The extension package curve2e

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Contents

Abstract

This file documents the `curve2e` extension package to the `pict2e` bundle implementation; the latter was described by Lamport himself in the 1994 second edition of his LaTeX handbook.

Please take notice that on April 2011 a new updated version of the package `pict2e` has been released that incorporates some of the commands defined in early versions of this package; apparently there are no conflicts, but only the advanced features of `curve2e` remain available for extending the above package.

This extension redefines some commands and introduces some more drawing facilities that allow to draw circular arcs and arbitrary curves with the minimum of user intervention. This version is open to the contribution of other users as well as it may be incorporated in other people's packages. Please cite the original author and the chain of contributors.

1 The configuration file

This package `curve2e` is distributed with a `ltxdoc.cfg` configuration file that contains, besides the preamble and the postamble comment lines, the following line of code:

```
\AtBeginDocument{\OnlyDescription}
```

If you want to type the whole documentation, comment out that code line in the `ltxdoc.cfg` file. This is the only modification allowed by the LPPL licence that does not require to change the file name.

For your information, the initial part is about 20 pages long; the whole documentation is about 80 pages long.

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2 Package `pict2e` and this extension `curve2e`

Package `pict2e` was announced in issue 15 of `latexnews` around December 2003; it was declared that the new package would replace the dummy one that has been accompanying every release of LaTeX 2_ε since its beginnings in 1994. The dummy package was just issuing an info message that simply announced the temporary unavailability of the real package.

Eventually Gäßlein and Niepraschk implemented what Lamport himself had already documented in the second edition of his LaTeX handbook, that is a LaTeX package that contained the macros capable of removing all the limitations contained in the standard commands of the original `picture` environment; specifically what follows.

1. The line and vector slopes were limited to the ratios of relative prime one-digit integers of magnitude not exceeding 6 for lines and 4 for vectors.
2. Filled and unfilled full circles were limited by the necessarily limited number of specific glyphs contained in the special LaTeX `picture` fonts.
3. Quarter circles were also limited in their radii for the same reason.
4. Ovals (rectangles with rounded corners) could not be too small because of the unavailability of small radius quarter circles, nor could be too large, in the sense that after a certain radius the rounded corners remained the same and would not increase proportionally to the oval size.
5. Vector arrows had only one possible shape and matched the limited number of vector slopes.
6. For circles and inclined lines and vectors just two possible thicknesses were available.

The package `pict2e` removes most if not all the above limitations.

1. Line and vector slopes are virtually unlimited; the only remaining limitation is that the direction coefficients must be three-digit integer numbers; they need not be relatively prime; with the 2009 upgrade even this limitation was removed and now slope coefficients can be any fractional number whose magnitude does not exceed 16 384, the maximum dimension in points that TeX can handle.
2. Filled and unfilled circles can be of any size.
3. Ovals can be designed with any specified corner curvature and there is virtually no limitation to such curvatures; of course corner radii should not exceed half the lower value between the base and the height of the oval.
4. There are two shapes for the arrow tips; the triangular one traditional with LaTeX vectors, or the arrow tip with PostScript style.

5. The `\linethickness` command changes the thickness of all lines, straight, curved, vertical, horizontal, arrow tipped, et cetera.

This specific extension package `curve2e` adds the following features.

1. Point coordinates my be specified in both cartesian and polar form: internally they are handled as cartesian coordinates, but the user can specify his/her points also in polar form. In order to avoid confusion with other graphic packages, `curve2e` uses the usual comma separated couple $\langle x, y\rangle$ of integer or fractional numbers for cartesian coordinates, and the couple $\langle\theta\rangle{:}\langle\rho\rangle$ for polar coordinates (the angle preceding the radius). All graphic object commands accept polar or cartesian coordinates at the choice of the user who may use for each object the formalism s/he prefers. Also the `put` and `\multiput` commands have been redefined so as to accept cartesian or polar coordinates.

   Of course the user must pay attention to the meaning of cartesian vs. polar coordinates. Both imply a displacement with respect the actual origin of the axes. So when a circle is placed at coordinates a, b with a normal `\put` command, the circle is placed exactly in that point; with a normal `\put` command the same happens if coordinates $\alpha{:}\rho$ are specified. But if the `\put` command is nested into another `\put` command, the current origin of the axes is displaced — this is obvious and the purpose of nesting `\put` commands is exactly that. But if a segment is specified so that its ending point is at a specific distance and in specific direction form its starting point, polar coordinates appear to be the most convenient to use; in this case, though, the origin of the axes become the stating point of the segment, therefore the segment might be drawn in a strange way. Attention has been paid to avoid such misinterpretation, but maybe some unusual situation may not have come to my mind; feedback is very welcome. Meanwhile pay attention when you use polar coordinates.

2. Most if not all cartesian coordinate pairs and slope pairs are treated as *ordered pairs*, that is *complex numbers*; in practice the user does not notice any difference from what s/he was used to, but all the mathematical treatment to be applied to these entities is coded as complex number operations, since complex numbers may be viewed non only as ordered pairs, but also as vectors or as roto-amplification operators.

3. Commands for setting the line terminations were introduced; the user can chose between square or rounded caps; the default is set to rounded caps; now this feature is directly available with `pict2e`.

4. Commands for specifying the way two lines or curves join to one another.

5. Originally the `\line` macro was redefined so as to allow large (up to three digits) integer direction coefficients, but maintaining the same syntax as in the original `picture` environment; now `pict2e` removes the integer number limitations and allows fractional values, initially implemented by `curve2e`.

6. A new macro `\Line` was originally by `curve2e` defined so as to avoid the need to specify the horizontal projection of inclined lines; now this functionality is available directly with `pict2e`; but this `curve2e` macro name now conflicts with `pict2e` 2009 version; therefore its name is changed to `\LIne` and supposedly it will not be used very often, if ever, by the end user (but it is used within this package macros).

7. A new macro `\LINE` was defined in order to join two points specified with their coordinates; this is now the normal behaviour of the `\Line` macro of `pict2e` so that in this package `\LINE` is now renamed `\segment`; there is no need to use the `\put` command with this line specification.

8. A new macro `\DashLine` (alias: `\Dline`) is defined in order to draw dashed lines joining any two given points; the dash length and gap (equal to one another) get specified through one of the macro arguments.The stating point mai be specified in cartesiano or polar form; the end point in cartesian format specifies the desired end point; while if the second point is in polar form it is meant *relative to the starting point*, not as an absolute end point. See the examples further on.

9. A similar new macro `\Dotline` is defined in order to draw dotted straight lines as a sequence of equally spaced dots, where the gap can be specified by the user; such straight line may have any inclination, as well as the above dashed lines.Polar coordinates for the second point have the same relative meaning as specified for the `\Dashline` macro.

10. Similar macros are redefined for vectors; `\vector` redefines the original macro but with the vector slope limitations removed; `\Vector` gets specified with its two horizontal and vertical components in analogy with `\LIne`; `\VECTOR` joins two specified points (without using the `\put` command) with the arrow pointing to the second point.

11. A new macro `\polyline` for drawing polygonal lines is defined that accepts from two vertices up to an arbitrary (reasonably limited) number of them (available now also in `pict2e`); here it is redefined so as to allow an optional specification of the way segments for the polyline are joined to one another. Vertices may be specified with polar coordinates and are always relative to the preceding point.

12. The `pict2e polygon` macro to draw closed polylines (in practice general polygons) has been redefined in such a way that it can accept the various vertices specified with (relative) polar coordinates. The `polygon*` macro produces a color filled polygon; the default color is black, but a different color may be specified with the usual `\color` command given within the same group where `\polygon*` is enclosed.

13. A new macro `\Arc` is defined in order to draw an arc with arbitrary radius and arbitrary aperture (angle amplitude); this amplitude is specified in sexagesimal degrees, not in radians; a similar functionality is now achieved with

the `\arc` macro of `pict2e`, which provides also the starred version `\arc*` that fills up the interior of the generated circular arc with the current color. It must be noticed that the syntax is slightly different, so that it's reasonable that these commands, in spite of producing identical arcs, might be more comfortable with this or that syntax.

14. Two new macros `\VectorArc` and `\VectorARC` are defined in order to draw circular arcs with an arrow at one or both ends.

15. A new macro `\Curve` is defined so as to draw arbitrary curved lines by means of cubic Bézier splines; the `\Curve` macro requires only the curve nodes and the directions of the tangents at each node. The starred version fills up the interior of the curve with the current color.

16. the above `\Curve` macro is a recursive macro that can draw an unlimited (reasonably limited) number of connected Bézier spline arcs with continuous tangents except for cusps; these arcs require only the specification of the tangent direction at the interpolation nodes. It is possible to use a lower level macro `\CbezierTo` that does the same but lets the user specify the control points of each arc; it is more difficult to use but it is more performant.

17. The basic macros used within the cumulative `\Curve` macro can be used individually in order to draw any curve, one cubic arc at the time; but they are intended for internal use, even if it is not prohibited to use them; by themselves such arcs are not different form those used by `Curve`, but the final command, `\FillCurve`, should be used in place of `\CurveFinish`, so as to fill up the closed path with the locally specified color; see figure **??**. It is much more convenient to use the starred version of the `\Curve` macro.

The `pict2e` package already defines macros such as `\moveto`, `\lineto`, `\curveto`, `\closepath`, `\fillpath`, and `\strokepath`; of course these macros can be used by the end user, and sometimes they perform better than the macros defined in this package, because the user has a better control on the position of the Bézier splines control points, while here the control points are sort of rigid. It would be very useful to resort to the `hobby` package, but its macros are compatible with those of the `tikz` and `pgf` packages, not with `curve2e`; an interface should be created in order to deal with the `hobby` package, but this has not been done yet. In any case they are redefined so as to accept symbolic names for the point coordinates in both the cartesian and polar form.

In order to make the necessary calculations many macros have been defined so as to use complex number arithmetics to manipulate point coordinates, directions (unit vectors, also known as 'versors'), rotations and the like. In the first versions of this package the trigonometric functions were also defined in a way that the author believed to be more efficient than those defined by the `trig` package; in any case the macro names were sufficiently different to accommodate both definition sets in the same LaTeX run. With the progress of the LaTeX 3 language, the `xfp` has recently become available, by which any sort of calculations can be done with floating point decimal numbers; therefore the most common algebraic, irrational

and transcendental functions can be computed in the background with the stable internal floating point facilities. We maintain some computation with complex number algebra, but use the `xfp` functionalities for other computations.

Many aspects of this extension could be fine tuned for better performance; many new commands could be defined in order to further extend this extension. If the new service macros are accepted by other TEX and LATEX programmers, this version could become the start for a real extension of the `pict2e` package or even become a part of it. Actually some macros have already been included in the `pict2e` package. The `\Curve` algorithm, as I said before, might be redefined so as to use the macros introduced in the `hobby` package, that implements for the `tikz` and `pgf` packages the same functionalities that John Hobby implemented for the METAFONT and METAPOST programs.

For these reasons I suppose that every enhancement should be submitted to Gäßlein, Niepraschk, and Tkadlec who are the prime maintainers of `pict2e`; they are the only ones who can decide whether or not to incorporate new macros in their package.

3 Summary and examples of new commands

This package `curve2e` extends the power of `pict2e` with the following modifications and the following new commands.

1. This package `curve2e` calls directly the LATEX packages `color` and `pict2e` to which it passes any possible option that the latter can receive; actually the only options that make sense for `pict2e` are those concerning the arrow tips, either LATEX or PostScript styled, because it is assumed that if you use this package you are not interested in using the original LATEX commands. See the `pict2e` documentation in order to see the correct options `pict2e` can receive. If the user wants to use the `xcolor` package, it has to load this one before `curve2e`.

2. The user is offered new commands in order to control the line terminators and the line joins; specifically:

   - `\roundcap`: the line is terminated with a semicircle;
   - `\squarecap`: the line is terminated with a half square;
   - `\roundjoin`: two lines are joined with a rounded join;
   - `\beveljoin`: two lines are joined with a bevel join;
   - `\miterjoin`: two lines are joined with a miter join.

   All the above commands should respect the intended range; but since they act at the PostScript or PDF level, not at TEX level, it might be necessary to issue the necessary command in order to restore the previous terminator or join.

3. The commands `\linethickness`, `\thicklines`, `\thinlines` together with `\defaultlinethickness` always redefine the internal `\@wholewidth` and `\@halfwidth` so that the latter always refer to a full width and to a half of it in this way: if you issue the command `\defaultlinewidth{2pt}` all thin lines will be drawn with a thickness of 1 pt while, if a drawing command directly refers to the internal value `\@wholewidth`, its line will be drawn with a thickness of 2 pt. If one issues the declaration `\thinlines` all lines will be drawn with a 1 pt width, but if a command refers to the internal value `\@halfwidth` the line will be drawn with a thickness of 0.5 pt. The command `\linethickness` redefines the above internals but does not change the default width value; all these width specifications apply to all lines, straight ones, curved ones, circles, ovals, vectors, dashed, et cetera. It's better to recall that `\thinlines` and `\thicklines` are declarations that do not take arguments; on the opposite the other two commands follow the standard syntax:

   `\linethickness`{⟨*dimensioned value*⟩}
   `\defaultlinewidth`{⟨*dimensioned value*⟩}

   where ⟨*dimensioned value*⟩ means a length specification complete of its units, or a dimensional expression.

4. Straight lines and vectors are redefined in such a way that fractional slope coefficients may be specified; the zero length line does not produce errors and is ignored; the zero length vectors draw only the arrow tips.

5. New line and vector macros are defined that avoid the necessity of specifying the horizontal component; `\put(3,4){\LIne(25,15)}` specifies a segment that starts at point $(3, 4)$ and goes to point $(3 + 25, 4 + 15)$; the command `\segment(3,4)(28,19)` achieves the same result without the need of using command `\put`.

   The same applies to the vector commands `\Vector` and `\VECTOR` and `\VVECTOR`; the latter command behaves as `\VECTOR` but draws a vector with arrow tips at both ends; furthermore this command is available only with this new release of the `curve2e` package. Experience has shown that the commands intended to join two specified points are particularly useful.

6. The `\polyline` command has been introduced: it accepts an unlimited list of point coordinates enclosed within round parentheses; the command draws a sequence of connected segments that join in order the specified points; the syntax is:

   `\polyline[`⟨*optional join style*⟩`](`⟨P_1⟩`)(`⟨P_2⟩`)...(`⟨P_n⟩`)`

   See figure **??** where a regular pentagon is drawn; usage of polar coordinates is also shown; please notice how relative polar coordinates act in this figure.

   Examples of using polar and cartesian coordinates are shown in figure **??**.

```
\unitlength=.5mm
\begin{picture}(60,20)
\put(0,0){\GraphGrid(80,20)}
\put(0,0){\vector(1.5,2.3){10}}
\put(20,0){\Vector(10,15.33333)}
\VECTOR(40,0)(50,15.33333)
\ifdefined\VVECTOR \VVECTOR(60,0)(80,10)\fi
\end{picture}
```

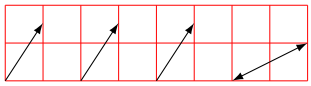


Figure 1: Three (displaced) identical vectors obtained with the three vector macros; a double tipped vector is also shown.

```
\unitlength=.5mm
\begin{picture}(40,32)(-20,-20)
\polyline(90:20)(162:20)(234:20)(306:20)(378:20)(90:20)
\end{picture}
```

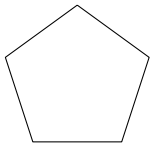


Figure 2: Polygonal line obtained by means of the `\polyline` command; vertex coordinates are in polar form.

