# SemanTEX: Semantic mathematics (v0.201 $\alpha$ ) 

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The SemanT ${ }_{E} X$ package for ${ }^{\mathrm{EA}} \mathrm{E}_{\mathrm{E}} \mathrm{X}$ delivers a more semantic, systematized way of writing mathematics, compared to the classical math syntax in $\mathrm{EAT}_{\mathrm{E}} \mathrm{X}$. The system uses keyval syntax and is highly customizable. At the same time, care has been taken to make it the syntax as simple, natural, practical, and lightweight as possible. Note: SemanTEX is still in its alpha stage and cannot be considered stable at this point. You are more than welcome to report bugs and come with suggestions!

Let us take an example from elementary analysis to demonstrate the idea of the package: Suppose we want to take the complex conjugate of a function $f$ and then derive it $n$ times, i.e. take $\bar{f}^{(n)}$. SemanTEX allows you to typeset this something like this:
\$ \vf[conj, der=\vn] \$


I shall explain the syntax in detail below, but some immediate comments are in order: First and foremost, the $v$ in the command names $\backslash v f$ and $\backslash v n$ stands for "variable", so these commands are the variables $f$ and $n$. In SemanTEX, it is usually best to create commands \va, \vA, \vb, \vB,...for each variable you are using, upper- and lowercase. However, it is completely up to the user how to do that and what to call them. Note also that all of the keys inv, res, etc. are defined by the user, and they can be modified and adjusted for all sorts of situations in any kinds of mathematics. In other words, for the most part, you get to choose your own syntax.

Next, suppose we want to invert a function $g$ and restrict it to a subset $U$, and then apply it to $x$, i.e. take $\left.g^{-1}\right|_{U}(x)$. This can be done by writing
\$ $\backslash v g[i n v, r e s=\backslash v U]\{\backslash v x\} \$$

$$
\left.g^{-1}\right|_{U}(x)
$$

Next, let us take an example from algebraic geometry: Suppose $\mathcal{F}$ is a sheaf and $h$ a map, and that we want to typeset the equation $\left(h^{-1} \mathcal{F}\right)_{p}=\mathcal{F}_{h(p)}$, saying that the stalk of the inverse image $h^{-1} \mathcal{F}$ at the point $p$ is $\mathcal{F}_{h(p)}$. This can be accomplished by typing

```
$ \vh[inverseimage]{\sheafF }[
spar, stalk=\vp] 
```

\sheafF[stalk=\vh\{\vp\}] \$

Here, spar (an abbreviation for "symbol parentheses") is the key that adds the parentheses around $h^{-1} \mathcal{F}$.

Let us see how you could set up all the above notation:

```
\documentclass{article}
\usepackage{amsmath, semantex}
\NewVariableClass\MyVar % creates a new class of variables,
called "|MyVar"
% Now we create a couple of variables of the class |MyVar:
\NewObject\MyVar\vf{f}
\New0bject\MyVar\vg{g}
\NewObject\MyVar\vh{h}
\New0bject\MyVar\vn{n}
\New0bject\MyVar\vp{p}
\NewObject\MyVar\vU{U}
\New0bject\MyVar\sheafF{\mathcal{F}}
% Now we set up the class |MyVar:
\SetupClass \MyVar{
    output=\MyVar, % This means that the output of an object
                    % of class \MyVar is also of class \MyVar
    % We add a few keys for use with the class \MyVar:
    singlekeys={ % keys taking no values
            {inv}{upper={-1}},
            {conj}{overline},
            {inverseimage } {upper={-1}, nopar},
    },
    valuekeys={ % keys taking a value
            {der}{upper={(#1)}},
            {stalk}{clower={#1}},
            % "clower" means "comma lower", i.e. lower index
            % separated from any previous lower index by a comma
            {res}{ return , symbolputright ={|}, lower ={#1} },
    },
}
\begin{document }
$ \vf[conj,der=\vn] $
$ \vg[inv,res=\vU]{x} $
$ \vh[inverseimage]{\sheafF}[spar, stalk=\vp]
    = \sheafF[stalk=\vh{\vp}] $
\end{document }
```


## 1 Getting started

To get started using SemanT ${ }_{E} \mathrm{X}$, load down the package with

```
\usepackage{semantex}
```

The SemanTEX system is object-oriented; all entities are objects of some class. When you load the package, there is only one class by default, which is simply
called \SemantexVariable．You should think of this as a low－level class，the parent of all other classes．Therefore，I highly advice against using it directly or modifying it．Instead，we create a new，more high－level variable class．We choose to call it $\backslash M y V a r$ ．It is best to always start class names with uppercase letters to separate them from objects．We could write \NewVariableClass \MyVar， but we choose to pass some options to it in［．．．］：
\NewVariableClass \MyVar［output＝\MyVar］
This output＝\MyVar option will be explained better below．Roughly speaking，it tells SemanT ${ }_{\mathrm{E}} \mathrm{X}$ that everything a variable outputs will also be a variable．For in－ stance，if the function $\backslash v f$（i．e．$f$ ）is of class $\backslash M y \operatorname{Var}$ ，then $\backslash v f\{\backslash v x\}$（i．e．$f(x)$ ）will also of class \MyVar．

Now we have a class，but we do not have any objects．To create the object $\backslash \mathrm{vf}$ of class $\backslash M y$ Var with symbol $f$ ，we write $\backslash \operatorname{NewObject~} \backslash M y \operatorname{Var} \backslash v f\{f\}$ ．In general， when you have class $\backslash\langle C l a s s\rangle$ ，you can create objects of that class wtih the syntax

```
\New0bject\\langleClass\rangle\\langleobject\rangle{\langleobject symbol\rangle}[<<options\rangle]
```

To distinguish objects from classes，it is a good idea to denote objects by lowercase letters．${ }^{1}$ So after writing，
$\backslash$ New0bject $\backslash M y \operatorname{Var} \backslash \operatorname{vf}\{\mathrm{f}\}$
$\backslash$ New0bject $\backslash \operatorname{MyVar\backslash vx\{ x\} }$
we get two variables \vf and \vx with symbols $f$ resp．$x$ ．Let us perform a stupid test to see if the variables work：
$\$ \backslash \mathrm{vf}$ ，$\$ \backslash \mathrm{vx} \$$

$$
f, x
$$

Th general syntax of a variable－type object is
$\backslash\langle$ object $\rangle[\langle$ options $\rangle]\{\langle$ argument $\rangle\}$
Both 〈options〉 and 〈argument〉 are optional arguments（they can be left out if you do not need them）．The 〈options〉 should consist of a list of options separated by commas，using keyval syntax．On the other hand，〈argument〉 is the actual argument of the function．By design，SemanTEX does not distinguish between variables and functions，so all variables can take arguments．This is a design choice to make the system easier to use；after all，it is fairly common in mathematics that something is first a variable and then a moment later takes an argument．So we may write：
$\$ \backslash v f\{1\} \$, \quad \$ \backslash v f\{\backslash v x\} \$$ ，
$\$ \backslash v x\{\backslash v x\} \$$

$$
f(1), f(x), x(x)
$$

So far，we do not have very many options to write in the 〈options〉 position， since we have not added any keys yet．However，we do have access to the most important of all options：the index．There is a simple shortcut for writing an index：You simply write the index itself in the options tag：
\＄\vf［1］\＄，\＄\vf［\vf］\＄，
$\$ \backslash v f[1,2, \backslash v f]\{2\} \$$

$$
f_{1}, f_{f}, f_{1,2, f}(2)
$$

[^0]As long as what you write in the options tag is not recognized as a predefined key, it will be printed as the index. Other than that, there are two important predefined keys: upper and lower which simply add something to the upper and lower index:

```
$\vf[upper=2]$,
$\vf[lower=3]$
```


