

Simplest form of the *Central Limit Theorem*: Let X_1, X_2, \dots be a sequence of iid random variables with mean 0 and variance 1 on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Then

$$\mathbb{P} \left(\frac{X_1 + \dots + X_n}{\sqrt{n}} \leq y \right) \rightarrow \mathfrak{N}(y) := \int_{-\infty}^y \frac{e^{-t^2/2}}{\sqrt{2\pi}} \, dt \quad \text{as } n \rightarrow \infty,$$

or, equivalently, letting $S_n := \sum_1^n X_k$,

$$\mathbb{E} f \left(S_n / \sqrt{n} \right) \rightarrow \int_{-\infty}^{\infty} f(t) \frac{e^{-t^2/2}}{\sqrt{2\pi}} \, dt \quad \text{as } n \rightarrow \infty, \text{ for every } f \in \mathfrak{b}\mathcal{C}(\mathbb{R}).$$