A similar example may be obtained with the `\polygon` macro that does not require to terminate the polyline at the starting point. Figure **??** shows how to get a coloured filled pentagon.

7. The new command `\Dashline` (alias: `\Dline` for backwards compatibility)

   `\Dashline(`⟨*first point*⟩`)(`⟨*second point*⟩`){`⟨*dash length*⟩`}`

   draws a dashed line containing as many dashes as possible, just as long as specified, and separated by a gap exactly the same size; actually, in order to make an even gap-dash sequence, the desired dash length is used to do some computations in order to find a suitable length, close to the one specified, such that the distance of the end points is evenly divided in equally sized

```
\begin{picture}(40,30)
\put(0,0){\GraphGrid(40,30)}
\Zbox(40,0)[l]{40,0}[1]
\Zbox(90:30)[bc]{90{:}30}[1]
\Zbox(60:30)[bc]{60{:}30}[1]
\Zbox(30,30)[bc]{30,30}[1]
\multiput(0,0)(30:10){5}%
   {\makebox(0,0){\rule{1.5mm}{1.5mm}}}
\end{picture}
```

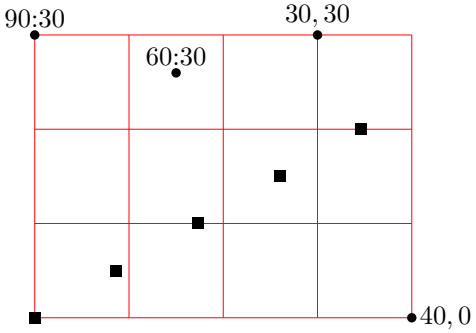


Figure 3: Use of cartesian and absolute polar coordinates. The `\Zbox` macro is just a shortcut to set a small dot with a (math) legend close to it; its definition by means of the `xparse` functionalities is straightforward.

```
\unitlength=.5mm
\begin{picture}(40,32)(-20,-20)
\color{magenta}
\polygon*(90:20)(162:20)(234:20)(306:20)(378:20)
\end{picture}
```

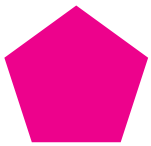


Figure 4: A pentagon obtained by means of the `\polygon*` command; vertex coordinates are in relative polar form.

dashes and gaps. The end points may be anywhere in the drawing area, without any constraint on the slope of the joining segment. The desired dash length is specified as a fractional multiple of `\unitlength`; see figure **??**.

```
\unitlength=1mm
\begin{picture}(40,40)
\put(0,0){\GraphGrid(40,40)}
\Dashline(0,0)(40,10){4}
\put(0,0){\circle*{2}}
\Dashline(40,10)(0,25){4}
\put(40,10){\circle*{2}}
\Dashline(0,25)(20,40){4}
\put(0,25){\circle*{2}}
\put(20,40){\circle*{2}}
\Dotline(0,0)(40,40){2}
 \put(40,40){\circle*{2}}
\end{picture}
```

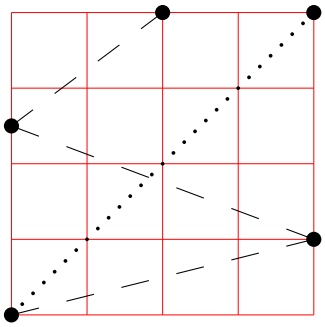


Figure 5: Dashed lines and graph grid

Another example of usage of cartesian and polar coordinates usage is shown in figure **??** together with its code.

Another

8. Analogous to `\Dashline`, a new command `\Dotline` draws a dotted line with the syntax:

   `\Dotline(`⟨*first point*⟩`)(`⟨*end point*⟩`){`⟨*dot gap*⟩`}`

   See figures **??** and **??** for examples.

9. `\GraphGrid` is a command that draws a red grid under the drawing with lines separated `10\unitlength`s apart; it is described only with a comma separated couple of numbers, representing the base and the height of the grid, see figure **??**; it's better to specify multiples of ten and the grid can be placed anywhere in the drawing canvas by means of `\put`, whose cartesian coordinates are multiples of 10; nevertheless the grid line distance is rounded to the nearest multiple of 10, while the point coordinates specified to `\put` are not rounded at all; therefore some care should be used to place the

```
\begin{picture}(40,30)
\put(0,0){\GraphGrid(40,30)}
\Dashline(0,0)(40,10){2}\Dashline(0,0)(40,20){2}
\Dashline(0,0)(40,30){2}\Dashline(0,0)(30,30){2}
\Dashline(0,0)(20,30){2}\Dashline(0,0)(10,30){2}
{\color{blue}%
\Dashline(40,0)(108:30){2}
\Dashline(40,0)(126:30){2}
\Dashline(40,0)(144:30){2}
\Dashline(40,0)(162:30){2}}
\end{picture}
```

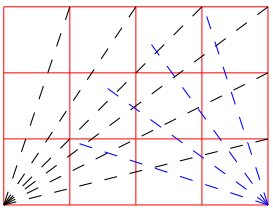


Figure 6: Different length dashed lines with the same nominal dash length; notice the relative polar coordinates used for the dashed lines starting at the grid lower right vertex.

```
\begin{picture}(40,30)
\put(0,0){\GraphGrid(40,30)}
\Dotline(0,0)(40,10){1.5}\Dotline(0,0)(40,20){1.5}
\Dotline(0,0)(40,30){1.5}\Dotline(0,0)(30,30){1.5}
\Dotline(0,0)(20,30){1.5}\Dotline(0,0)(10,30){1.5}
{\color{red}\Dotline(40,0)(108:30){1.5}
\Dotline(40,0)(126:30){1.5}
\Dotline(40,0)(144:30){1.5}
\Dotline(40,0)(162:30){1.5}}%
\end{picture}
```

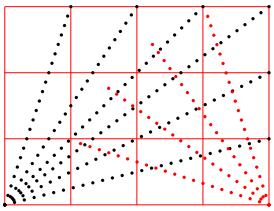


Figure 7: Different length dotted lines with the same nominal dot gap; again notice the relative polar coordinates for the dotted lines starting at thhe grid lower right vertex.

working grid on the drawing canvas. This grid is intended as an aid while drawing; even if you sketch your drawing on millimetre paper, the drawing grid turns out to be very useful; one must only delete or comment out the command when the drawing is finished. Several examples of usage of such grid are shown in several figures.

10. New trigonometric function macros have been computed by means of the functionalities of the `xfp` package. The difference woth the other existing macros is that angles are specified in sexagesimal degrees, so that the users need not transform to radians. The computations are done taking into account that "abnormal" values can occasionally be avoided, for example $\tan 90°$ must be avoided and replaced with a suitably large number, because the TeX system does not handle "infinity".

    These trigonometric functions are used within the complex number macros; but if the user wants to use them the syntax is the following:

    `\SinOf`⟨*angle*⟩`to`⟨*control sequence*⟩

```
\unitlength=0.5mm
\begin{picture}(60,40)
\put(0,0){\GraphGrid(60,40)}
\Arc(0,20)(30,0){60}
\VECTOR(0,20)(30,0)\VECTOR(0,20)(32.5,36)
\VectorArc(0,20)(15,10){60}
\put(20,20){\makebox(0,0)[l]{$60^\circ$}}
\VectorARC(60,20)(60,0){-180}
\end{picture}
```

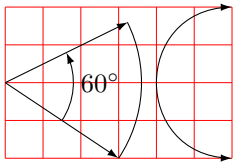


Figure 8: Arcs and curved vectors

`\CosOf`⟨*angle*⟩`to`⟨*control sequence*⟩
`\TanOf`⟨*angle*⟩`to`⟨*control sequence*⟩

The ⟨*control sequence*⟩ may then be used, for example, as a multiplying factor of a length.

11. Arcs can be drawn as simple circular arcs, or with one or two arrows at their ends (curved vectors); the syntax is:

    `\Arc(`⟨*center*⟩`)(`⟨*starting point*⟩`){`⟨*angle*⟩`}`
    `\VectorArc(`⟨*center*⟩`)(`⟨*starting point*⟩`){`⟨*angle*⟩`}`
    `\VectorARC(`⟨*center*⟩`)(`⟨*starting point*⟩`){`⟨*angle*⟩`}`

    If the angle is specified numerically it must be enclosed in braces, while if it is specified with a control sequence the braces (curly brackets) are not necessary. The above macro `\Arc` draws a simple circular arc without arrows; `\VectorArc` draws an arc with an arrow tip at the ending point; `\VectorARC` draws an arc with arrow tips at both ends; see figure **??**.

12. A multitude of commands have been defined in order to manage complex numbers; actually complex numbers are represented as a comma separated pair of fractional numbers (here we use only cartesian coordinates). They are used to address specific points in the drawing plane, but also as operators so as to scale and rotate other objects. In the following ⟨*vector*⟩ means a comma separated pair of fractional numbers, ⟨*vector macro*⟩ means a macro that contains a comma separated pair of fractional numbers; ⟨*angle macro*⟩ means a macro that contains the angle of a vector in sexagesimal degrees; ⟨*argument*⟩ means a brace delimited numeric value, even a macro; ⟨*numeric macro*⟩ means a macro that contains a fractional number; *macro* is a valid macro name, i.e. a backslash followed by letters, or anything else that can receive a definition. A *direction* of a vector is its versor; the angle of a vector is the angle between the vector and the positive x axis in counterclockwise direction, as it is used in the Euler formula $\vec{v} = Me^{j\varphi}$.

    - `\MakeVectorFrom`⟨*numeric macro*⟩⟨*numeric macro*⟩`to`⟨*vector macro*⟩
    - `\CopyVect`⟨*first vector*⟩`to`⟨*second vector macro*⟩

- `\ModOfVect`⟨*vector*⟩`to`⟨*modulus macro*⟩
- `\DirOfvect`⟨*vector*⟩`to`⟨*versor macro*⟩
- `\ModAndDirOfVect`⟨*vector*⟩`to`⟨*modulus macro*⟩`and`⟨*versor macro*⟩
- `\ModAndAngleOfVect`⟨*vector*⟩`to`⟨*modulus macro*⟩`and`⟨*angle macro*⟩
- `\DistanceAndDirOfVect`⟨*1st vector*⟩`minus`⟨*2nd vector*⟩`to`⟨*distance macro*⟩`and`⟨*versor macro*⟩
- `\XpartOfVect`⟨*vector*⟩`to`⟨*macro*⟩
- `\YpartOfVect`⟨*vector*⟩`to`⟨*macro*⟩
- `\DirFromAngle`⟨*angle*⟩`to`⟨*versor macro*⟩
- `\ArgOfVect`⟨*vector*⟩`to`⟨*angle macro*⟩
- `\ScaleVect`⟨*vector*⟩`by`⟨*scaling factor*⟩`to`⟨*vector macro*⟩
- `\ConjVect`⟨*vector*⟩`to`⟨*conjugate vector macro*⟩
- `\SubVect`⟨*subtrahend vector*⟩`from`⟨*minuend vector*⟩`to`⟨*vector macro*⟩
- `\AddVect`⟨*first vector*⟩`and`⟨*second vector*⟩`to`⟨*vector macro*⟩
- `\Multvect`{⟨*first vector*⟩}*{⟨*second vector*⟩}*{⟨*vector macro*⟩} (the asterisks are optional; either one changes the second vector into its complex conjugate)
- `\MultVect`⟨*first vector*⟩`by`⟨*second vector*⟩`to`⟨*vector macro*⟩ (discouraged; maintained for backwards compatibility)
- `\MultVect`⟨*first vector*⟩`by*`⟨*second vector*⟩`to`⟨*vector macro*⟩ (discouraged; maintained for backwards compatibility)
- `\Divvect`{⟨*dividend vector*⟩}{⟨*divisor vector*⟩}{⟨ *vector macro*⟩}
- `\DivVect`⟨*dividend vector*⟩`by`⟨*divisor vector*⟩`to`⟨*vector macro*⟩ (maintained for backwards compatibility)

13. General curves can be drawn with the `pict2e` macro `\curve` but it requires the specification of the third-order Bézier-spline control points; sometimes it's better to be very specific with the control points and there is no other means to do a decent graph; sometimes the curves to be drawn are not so tricky and a general set of macros can be defined so as to compute the control points, while letting the user specify only the nodes through which the curve must pass, and the tangent direction of the curve in such nodes. Such commands are the following:

    - `\Curve` to draw a sequence of arcs as explained above, using third order (cubic) Bézier splines. The starred version of this command fills the internal part of the curve with the current color; if the last arc finishes where the fist arc starts, it is clear what is the interior; if it does not, the driver (not the code of this package, but the driver between this code and the physical representation on paper or screen) assumes a straight line closure of the whole path.
    - `\Qurve` similar to `\Curve`, but with second order (quadratic) Bézier splines. The starred version fills the interior with the current color.
    - `\CurveBetween` draws a single cubic Bézier spline between two given nodes and with two given directions vectors.

- `\CBezierBetween` draws a single cubic Bézier spline between two given nodes, with two given directions versors along which the control node distances are specified. This is the most general macro (rather difficult to use) with which not only the arc end points are specified but also the control nodes coordinates are given.

The main macro is `\Curve` and must be followed by an "unlimited" sequence of node-direction coordinates as a quadruple defined as

$$\texttt{(}\langle \textit{node coordinates} \rangle\texttt{)<}\langle \textit{direction vector} \rangle\texttt{>}$$

Possibly if a sudden change of direction has to be performed (cusp) another item can be inserted after one of those quadruples in the form

$$\ldots\texttt{(}\langle\ldots\rangle\texttt{)<}\langle\ldots\rangle\texttt{>[}\langle \textit{new direction vector} \rangle\texttt{](}\langle\ldots\rangle\texttt{)<}\langle\ldots\rangle\texttt{>}\ldots$$

Possibly it is necessary to specify the "tension" or the "looseness" of a specific Bézier arc; such tension parameters range from 0 (zero) to 4; the zero value implies a very stiff arc, as if it was a string subject to a high tension (i.e. with zero looseness); a value of 4 implies a very low tension (very high looseness), almost as if the string was not subject to any tension. In METAFONT or METAPOST language such a concept is used very often; in this package, where the Hobby algorithms are not used, the parameter value appears to mean the opposite of tension. A couple of comma separated tension values may be optionally used, they are separated with a semicolon form the direction vector, and they apply to the arc terminating with the last node; their specification must precede any possible change of tangent according to this syntax[1]:

$$\ldots\texttt{(}\langle\ldots\rangle\texttt{)<}\langle \textit{direction vector}\texttt{;}\textit{start tension}\texttt{,}\textit{end tension} \rangle\texttt{>(}\langle\ldots\rangle\texttt{)<}\langle\ldots\rangle\texttt{>}\ldots$$

The `\Curve` macro does not (still) have facilities for cycling the path, that is to close the path from the last specified node-direction to the first specified node-direction; but, as already mentioned, if the ending node of the last arc does not coincide with the starting node of the first arc, a straight line is assumed to join such nodes; this line does not get drawn, but with starred commands no lines are drawn because only the interior is coloured. The tangent direction need not be specified with a unit vector, although only its direction is relevant; the scaling of the specified direction vector to a unit vector is performed by the macro itself. Therefore one cannot specify the fine tuning of the curve convexity as it can be done with other programs or commands, as, for example, with METAFONT or the `pgf/tikz` package and environment. See figure **??** for an example.