## $f^{2}, f_{3}$

We are soon going to need more variables than just $f$ and $x$. In fact, I advise you to create a variable for each letter in the Latin and Greek alphabets, both uppercase and lowercase. This is pretty time-consuming, so I did it for you already:

```
\(\backslash\) NewObject \(\backslash \operatorname{MyVar} \backslash\) va \(\{a\}\)
\NewObject \(\backslash M y \operatorname{Var} \backslash \mathrm{vb}\{\mathrm{b}\}\)
\NewObject \MyVar\vc\{c\}
\NewObject \MyVar\vd\{d\}
\NewObject \(\backslash M y \operatorname{Var} \backslash \operatorname{ve}\{e\}\)
\New0bject \MyVar\vf\{f\}
\NewObject \MyVar\vg\{g\}
\NewObject \MyVar\vh\{h\}
\New0bject \MyVar\vi\{i\}
\NewObject \MyVar\vj\{j\}
\NewObject \(\backslash M y \operatorname{Var} \backslash \mathrm{vk}\{\mathrm{k}\}\)
\(\backslash\) NewObject \MyVar\vl\{l\}
\NewObject \MyVar\vm\{m\}
\NewObject \MyVar\vn\{n\}
\New0bject \MyVar\vo\{o\}
\NewObject \MyVar\vp\{p\}
\(\backslash\) NewObject \MyVar\vq\{q\}
\NewObject \MyVar\vr\{r\}
\NewObject \MyVar\vs\{s\}
\NewObject \MyVar\vt\{t\}
\New0bject \MyVar\vu\{u\}
\NewObject \MyVar\vv\{v\}
\New0bject \MyVar\vw\{w\}
\New0bject \MyVar\vx\{x\}
\NewObject \MyVar\vy\{y\}
\New0bject\MyVar\vz\{z\}
\NewObject \MyVar\vA\{A\}
\(\backslash\) New0bject \(\backslash M y \operatorname{Var} \backslash \mathrm{vB}\{\mathrm{B}\}\)
\NewObject \(\backslash M y \operatorname{Var} \backslash v C\{C\}\)
\NewObject \MyVar\vD\{D\}
\New0bject \MyVar\vE\{E\}
\(\backslash\) New0bject \(\backslash M y \operatorname{Var} \backslash v F\{F\}\)
\NewObject \MyVar\vG\{G\}
\New0bject \(\backslash M y \operatorname{Var} \backslash \mathrm{vH}\{\mathrm{H}\}\)
\NewObject \MyVar\vI\{I\}
\NewObject \MyVar\vJ\{J\}
\New0bject \MyVar\vK\{K\}
\NewObject \MyVar\vL\{L\}
\NewObject \MyVar\vM\{M\}
\New0bject \MyVar\vN\{N\}
\(\backslash\) New0bject \MyVar\v0\{0\}
```

```
\New0bject\MyVar\vP{P}
\NewObject\MyVar\vQ{Q}
\New0bject\MyVar\vR{R}
\New0bject\MyVar\vS{S}
\NewObject\MyVar\vT{T}
\NewObject\MyVar\vU{U}
\New0bject\MyVar\vV{V}
\NewObject\MyVar\vW{W}
\NewObject\MyVar\vX{X}
\New0bject\MyVar\vY{Y}
\NewObject\MyVar\vZ{Z}
\New0bject\MyVar\valpha{\alpha}
\New0bject\MyVar\vvaralpha{\varalpha}
\New0bject\MyVar\vbeta{\beta}
\NewObject\MyVar\vgamma{\gamma}
\New0bject\MyVar\vdelta{\delta}
\NewObject\MyVar\vepsilon{\epsilon}
\New0bject\MyVar\vvarepsilon{\varepsilon}
\NewObject\MyVar\vzeta{\zeta}
\NewObject\MyVar\veta{\eta}
\New0bject\MyVar\vtheta{0}
\NewObject\MyVar\viota{\iota}
\New0bject\MyVar\vkappa{\kappa}
\NewObject\MyVar\vlambda{\lambda}
\NewObject\MyVar\vmu{\mu}
\New0bject\MyVar\vnu{\nu}
\New0bject\MyVar\vxi{\xi}
\NewObject\MyVar\vpi{\pi}
\NewObject\MyVar\vvarpi{\varpi}
\New0bject\MyVar\vrho{\rho}
\New0bject\MyVar\vsigma{\sigma}
\NewObject\MyVar\vtau{\tau}
\NewObject\MyVar\vupsilon{\upsilon}
\NewObject\MyVar\vphi{\phi}
\NewObject\MyVar\vvarphi{\varphi}
\NewObject\MyVar\vchi{\chi}
\NewObject\MyVar\vpsi{\psi}
\New0bject\MyVar\vomega{\omega}
\NewObject\MyVar\vGamma{\Gamma }
\NewObject\MyVar\vDelta{\Delta}
\NewObject\MyVar\vTheta{\Theta}
\NewObject\MyVar\vLambda{\Lambda}
\NewObject\MyVar\vXi{\Xi}
\New0bject\MyVar\vPi{\Pi}
\NewObject\MyVar\vSigma{\Sigma}
\NewObject\MyVar\vUpsilon{\Upsilon}
\NewObject\MyVar\vPhi{\Phi}
\NewObject\MyVar\vPsi{\Psi}
\NewObject\MyVar\v0mega{\0mega }
```

Just like $\backslash v f$, these can all be regarded as functions, so $\backslash \mathrm{va}\{\backslash \mathrm{vb}\}$ produces $a(b)$. Importantly, parentheses can be scaled. To make parentheses
bigger, use the following keys:

```
$\vf{\vx}$,
$\vf[par=\big]{\vx}$,
$\vf[par=\Big]{\vx}$,
$\vf[par=\bigg]{\vx}$,
$\vf[par=\Bigg]{\vx}$,
$\vf[par=auto]{\frac{1}{2}}$
```

Using par=auto corresponds to using \left... \right. Just as for ordinary math, I advice you to use manual scaling rather than automatic scaling, as $\mathrm{T}_{\mathrm{E}} \mathrm{X}$ has a tendency to scale things wrong. If you do not want parentheses at all, you can pass the key nopar (it will still print parentheses if there is more than one argument, though; to exclude this behaviour, run neverpar instead):
\$\vf[nopar]\{\vx\}\$

```
fx
```

Primes are added via the key prime or the keys ', '' and ''':

```
$\vf['] = \vf[prime]$,
$\vf[''] = \vf[prime,prime]$
$\vf['''] = \vf[prime,prime,
f}=\mp@subsup{f}{}{\prime},\mp@subsup{f}{}{\prime\prime}=\mp@subsup{f}{}{\prime\prime}\mp@subsup{f}{}{\prime\prime\prime}=\mp@subsup{f}{}{\prime\prime\prime
prime]$
```

So far, so good, but our variables cannot really do anything yet. For this, we need to assign keys to them. The more pieces of math notation you need, the more keys you will have to define. Keys are being added via two different keys:
singlekeys and valuekeys.

In short, singlekeys is for keys that do not take a value (i.e. keys using the syntax $\backslash\langle$ object $\rangle[$ key ]), and valuekeys is for keys that do take a value (i.e. keys using the syntax $\backslash\langle o b j e c t\rangle[k e y=v a l u e])$ ). We explain the syntax for using them in the next section where we show how to make keyval syntax for elementary calculus.

For the rest of the manual, we assume that you have already defined a class $\backslash \mathrm{My}$ Var and the variables $\backslash \mathrm{va}, \backslash \mathrm{vA}, \backslash \mathrm{vb}, \backslash \mathrm{vB}, \ldots$, as above.

