]{SB}{\Snodecount}%
\thickBspline[plotpoints=50,linestyle=none]{S}{3}{20pt}%
{\psframe[fillstyle=crosshatch](-1,-1)(6,6)}%
\end{pspicture}
\end{document}
```

The \thickBspline macro works as expected in the closed (periodic) case, taking advantage of automatic incrementing of the nodecount. Note that \thickBspline interprets thickness as visual, unaffected by possible differences between xunit and yunit.
\documentclass\{article\}

```
\usepackage{pstricks}
\usepackage{pst-bspline,pstricks-add}
\begin{document}
\psset{yunit=1.5cm}
\begin{pspicture}[showgrid=true](-.5,-.5)(6,5)
\pnodes{S}(1.5,0)(5,1)(4,4)(1,3)%
\psBsplineInterpC{S}{3}%
% defines nodes SB0, SB1, SB2, SB3, SB4 --- the Bspline control points
% increments \Snodecount by 1 for future macros
% Don't use C form of \psBsplineNodes with this new \Snodecount
\psBsplineNodes[linestyle=none, showpoints=false]{SB}{\Snodecount}%
% Constructs the Bezier control points SBR0, SBL1, SBR1, etc
\thickBspline[linestyle=none]{S}{\Snodecount}{22pt}%
{\psframe[fillstyle=vlines](-1,-1)(6,6)}%
\end{pspicture}
\end{document}
```

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