[1] The tension may be specified only for cubic splines, because the quadratic ones do not use enough parameters to control the tension; not all commands for drawing cubic splines accept this optional tension specification.

```
\unitlength=8mm\relax
\begin{picture}(5,5)
\put(0,0){\framebox(5,5){}}\thicklines\roundcap
\Curve(2.5,0)<1,1>(5,3.5)<0,1>%
  (4,5)<-1,0>(2.5,3.5)<-.5,-1.2>[-.5,1.2]%
  (1,5)<-1,0>(0,3.5)<0,-1>(2.5,0)<1,-1>
\end{picture}
```

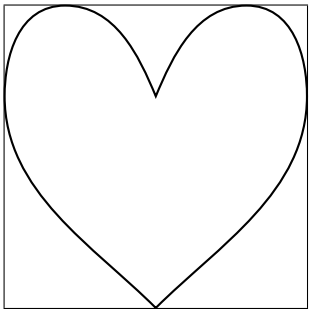


Figure 9: A heart shaped curve with cusps drawn with `\Curve`

```
\unitlength=8mm\relax
\begin{picture}(5,5)
\put(0,0){\framebox(5,5){}}\thicklines\roundcap
   \color{green}\relax
\Curve*(2.5,0)<1,1>(5,3.5)<0,1>%
  (4,5)<-1,0>(2.5,3.5)<-.5,-1.2>[-.5,1.2]%
  (1,5)<-1,0>(0,3.5)<0,-1>(2.5,0)<1,-1>
\end{picture}
```

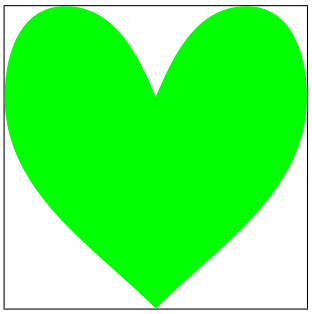


Figure 10: Coloring the inside of a closed path drawn with `\Curve*`

With the starred version of `\Curve`, instead of stroking the contour, the macro fills up the contour with the selected current color, figure **??**.

Figure **??** shows a geometric construction that contains the geometric elements and symbols used to determine the parameters of a cubic spline required to draw a quarter circle. This construction contains many of the commands described so far.

To show what you can do with `\CurveBetween` see the code and result shown in figure **??**. Notice the effect of changing the directions at both or a the end nodes of a single cubic spline. The directions are conveniently expressed with unit vectors described by polar coordinates.

A little more complicated is the use of the `\CBezierBetween` macro, figure **??**. The directions are specified with unit vectors in polar form; the control points are specified by adding their distances from their neighbouring nodes; actually the right distance is maintained to the value 1, while the left one increases from 4 to 10. The black line corresponds to the standard `\CurveBetween` where the default distance is computed to trace an arc of a circle and is approximately 3.5.

In figure **??** the effect of tension specification is shown. The red line corre-

```
\unitlength=0.007\textwidth
\begin{picture}(100,90)(-50,-50)
\put(-50,0){\vector(1,0){100}}\put(50,1){\makebox(0,0)[br]{$x$}}%
\put(20,-1){\makebox(0,0)[t]{$s$}}%
\put(0,0){\circle*{2}}\put(-1,-1){\makebox(0,0)[tr]{$M$}}%
\legenda(12,-45){s=\overline{MP_2}=R\sin\theta}%
\put(0,-50){\vector(0,1){90}}%
\put(1,40){\makebox(0,0)[tl]{$y$}}%
\put(0,-40){\circle*{2}}\put(1,-41){\makebox(0,0)[lt]{$C$}}%
\segment(0,-40)(-40,0)\segment(0,-40)(40,0)%
\put(-41,1){\makebox(0,0)[br]{$P_1$}}\put(-40,0){\circle*{2}}%
\put(41,1){\makebox(0,0)[bl]{$P_2$}}\put(40,0){\circle*{2}}%
\put(0,0){\linethickness{1pt}\Arc(0,-40)(40,0){90}}%
\segment(-40,0)(-20,20)\put(-20,20){\circle*{2}}%
\put(-20,21.5){\makebox(0,0)[b]{$C_1$}}%
\segment(40,0)(20,20)\put(20,20){\circle*{2}}%
\put(20,21.5){\makebox(0,0)[b]{$C_2$}}%
\put(0,-40){\put(0,56.5685){\circle*{2}}%
\put(1,58){\makebox(0,0)[bl]{$P$}}}%
\VectorARC(0,-40)(15,-25){45}\put(10,-18){\makebox(0,0)[c]{$\theta$}}%
\VectorARC(40,0)(20,0){-45}\put(19,5){\makebox(0,0)[r]{$\theta$}}%
\VectorARC(-40,0)(-20,0){45}\put(-19,5){\makebox(0,0)[l]{$\theta$}}%
\put(-20,-18){\makebox(0,0)[bl]{$R$}}%
\put(-32,13){\makebox(0,0)[bl]{$K$}}%
\put(32,13){\makebox(0,0)[br]{$K$}}%
\end{picture}
```

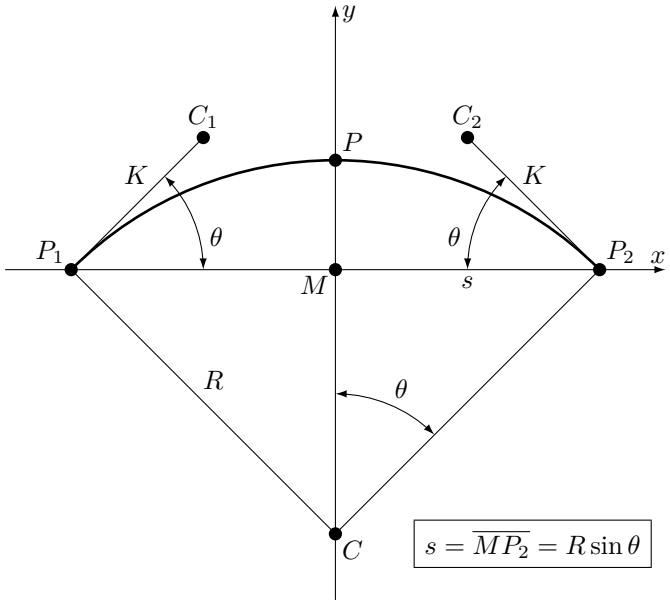


Figure 11: The code to display the nodes and control points for an arc to be approximated with a cubic Bézier spline

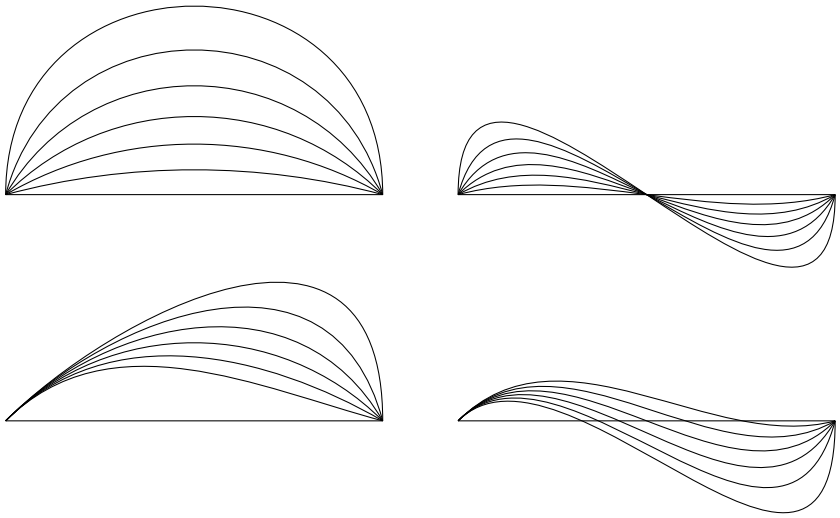
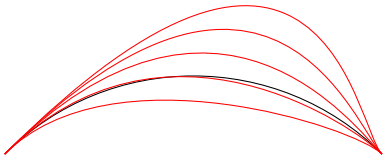


Figure 12: Curves between two points with different start and end slopes



```
\unitlength=0.1\textwidth
\begin{picture}(10,3)
\CurveBetween0,0and10,0WithDirs1,1and{1,-1}
\color{red}%
\CbezierBetween0,0And10,0 WithDirs45:1And-45:1UsingDists4And{1}
\CbezierBetween0,0And10,0 WithDirs45:1And-45:1UsingDists6And{1}
\CbezierBetween0,0And10,0 WithDirs45:1And-45:1UsingDists8And{1}
\CbezierBetween0,0And10,0 WithDirs45:1And-45:1UsingDists10And{1}
\CbezierBetween0,0And10,0 WithDirs45:1And-45:1UsingDists12And{1}
\end{picture}
```

Figure 13: Comparison between similar arcs drawn with `\CurveBetween` (black) and `\CbezierTo` (red)

sponds to the default tension, since the tension values are not specified. The black lines correspond to the various values used in the various commands to the `\Curve` macro. With a tension of zero, the spline is almost coincident with the horizontal base line of the frame. Increasing the parameter value to 4.5, the curved becomes taller and taller, until it wraps itself displaying an evident loop. We would say that the value of 2 is a reasonable maximum and increasing that value is just to obtain special effects.

Figure **??** displays two approximations of a sine wave; Bézier splines can approximate transcendental curves, but the approximation may be a poor one, depending on the approximated curve, when few arcs are used to draw it. With arcs specified with more complicated macros the approximation is better even with a lower number of arcs. With many arcs it is possible to approximate almost anything. On the left side a modest approxima-

```
\raggedleft\unitlength=0.01\textwidth
\begin{picture}(70,70)
\put(0,0){\color{blue}\frame(70,70){}}
\put(0,0){\color{red}\Curve(0,0)<1,1>(70,0)<1,-1>}
\Curve(0,0)<1,1>(70,0)<1,-1;0,0>
\Curve(0,0)<1,1>(70,0)<1,-1;0.2,0.2>
\Curve(0,0)<1,1>(70,0)<1,-1;2,2>
\Curve(0,0)<1,1>(70,0)<1,-1;4.5,4.5>
\Curve(0,0)<1,1>(70,0)<1,-1;0,3>
\Curve(0,0)<1,1>(70,0)<1,-1;3,0>
\end{picture}
```

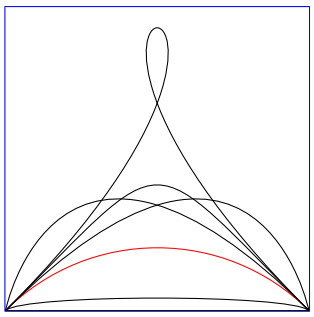


Figure 14: The effects of tension factors

tion is obtained with just three standard arcs obtained with `\Curve` and four node specifications; on the right we have just two arcs created with `CBezierBetween` with tension specification and control point distances; this drawing is almost undistinguishable from a real sinusoid.

In figure **??** some lines drawn are shown; they are drawn with quadratic splines by means of the `\Qurve` macro. In the left there are some open and closed curves inscribed within a square. On the right a "real" circle is compared to a quadratic spline circle; the word "real" is emphasised because it actually is an approximation with four quarter-circle cubic splines that, in spite of being drawn with third degree parametric polynomials, approximate very well a real circle; on the opposite the quadratic spline circle is clearly a poor approximation even if the maximum radial error amounts just to about 6% of the radius.

Notice that the previous version of `curve2e` contained an error and would color the outside of the green four-pointed star. The `curve2e-v161`, attached to this bundle, has been corrected; therefore it is not actually identical to the previous version, although the latter one performed correctly for everything else except for color-filled quadratic paths.

14. The new version of `\multiput` is backwards compatibile with the original version contained in the LaTeX kernel. The new macro adds the handling of the coordinate increments from one position to the next for the ⟨*object*⟩ to include in the drawing.

    On page **??** we show the code for the figure shown there. The red grid is nothing new, except that it displays the traditional `\multiput` used in this code, shown in a previous example, produces exactly the same result. But the for "graphs" on the grid, it display an alignment of black dots along the diagonal of the grid (again traditional `\multiput` rendered with the new version); a number of blue dots along a parabola; another number of magenta dots alined along a half sine wave; a number of little green squares aligned along a -15 ∘ line starting from the center of the grid; notice the polar values that are used as polar relative coordinate increments.

```
\unitlength=0.01\textwidth
\begin{picture}(100,50)(0,-25)
\put(0,0){\VECTOR(0,0)(45,0)\VECTOR(0,-25)(0,25)
\Zbox(45,0)[br]{x}\Zbox(0,26)[tl]{y}
\Curve(0,0)<77:1>(10,20)<1,0;2,0.4>(30,-20)<1,0;0.4,0.4>(40,0)<77:1;0.4,2>
}
\put(55,0){\VECTOR(0,0)(45,0)\VECTOR(0,-25)(0,25)
\Zbox(45,0)[br]{x}\Zbox(0,26)[tl]{y}
\CbezierBetween0,0And20,0WithDirs77:1And-77:1UsingDists28And{28}
\CbezierBetween20,0And40,0WithDirs-77:1And77:1UsingDists28And{28}}
\end{picture}
```

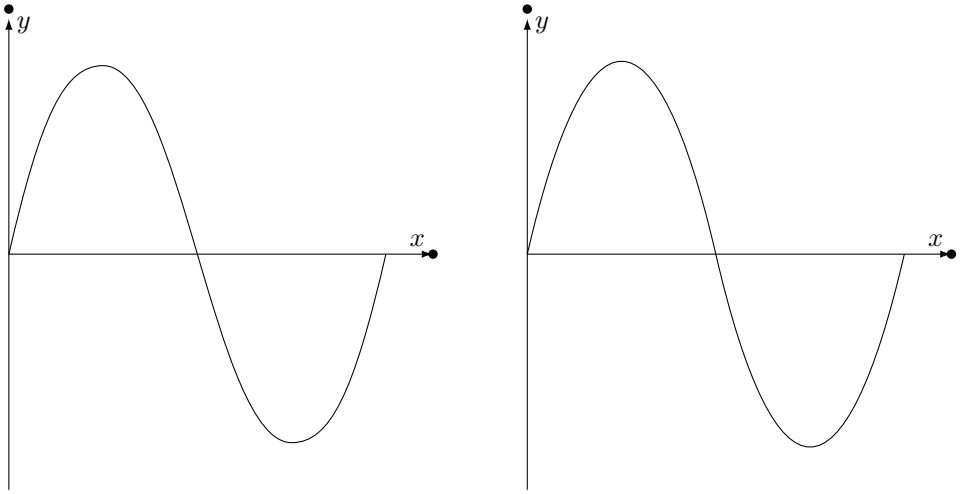


Figure 15: A sequence of arcs; the left figure has been drawn with the `\Curve` command with a sequence of four couples of point-direction arguments; the right figure has been drawn with two commands `\CbezierBetween` that include also the specification of the control points

A new command `\xmultiput` (not available with the previous versions of `curve2e`) extended with respect to the original `\multiput` is defined by using some L3 functions; in particular the cycling counter is accessible to the LaTeX commands and it is stepped forward from 1 to the value specified in the proper command argument (in the original command it starts from that value and is stepped down to zero). See the figure on page **??** to inspect its usage. It is important to notice that if the command`\rotatebox` has to be used, as in the example of figure **??**, the package `graphics` should be also loaded, because `curve2e` does not do it.