## 2 Example: Elementary calculus

One thing we might want to do to a variable is invert it. We therefore add a key inv that adds an upper index -1 to the symbol. We add this key using the key singlekeys, which is for keys that do not take a value:

```
\SetupClass\MyVar{
    singlekeys={
        {inv}{ upper={-1} },
    },
}
```

Now the key inv has been defined to be equivalent to upper=\{-1\}. Now we can do the following:

```
$\va[inv]$, $\vf[inv]$,
$\vg[1,2,inv]$,
$\vh[\va,\vb,inv]$
```

$a^{-1}, f^{-1}, g_{1,2}^{-1}, h_{a, b}^{-1}$

Other keys might need to take a value. For defining such, we have the command valuekeys. For instance, suppose we want a command for deriving a function $n$ times. For this, we add the key der:

```
\SetupClass\MyVar{
    singlekeys={
        {inv}{ upper={-1} },
    },
    valuekeys={
        {der}{ upper={(#1)} },
    },
}
```

The \#1 will contain whatever the user wrote as the value of the key. Now we can write:

$$
\$ \backslash v f[\operatorname{der}=\backslash \mathrm{vn}]\{\backslash \mathrm{vx}\} \$ \quad f^{(n)}(x)
$$

Maybe we also want a more elementary key power for raising a variable to a power:

```
\SetupClass\MyVar{
    singlekeys={
        {inv}{ upper={-1} },
    },
    valuekeys={
        {der}{ upper={(#1)} },
        {power}{ upper={#1} },
    },
}
```

This allows us to write
\$ $\backslash \mathrm{vx}[$ power=2] ,
$\$ \backslash \mathrm{vy}\left[1\right.$, power=2] $+\operatorname{lvy}\left[2\right.$, power $\quad x^{2}, y_{1}^{2}+y_{2}^{2}$
$=2] \$$
Let us try doing something a bit more complicated: adding a key for restricting a function to a smaller subset. For this, we do the following:

```
\SetupClass \MyVar{
    singlekeys={
        {inv}{ upper={-1} },
    },
    valuekeys={
        {der}{ upper={(#1)} },
        {power}{ upper={#1} },
        {res}{ return,symbolputright={|}, lower={#1} },
    },
}
```

This adds a horizonal line " $\mid$ " to the right of the symbol followed by a lower index containing whatever you passed to the key (contained in the command \#1). (There is also an extra key, return, which is a bit more advanced and should be taken for granted for now. Roughly speaking, it is there to make sure that the restriction symbol is printed after all indices that you might have added before. More details in chapter 6.) Now we may write the following:
$\$ \backslash v f[r e s=\backslash v U]\{\backslash v x\} \$$,
$\$ \backslash \mathrm{vg}[1, \mathrm{res}=\backslash \mathrm{vY}]\{\backslash \mathrm{vy}\} \$$,
\$ $\backslash \mathrm{vh}[i n v, r e s=\backslash \mathrm{vT}]\{\backslash \mathrm{vz}\} \$$

$$
\left.f\right|_{U}(x),\left.g_{1}\right|_{Y}(y),\left.h^{-1}\right|_{T}(z)
$$

If the reader starts playing around with the SemanT $\mathrm{E}_{\mathrm{E}} \mathrm{X}$ functions, they will discover that whenever you apply a function to something, the result becomes a new function that can take an argument itself (this is why we wrote output $=\backslash$ MyVar in the definition of the class $\backslash M y V a r$ ). This behaviour is both useful and extremely necessary in order for the package to be useful in practice. For instance, you may write

```
$\vf[der=\vn]{\vx}{\vy}{\vz}
= \vg{\vu,\vv,\vw}[3]{
        \vt}$
```

    \(f^{(n)}(x)(y)(z)=g(u, v, w)_{3}\left(x_{1}, x_{2}\right)_{8,1}^{(2)}(t)\)
    Some people prefer to be able to scale the vertical line in the restriction notation. I rarely do that, but for this purpose, we could do the following:

```
\(\backslash\) SetupClass \MyVar\{
    valuekeys=\{
        \{bigres \(\}\) \{ return, symbolputright=\big\vert, lower=\{\#1\} \},
        \(\{\) Bigres \(\}\{\) return, symbolputright=\Big\vert, lower=\{\#1\} \},
        \(\{b i g g r e s\}\{r e t u r n, ~ s y m b o l p u t r i g h t=\backslash b i g g \backslash v e r t, ~ l o w e r=\{\# 1\}\}\),
        \(\{B i g g r e s\}\{r e t u r n, ~ s y m b o l p u t r i g h t=\backslash B i g g \backslash v e r t, ~ l o w e r=\{\# 1\}\}\),
        \{autores\}\{ return, Otherspar=\{.\}\{\vert\}\{auto\},
        lower \(=\{\# 1\}\),
        \% This auto scales the vertical bar. See the chapter on the
        \% spar key for information about sparsize and Otherspar
    \},
\}
```

So to sum up, we first defined a class \MyVar via \NewVariableClass and then used \SetupClass to add keys to it. In fact, we could have done it all at once by passing these options directly to \NewVariableClass:

```
\NewVariableClass \MyVar[
    output=\MyVar, % This means that the output of an object
        % of class |MyVar is also of class |MyVar
    singlekeys={
        {inv}{ upper={-1} },
    },
    valuekeys={
        {der}{ upper={(#1)} },
        {power}{ upper={#1} },
        {res}{ rightreturn, symbolputright={|},
            lower={#1} },
    },
]
```

As we proceed in this guide, we shall use $\backslash$ SetupClass to add more and more keys to \MyVar. However, when you set up your own system, you may as well just add all of the keys at once like this when you create the class and then be done with it.

Let me add that it is possible to create subclasses of existing classes. You just write parent= $\backslash\langle$ Class $\rangle$ in the class declaration to tell that $\backslash\langle$ Class $\rangle$ is the
parent class. But a word of warning: It is a natural idea to create different classes for different mathematical entities, each with their own keyval syntax that fits whatever class you are in; for instance, you could have one class for algebraic structures like rings and modules with keys for opposite rings and algebraic closure, and you could have another class for topological spaces with keys for closure and interior. However, as the reader can probably imagine, this becomes extremely cumbersome to work with in practice since an algebraic structure might very well also carry a topology. So at the end of the day, I advice you to use a single superclass $\backslash M y V a r$ that has all the keyval syntax and mainly use subclasses for further customization. We shall see examples of this below.

## 3 Example: Elementary algebra

Let us try to use SemanTEX to build some commands for doing algebra. As an algebraist, one of the first things you might want to do is to create polynomial rings $k[x, y, z]$. Since all variables can already be used as functions (this is a design choice we discussed earlier), all we need to do is find a way to change from using parentheses to square brackets. This can be done the following way:

```
\SetupClass\MyVar{
    singlekeys={
        {poly}{
            par, % This tells semantex to use parentheses around
                    % the argument in the first place, in case this
                    % had been turned off
                leftpar=[,rightpar=],
        },
    },
}
```

Now we may write

$$
\text { \$\vk[poly ]\{\vx, \vy, \vz\}\$ } k[x, y, z]
$$

It is straightforward how to do adjust this to instead write the field generated by the variables $x, y, z$ :

```
\SetupClass\MyVar{
    singlekeys={
        {poly}{
            par, % This tells semantex to use parentheses around
                    % the argument in the first place, in case this
                    % had been turned off
            leftpar=[,rightpar=],
        },
        {field}{
            par,
            leftpar=(,rightpar=),
        },
    },
}
```

Now $\backslash v k[f i e l d]\{\backslash v x, \backslash v y, \backslash v z\}$ produces $k(x, y, z)$. Of course, leaving out the field key would produce the same result with the current configuration of the class \MyVar. However, it is still best to use a key for this, both because this makes the semantics more clear, but also because you might later change some settings that would cause the default behaviour to be different.

Adding support for free algebras, power series, and Laurent series is almost as easy, but there is a catch:

```
\SetupClass\MyVar{
    singlekeys={
        {poly}{
            par, % This tells semantex to use parentheses around
                    % the argument in the first place, in case this
                    % had been turned off
            leftpar=[,rightpar=],
        },
        {field}{
            par,
            leftpar=(,rightpar=),
        },
        {freealg}{
            par,
            leftpar=\noexpand\langle,
            rightpar=\noexpand\rangle,
        },
        {powerseries}{
            par,
            leftpar=\noexpand\llbracket,
            rightpar=\noexpand\rrbracket,
        },
        {laurent}{
            par,
            leftpar=(, rightpar=),
            prearg={\!\!\noexpand\SemantexDelimiterSize(},
            postarg={\noexpand\SemantexDelimiterSize)\!\!},
            % These are printed before and after the argument.
            % The command "|SemantexDelimiterSize" is substituted
            % by lbig, \Big, ..., or whatever size the
            % argument delimiters have
        },
    },
}
```