In spite of the relative simplicity of the macros contained in this package, the described macros, as well as the original ones included in the `pict2e` package, allow to produce fine drawings that were unconceivable with the original LaTeX picture

```
%\unitlength=0.0045\textwidth
%\begin{picture}(100,100)
%\put(0,0){\framebox(100,100){}}
%\put(50,50){%
%  \Qurve(0,-50)<1,0>(50,0)<0,1>(0,50)<-1,0>(-50,0)<0,-1>(0,-50)<1,0>
%\color{green}
%  \Qurve*(0,-50)<0,1>(50,0)<1,0>[-1,0](0,50)<0,1>[0,-1](-50,0)<-1,0>[1,0](0,-50)<0,-1>
%}
%\Qurve(0,0)<1,4>(50,50)<1,0>(100,100)<1,4>
%\put(5,50){\Qurve(0,0)<1,1.5>(22.5,20)<1,0>(45,0)<1,-1.5>%
%(67.5,-20)<1,0>(90,0)<1,1.5>}
%\Zbox(0,0)[tc]{0,0}\Zbox(100,0)[tc]{100,0}
%\Zbox(100,100)[bc]{100,100}\Zbox(0,100)[bc]{0,100}
%\Pall[2](0,0)\Pall[2](100,0)\Pall[2](100,100)\Pall[2](0,100)
%\end{picture}
%\hfill
%\begin{picture}(100,100)
%\put(0,0){\framebox(100,100){}}
%\put(50,50){%
%\Qurve(0,-50)<1,0>(50,0)<0,1>(0,50)<-1,0>(-50,0)<0,-1>(0,-50)<1,0>
%\Curve(0,-50)<1,0>(50,0)<0,1>(0,50)<-1,0>(-50,0)<0,-1>(0,-50)<1,0>}
%\Zbox(50,50)[t]{0}\Pall[2](50,50)\put(50,50){\Vector(45:50)}\Zbox(67,70)[tl]{R}
%\end{picture}
```

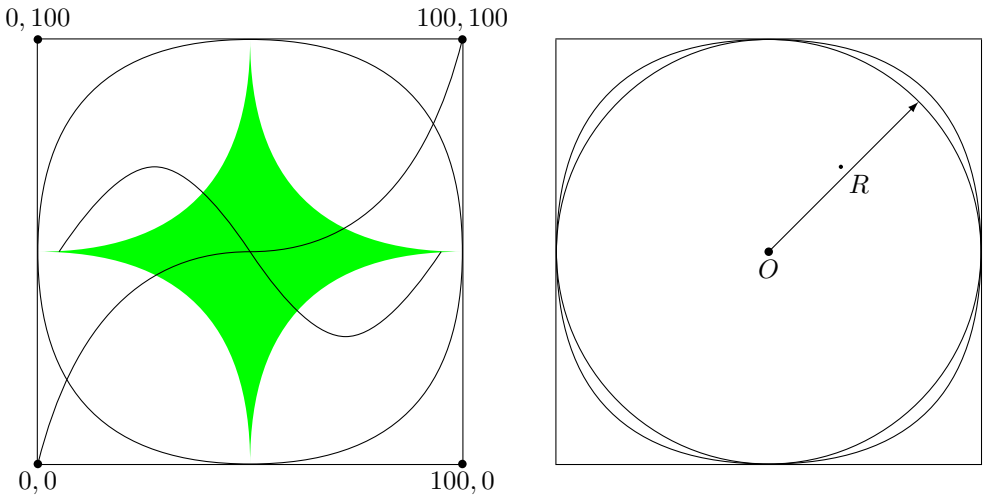


Figure 16: Several graphs drawn with quadratic Bézier splines. On the right a quadratic spline circle is compared with a cubic line circle.

environment. Leslie Lamport himself announced an extension to his environment when LaTeX 2_ε was first released in 1994; in the `latexnews` news-letter of December 2003; the first implementation was announced; the first version of this package `curve2e` was issued in 2006. It was time to have a better drawing environment; this package is a simple attempt to follow the initial path while extending the drawing facilities; but Till Tantau's `pgf` package has gone much farther.

```
%\unitlength=0.01\linewidth
%\begin{picture}(100,100)
%\put(0,0){\GraphGrid(100,100)}
%\multiput(0,0)(10,10){11}{\circle*{2}}
%\color{blue!70!white}
%\multiput(0,0)(10,0){11}{\circle*{2}}%
%  [\GetCoord(\R)\X\Y
%  \edef\X{\fpeval{\X+10}}
%  \edef\Y{\fpeval{(\X/10)**2}}
%  \CopyVect\X,\Y to\R]
%\color{magenta}
%\multiput(0,0)(10,1){11}{\circle*{2}}%
%  [\GetCoord(\R)\X\Y
%   \edef\X{\fpeval{\X+10}}
%   \edef\Y{\fpeval{sind(\X*1.8)*100}}
%   \CopyVect\X\Y to\R]
%\color{green!60!black}
%\multiput(50,50)(-15:5){11}{%
%\polygon*(-1,-1)(1,-1)(1,1)(-1,1)}
%\end{picture}
```

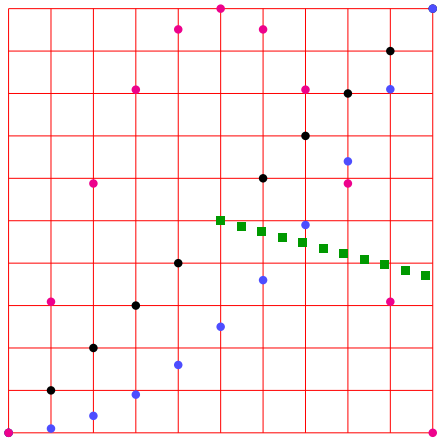


Figure 17: Some examples of the ⟨*handler*⟩ optional argument

```
\unitlength=0.0095\linewidth
\begin{picture}(100,100)
\put(0,0){\GraphGrid(100,100)}
\put(50,50){\thicklines\circle{100}}
\xmultiput[50,50](60:40)(-30:1){12}%
  {\makebox(0,0){\circle*{2}}}%
  [\MultVect\R by\D to\R]%
\xmultiput[50,50](60:46)(-30:1){12}%
  {\ArgOfVect\R to\Ang
  \rotatebox{\fpeval{\Ang-90}}%
  {\makebox(0,0)[b]{\Roman{multicnt}}}}%
  [\Multvect{\R}{\D}\R]
\end{picture}
```

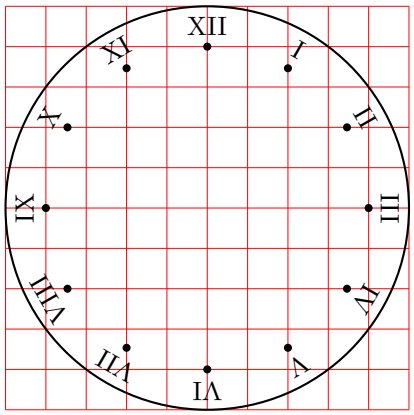


Figure 18: Usage example of the `\xmultiput` command

4 Remark

There are other packages in the CTAN archives that deal with tracing curves of various kinds. `PSTricks` and `tikz/pgf` are the most powerful ones. But there is also the package `curves` that is intended to draw almost anything by using little dots or other symbols partially superimposed to one another. It uses only quadratic Bézier curves and the curve tracing is eased by specifying only the curve nodes, without specifying the control nodes; with a suitable option to the package call it is possible to reduce the memory usage by using short straight segments drawn with the PostScript facilities offered by the `dvips` driver.

Another package `ebezier` performs about the same as `curve2e` but draws its Bézier curves by using little dots partially superimposed to one another. The documentation is quite interesting since it explains very clearly what exactly are the Bézier splines. Apparently `ebezier` should be used only for DVI output without recourse to PostScript or PDF machinery.

The `picture` package extends the performance of the `picture` environment (extended with `pict2e`) by accepting coordinates and lengths in real absolute dimensions, not only as multiples of `\unitlength`; it provides commands to extend that functionality to other packages. In certain circumstances it is very useful.

Package `xpicture` builds over the `picture` LaTeX environment so as to allow to draw the usual curves that are part of an introductory analytic geometry course; lines, circles, parabolas, ellipses, hyperbolas, and polynomials; the syntax is very comfortable; for all these curves it uses the quadratic Bézier splines.

Package `hobby` extends the cubic Bézier spline handling with the algorithms John Hobby created for METAFONT and METAPOST. But by now this package interfaces very well with `tikz`; it has not (yet) been adapted to the common `picture` environment, even when extended with `pict2e`, and, why not, with `curve2e`.

5 Acknowledgements

I wish to express my deepest thanks to Michel Goosens who spotted some errors and very kindly submitted them to me so that I was able to correct them.

Josef Tkadlec and the author collaborated extensively in order to make a better real long division so as to get correctly the quotient fractional part and to avoid as much as possible any numeric overflow; many Josef's ideas are incorporated in the macro that was implemented in the previous version of this package, although the macro used by Josef was slightly different. Both versions aim/aimed at a better accuracy and at widening the operand ranges. In this version we abandoned the long division macro, and substituted it with the floating point division provided by the `xfp` package.

Daniele Degiorgi spotted a fault in the kernel definition of `\linethickness` that heavily influenced also `curve2e`; see below in the code documentation part.

Thanks also to Jin-Hwan Cho and Juho Lee who suggested a small but crucial modification in order to have `curve2e` work smoothly also with XeTeX (XeLaTeX). Actually if `pict2e`, version 0.2x or later, dated 2009/08/05 or later, is being used, such modification is not necessary any more, but it's true that it becomes imperative if older versions are used.

6 Source code

6.1 Some preliminary extensions to the `pict2e` package

The necessary preliminary code has already been introduced. Here we require the `color` package and the `pict2e` one; for the latter one we make sure that a

sufficiently recent version is used. If you want to use package `xcolor`, load it *after* `curve2e`.

Here we load also the `xparse` and `xfp` packages because we use their functionalities; but we do load them only if they are not already loaded with or without options; nevertheless we warn the user who wants to load them explicitly, to do this action before loading `curve2e`. The `xfp` package is absolutely required; if this package is not found in the TEX system installation, the loading of this new `curve2e` is aborted, and the previous version 1.61 is loaded in its place; the overall functionalities should non change much, but the functionalities of `xfp` are not available.

```
\IfFileExists{xfp.sty}{%
  \RequirePackage{color}
  \RequirePackageWithOptions{pict2e}[2014/01/01]
  \@ifl@aded{sty}{xparse}{}{\RequirePackage{xparse}}
  \@ifl@aded{sty}{xfp}{}{\RequirePackage{xfp}}%
}{%
    \RequirePackage{curve2e-v161}%
    \PackageWarningNoLine{curve2e}{%
      Package xfp is required, but apparently\MessageBreak%
      such package cannot be found in this \MessageBreak%
      TeX system installation\MessageBreak%
      Either your installation is not complete \MessageBreak%
      or it is older than 2018-10-17.\MessageBreak%
      \MessageBreak%
      **************************************\MessageBreak%
      Version 1.61 of curve2e has been loaded\MessageBreak%
      instead of the current version\MessageBreak%
      **************************************\MessageBreak}%
      \endinput
}
```

Since we already loaded package`xfp` or at least we explicitly load it in our preamble, we add, if not already defined by the package, the two new commands that allow to make floating point tests, and to implement a "while" cycle[2]

```
%
\ExplSyntaxOn
\AtBeginDocument{%
\ProvideExpandableDocumentCommand\fptest{m m m}{%
   \fp_compare:nTF{#1}{#2}{#3}}
\ProvideExpandableDocumentCommand\fpdowhile{m m}{%
   \fp_do_while:nn{#1}{#2}}
}
\ExplSyntaxOff

```

The next macros are just for debugging. With the `trace` package it would probably be better to define other macros, but this is not for the users, but for the developers.

[2]Thanks to Brian Dunn who spotted a bug in the previous definitions.

```
\def\TRON{\tracingcommands\tw@ \tracingmacros\tw@}%
\def\TROF{\tracingcommands\z@ \tracingmacros\z@}%
```

Next we define some new dimension registers that will be used by the subsequent macros; should they be already defined, there will not be any redefinition; nevertheless the macros should be sufficiently protected so as to avoid overwriting register values loaded by other macro packages.

```
\ifx\undefined\@tdA \newdimen\@tdA \fi
\ifx\undefined\@tdB \newdimen\@tdB \fi
\ifx\undefined\@tdC \newdimen\@tdC \fi
\ifx\undefined\@tdD \newdimen\@tdD \fi
\ifx\undefined\@tdE \newdimen\@tdE \fi
\ifx\undefined\@tdF \newdimen\@tdF \fi
\ifx\undefined\defaultlinewidth \newdimen\defaultlinewidth \fi
```

6.2 Line thickness macros

It is better to define a macro for setting a different value for the line and curve thicknesses; the '`\defaultlinewidth` should contain the equivalent of `\@wholewidth`, that is the thickness of thick lines; thin lines are half as thick; so when the default line thickness is specified to, say, 1pt, thick lines will be 1pt thick and thin lines will be 0.5pt thick. The default whole width of thick lines is 0,8pt, but this is specified in the kernel of LaTeX and/or in `pict2e`. On the opposite it is necessary to redefine `\linethickness` because the LaTeX kernel global definition does not hide the space after the closed brace when you enter something such as `\linethickness{1mm}` followed by a space or a new line.[3]

```
\gdef\linethickness#1{%
\@wholewidth#1\@halfwidth.5\@wholewidth\ignorespaces}%
\newcommand\defaultlinethickness[1]{\defaultlinewidth=#1\relax
\def\thicklines{\linethickness{\defaultlinewidth}}%
\def\thinlines{\linethickness{.5\defaultlinewidth}}\thinlines
   \ignorespaces}%
```

The `\ignorespaces` at the end of these macros is for avoiding spurious spaces to get into the picture that is being drawn, because these spaces introduce picture deformities often difficult to spot and eliminate.

6.3 Improved line and vector macros

The macro `\LIne` allows to draw a line with arbitrary inclination as if it was a polygonal with just two vertices; actually it joins the canvas coordinate origin with the specified relative coordinate; therefore this object must be set in place by means of a `\put` command. Since its starting point is always at a relative 0,0 coordinate point inside the box created with `\put`, the two arguments define the horizontal and the vertical component respectively.

```
\def\LIne(#1){{\GetCoord(#1)\@tX\@tY
  \moveto(0,0)
```

[3] Thanks to Daniele Degiorgi `degiorgi@inf.ethz.ch`).

```
\pIIe@lineto{\@tX\unitlength}{\@tY\unitlength}\strokepath}\ignorespaces
}%
```

A similar macro `\segment` operates between two explicit points with absolute coordinates, instead of relative to the position specified by a `\put` command; it resorts to the `\polyline` macro that shall be defined in a while. The `\@killglue` command might be unnecessary, but it does not harm; it eliminates any explicit or implicit spacing that might precede this command.

```
\def\segment(#1)(#2){\@killglue\polyline(#1)(#2)}%
```

By passing its ending points coordinates to the `\polyline` macro, both macro arguments are a pair of coordinates, not their components; in other words, if $P_1 = (x_1, y_2)$ and $P_2 = (x_2, y_2)$, then the first argument is the couple x_1, y_1 and likewise the second argument is x_2, y_2. Notice that since `\polyline` accepts also the vertex coordinates in polar form, also`\segment` accepts the polar form. Please remember that the decimal separator is the decimal *point*, while the *comma* acts as coordinate separator. This recommendation is particularly important for non-English speaking users, since in all other languages the comma is or must be used as the decimal separator.

The `\line` macro is redefined by making use of a division routine performed in floating point arithmetics; for this reason the LaTeX kernel and the overall TeX system installation must be as recent as the release date of the `xfp` package, i.e. 2018-10-17. The floating point division macro receives in input two fractional numbers and yields on output their fractional ratio. Notice that this command `\line` should follow the same syntax as the original pre 1994 LaTeX version; but the new definition accepts the direction coefficients in polar mode; that is, instead of specifying a slope of 30° with the actual sine and cosine (or values proportional to such functions), for example `(0.5,0.866025)`, you may specify it as `(30:1)`, i.e. as a unit vector with the required slope of 30°.

The beginning of the macro definition is the same as that of `pict2e`:

```
\def\line(#1)#2{\begingroup
  \@linelen #2\unitlength
  \ifdim\@linelen<\z@\@badlinearg\else
```

but as soon as it is verified that the line length is not negative, things change remarkably; in facts the machinery for complex numbers is invoked. This makes the code much simpler, not necessarily more efficient; nevertheless `\DirOfVect` takes the only macro argument (that actually contains a comma separated pair of fractional numbers) and copies it to `\Dir@line` (an arbitrarily named control sequence) after re-normalizing to unit magnitude; this is passed to `GetCoord` that separates the two components into the control sequences `\d@mX` and `\d@mY`; these in turn are the values that are actually operated upon by the subsequent commands.

```
    \expandafter\DirOfVect#1to\Dir@line
    \GetCoord(\Dir@line)\d@mX\d@mY
```

The normalised vector direction is actually formed with the directing cosines of the line direction; since the line length is actually the horizontal component for non vertical lines, it is necessary to compute the actual line length for non vertical

lines by dividing the given length by the magnitude of the horizontal cosine `\d@mX`, and the line length is accordingly scaled:

```
    \ifdim\d@mX\p@=\z@\else
      \edef\sc@lelen{\fpeval{1 / abs(\d@mX)}}\relax
      \@linelen=\sc@lelen\@linelen
    \fi
```

Of course, if the line is vertical this division must not take place. Finally the `moveto`, `lineto` and `stroke` language keywords are invoked by means of the internal `pict2e` commands in order to draw the line. Notice that even vertical lines are drawn with the PDF language commands instead of resorting to the DVI low level language that was used in both `pict2e` and the original (pre 1994) `picture` commands; it had a meaning in the old times, but it certainly does not have any nowadays, since lines are drawn by the driver that produces the output in a human visible document form, not by TEX the program.

```
    \moveto(0,0)\pIIe@lineto{\d@mX\@linelen}{\d@mY\@linelen}%
    \strokepath
  \fi
\endgroup\ignorespaces}%
```

The new definition of the command `\line`, besides the ease with which is readable, does not do different things from the definition of `pict2e` 2009, even if it did perform in a better way compared to the 2004 version that was limited to integer direction coefficients up to 999 in magnitude. Moreover this `curve2e` version accepts polar coordinates as slope pairs, making it much simpler to draw lines with specific slopes.

It is necessary to redefine the low level macros `\moveto`, `\lineto`, and `\curveto`, because their original definition accepts only cartesian coordinates. We proceed the same as for the `\put` command.

```
\let\originalmoveto\moveto
\let\originallineto\lineto
\let\originalcurveto\curveto

\def\moveto(#1){\GetCoord(#1)\MTx\MTy
  \originalmoveto(\MTx,\MTy)\ignorespaces}
\def\lineto(#1){\GetCoord(#1)\LTx\LTy
  \originallineto(\LTx,\LTy)\ignorespaces}
\def\curveto(#1)(#2)(#3){\GetCoord(#1)\CTpx\CTpy
  \GetCoord(#2)\CTsx\CTsy\GetCoord(#3)\CTx\CTy
  \originalcurveto(\CTpx,\CTpy)(\CTsx,\CTsy)(\CTx,\CTy)\ignorespaces}
```

6.4 Dashed and dotted lines

Dashed and dotted lines are very useful in technical drawings; here we introduce two macros that help drawing them in the proper way; besides the obvious difference between the use of dashes or dots, they may refer in a different way to the end points that must be specified to the various macros.

The coordinates of the first point P_1, where le line starts, are always referred to the origin of the coordinate axes; the end point P_2 coordinates are referred to the origin of the axes if in cartesian form, while with the polar form they are referred to P_1; both coordinate types have their usefulness and figures **??** on page **??** and **??** on page **??** show how to use such coordinate types.

The above mentioned macros create dashed lines between two given points, with a dash length that must be specified, or dotted lines, with a dot gap that must be specified; actually the specified dash length or dot gap is a desired one; the actual length or gap is computed by integer division between the distance of the given points and the desired dash length or dot gap; when dashes are involved, this integer is tested in order to see if it is an odd number; if it's not, it is increased by unity. Then the actual dash length or dot gap is obtained by dividing the above distance by this number.

Another vector $P_2 - P_1$ is created by dividing it by this number; then, when dashes are involved, it is multiplied by two in order to have the increment from one dash to the next; finally the number of patterns is obtained by integer division of this number by 2 and increasing it by 1. Since the whole dashed or dotted line is put in position by an internal `\put` command, is is not necessary to enclose the definitions within groups, because they remain interna to the `\put` argument box.

Figure **??** on page **??** shows the effect of the slight changing of the dash length in order to maintain approximately the same dash-space pattern along the line, irrespective of the line length. The syntax is the following:

`\Dashline(`⟨*first point*⟩`)(`⟨*second point*⟩`){`⟨*dash length*⟩`}`

where ⟨*first point*⟩ contains the coordinates of the starting point and ⟨*second point*⟩ the absolute (cartesian) or relative (polar) coordinates of the ending point; of course the ⟨*dash length*⟩, which equals the dash gap, is mandatory. An optional asterisk used to play a specific role with previous implementations; it is maintained for backwards compatibility, but its use is now superfluous; with the previous implementation of the code, in facts, if coordinates were specified in polar form, without the optional asterisk the dashed line was misplaced, while if the asterisk was specified, the whole object was put in the proper position. With this new implementation, both the cartesian and polar coordinates always play the role they are supposed to play independently from the asterisk. The `\IsPolar` macro is introduced to analyse the coordinate type used for the second argument, and uses such second argument accordingly.

```
\def\IsPolar#1:#2?{\def\@TempOne{#2}\unless\ifx\@TempOne\empty
   \expandafter\@firstoftwo\else
   \expandafter\@secondoftwo\fi}

\ifx\Dashline\undefined
  \def\Dashline{\@ifstar{\Dashline@}{\Dashline@}}% bckwd compatibility
  \let\Dline\Dashline

  \def\Dashline@(#1)(#2)#3{\put(#1){%
   \GetCoord(#1)\@tA\@tB \MakeVectorFrom\@tA\@tB to\V@ttA
```

```
  \GetCoord(#2)\@tA\@tB \MakeVectorFrom\@tA\@tB to\V@ttB
  \IsPolar#2:?{%                        Polar
     \Dashline@@(0,0)(\V@ttB){#3}}%
  {%                                    Cartesian
    \SubVect\V@ttA from\V@ttB to\V@ttC
    \Dashline@@(0,0)(\V@ttC){#3}%
  }
}}

 \def\Dashline@@(#1)(#2)#3{%
  \countdef\NumA3254\countdef\NumB3252\relax
  \GetCoord(#1)\@tA\@tB \MakeVectorFrom\@tA\@tB to\V@ttA
  \GetCoord(#2)\@tA\@tB \MakeVectorFrom\@tA\@tB to\V@ttB
  \SubVect\V@ttA from\V@ttB to\V@ttC
  \ModOfVect\V@ttC to\DlineMod
  \DivideFN\DlineMod by#3 to\NumD
  \NumA=\fpeval{trunc(\NumD,0)}\relax
  \unless\ifodd\NumA\advance\NumA\@ne\fi
  \NumB=\NumA \divide\NumB\tw@
  \DividE\DlineMod\p@ by\NumA\p@ to\D@shMod
  \DividE\p@ by\NumA\p@ to \@tempa
  \Multvect{\V@ttC}{\@tempa,0}\V@ttB
  \Multvect{\V@ttB}{2,0}\V@ttC
  \advance\NumB\@ne
  \put(\V@ttA){\multiput(0,0)(\V@ttC){\NumB}{\LIne(\V@ttB)}}
  \ignorespaces}
\fi
```

A simpler `\Dotline` macro can draw a dotted line between two given points; the dots are rather small, therefore the inter dot distance is computed in such a way as to have the first and the last dot at the exact position of the dotted-line end-points; again the specified dot distance is nominal in the sense that it is recalculated in such a way that the first and last dots coincide with the line end points. Again if the second point coordinates are in polar form they are considered as relative to the first point. The syntax is as follows:

`\Dotline(`⟨*start point*⟩`)(`⟨*end point*⟩`){`⟨*dot distance*⟩`}`

```
\ifx\Dotline\undefined
  \def\Dotline{\@ifstar{\Dotline@}{\Dotline@}}% backwards compatibility
  \def\Dotline@(#1)(#2)#3{\put(#1){%
  \IsPolar#2:?{%                    Polar
     \Dotline@@(0,0)(#2){#3}}
   {%                               Cartesian
   \CopyVect#1to\V@ttA
   \CopyVect#2to\V@ttB
   \SubVect\V@ttA from\V@ttB to\V@ttC
   \Dotline@@(0,0)(\V@ttC){#3}}%
   }}

  \def\Dotline@@(#1)(#2)#3{%
```

```
  \countdef\NumA 3254\relax
  \countdef\NumB 3255\relax
  \CopyVect#1to\V@ttA
  \CopyVect#2to\V@ttB
  \SubVect\V@ttA from\V@ttB to\V@ttC
  \ModOfVect\V@ttC to\DotlineMod
  \DivideFN\DotlineMod by#3 to\NumD
  \NumA=\fpeval{trunc(\NumD,0)}\relax
  \Divvect{\V@ttC}{\NumA,0}\V@ttB
  \advance\NumA\@ne
  \put(\V@ttA){\multiput(0,0)(\V@ttB){\NumA}{\makebox(0,0)%
              {\circle*{0.5}}}}
  \ignorespaces
 }%
\fi
```

Notice that vectors as complex numbers in their cartesian and polar forms always represent a point position referred to a local origin of the axes; this is why in figures ?? on page ?? and ?? on page ?? the dashed and dotted lines that start from the lower right corner of the graph grid, and that use polar coordinates, are correctly put in their correct position thanks to the different behaviour obtained with the `\IsPolar` macro.

6.5 Coordinate handling

The new macro `\GetCoord` splits a vector (or complex number) specification into its components; in particular it distinguishes the polar from the cartesian form of the coordinates. The latter have the usual syntax $\langle x,y\rangle$, while the former have the syntax $\langle angle{:}radius\rangle$. The `\put` and `\multiput` commands are redefined to accept the same syntax; the whole work is done by `\SplitNod@` and its subsidiaries.

Notice that package `eso-pic` uses `picture` macros in its definitions, but its original macro `\LenToUnit` is incompatible with this `\GetCoord` macro; its function is to translate real lengths into coefficients to be used as multipliers of the current `\unitlength`; in case that the `eso-pic` had been loaded, at the `\begin{document}` execution, the `eso-pic` macro is redefined using the e-TEX commands so as to make it compatible with these local macros.[4]

```
\AtBeginDocument{\@ifpackageloaded{eso-pic}{%
\renewcommand\LenToUnit[1]{\strip@pt\dimexpr#1*\p@/\unitlength}}{}}%
```

The above redefinition is delayed at `\AtBeginDocument` in order to have the possibility to check the the `eso-pic` package had actually been loaded. Nevertheless the code is defined here just because the original `eso-pic` macro was interfering with the algorithms of coordinate handling.

But let us come to the real subject of this section. We define a `\GettCoord` macro that passes control to the service macro with the expanded arguments; expanding arguments allows to use macros to named points, instead of explicit coordinates; with this version of `curve2e` this facility is not fully exploited, but a

[4] Thanks to Franz-Joseph Berthold who was so kind to spot the bug.

creative user can use this feature. Notice the usual trick to pass through a dummy macro that is defined within a group with expanded arguments, but where the group is closed by the macro itself, so that no traces remain behind after its expansion.

```
\def\GetCoord(#1)#2#3{\bgroup\edef\x{\egroup\noexpand\IsPolar#1:?}\x
{% Polar
  \bgroup\edef\x{\egroup\noexpand\SplitPolar(#1)}\x\SCt@X\SCt@Y}%
{% Cartesian
  \bgroup\edef\x{\egroup\noexpand\SplitCartesian(#1)}\x\SCt@X\SCt@Y}%
  \edef#2{\SCt@X}\edef#3{\SCt@Y}\ignorespaces}

\def\SplitPolar(#1:#2)#3#4{%
  \edef#3{\fpeval{#2 * cosd#1}}\edef#4{\fpeval{#2 * sind#1}}}

\def\SplitCartesian(#1,#2)#3#4{\edef#3{#1}\edef#4{#2}}

```

The macro that detects the form of the coordinates is `\IsPolar`; it examines the parameter syntax in order to see if it contains a colon; it has already been used with the definition of dashed and dotted lines.