For expansion reasons (which I am not completely sure of), we need \noexpand before these commands. In general, whenever something fails, try throwing in Inoexpand's in front of suspicious-looking commands, and things will usually work out just fine. See for yourself:
\$\vk[freealg]\{\vx\}\$,
$\$ \backslash \mathrm{vk}[$ powerseries $]\{\backslash \mathrm{vy}\} \$, \quad k\langle x\rangle, k \llbracket y \rrbracket, k((z))$
\$\vk[laurent]\{\vz\}\$
Let us look at some other algebraic operations that we can control via SemanTEX:

```
\SetupClass \MyVar{
    singlekeys={
        {op}{upper={\noexpand\mathrm{op}}},
            % opposite groups, rings, categories, etc.
            {algclosure}{overline},
                % algebraic closure
            {conj}{overline},
                % complex conjugation
            {dual}{upper=*} ,
                % dual vector space
            {perp}{upper=\perp},
                % orthogonal complement
    },
    valuekeys={
        {mod}{symbolputright={/#1}},
                % for modulo notation like R/I
            {dom}{symbolputleft={#1\backslash}},
                % for left modulo notation like I\R
                % "dom" is "mod" spelled backwards
            {oplus}{upper={\oplus#1}},
                % for notation like R^{loplus n}
            {tens}{upper={\otimes#1}},
                % for notation like R^{lotimes n}
            {localize}{symbolputright={ \relax [#1^{-1}] }},
                % localization at a multiplicative subset;
                % the Irelax is necessary becauese, in some cases,
                % the [...] can be interpreted as an optional argument
            {localizeprime}{lower={#1}},
                % for localization at a prime ideal
    },
}
```

Let us see it in practice:

```
$\vR[op]$, $\vk[algclosure]$,
$\vz[conj]$, $\vV[dual]$,
$\vR[mod=\vI]$,$\vR[dom=\vJ
]$ ,
$\vR[oplus=\vn]$,
$\vV[tens=\vm]$,
```

$\$ \backslash \mathrm{vR}[$ localize $=\backslash \mathrm{vS}] \$$,
\$ \vR[localizeprime=\vI]\$,
\$ $\backslash v k[f r e e a l g]\{\backslash v S\}[o p] \$$,
\$ $\backslash \mathrm{vV}[$ perp] $\$$

## 4 The spar key

The spar key is one of the most important commands in SemanTEX at all. To understand why we need it, imagine you want to derive a function $n$ times and then invert it. Writing something like
\$\vf[der=\vn,inv]\$

```
f(n)-1
```

does not yield a satisfactory result. However, the spar key saves the day:

```
$\vf[der=\vn,spar,inv]$
```

$$
\left(f^{(n)}\right)^{-1}
$$

So spar simply adds a pair of parentheses around the current symbol, complete with all indices that you may have added to it so far. The name spar stands for "symbol parentheses". You can add as many as you like:

```
$ \vf[1,res=\vV,spar,conj,op,
spar,0,inv,spar,mod=\vI,spar,
dual]{\vx} $
\[
\left(\left(\left({\overline{\left(\left.f_{1}\right|_{V}\right)}}^{\mathrm{op}}\right)_{0}^{-1}\right) / I\right)^{*}(x)
\]
```

If it becomes too messy, you can scale the parentheses, too. Simply use the syntax spar=\big, spar=\Big, etc. You can also get auto-scaled parentheses base on \left... \right, using the key spar=auto:

```
$\vf[spar]$,
```

\$\vf[spar=\big]\$,
\$\vf[spar=\Big]\$,
$\$ \backslash v f[s p a r=\backslash b i g g] \$$,
\$\vf[spar=\Bigg]\$,
\$\vf[spar=auto]\$

$$
(f),(f),(f),(f),(f),(f)
$$

\$\vf[spar=auto]\$
So returning to the above example, we can write
$\$ \backslash v f[1, r e s=\backslash v V, s p a r, c o n j, o p$, spar=\big,0,inv, spar=\Big,mod $=\backslash v I$, spar $=\backslash$ big, dual $]\{\backslash v x\} \$$

$$
\left(\left(\left({\overline{\left(\left.f_{1}\right|_{V}\right)}}^{\mathrm{op}}\right)_{0}^{-1}\right) / I\right)^{*}(x)
$$

To adjust the type of brackets, use the leftspar and rightspar keys:
$\$ \backslash v f[l e f t s p a r=\{[ \}, r i g h t s p a r$
$=\{\backslash\}\}$, spar, spar $=\backslash$ Big $] \$$


Occassionally, it is useful to be able to input a particular kind of brackets just once, without adjusting any settings. For this purpose, we have the otherspar and 0 therspar keys. They use the syntax

```
otherspar={\langleopening bracket\rangle}{\langleclosing bracket\rangle}
Otherspar={\langleopening bracket\rangle}{\langleclosing bracket\rangle}{\langlesize\rangle}
```

Let us see them in action:

```
$\vf[otherspar={[}{)},
otherspar = {\{}{\rangle},
    Otherspar={\langle}{\rangle
    }{\Bigg},spar]$
```



## 5 The $\backslash\langle$ Class $\rangle$ command

So far, we have learned that every mathematical entity should be treated as an object of some class. However, then we run into issues the moment we want to write expressions like

$$
(f \circ g)^{(n)}(x) .
$$

We do not want to have to define a new variable with symbol $f \circ g$ just to write something like this．Fortunately，once you have created the class $\backslash \mathrm{MyVar}$ ， you can actually use $\backslash M y$ Var as a command to create a quick instance of the class．More precisely $\backslash \operatorname{MyVar}\{\langle$ symbol $\rangle\}$ creates a variable on the spot with symbol 〈symbol〉．So the above equation can be written
$\$ \backslash \operatorname{MyVar}\{\backslash \mathrm{vf} \backslash \mathrm{circ} \backslash \mathrm{vg}\}[$
spar，der＝\vn］\｛\vx\}\$

$$
(f \circ g)^{(n)}(x)
$$

More generally，when you crate the class $\backslash\langle$ Class $\rangle$ ，you can use it as a command with the following syntax：
\〈Class $\rangle\{\langle$ symbol $\rangle\}[\langle$ options $\rangle]\langle u s u a l$ syntax of class $\rangle$

## 6 The return keys

Suppose you want to take the complex conjugate of the variable $z_{1}$ ．Then you might write something like
\＄\vz［1，conj］\＄

$$
\bar{z}_{1}
$$

Notice that the bar has only been added over the $z$ ，as is standard mathematical typography；you normally do not write $\overline{z_{1}}$ ．This reveals a design choice that has been made in SemanTEX：When you add an index or a command via the command key，it is not immediately applied to the symbol．Rather，both commands and indices are added to a register and are then applied at the very last，right before the symbol is printed．This allows us to respect standard mathematical typography，as shown above．

However，there are other times when this behaviour is not what you want． For instance，if you want to comjugate the inverse of a function，the following looks wrong：
\＄\vf［inv，conj］\＄
$\bar{f}^{-1}$
Therefore，there is a command return that can be applied at any point to invoke the routine of adding all indices and commands to the symbol．Let us try it out：
\＄\vf［inv，return，conj］\＄

```
\mp@subsup{f}{}{-1}
```

In fact，return is an umbrella key that invokes three different return routines： leftreturn，innerreturn，and rightreturn．The command leftreturn adds the left indices to the symbol（we have not discussed left indices yet，though）．The command innerreturn adds all commands to the symbol（those defined using the command key）．Finally，rightreturn adds all right indices and arguments to the symbol．In general，the user should probably be satisfied with just using return．

## 7 The command key

Above，we used the key overline a couple of times：
\$\va[overline] $\$$,
\$ VH [overline] $\$$
This command applies the command \overline to the symbol. In fact, you can create similar commands yourself via the command key. In fact, you could have defined the overline yourself as follows:

```
\SetupClass \MyVar{
    singlekeys={
        {overline}{command=\noexpand\overline},
    },
}
```

This is how the key overline is defined internally, except it is defined on the level of the superclass \SemantexVariable instead. We need the key \noexpand in order for everything to expand properly. This is only necessary for some commands, and to tell the truth, I haven't quite figured out the system of which commands need it and which ones do not. However, as usual, if something does not work, try throwing in some \noexpand's and see if it solves the problem. Here are some more examples of predefined keys that use the command key:

```
\SetupClass\MyVar{ % do not add these -- they are already
predefined!
    novalueskeys={
        {smash}{command=\noexpand\smash},
        {tilde}{command=\noexpand\tilde},
        {widetilde}{command=\widetilde},
        {bar}{command=\ noexpand\bar},
        {bold}{command=\ noexpand\mathbf},
        {roman}{command=\noexpand\mathrm},
    },
}
```