In order to accept polar coordinates with `\put` and `\multiput` we resort to using `\GetCoord`; therefore the redefinition of `\put` is very simple because it suffices to save the original meaning of that macro and redefine the new one in terms of the old one.

```
\let\originalput\put
\def\put(#1){\bgroup\GetCoord(#1)\@tX\@tY
\edef\x{\noexpand\egroup\noexpand\originalput(\@tX,\@tY)}\x}
```

For `\multiput` it is more complicated, because the increments from one position to the next cannot be done efficiently because the increments in the original definition are executed within boxes, therefore any macro instruction inside these boxes is lost. It is a good occasion to modify the `\multiput` definition by means of the advanced macro definitions provided by package `xparse`; we can add also some error messages for avoiding doing anything when some mandatory parameters are missing ore are empty, or do not contain anything different from an ordered pair or a polar form. We add also an optional argument to handle the increments outside the boxes. The new macro has the following syntax:

`\multiput[`⟨*displacement*⟩`](`⟨*initial*⟩`)(`⟨*increment*⟩`){`⟨*number*⟩`}{`⟨*object*⟩`}[`⟨*handler*⟩`]`

where the optional ⟨*displacement*⟩ is used to displace to whole set of ⟨*object*⟩s from their original position; ⟨*initial*⟩ contains the cartesian or polar coordinates of the initial point; ⟨*increment*⟩ contains the cartesian or polar increment for the coordinates to be used from the second position to the last; ⟨*number*⟩ il the total number of ⟨*object*⟩s to be drawn; ⟨*object*⟩ is the object to be put in position at each cycle repetition; the optional ⟨*handler*⟩ may be used to control the current values of the horizontal and vertical increments. The new definition contains two `\put` commands where the second is nested within a while-loop which in turn is within the argument of the first `\put` command. Basically it is the same idea that

the original macros, but now the increments are computed within the while loop, but outside the argument of the inner `\put` command. If the optional ⟨*handler*⟩ is specified the increments are computed from the macros specified by the user.

The two increments components inside the optional argument may be set by means of mathematical expressions operated upon by the `\fpeval` function given by the `\xfp` package already loaded by `curve2e`. Of course it is the user responsibility to pay attention to the scales of the two axes and to write meaningful expressions; the figure and code shown in the first part of this documentation show some examples: see pages **??** and **??**.

```
\RenewDocumentCommand{\multiput}{O{0,0} d() d() m m o }{%
  \IfNoValueTF{#2}{\PackageError{curve2e}%
      {\string\multiput\space initial point coordinates missing}%
      {Nothing done}
    }%
    {\IfNoValueTF{#3}{\PackageError{curve2e}
      {\string\multiput\space Increment components missing}%
      {Nothing done}
    }
    {\put(#1){\let\c@multicnt\@multicnt
      \CopyVect #2 to \R
      \CopyVect#3 to\D
        \@multicnt=#4\relax
        \@whilenum \@multicnt > \z@\do{%
          \put(\R){#5}%
          \IfValueTF{#6}{#6}{\AddVect#3 and\R to \R}%
          \advance\@multicnt\m@ne
        }
      }
    }
  }\ignorespaces
}
```

And here it is the new `\xmultiput` command; remember: the internal cycling TEX counter `\@multicn` is now accessible as it was a LATEX counter, in particular the user can access its contents with a command such as `\value{multicnt}`. Such counter is stepped up by one at each cycle, instead of being stepped down as in the original `\multiput` command. The code is not so different from the one used for the new version of `\multiput`, but it appears more efficient and its code logically more readable.

```
\NewDocumentCommand{\xmultiput}{O{0,0} d() d() m m o }{%
\IfNoValueTF{#2}{\PackageError{curve2e}{%
  \string\Xmultiput\space initial point coordinates missing}%
  {Nothing done}}%
  {\IfNoValueTF{#3}{\PackageError{curve2e}{%
    \string\Xmultiput\space Increment components missing}%
    {Nothing done}}%
  {\put(#1)%
  {\let\c@multicnt\@multicnt
  \CopyVect #2 to \R
```

```
  \CopyVect #3 to \D
    \@multicnt=\@ne
      \fpdowhile{\value{multicnt} < \inteval{#4+1}}% Test
        {%
        \put(\R){#5}
        \IfValueTF{#6}{#6}{%
          \AddVect#3 and\R to \R}
          \advance\@multicnt\@ne
      }
    }
  }}\ignorespaces
}
```

Notice that the internal macros `\R` and `\D`, (respectively the current point coordinates, in form of a complex number, where to put the⟨*object*⟩, and the current displacement to find the next point) are accessible to the user both in the ⟨*object*⟩ argument field and the ⟨*handler*⟩ argument field. The code used in page **??** shows how to create the hour marks of a clock together with the rotated hour roman numerals.

6.6 Vectors

The redefinitions and the new definitions for vectors are a little more complicated than with segments, because each vector is drawn as a filled contour; the original `pict2e` 2004 macro checks if the slopes are corresponding to the limitations specified by Lamport (integer three digit signed numbers) and sets up a transformation in order to make it possible to draw each vector as an horizontal left-to-right arrow and then to rotate it by its angle about its tail point; with `pict2e` 2009, possibly this redefinition of `\vector` is not necessary, but we do it as well and for the same reasons we had for redefining `\line`; actually there are two macros for tracing the contours that are eventually filled by the principal macro; each contour macro draws the vector with a LaTeX or a PostScript styled arrow tip whose parameters are specified by default or may be taken from the parameters taken from the`PSTricks` package if this one is loaded before `pict2e`; in any case we did not change the contour drawing macros because if they are modified the same modification is passed on to the arrows drawn with the `curve2e` package redefinitions.

Because of these features the new macros are different from those used for straight lines.

We start with the redefinition of `\vector` and we use the machinery for vectors (as complex numbers) we used for `\line`.

```
\def\vector(#1)#2{%
  \begingroup
    \GetCoord(#1)\d@mX\d@mY
    \@linelen#2\unitlength
```

As in `pict2e` we avoid tracing vectors if the slope parameters are both zero.

```
    \ifdim\d@mX\p@=\z@\ifdim\d@mY\p@=\z@\@badlinearg\fi\fi
```

But we check only for the positive nature of the l_x component; if it is negative, we simply change sign instead of blocking the typesetting process. This is useful also for macros `\Vector`, `\VECTOR`, and `\VVECTOR` to be defined in a while.

```
\ifdim\@linelen<\z@ \@linelen=-\@linelen\fi
```

We now make a vector with the given slope coefficients even if one or the other is zero and we determine its direction; the real and imaginary parts of the direction vector are also the values we need for the subsequent rotation.

```
\MakeVectorFrom\d@mX\d@mY to\@Vect
\DirOfVect\@Vect to\Dir@Vect
```

In order to be compatible with the original `pict2e` we need to transform the components of the vector direction in lengths with the specific names `\@xdim` and `\@ydim`

```
    \YpartOfVect\Dir@Vect to\@ynum \@ydim=\@ynum\p@
    \XpartOfVect\Dir@Vect to\@xnum \@xdim=\@xnum\p@
```

If the vector is really sloping we need to scale the l_x component in order to get the vector total length; we have to divide by the cosine of the vector inclination which is the real part of the vector direction. We use the floating point division function; since it yields a "factor" We directly use it to scale the length of the vector. We finally memorise the true vector length in the internal dimension `@tdB`

```
    \ifdim\d@mX\p@=\z@
    \else\ifdim\d@mY\p@=\z@
      \else
        \edef\sc@lelen{\fpeval{1 / abs(\@xnum)}}\relax
        \@linelen=\sc@lelen\@linelen
      \fi
    \fi
  \@tdB=\@linelen
```

The remaining code is definitely similar to that of `pict2e`; the real difference consists in the fact that the arrow is designed by itself without the stem; but it is placed at the vector end; therefore the first statement is just the transformation matrix used by the output driver to rotate the arrow tip and to displace it the right amount. But in order to draw only the arrow tip we have to set the `\@linelen` length to zero.

```
\pIIe@concat\@xdim\@ydim{-\@ydim}\@xdim{\@xnum\@linelen}{\@ynum\@linelen}%
    \@linelen\z@
    \pIIe@vector
    \fillpath
```

Now we can restore the stem length that must be shortened by the dimension of the arrow; by examining the documentation of `pict2e` we discover that we have to shorten it by an approximate amount of AL (with the notations of `pict2e`, figs 10 and 11); the arrow tip parameters are stored in certain variables with which we can determine the amount of the stem shortening; if the stem was too short and the new length is negative, we avoid designing such a stem.

```
  \@linelen=\@tdB
  \@tdA=\pIIe@FAW\@wholewidth
```

```
      \@tdA=\pIIe@FAL\@tdA
      \advance\@linelen-\@tdA
      \ifdim\@linelen>\z@
        \moveto(0,0)
        \pIIe@lineto{\@xnum\@linelen}{\@ynum\@linelen}%
        \strokepath\fi
    \endgroup}
```

We define the macro that does not require the specification of the length or the l_x length component; the way the new `\vector` macro works does not actually require this specification, because TEX can compute the vector length, provided the two direction components are exactly the horizontal and vertical vector components. If the horizontal component is zero, the actual length must be specified as the vertical component. The object defined with `\Vector`, as well as `\vector`, must be put in place by means of a `\put` command.

```
\def\Vector(#1){{%
\GetCoord(#1)\@tX\@tY
\ifdim\@tX\p@=\z@
  \vector(\@tX,\@tY){\@tY}%
\else
  \vector(\@tX,\@tY){\@tX}%
\fi}}
```

On the opposite the next macro specifies a vector by means of the coordinates of its end points; the first point is where the vector starts, and the second point is the arrow tip side. We need the difference of these two coordinates, because it represents the actual vector.

```
\def\VECTOR(#1)(#2){\begingroup
\SubVect#1from#2to\@tempa
\expandafter\put\expandafter(#1){\expandafter\Vector\expandafter(\@tempa)}%
\endgroup\ignorespaces}
```

The double tipped vector is built on the `\VECTOR` macro by simply drawing two vectors from the middle point of the double tipped vector.

```
\def\VVECTOR(#1)(#2){{\SubVect#1from#2to\@tempb
\ScaleVect\@tempb by0.5to\@tempb
\AddVect\@tempb and#1to\@tempb
\VECTOR(\@tempb)(#2)\VECTOR(\@tempb)(#1)\ignorespaces}}
```

The `pict2e` documentation says that if the vector length is zero the macro draws only the arrow tip; this may work with macro `\vector`, certainly not with `\Vector` and `\VECTOR`. This might be useful for adding an arrow tip to a circular arc. See examples in figure **??** on page **??**.

6.7 Polylines and polygons

We now define the polygonal line macro; its syntax is very simple:

`\polygonal[`⟨*join*⟩`](`⟨P_0⟩`)(`⟨P_1⟩`)(`⟨P_2⟩`) ...(`⟨P_n⟩`)`

Remember: `\polyline` has been incorporated into `pict2e` 2009, but we redefine it so as to allow an optional argument to specify the line join type.

In order to write a recursive macro we need aliases for the parentheses; actually we need only the left parenthesis, but some editors complain about unmatched delimiters, so we define an alias also for the right parenthesis.

```
\let\lp@r( \let\rp@r)
```

The first call to `\polyline`, besides setting the line joints, examines the first point coordinates and moves the drawing position to this point; afterwards it looks for the second point coordinates; they start with a left parenthesis; if this is found the coordinates should be there, but if the left parenthesis is missing (possibly preceded by spaces that are ignored by the `\@ifnextchar` macro) then a warning message is output together with the line number where the missing parenthesis causes the warning: beware, this line number might point to several lines further on along the source file! In any case it's necessary to insert a `\@killglue`command, because `\polyline` refers to absolute coordinates, and not necessarily is put in position through a `\put` command that provides to eliminate any spurious spaces preceding this command.

```
\unitlength=0.07\hsize
\begin{picture}(8,8)(-4,-4)\color{red}
\polygon*(45:4)(135:4)(-135:4)(-45:4)
\end{picture}
```

Figure 19: The code and the result of defining a polygon with its vertex polar coordinates

In order to allow a specification for the joints of the various segments of a polyline it is necessary to allow for an optional parameter; the default is the bevel join.

```
\renewcommand*\polyline[1][\beveljoin]{\p@lylin@[#1]}

\def\p@lylin@[#1](#2){\@killglue#1\GetCoord(#2)\d@mX\d@mY
    \pIIe@moveto{\d@mX\unitlength}{\d@mY\unitlength}%
    \@ifnextchar\lp@r{\p@lyline}{%
    \PackageWarning{curve2e}%
    {Polylines require at least two vertices!\MessageBreak
     Control your polyline specification\MessageBreak}%
    \ignorespaces}}

```

But if there is a second or further point coordinate, the recursive macro `\p@lyline` is called; it works on the next point and checks for a further point; if such a point exists the macro calls itself, otherwise it terminates the polygonal line by stroking it.

```
\def\p@lyline(#1){\GetCoord(#1)\d@mX\d@mY
    \pIIe@lineto{\d@mX\unitlength}{\d@mY\unitlength}%
    \@ifnextchar\lp@r{\p@lyline}{\strokepath\ignorespaces}}
```

The same treatment must be done for the `\polygon` macros; we use the defining commands of package `xparse`, in order to use an optional asterisk; as it is usual with `picture` convex lines, the command with asterisk does not trace the contour, but fills the contour with the current color.The asterisk is tested at the beginning and, depending on its presence, a temporary switch is set to `true`; this being the case the contour is filled, otherwise it is simply stroked.

```
\providecommand\polygon{}
\RenewDocumentCommand\polygon{s O{\beveljoin} }{\@killglue\begingroup
\IfBooleanTF{#1}{\@tempswatrue}{\@tempswafalse}%
\@polygon[#2]}

\def\@polygon[#1](#2){\@killglue#1\GetCoord(#2)\d@mX\d@mY
    \pIIe@moveto{\d@mX\unitlength}{\d@mY\unitlength}%
    \@ifnextchar\lp@r{\@@polygon}{%
    \PackageWarning{curve2e}%
    {Polygons require at least two vertices!\MessageBreak
     Control your polygon specification\MessageBreak}%
    \ignorespaces}}

  \def\@@polygon(#1){\GetCoord(#1)\d@mX\d@mY
    \pIIe@lineto{\d@mX\unitlength}{\d@mY\unitlength}%
    \@ifnextchar\lp@r{\@@polygon}{\pIIe@closepath
      \if@tempswa\pIIe@fillGraph\else\pIIe@strokeGraph\fi
      \endgroup
      \ignorespaces}}
```

Now, for example, a filled polygon can be drawn using polar coordinates for its vertices; see figure **??** on page **??**.

Remember; the polygon polar coordinates are relative to the origin of the local axes; therefore in order to put a polygon in a different position, it is necessary to do it through a `\put` command.