Let us test:
\$lva[widetilde] ,
\$\va[bold]\$,
\$ iva[roman] $\$$
\$lva[bar]\$

## 8 Example: Algebraic geometry

Let us discuss how to typeset sheaves and operations on morphisms in algebraic geometry. First of all, adding commands for sheaves is not a big deal:

```
\New0bject\MyVar\sheafF {\mathcal{F}}
\New0bject\MyVar\sheafG}{\mathcal{G}}
\New0bject\MyVar\sheafH{\mathcal{H}}
\New0bject\MyVar\sheafreg{\mathcal {0}}
    % sheaf of regular functions
\NewObject\MyVar\sheafHom{\mathop{\mathcal{H}om}}
```

You can of course add as many sheaf commands as you need.
Next, for morphisms of schemes $f: X \rightarrow Y$, we need to be able to typeset comorphisms as well as the one hundred thousand different pullback and pushforward operations. For this, we add some keys to the \MyVar key:

```
\SetupClass\MyVar{
    singlekeys={
        {comorphism}{upper=\#},
            % comorphisms, i.e. f^{\#}
        {inverseimage}{upper={-1},nopar},
            % inverse image of sheaves
        {sheafpull}{upper=*,nopar},
            % sheaf *-pullback
        {sheafpush}{lower=*,nopar},
            % sheaf *-pushforward
        {sheaf!pull}{upper=!,nopar},
            % sheaf !-pullback
        {sheaf!push}{lower=!,nopar},
            % sheaf !-pushforward
    },
}
```

We have added the command nopar to all pullback and pushforward commands since it is custom to write, say, $f^{*} \mathcal{F}$ rather than $f^{*}(\mathcal{F})$. Of course, you can decide that for yourself, and in any case, you can write $\backslash v f[$ sheafpull, par]\{\sheaff \} if you want to force it to use parentheses in a particular case. Of course, since all SemanTEX variables can be used as functions, so can whatever these pullback and pushforward operations output. So we may write:

```
For a morphism~$ \vf \colon
IvX \to \vY $ with
comorphism~$ \vf[comorphism]
\colon \sheafreg[\vY] \to
\vf[sheafpush]{\sheafreg[\vX
]} $,
and for a sheaf~$ \sheafF $
on~$ lvY $, we can define the
pullback~$ \vf[sheafpull]{
\sheafF} $ by letting~$
\vf[sheafpull]{ \sheafF}{\vU}
    = \cdots $ and the $ ! $-
pullback by letting~$
\vf[sheaf!pull]{\sheafF }{\vU}
= \cdots $.
```

For a morphism $f: X \rightarrow Y$ with comorphism $f^{\#}: \mathcal{O}_{Y} \rightarrow f_{*} \mathcal{O}_{X}$, and for a sheaf $\mathcal{F}$ on $Y$, we can define the pullback $f^{*} \mathcal{F}$ by letting $f^{*} \mathcal{F}(U)=\cdots$ and the !-pullback by letting $f^{!} \mathcal{F}(U)=\cdots$.

Maybe some people would write pull, push, etc. instead, but there are many different kinds of pullbacks in mathematics, so I prefer to use the sheaf prefix to show that this is for sheaves. Probably, in the long run, an algebraic geometer might also want to abbreviate inverseimage to invim.

There are a number of other operations we might want to do for sheaves. We already defined the key res for restriction, so there is no need to define this again. However, we might need to stalk, sheafify, take dual sheaves, and twist sheaves. Let us define keys for this:

```
\SetupClass\MyVar{
    valuekeys={
        {stalk}{clower={#1}},
        % "clower" means "comma lower", i.e. lower index
        % separated from any previous lower index by a comma
        {sheaftwist}{return, symbolputright={(#1)}},
```

```
    },
    singlekeys={
        {sheafify}{upper=+},
        {sheafdual}{upper=\vee},
    },
}
```

The key clower stands for "comma lower". It is like lower, except that it checks whether the index is already non-empty, and if so, it separates the new index from the previous index by a comma. There is, of course, a cupper key that does the same with the upper index.

```
$\sheafF[res=\vU,stalk=\vp]$,
$\sheafF[res=\vU, spar, stalk=
lvp]$,
$\sheafreg[\vX, stalk=\vp]$,
$\sheafG[sheafify]$,
$\vf[inverseimage]{\sheafreg[
\vY]}[spar, stalk=\vx]$
$\sheafG[sheafdual]$,
$\sheafreg[\vX][sheaftwist
=-1]$,
$\sheafreg[sheaftwist=1,
sheafdual]$
```


## 9 Example: Homological algebra

Before you venture into homological algebra, you should probably define some keys for the standard constructions:

```
\NewObject \MyVar \(\backslash \operatorname{Hom}\{\) \operatorname \(\{\) Hom \(\}\}\)
\NewObject \(\backslash M y V a r \backslash E x t\{\backslash o p e r a t o r n a m e\{E x t\}\}\)
\NewObject \MyVar\Tor \(\{\backslash\) operatorname \(\{\) Tor \(\}\}\)
```

Now the ability to easily print indices via the options key will come in handy:
\$ $\backslash \operatorname{Hom}[\backslash v A]\{\backslash v M, \backslash v N\} \$$,
\$\Ext[\vA]\{\vM, \vN\}\$
$\operatorname{Hom}_{A}(M, N), \operatorname{Ext}_{A}(M, N)$
You will probably need several keyval interfaces, some of which will be covered below. But right now, we shall implement a shift operation $X \mapsto X[n]$ :

```
\SetupClass\MyVar{
    valuekeys={
            {shift}{ return,symbolputright={ \relax [ {#1} ] } },
            % |relax is necessary since otherwise [...] can
            % occasionally be interpreted as an optional argument
    },
}
```

Let us see that it works:
$\$ \backslash v X \backslash$ mapsto $\backslash v X[s h i f t=\backslash v n] \$$

$$
X \mapsto X[n]
$$

Finally, let us define a command for the differential (in the homolgoical algebra sense):
\New0bject $\backslash$ MyVar $\backslash$ dif $\{d\}[$ nopar ]
$\$ \backslash \operatorname{dif}\{\backslash v x\}=0 \$$

$$
d x=0
$$

### 9.1 The keys $\mathrm{i}=$ index and $\mathrm{d}=$ deg $=$ degree

Homological algebra is a place where people have very different opinions about the positions of the gradings. As an algebraist, I am used to upper gradings ("cohomological" grading), whereas many topologists prefer lower gradings ("homological" grading). The SemanTEX system supports both, but the default is upper gradings (the package author has the privilege to decide). You can adjust this by writing gradingposition=upper or gradingposition=lower.

We already learned about the keys upper and lower. There are two more, "relative" keys that print the index either as an upper index or as a lower index, depending on your preference for cohomological or homological grading. They are called
index and degree

The degree is the actual grading in the homological algebra sense. The index is an additional index where you can put extra information that you might need. To understand the difference, keep the following two examples in mind: the hom complex $\operatorname{Hom}_{A}^{-}$and the simplicial homology $H^{\Delta}$ (we will define the command \ho for homology in the next section):

| $\$ \backslash \operatorname{Hom}[$ inde $=\backslash v A$, degree $=0] \$$, | $\operatorname{Hom}_{A}^{0}, H_{1}^{\Delta}$ |
| :--- | :--- |
| $\$ \backslash$ ho $[$ index $=\backslash$ vDelta, degree $=1] \$$ |  |

These names are not perfect; many people would say that the degree is also an index, but feel free to come up with a more satisfactory naming principle, and I shall be happy to consider it. These names probably become a bit too heavy to write in the long run, so both keys have abbreviated equivalents:

$$
i=\text { index } \quad \text { and } \quad d=d e g=\text { degree }
$$

Let us see them in action:

```
$ \vX[d=3,i=\vk] $
\Setup0bject\vX{
    gradingposition=lower
}
```

| $X_{k}^{3}$ |
| :--- |
| $X_{3}^{k}$ |

\$ $\backslash v X[d=3, i=\backslash v k] \$$
(We haven't seen the command \Setup0bject before, but I imagine you can guess what it does). If you want to print a bullet as the degree, there is the predefined key * for this:
\$ $\operatorname{lvX}[*] \$$

```
\Setup0bject\vX{
    gradingposition=lower
}
X.
```

```
\$ \(\operatorname{lvX}[*] \$\)
```

I guess it is also time to reveal that the previously mentioned shorthand notation $\operatorname{lvx}[1]$ for indices always prints the 1 on the index position. So changing the grading position changes the position of the index:

```
$ \vX[1] $
```

```
\Setup0bject\vX{
    gradingposition=lower
}
```

$X_{1}$
$X^{1}$
\$ \vX[1] \$

In other words, in the first example above, we could have written
\$ $\backslash \operatorname{Hom}[\backslash v A, d=0] \$$,
\$ $\backslash$ ho [ \vDelta, $d=1] \$$

$$
\operatorname{Hom}_{A}^{0}, H_{1}^{\Delta}
$$

Note that the use of the short notations $d$ and i does not prevent you from writing \vx[d] and \vx[i]. This still works fine:

```
$\vf[i]$, $\vf[i=]$,
$\vf[d]$, $\vf[d=]$
```

$$
f_{i}, f, f_{d}, f
$$

As we see, it is only when a d or i key is followed by an equality sign = that the routines of these keys are invoked. In fact, SemanTEX carefully separates valuekeys from singlekeys.