6.8 The red service grid

The next command is very useful for debugging while editing one's drawing; it draws a red grid with square meshes that are ten drawing units apart; there is no graduation along the grid, since it is supposed to be a debugging aid and the user should know what he/she is doing; nevertheless it is advisable to displace the grid by means of a `\put` command so that its grid lines coincide with graph coordinates that are multiples of 10. Missing to do so the readings become cumbersome. The `\RoundUp` macro provides to increase the grid dimensions to integer multiples of ten.

```
\def\GraphGrid(#1,#2){\bgroup\textcolor{red}{\linethickness{.1\p@}%
\RoundUp#1modulo10to\@GridWd \RoundUp#2modulo10to\@GridHt
\@tempcnta=\@GridWd \divide\@tempcnta10\relax \advance\@tempcnta\@ne
```

```
\multiput(0,0)(10,0){\@tempcnta}{\line(0,1){\@GridHt}}%
\@tempcnta=\@GridHt \divide\@tempcnta10\advance\@tempcnta\@ne
\multiput(0,0)(0,10){\@tempcnta}{\line(1,0){\@GridWd}}\thinlines}%
\egroup\ignorespaces}
```

Rounding up is useful because also the grid margins fall on coordinates multiples of 10. It resorts to the `\Integer` macro that will be described in a while.

```
\def\RoundUp#1modulo#2to#3{\expandafter\@tempcnta\Integer#1.??%
\count254\@tempcnta\divide\count254by#2\relax
\multiply\count254by#2\relax
\count252\@tempcnta\advance\count252-\count254
\ifnum\count252>0\advance\count252-#2\relax
\advance\@tempcnta-\count252\fi\edef#3{\number\@tempcnta}\ignorespaces}%
```

The `\Integer` macro takes a possibly fractional number whose decimal separator, if present, *must* be the decimal point and uses the point as an argument delimiter. If one has the doubt that the number being passed to `\Integer` might be an integer, he/she should call the macro with a further point; if the argument is truly integer this point works as the delimiter of the integer part; if the argument being passed is fractional this extra point gets discarded as well as the fractional part of the number.

```
\def\Integer#1.#2??{#1}%
```

7 Math operations on fractional operands

This is not the place to complain about the fact that all programs of the TEX system used only integer arithmetics; now, with the 2018 distribution of the modern TEX system, package `xfp` is available: this package resorts in the background to language LATEX 3; this language now can compute fractional number operations coded in decimal, not in binary, and accepts also numbers written in the usual way in computer science, that is as a fractional, possibly signed, number followed by an expression that contains the exponent to 10 necessary to (ideally) move the fractional separator in one or the other direction according to the sign of the exponent of 10; in other words the L3 library for floating point calculations accepts such expressions as `123.456`, `0.12345e3`, and `12345e-3`, and any other equivalent expression. If the first number is integer, it assumes that the decimal separator is to the right of the rightmost digit of the digit string.

Floating point calculations may be done through the `\fpeval` L3 function with a very simple syntax:

`\fpeval`{⟨*mathematical expression*⟩}

where ⟨*mathematical exression*⟩ can contain the usual algebraic operation sings, `= - * / ** ^` and the function names of the most common algebraic, trigonometric, and transcendental functions; for direct and inverse trigonometric functions it accepts arguments in radians and in sexagesimal degrees; it accepts the group of rounding/truncating operators; it can perform several kinds of comparisons; as to now (Nov. 2019) the todo list includes the direct and inverse hyperbolic

functions. The mantissa length of the floating point operands amounts to 16 decimal digits. Further details may be read in the documentations of the `xfp` and `interface3` packages, just by typing into a command line window the command `texdoc` ⟨*document*⟩, where ⟨*document*⟩ is just the name of the above named files without extension.

Furthermore we added a couple of more interface macros with the internal L3 floating point functions; `\fptest` and `\fpdowhile`. They have the following syntax:

`\fptest`{⟨*logical expression*⟩}{⟨*true code*⟩}{⟨*false code*⟩}
`\fpdowhile`{⟨*logical expression*⟩}{⟨*code*⟩}

The ⟨*logical expression*⟩ compares the values of any kind by means of the usual `>`, `=`, and `<` operators that may be negated with the "not' operator `!`; furthermore the logical results of these comparisons may be acted upon with the "and" operator `&&` and the "or" operator . The ⟨*true code*⟩, and ⟨*code*⟩ are executed if or while the ⟨*logical expression*⟩ is true, while the ⟨*false code*⟩ is executed if the ⟨*logical expression*⟩ is false

Before the availability of the `xfp` package, it was necessary to fake fractional number computations by means of the native e-TEX commands `\dimexpr`, i.e. to multiply each fractional number by the unit `\p@` (1 pt) so as to get a length; operate un such lengths, and then stripping off the 'pt' component from the result; very error prone and with less precision as the one that the modern decimal floating point calculations can do. Of course it is not so important to use fractional numbers with more that 5 or 6 fractional digits, because the other TEX and LATEX macros cannot handle them, but it is very convenient to have simpler and more readable code. We therefore switched to the new floating point functionality, even if this maintains the `curve2e` functionality, but renders this package unusable with older LATEX kernel installations. It has already been explained that the input of this up to date version of `curve2e` is aborted if the `xfp` package is not available, but the previous version 1.61 version is loaded; very little functionality is lost, but, evidently, this new version performs in a better way.

7.1 The division macro

The most important macro is the division of two fractional numbers; we seek a macro that gets dividend and divisor as fractional numbers and saves their ratio in a macro; this is done in a simple way with the following code.

```
294 \def\DividE#1by#2to#3{\edef#3{\fpeval{#1 / #2}}}
```

In order to avoid problems with divisions by zero, or with numbers that yield results to large to be used as multipliers of lengths, it would be preferable that the above code be preceded or followed by some tests and possible messages. Actually we decided to avoid such tests and messages, because the internal L3 functions already provide some. This was done in the previous versions of this package, when the `\fpeval` L3 function was not available.

Notice that operands `#1` and `#2` may be integer numbers or fractional, or mixed numbers. They may be also dimensions, but while dimensions in printer points

(72.27pt=1in) are handled as assumed, when different units are used, the length must be enclosed in parentheses:

```
\DividE(1mm)by(3mm) to\result
```

yields correctly `\result=0.33333333`. Without parentheses the result is unpredictable.

For backwards compatibility we need an alias.

```
\let\DivideFN\DividE
```

We do the same in order to multiply two integer o fractional numbers held in the first two arguments and the third argument is a definable token that will hold the result of multiplication in the form of a fractional number, possibly with a non null fractional part; a null fractional part is stripped away

```
\def\MultiplY#1by#2to#3{\edef#3{\fpeval{#1 * #2}}}\relax
\let\MultiplyFN\MultiplY
```

but with multiplication it is better to avoid computations with lengths.

The next macro uses the `\fpeval` macro to get the numerical value of a measure in points. One has to call `\Numero` with a control sequence and a dimension; the dimension value in points is assigned to the control sequence.

```
\unless\ifdefined\Numero
  \def\Numero#1#2{\edef#1{\fpeval{round(#2,6)}}\ignorespaces}%
\fi
```

The numerical value is rounded to 6 fractional digits that are more than sufficient for the graphical actions performed by `curve2e`.

The `\ifdefined` primitive command is provided by the e-TEX extension of the typesetting engine; the test does not create any hash table entry; it is a different way than the `\ifx\csname...\endcsname` test, because the latter first possibly creates a macro meaning `\relax` then executes the test; therefore an undefined macro name is always defined to mean `\relax`.

7.2 Trigonometric functions

We now start with trigonometric functions. In previous versions of this package we defined the macros `\SinOf`, `\CosOf` and `\TanOf` (`\CotOf` does not appear so essential) by means of the parametric formulas that require the knowledge of the tangent of the half angle. We wanted, and still want, to specify the angles in sexagesimal degrees, not in radians, so that accurate reductions to the main quadrants are possible. The formulas are

$$\begin{aligned}\sin\theta &= \frac{2}{\cot x+\tan x}\\ \cos\theta &= \frac{\cot x-\tan x}{\cot x+\tan x}\\ \tan\theta &= \frac{2}{\cot x-\tan x}\end{aligned}$$

where

$$x \quad = \quad \theta/114.591559$$

is the half angle in degrees converted to radians.

But now, in this new version, the availability of the floating point computations with the specific L3 library makes all the above superfluous; actually the above approach gave good results but it was cumbersome and limited by the fixed radix computations of the TEX system programs.

Matter of facts, we compared the results (with 6 fractional digits) the computations executed with the `sind` function name, in order to use the angles in degrees, and a table of trigonometric functions with the same number of fractional digits, and we di not find any difference, not even one unit on the sixth decimal digit. Probably the `\fpeval` computations, without rounding before the sixteenth significant digit, are much more accurate, but it is useless to have a better accuracy when the other TEX and LATEX macros would not be able to exploit them.

Having available such powerful instrument, even the tangent appears to be of little use for the kind of computations that are supposed to be required in this package.

The codes for the computation of `\SinOf` and `\CosOf` of the angle in degrees is now therefore the following

```
\def\SinOf#1to#2{\edef#2{\fpeval{round(sind#1,6)}}}\relax
\def\CosOf#1to#2{\edef#2{\fpeval{round(cosd#1,6)}}}\relax
```

As of today the anomaly (angle) of a complex number may not be necessary, but it might become useful in the future; therefore with macro `\ArgOfVect` we calculate the four quadrant arctangent (in degrees) of the given vector taking into account the sings of the vector components. We use the `xfp atand` with two arguments, so that it automatically takes into account all the signs for determining the argument of vector x, y by giving the values x and y in the proper order to the function `atan`:

$$\text{if} \quad x + \mathrm{i}y = M\mathrm{e}^{\mathrm{i}\varphi} \quad \text{then} \quad \varphi = \texttt{\textbackslash fpeval}\{\texttt{atand}(y, x)\}$$

The `\ArgOfVect` macro receives on input a vector; from the signs of the horizontal and vertical components it determines the ratio and from this ratio the arctangent; but before doing this it tests the components in order to determine the quadrant of the vector tip; depending on signs it possibly adds what is necessary to determine the angle in the range $-180° < \varphi \leq +180°$. If both components are zero, the angle is undefined, but for what concerns `curve2e` it is assigned the angle $0^c irc$.

```
\def\ArgOfVect#1to#2{\GetCoord(#1){\t@X}{\t@Y}%
\fptest{\t@X=\z@ && \t@Y=\z@}{\edef#2{0}}{%
\edef#2{\fpeval{round(atand(\t@Y,\t@X),6)}}}\ignorespaces}
```

The anomaly of a null vector is meaningless, but we set it to zero in case that input data are wrong. Computations go on anyway, but the results my be worthless; such strange results are an indications that some controls on the code should be done.

It is worth examining the following table, where the angles of nine vectors $45°$ degrees apart from one another are computed from this macro.

Vector	0,0	1,0	1,1	0,1	-1,1	-1,0	-1,-1	0,-1	1,-1
Angle	0	0	45	90	135	180	-135	-90	-45

Real computations with the `\ArgOfVect` macro produce those very numbers without the need of rounding; `\fpeval` produces all trimming of lagging zeros and rounding by itself.

7.3 Arcs and curves preliminary information

We would like to define now a macro for drawing circular arcs of any radius and any angular aperture; the macro should require the arc center, the arc starting point and the angular aperture. The arc has its reference point in its center, therefore it does not need to be put in place by the command `\put`; nevertheless if `\put` is used, it may displace the arc into another position.

The command should have the following syntax:

`\Arc(`⟨*center*⟩`)(`⟨*starting point*⟩`){`⟨*angle*⟩`}`

which is totally equivalent to:

`\put(`⟨*center*⟩`){\Arc(0,0)(`⟨*starting point*⟩`){`⟨*angle*⟩`}}`

If the ⟨*angle*⟩, i.e. the arc angular aperture, is positive the arc runs counterclockwise from the starting point; clockwise if it is negative. Notice that since the ⟨*starting point*⟩ is relative to the ⟨*center*⟩ point, its polar coordinates are very convenient, since they become `(`⟨⟨*start angle*⟩`:`⟨*radius*⟩⟩`)`, where the ⟨*start angle*⟩ is relative to the arc center. Therefore you can think about a syntax such as this one:

`\Arc(`⟨⟨*center*⟩⟩`)(`⟨*start angle*`:`*radius*⟩`){`⟨*angle*⟩`}`

The difference between the `pict2e` `\arc` definition consists in a very different syntax:

`\arc[`⟨*start angle*⟩`,`⟨*end angle*⟩`]{`⟨*radius*⟩`}`

and the center is assumed to be at the coordinate established with a required `\put` command; moreover the difference in specifying angles is that ⟨*end angle*⟩ equals the sum of ⟨*start angle*⟩ and ⟨*angle*⟩. With the definition of this `curve2e` package use of a `\put` command is not prohibited, but it may be used for fine tuning the arc position by means of a simple displacement; moreover the ⟨*starting point*⟩ may be specified with polar coordinates (that are relative to the arc center).

It's necessary to determine the end point and the control points of the Bézier spline(s) that make up the circular arc.

The end point is obtained from the rotation of the starting point around the center; but the `pict2e` command `\pIIe@rotate` is such that the pivoting point appears to be non relocatable. It is therefore necessary to resort to low level TEX commands and the defined trigonometric functions and a set of macros that operate on complex numbers used as vector roto-amplification operators.

7.4 Complex number macros

In this package *complex number* is a vague phrase; it may be used in the mathematical sense of an ordered pair of real numbers; it can be viewed as a vector joining the origin of the coordinate axes to the coordinates indicated by the ordered pair; it can be interpreted as a roto-amplification operator that scales its operand and rotates it about a pivot point; besides the usual conventional representation used by the mathematicians where the ordered pair is enclosed in round parentheses (which is in perfect agreement with the standard code used by the `picture` environment) there is the other conventional representation used by the engineers that stresses the roto-amplification nature of a complex number:

$$(x, y) = x + \mathrm{j}y = M\mathrm{e}^{\mathrm{j}\theta}$$

Even the imaginary unit is indicated with i by the mathematicians and with j by the engineers. In spite of these differences, these objects, the *complex numbers*, are used without any problem by both mathematicians and engineers.

The important point is that these objects can be summed, subtracted, multiplied, divided, raised to any power (integer, fractional, positive or negative), be the argument of transcendental functions according to rules that are agreed upon by everybody. We do not need all these properties, but we need some and we must create the suitable macros for doing some of these operations.

In facts we need macros for summing, subtracting, multiplying, dividing complex numbers, for determining their directions (unit vectors or versors); a unit vector is the complex number divided by its magnitude so that the result is the cartesian or polar form of the Euler's formula

$$\mathrm{e}^{\mathrm{j}\phi} = \cos\phi + \mathrm{j}\sin\phi$$

The magnitude of a vector is determined by taking the square root of a function of the real and the imaginary parts; see further on.

It's better to represent each complex number with one control sequence; this implies frequent assembling and disassembling the pair of real numbers that make up a complex number. These real components are assembled into the defining control sequence as a couple of coordinates, i.e. two comma separated integer or fractional signed decimal numbers.

For assembling two real numbers into a complex number we use the following elementary macro:

```
\def\MakeVectorFrom#1#2to#3{\edef#3{#1,#2}\ignorespaces}%
```

Another elementary macro copies a complex number into another one:

```
\def\CopyVect#1to#2{\edef#2{#1}\ignorespaces}%
```

The magnitude is determined with the macro `\ModOfVect` with delimited arguments; as usual it is assumed that the results are retrieved by means of control sequences, not used directly.