### 9.2 The Cohomology class type

Now homological algebra is hard unless we can do cohomology and homology. In principle, this is not hard to do, as we can write e.g. $\backslash \mathrm{vH}[\mathrm{d}=0]\{\backslash \mathrm{vX}\}$ to get $H^{0}(X)$. However, some people might find it cumbersome to have to write d= every time you want to print an index. This is probably the right time to reveal that SemanTEX supports multiple class types. So far, we have been exclusively using the Variable class type, which is what you create when you apply the command \NewVariableClass. The first other class type we shall need is the Cohomology class type, which has a different input syntax that fits cohomology. Let us try to use it:

```
\NewCohomologyClass \MyCohomology[
    parent=\MyVar,gradingposition=upper
]
\New0bject\MyCohomology\co{H}
\NewCohomologyClass \MyHomology[
    parent=\MyCohomology, gradingposition=lower
```

$\backslash$ NewObject $\backslash$ MyHomology $\backslash$ ho $\{H\}$
The cohomology command $\backslash c o$ in general works very much like a command of Variable type. However, the input syntax is a bit different:

```
\co[\langleoptions\rangle]{\langledegree\rangle}{\langleargument\rangle}
```

All three arguments are optional. Let us see it in practice:

```
$\co{0}$, $\co{*}$,
$\co{\vi}{\vX}$,
$\co[\vG]{0}$,
$\co[\vH]{*}$,
$\co[\vDelta]{\vi}{\vX}$
$\ho{0}$, $\ho{*}$,
$\ho{\vi}{\vX}$,
$\ho[\vG]{0}$,
$\ho[\vH]{*}$,
```

\$\ho[\vDelta]\{\vi\}\{\vX\}\$

Of course, you can define similar commands for cocycles, coboundaries, and all sorts of other entities that show up in homological algebra.

You might also want to implement feature like reduced cohomology, Čech cohomology, and hypercohomology. This is quite easy with the command key:

```
\SetupClass\MyVar{
    singlekeys={
        {reduced}{command=\widetilde},
        {cech}{command=\ noexpand\check},
        {hyper{command=\ noexpand \mathbb},
    },
}
$\co[reduced]{i}$,
$\co[cech]{*}$,
$\co[hyper, cech ] {0}{\vX}$
\[
\widetilde{H}^{i}, \check{H}^{\cdot}, \check{H}^{0}(X)
\]
```

The Cohomology class type also provides a nice way to implement derived functors:
$\backslash$ New0bject $\backslash$ MyCohomology $\backslash \operatorname{Lder}\{\backslash$ mathbb $\{L\}\}$ [nopar]
\NewObject \MyCohomology \Rder $\{\backslash$ mathbb $\{R\}\}$ [nopar]
For instance, we can write
$\$ \backslash \operatorname{Lder}\{\backslash \mathrm{vi}\}\{\backslash \mathrm{vf}\} \$$,
$\$ \backslash \operatorname{Rder}\{0\}\{\backslash \mathrm{vf}\} \$$
$\mathbb{L}^{i} f, \mathbb{R}^{0} f$
Alternatively, the user might prefer to use keyval syntax on the level of the function itself ( $f$ in this case). This can be done the following way:

```
\SetupClass\MyVar{
    valuekeys={
        {Lder} {
                innerreturn,leftreturn,
                symbolputleft=\noexpand\mathbb{L}^{#1},
        },
        {Rder} {
            innerreturn,leftreturn,
            symbolputleft=\noexpand\mathbb{R}^{#1},
        },
    },
    singlekeys={
        {Lder} {
            innerreturn,leftreturn,
            symbolputleft=\noexpand\mathbb{L},
        },
        {Rder} {
            innerreturn,leftreturn,
            symbolputleft=\noexpand\mathbb {R},
        },
    },
}
```

Then the syntax becomes:

```
$\vF[Lder=\vi]$,
$\vF[Lder]{\vX[*]}$,
$\vF[Rder]{\vX[*]}$,
L i
$\Hom[Rder]{\vX,\vY}$
```

If you get tired of having to write \Hom[Rder] all the time, you can create a shortcut:
\NewObject $\backslash$ MyVar $\backslash$ RHom [ clone $=$ \Hom, Rder]
The clone key is like the parent key, except it allows you to inherit the settings from an object rather than a class. Notice that we did not specify a symbol; the symbol argument is optional, and in this case, it was unnecessary, as the symbol was inherited from $\backslash$ Hom. Let us see it in action:

| $\$ \backslash \operatorname{Hom}\{\backslash \mathrm{~V}, \backslash \mathrm{Y}\}$ |  |
| :--- | :--- | :--- |
| $\$$ | $\mathbb{R} \operatorname{Hom}(X, Y)$ |

## 10 Keyval syntax in arguments (Example: Cohomology with coefficients)

Imagine we want to do cohomology with coefficients in some ring $R$. It is common to write this as $H^{\bullet}(X ; R)$ with a semicolon instead of a comma. This can be implemented, too, with the syntax
$\$ \backslash \operatorname{co}\{*\}\{\backslash v X, \operatorname{coef}=\backslash v R\} \$$
$H^{\cdot}(X ; R)$
This shows that arguments of functions also support keyval syntax. In order to customize this, there are two extra keys:
argsinglekeys and argvaluekeys

These work exactly like singlekeys and valuekeys.

```
\SetupClass\MyVar{
    argvaluekeys={
        {coef}{ othersep={;}{#1} },
    },
}
```

(But it will not quite work yet - stay tuned for a moment!) The key othersep is a key that controls the separator between the current argument and the previous argument (it will only be printed if there was a previous argument). By default, this separator is a comma. So in the syntax $\backslash \operatorname{co}\{*\}\{\backslash v X, \operatorname{coef}=\backslash v R\}$, there are two arguments, $\backslash v X$ and $\backslash v R$, and the separator is a semicolon.

However, even with the above setup, the notation $\backslash \operatorname{co}\{*\}\{\backslash v X, \operatorname{coef}=\backslash \mathrm{vR}\}$ will not work just yet. For the keys you define using argvaluekeys are turned off by default. To turn them on for the object \co, run the following code:

```
\Setup0bject\co{
    valuekeys={
        {arg}{argwithkeyval={#1}},
    },
}
```

The reason the keys are turned off by default is that keys in arguments that support values are only used in very rare cases, like cohomology with coefficients. If such keys were turned on in general, it would mess up every occurrence of an equality sign in arguments, and the following would not work:

```
$\Hom[\sheafreg[\vU]]{
    \sheafG[res=\vU]
}$
```

    IsheafF[res=\vU], \(\quad \operatorname{Hom}_{\mathcal{O}_{U}}\left(\left.\mathcal{F}\right|_{U},\left.\mathcal{G}\right|_{U}\right)\)
    It should be noted that there are several predefined keys (of type singlekey) which are defined on the level of the class \SemantexVariable. The full list is:

- slot, ...

We should also talk about the arg key.

## 11 Multi-value keys

Sometimes, a key with one value is simply not enough. For instance, if you work in GIT, you will eventually have to take the proj quotient $X / /{ }_{\chi} G$ of $X$ with respect to the action of the group $G$ and the character $\chi$. In other words, the proj quotient depends on two parameters, $\chi$ and $G$. For this purpose, we have the key $2 v a l u e k e y s$. It works exactly like valuekeys, except you get to use two arguments instead of one:

```
\SetupClass \MyVar\{
    2valuekeys=\{
        \{projquotient\}\{ symbolputright=\{ ハ!!!/ _ \{ \#1 \} \#2 \} \},
    \}
\}
```

```
$\vX[projquotient={\vchi}{\vG
}] $
X// XG
```

There are also commands 3valuekeys，4valuekeys，．．．，9valuekeys．The syntax for these is completely analoguous．There are also commands arg2valuekeys， arg3valuekeys，．．．，arg9valuekeys for keys in arguments with multiple values．

## 12 The Simple class type（Example：Derived tensor products and fibre products）

The SemanTEX system has facilities for printing tensor products $\otimes$ as well as derived tensor products $\otimes^{L}$ ．For this，we need the Simple class type．This has exactly the same syntax as the Variable class type，except that it cannot take an argument．In other words，its syntax is

```
\〈object〉[〈options〉]
```

（You should normally only use it for special constructions like binary operators and not for e．g．variables－the ability to add arguments to variables comes in handy much more often than one might imagine．）Let us try to use it to define tensor products and fibre products：

```
\NewSimpleClass \MyBinary0perator[
    singlekeys={
        {Lder}{upper=L},
        {Rder} {upper=R },
    },
    mathbin,
    % this makes sure that the output is wrapped in |mathbin
]
\New0bject\MyBinary0perator\tensor{\otimes}[
    singlekeys={
        {der}{Lder},
    },
]
\New0bject\MyBinary0perator\fibre{\times }[
    % Americans are free to call it |fiber instead
    singlekeys={
        {der}{Rder},
    },
]
```

As you see，this is one of the few cases where I recommend adding keyval syntax to other classes than your superclass \MyVar．Also，notice that it does not have any parent＝$=\backslash M y$ Var，as I do not really see any reason to inherit all the keyval syntax from the \MyVar class．Now we first define keys Lder and Rder for left and right derived binary operators．Next，we build in a shortcut in both Itensor and \fibre so that we can write simply der and get the correct notion of derived functor．Let us see it in action：

```
$\vA \tensor \vB$,
$\vX[*] \tensor[\vR] \vY[*]$
$\vk \tensor[\vA,der] \vk$,
A\otimesB,X`\otimes}\mp@subsup{\otimes}{R}{}\mp@subsup{Y}{}{`}k\mp@subsup{\otimes}{A}{L}k,X\mp@subsup{\times}{Y}{R}
```

\$\vX \fibre[\vY,der] \vX\$

## 13 Class types

The SemanT ${ }_{\mathrm{E}} \mathrm{X}$ system uses several different class types．We have been almost exclusively using the Variable class type（which is by far the most important one），but in the last chpaters，we were introduced to the Cohomology and the Simple class types．

In fact，all class types are identical internally；the low－level machinery of SemanTEX does not＂know＂what type a class has．The only difference between the class types is the input syntax．In other words，it determines which arguments an object of that class can take．The syntax for creating new objects also varies．

The current implementation has the following class types：
－Variable：A new class is declared with the syntax
\NewVariableClass $\{\backslash\langle$ Class $\rangle\}[\langle$ options $\rangle]$
A new object is declared by
\NewObject\〈Class〉\〈object〉\｛〈symbol〉\}[〈options〉]
The syntax for this object is
\〈object〉［〈options〉］\｛〈argument〉\}
－Cohomology：A new class is declared with the syntax
\NewCohomologyClass \＜Class〉［〈options〉］
A new object is declared by
\New0bject $\backslash\langle$ Class $\rangle \backslash\langle$ object $\rangle\{\langle$ symbol $\rangle\}[\langle$ options $\rangle]$
The syntax for this object is
\〈object〉［〈options $\rangle$ \｛ $\langle$ degree $\rangle\}\{\langle$ argument $\rangle\}$
－Simple：A new class is declared with the syntax
\NewSimpleClass \＜Class〉［〈options〉］
A new object is declared by
\NewObject $\backslash\langle C l a s s\rangle \backslash\langle o b j e c t\rangle\{\langle$ symbol $\rangle\}[\langle$ options $\rangle]$
The syntax for this object is
\〈object〉［〈options〉］\｛〈argument〉\}
－Delimiter：A new class is declared with the syntax
\NewDelimiterClass \＜Class〉［〈options〉］
A new object is declared by
\New0bject $\backslash\langle C l a s s\rangle \backslash\langle o b j e c t\rangle\{\langle l e f t ~ b r a c k e t\rangle\}\{\langle r i g h t$ bracket〉\}[〈options〉]

The syntax for this object is

$$
\backslash\langle\text { object }\rangle[\langle\text { options }\rangle]\{\langle\operatorname{argument}\rangle\}
$$

Let me add that SemanTEX uses a very clear separation between the input syntax and the underlying machinery. Because of this, if the user needs a different kind of class type, it is not very hard to create one. You must simply open the source code of $\mathrm{SemanT}_{\mathrm{E}} \mathrm{X}$, find the class you want to modify, and then copy the definition of the command $\backslash \mathrm{New}\langle\mathrm{Cl}$ ass type〉Class and modify it in whatever way you want.

## 14 The Delimiter class type

Delimiters are what they sound like: functions like $\|-\|$ and $\langle-,-\rangle$ that are defined using brackets only. Let us define a class of type Delimiter:
\NewDelimiterClass \MyDelim[parent=\MyVar]
Now we can create instances of the class \MyDelim with the following syntax:
\NewObject $\backslash$ MyDelim $\backslash\langle$ object $\rangle\{\langle$ left bracket $\rangle\}\{\langle$ right bracket $\rangle\}[\langle$ options

Now we can do the following:

```
\NewObject\MyDelim\norm{\IVert}{\rVert}
\NewObject\MyDelim\inner{\langle}{\rangle}
```

Indeed:

```
$\norm{\va}$,
```

\$ $\backslash \mathrm{inner}\{\backslash \mathrm{va}, \backslash \mathrm{vb}\} \$, \quad\|a\|,\langle a, b\rangle,\langle-,-\rangle$
\$ $\backslash$ inner $\{$ slot, slot $\} \$$

We can also use it for more complicated constructions, like sets. The following is inspired from the mathtools package where a similar construction is created using the commands from that package. My impression is that Lars Madsen is the main mastermind behind the code I use for the Iwhere command:

```
\newcommand\where{
    \nonscript\:
    \SemantexDelimiterSize\vert
    \allowbreak
    \nonscript\:
    \mathopen{}
}
\NewObject\MyDelim\Set{\lbrace}{\rbrace}[
    prearg={\,},postarg={\,},
    % adds l, inside {...}, as recommended by D. Knuth
    valuekeys={
        {arg}{argwithoutkeyval={#1}},
        % this turns off all keyval syntax in the argument
    }
]
```

Now you can use
\$\Set\{ Ivx \in $\backslash v Y$ Iwhere Ivx |ge 0$\} \$$

$$
\{x \in Y \mid x \geq 0\}
$$

Don't forget that anything created with SemanTEX outputs as a variable-type object. So you can do stuff like

```
$\Set{
    Ivx \in \vY[\vi]
    Iwhere
    lvx \ge 0
}[conj,\vi\in\vI]$
```

Tuple-like commands are also possible:

```
\New0bject\MyDelim\tup{(}{)} % tuples
\NewObject\MyDelim\pcoor{[}{]}[ % projective coordinates
    setargsep=\mathpunct {:},
        % changes the argument separator to colon
    setargdots=\cdots,
        % changes what is inserted if you write "..."
]
```

Let us see them in action:
\$\tup $\{\backslash \mathrm{va}, \backslash \mathrm{vb}, \ldots, \backslash \mathrm{vz}\} \$$,
\$\pcoor\{\va, |vb, ..., \vz\}\$

$$
(a, b, \ldots, z),[a: b: \cdots: z]
$$

One can also use tuples for other, less obvious purposes, like calculus differentials:

```
\NewDelimiterClass\CalculusDifferential[
    parent=\MyVar,
    argvaluekeys={
        {default}{s={d\!#1}},
        % default is the key that is automatically applied by the
        % system to anything you write in the argument. The s key
        % is a key that prints the value of the key with the
        % standard argument separator in front.
    },
    setargdots=\cdots,
    neverpar,
    % neverpar is like nopar, except nopar will still print
    % parentheses when there is more than one argument
    % -- neverpar does not even print parentheses in this case
]
\NewObject\CalculusDifferential\intD{(}{)}[setargsep={\,},
iffirstarg=false]
\New0bject\CalculusDifferential\wedgeD{(}{)}[setargsep=\wedge]
$\int \vf \intD{\vx[1],
    \vx[2],...,\vx[n]}$,
$\int \vf \wedgeD{\vx[1],
\intfd\mp@subsup{x}{1}{}d\mp@subsup{x}{2}{}\cdotsd\mp@subsup{x}{n}{},
\intfd\mp@subsup{x}{1}{}\wedged\mp@subsup{x}{2}{}\wedge\cdots\wedged\mp@subsup{x}{n}{}
```

    \vx[2],..., |vx[n]\}\$
    
## 15 The parse routine

As you can see above, SemanTEX has a "waterfall-like" behaviour. It runs keys in the order it receives them. This works fine most of the time, but for some more complicated constructions, it is useful to be able to provide a data set in any order and have them printed in a fixed order. For this purpose, we have the parse routine.

Suppose we want to be able to write the set of $n \times m$-matrices with entries in $k$ as $\operatorname{Mat}_{n \times m}(k)$. We can in principle do the following:
\New0bject \MyVar\Mat\{ loperatorname $\{$ Mat $\}$
\$ \Mat[\vn\times\vm]\{\vk\} \$.

$$
\operatorname{Mat}_{n \times m}(k) .
$$

However, this is not quite as systematic and semantic as we might have wanted. Indeed, what if later you would like to change the notation to Mat ${ }_{n, m}(k)$ ? Therefore, we do something like the following instead (we explain the notation below):

```
\New0bject\MyVar\Mat{\operatorname{Mat}}[
    execute={
        \SemantexDataProvide{rows}
        \SemantexDataProvide{columns}
            % provides data sets for number of rows and columns
            % for this object
    },
    valuekeys={
        {rows}{
            execute={
                \SemantexDataSet{rows}{#1}
            },
        },
        {columns}{
            execute={
                \SemantexDataSet{columns}{#1}
            },
        },
    },
    parseoptions={
        execute={
            \SemantexStrIfEqTF{\SemantexDataGetExpNot{columns}}
            {\SemantexDataGetExpNot {rows}}
            % tests if rows = columns
            {
            \SemantexKeysSetx{
                        lower={
                                \SemantexDataGetExpNot{columns}
                    }
            }
                }
                {
                    \SemantexKeysSetx{
                    lower={
                            \SemantexDataGetExpNot{rows}
```

```
                    \times
                    \SemantexDataGetExpNot{columns}
                    }
                    }
            }
        },
    },
]
```

Now we can do the following:
\$ \Mat[rows=\vn,columns=\vm
]\{\vk\} \$, \$ \Mat[rows=\vn,

$$
\operatorname{Mat}_{n \times m}(k), \operatorname{Mat}_{n}(k)
$$

columns=\vn〕\{\vk\} \$
The key execute is a key that basically just executes code. You can in principle write any $\mathrm{T}_{\mathrm{E}} \mathrm{X}$ code there, and it will be applied right at the spot. However, inside the execute key, you can also use the following locally defined commands. These can be used to handle the data that is associated with the object in question. I don't have time to document them right now, so you'll have to guess what they do from the name for now, or you can find their definition in the source code of the package.
\SemantexDataProvide
ISemantexDataSet
\SemantexDataSetx
\SemantexDataPutRight
SemantexDataPutRightx
\SemantexDataPutLeft
S SemantexDataPutLeftx
\SemantexDataGet
\SemantexDataGetExpNot
\SemantexDataClear
\SemantexKeysSet
\SemantexKeysSetx
\SemantexStrIfEqTF
\SemantexStrIfEqT
\SemantexStrIfEqF
\SemantexIfBlankTF
\SemantexIfBlankT
\SemantexIfBlankF
\SemantexBoolProvide
\SemantexBoolSetTrue
\SemantexBoolSetFalse
\SemantexBoolIfTF
\SemantexBoolIfT
\SemantexBoolIfF
\SemantexIntProvide
\SemantexIntGet
SemantexIntClear
\SemantexIntIncr
S SemantexIntSet
SemantexIntIfPositiveTF
\SemantexIntIfPositiveT
\SemantexIntIfPositiveF
\SemantexIntIfGreaterThan0neTF
\SemantexIntIfGreaterThanOneT
SemantexIntIfGreaterThan0neF
ISemantexExpNot
The key parseoptions is a key that is executed right before rendering the object. This is where you write whatever the system is supposed to do with the data sets you provide. You can also force it to be applied at any point by using the parse key.

Let us look at a more complicated example: Let us create a command for partial derivatives:

```
\New0bject\MyVar\partialdif[
    nopar,
    execute={
    \SemantexBoolProvide{raisedfunction}
    \SemantexBoolSetTrue{raisedfunction}
    },
    setidots=\cdots,
    setisep=\,
    valuekeys={
    {default}{
        si={\partial #1},
            },
    },
    singlekeys={
        {raisedfunction}{
            execute={
                \SemantexBoolSetTrue{raisedfunction}
            },
            },
            {noraisedfunction}{
                execute={
                    \SemantexBoolSetFalse{raisedfunction}
            },
            },
    },
    parseoptions={
        execute={
            \SemantexIfBlankTF{ \SemantexDataGet{upper} }
            {
            \SemantexKeysSetx{
                    symbol={
                            \fac
                            {
                            \partial ^ { \SemantexIntGet{numberoflowerindices
                                } }
                                \SemantexBoolIfT{raisedfunction}
                                {
                                \SemantexDataGetExpNot{arg}
                                }
                                }
                                {
                                    \SemantexDataGetExpNot{lower}
                }
```

```
                    },
                    }
            }
            {
                        \SemantexKeysSetx{
                    symbol={
                        \frac
                                {
                    \partial ^ { \SemantexDataGet{upper} }
                        \SemantexBoolIfT{raisedfunction}
                            {
                        \SemantexDataGetExpNot{arg}
                                }
                }
                {
                    \SemantexDataGetExpNot{lower}
                }
            },
            }
            }
            \SemantexDataClear{lower}
            \SemantexDataClear{upper}
            \SemantexBoolIfT{raisedfunction}
            {
                \SemantexDataClear{arg}
                \SemantexIntClear{numberofarguments}
            }
        },
    },
]
```

Let us see it in action:
\
\partialdif[\vx, \vy, \vz]\{
|vf \},
|partialdif [\vu^2, \vv^2,
d=4]\{ \vf \},
|partialdif[\vx[1],
Ivx[2],..., Ivx[Ivn],
$\mathrm{d}=\backslash \mathrm{vn}\}\{\backslash \mathrm{vf}\}$
1]
\
\partialdif[\vx,|vy,|vz,
noraisedfunction]\{ $\backslash v f$ \},
|partialdifl\vu^2, \vv^2,
d=4, noraisedfunction]\{
lvf \},
\partialdif[\vx[1],
Ivx[2],..., Ivx[|vn],
$d=\$ vn, noraisedfunction]\{
|vf \}
I]

As you see, we use the d key to tell the command what superscript it should put on the $\partial$ in the enumerator. If it does not receive a $d$, it counts the number
of variables you wrote and prints that. That is why the following would give the wrong result:

```
\[
    \partialdif[\vu^2,\vv^2]{
        \vf },
    \partialdif[\vx[1],
        \vx[2],...,\vx[\vn]]{
        |vf }
\]
```


## 16 Bugs

Lots of things can be improved in the system, including the order in which things are being expanded internally. I am not going to explain this is detail for now, but hope to correct this in the future. For now, the system seems to work fine as long as you do "normal" things and insert \noexpand's whenever something goes wrong. The only real bug that I currently know of occurs if you use the key 0 therspar in a heading. Then it all dies painfully. Then again, why the heck would you do that in the first place? Who scales parentheses in headings?


[^0]:    ${ }^{1}$ We shall not follow this convention strictly，as we shall later create objects with names like \Hom；using lowercase letters for these would just look weird．