In the preceding version of package `curve2e` the magnitude M was determined by taking the moduli of the real and imaginary parts, by changing their signs if

necessary; the larger component was then taken as the reference one, so that, if a is larger than b, the square root of the sum of their squares is computed as such:

$$M = \sqrt{a^2 + b^2} = |a|\sqrt{1 + (b/a)^2}$$

In this way the radicand never exceeds 2 and it was quite easy to get its square root by means of the Newton iterative process; due to the quadratic convergence, five iterations were more than sufficient. When one of the components was zero, the Newton iterative process was skipped. With the availability of the `xfp` package and its floating point algorithms it is much easier to compute the magnitude of a complex number; since these algorithms allow to use very large numbers, it is not necessary to normalise the complex number components to the largest one; therefore the code is much simpler than the one used for implementing the Newton method in the previous versions of this package.

```
\def\ModOfVect#1to#2{\GetCoord(#1)\t@X\t@Y
\edef#2{\fpeval{round(sqrt(\t@X*\t@X + \t@Y*\t@Y),6)}}%
\ignorespaces}%
```

Since the macro for determining the magnitude of a vector is available, we can now normalise the vector to its magnitude, therefore getting the Cartesian form of the direction vector. If by any chance the direction of the null vector is requested, the output is again the null vector, without normalisation.

```
\def\DirOfVect#1to#2{\GetCoord(#1)\t@X\t@Y
\ModOfVect#1to\@tempa
\fptest{\@tempa=\z@}{}{%
  \edef\t@X{\fpeval{round(\t@X/\@tempa,6)}}%
  \edef\t@Y{\fpeval{round(\t@Y/\@tempa,6)}}%
}\MakeVectorFrom\t@X\t@Y to#2\ignorespaces}%
```

A cumulative macro uses the above ones to determine with one call both the magnitude and the direction of a complex number. The first argument is the input complex number, the second its magnitude, and the third is again a complex number normalised to unit magnitude (unless the input was the null complex number); remember always that output quantities must be specified with control sequences to be used at a later time.

```
\def\ModAndDirOfVect#1to#2and#3{%
\ModOfVect#1to#2%
\DirOfVect#1to#3\ignorespaces}%
```

The next macro computes the magnitude and the direction of the difference of two complex numbers; the first input argument is the minuend, the second is the subtrahend; the output quantities are the third argument containing the magnitude of the difference and the fourth is the direction of the difference. The service macro `\SubVect` executes the difference of two complex numbers and is described further on.

```
\def\DistanceAndDirOfVect#1minus#2to#3and#4{%
\SubVect#2from#1to\@tempa
\ModAndDirOfVect\@tempa to#3and#4\ignorespaces}%
```

We now have two macros intended to fetch just the real or, respectively, the imaginary part of the input complex number.

```
\def\XpartOfVect#1to#2{%
\GetCoord(#1)#2\@tempa\ignorespaces}%
%
\def\YpartOfVect#1to#2{%
\GetCoord(#1)\@tempa#2\ignorespaces}%
```

With the next macro we create a direction vector (second argument) from a given angle (first argument, in degrees).

```
\def\DirFromAngle#1to#2{%
\edef\t@X{\fpeval{round(cosd#1,6)}}%
\edef\t@Y{\fpeval{round(sind#1,6)}}%
\MakeVectorFrom\t@X\t@Y to#2\ignorespaces}%
```

Sometimes it is necessary to scale a vector by an arbitrary real factor; this implies scaling both the real and imaginary part of the input given vector.

```
\def\ScaleVect#1by#2to#3{\GetCoord(#1)\t@X\t@Y
\edef\t@X{\fpeval{#2 * \t@X}}%
\edef\t@Y{\fpeval{#2 * \t@Y}}%
\MakeVectorFrom\t@X\t@Y to#3\ignorespaces}%
```

Again, sometimes it is necessary to reverse the direction of rotation; this implies changing the sign of the imaginary part of a given complex number; this operation produces the complex conjugate of the given number.

```
\def\ConjVect#1to#2{\GetCoord(#1)\t@X\t@Y
\edef\t@Y{-\t@Y}%
\MakeVectorFrom\t@X\t@Y to#2\ignorespaces}%
```

With all the low level elementary operations we can now proceed to the definitions of the binary operations on complex numbers. We start with the addition:

```
\def\AddVect#1and#2to#3{\GetCoord(#1)\tu@X\tu@Y
\GetCoord(#2)\td@X\td@Y
\edef\t@X{\fpeval{\tu@X + \td@X}}%
\edef\t@Y{\fpeval{\tu@Y + \td@Y}}%
\MakeVectorFrom\t@X\t@Y to#3\ignorespaces}%
```

Then the subtraction:

```
\def\SubVect#1from#2to#3{\GetCoord(#1)\tu@X\tu@Y
\GetCoord(#2)\td@X\td@Y
\edef\t@X{\fpeval{\td@X - \tu@X}}%
\edef\t@Y{\fpeval{\td@Y - \tu@Y}}%
\MakeVectorFrom\t@X\t@Y to#3\ignorespaces}%
```

For the multiplication we need to split the operation according to the fact that we want to multiply by the second operand or by the complex conjugate of the second operand; it would be nice if we could use the usual postfixed asterisk notation for the complex conjugate, but in the previous versions of this package we could not find a simple means for doing so. Therefore the previous version contained e definition of the `\MultVect` macro that followed a simple syntax with an optional asterisk *prefixed* to the second operand. Its syntax, therefore, allowed the following two forms:

`\MultVect`⟨*first factor*⟩ `by` ⟨*second factor*⟩ `to` ⟨*output macro*⟩
`\MultVect`⟨*first factor*⟩ `by` `*` ⟨*second factor*⟩ `to` ⟨*output macro*⟩

With the availability of the `xparse` package and its special argument descriptors for the arguments, we were able to define a different macro, `\Multvect`, with both optional positions for the asterisk: *after* and *before*; its syntax allows the following four forms:

`\Multvect`{⟨*first factor*⟩}{⟨*second factor*⟩}⟨*output macro*⟩ `\Multvect`{⟨*first factor*⟩}`*`{⟨*second factor*⟩}⟨*output macro*⟩ `\Multvect`{⟨*first factor*⟩}{⟨*second factor*⟩}`*`⟨*output macro*⟩ `\Multvect`{⟨*first factor*⟩}`*`{⟨*second factor*⟩}⟨*output macro*⟩ `\Multvect`{⟨*first factor*⟩}`*`{⟨*second factor*⟩}`*`⟨*output macro*⟩

Nevertheless we maintain a sort of interface between the old syntax and the new one, so that the two old forms can be mapped to two suitable forms of the new syntax. Old documents are still compilable; users who got used to the old syntax can maintain their habits.

First we define the new macro: it receives the three arguments, the first two as balanced texts; the last one must always be a macro, therefore a single (complex) token and doe not require braces, even if it is not forbidden to use them. Asterisks are optional. The input arguments are transformed into couples of anomaly and modulus; this makes multiplication much simpler as the output modulus is just the product of the input moduli, while the output anomaly is just the sum of input anomalies; eventually it is necessary to transform this polar version of the result into an ordered couple of cartesian values to be assigned to the output macro. In order to maintain the single macros pretty simple we need a couple of service macros and a named counter. We use `\ModOfVect` previously defined, and a new macro `\ModAndAngleOfVect` with the following syntax:

`\ModAndAngleOfVect`⟨*input vector*⟩ `to` ⟨*output modulus*⟩ `and` ⟨*output angle in degrees*⟩

The output quantities are always macros, so they do not need balanced bracing; angles in degrees are always preferred because, on case of necessity, they are easy to reduce to the range $-180° < \alpha \leq +180°$.

```
\def\ModAndAngleOfVect#1to#2and#3{\ModOfVect#1to#2\relax
\ArgOfVect#1to#3\ignorespaces}
```

We name a counter in the upper range accessible with all the modern three typesetting engines, `pdfLaTeX`, `LuaLaTeX` and `XeLaTeX`.

```
\countdef\MV@C=2560\relax
```

The user is warned; The counter register number is sort of casual, but it is not excluded that its name or number get in conflict with other macros from other packages. I would be grateful is such an event takes place.

Nov comes the real macro[5]:

[5] A warm thank-you to Enrico Gregorio, who kindly attracted my attention on the necessity of braces when using this kind of macro; being used to the syntax with delimited arguments I had taken the bad habit of avoiding braces. Braces are very important, but the syntax of the original TEX language, that did not have available the L3 one, spoiled me with the abuse of delimited arguments.

```
\NewDocumentCommand\Multvect{m s m s m}{%
\MV@C=0
\ModAndAngleOfVect#1to\MV@uM and\MV@uA
\ModAndAngleOfVect#3to\MV@dM and\MV@dA
\IfBooleanT{#2}{\MV@C=1}\relax
\IfBooleanT{#4}{\MV@C=1}\relax
\unless\ifnum\MV@C=0\edef\MV@dA{-\MV@dA}\fi
\edef\MV@rM{\fpeval{round((\MV@uM * \MV@dM),6)}}%
\edef\MV@rA{\fpeval{round((\MV@uA + \MV@dA),6)}}%
\GetCoord(\MV@rA:\MV@rM)\t@X\t@Y
\MakeVectorFrom\t@X\t@Y to#5}
```

The macro to remain backward compatible, reduce to two simple macros that take the input delimited arguments and passes them in braced form to the above general macro:

```
\def\MultVect#1by{\@ifstar{\let\MV@c\@ne\@MultVect#1by}%
  {\let\MV@c\empty\@MultVect#1by}}

\def\@MultVect#1by#2to#3{%
  \unless\ifx\MV@c\empty\Multvect{#1}{#2}*{#3}\else
    \Multvect{#1}{#2}{#3}\fi}
```

Testing of both the new and the old macros show that they behave as expected, although, using real numbers for trigonometric functions, some small rounding unit on the sixth decimal digit still remain; nothing to worry about with a package used for drawing.

The division of two complex numbers implies scaling down the dividend by the magnitude of the divisor and by rotating the dividend scaled vector by the conjugate versor of the divisor:

$$\frac{\vec{N}}{\vec{D}} = \frac{\vec{N}}{M\vec{u}} = \frac{\vec{N}}{M}\vec{u}^{\star}$$

therefore:

```
\def\DivVect#1by#2to#3{\Divvect{#1}{#2}{#3}}

\NewDocumentCommand\Divvect{ m m m }{%
\ModAndDirOfVect#2to\@Mod and\@Dir
\edef\@Mod{\fpeval{1 / \@Mod}}%
\ConjVect\@Dir to\@Dir
\ScaleVect#1by\@Mod to\@tempa
\Multvect{\@tempa}{\@Dir}#3\ignorespaces}%
```

Macros `\DivVect` and `\Divvect` are almost equivalent; the second is possibly slightly more robust. They match the corresponding macros for multiplying two vectors.

7.5 Arcs and curved vectors

We are now in the position of really doing graphic work.

7.5.1 Arcs

We start with tracing a circular arc of arbitrary center, arbitrary starting point and arbitrary aperture; the first macro checks the aperture; if this is not zero it actually proceeds with the necessary computations, otherwise it does nothing.

```
\def\Arc(#1)(#2)#3{\begingroup
\@tdA=#3\p@
\unless\ifdim\@tdA=\z@
  \@Arc(#1)(#2)%
\fi
\endgroup\ignorespaces}%
```

The aperture is already memorised in `\@tdA`; the `\@Arc` macro receives the center coordinates in the first argument and the coordinates of the starting point in the second argument.

```
\def\@Arc(#1)(#2){%
\ifdim\@tdA>\z@
  \let\Segno+%
\else
  \@tdA=-\@tdA \let\Segno-%
\fi
```

The rotation angle sign is memorised in `\Segno` and `\@tdA` now contains the absolute value of the arc aperture.

If the rotation angle is larger than 360° a message is issued that informs the user that the angle will be reduced modulo 360°; this operation is performed by successive subtractions rather than with modular arithmetics on the assumption that in general one subtraction suffices.

```
\Numero\@gradi\@tdA
\ifdim\@tdA>360\p@
 \PackageWarning{curve2e}{The arc aperture is \@gradi\space degrees
      and gets reduced\MessageBreak%
      to the range 0--360 taking the sign into consideration}%
 \@whiledim\@tdA>360\p@\do{\advance\@tdA-360\p@}%
\fi
```

Now the radius is determined and the drawing point is moved to the starting point.

```
\SubVect#2from#1to\@V \ModOfVect\@V to\@Raggio
\CopyVect#2to\@pPun
\CopyVect#1to\@Cent \GetCoord(\@pPun)\@pPunX\@pPunY
```

From now on it's better to define a new macro that will be used also in the subsequent macros that draw arcs; here we already have the starting point coordinates and the angle to draw the arc, therefore we just call the new macro, stroke the line and exit.

```
\@@Arc\strokepath\ignorespaces}%
```

And the new macro `\@@Arc` starts with moving the drawing point to the first point and does everything needed for drawing the requested arc, except stroking it; we leave the `stroke` command to the completion of the calling macro and nobody forbids to use the `\@@Arc` macro for other purposes.

```
\def\@@Arc{%
\pIIe@moveto{\@pPunX\unitlength}{\@pPunY\unitlength}%
```

If the aperture is larger than 180° it traces a semicircle in the right direction and correspondingly reduces the overall aperture.

```
\ifdim\@tdA>180\p@
  \advance\@tdA-180\p@
  \Numero\@gradi\@tdA
  \SubVect\@pPun from\@Cent to\@V
  \AddVect\@V and\@Cent to\@sPun
  \Multvect{\@V}{0,-1.3333333to}\@V
  \if\Segno-\ScaleVect\@V by-1to\@V\fi
  \AddVect\@pPun and\@V to\@pcPun
  \AddVect\@sPun and\@V to\@scPun
  \GetCoord(\@pcPun)\@pcPunX\@pcPunY
  \GetCoord(\@scPun)\@scPunX\@scPunY
  \GetCoord(\@sPun)\@sPunX\@sPunY
  \pIIe@curveto{\@pcPunX\unitlength}{\@pcPunY\unitlength}%
               {\@scPunX\unitlength}{\@scPunY\unitlength}%
               {\@sPunX\unitlength}{\@sPunY\unitlength}%
  \CopyVect\@sPun to\@pPun
\fi
```

If the remaining aperture is not zero it continues tracing the rest of the arc. Here we need the extrema of the arc and the coordinates of the control points of the Bézier cubic spline that traces the arc. The control points lay on the perpendicular to the vectors that join the arc center to the starting and end points respectively.

With reference to figure **??** on page **??**, the points P_1 and P_2 are the arc end-points; C_1 and C_2 are the Bézier-spline control-points; P is the arc mid-point, that should be distant from the center of the arc the same as P_1 and P_2. Choosing a convenient orientation of the arc relative to the coordinate axes, the coordinates of these five points are:

$$\begin{aligned} P_1 &= (-R\sin\theta, 0) \\ P_2 &= (R\sin\theta, 0) \\ C_1 &= (-R\sin\theta + K\cos\theta, K\sin\theta) \\ C_2 &= (R\sin\theta - K\cos\theta, K\sin\theta) \\ P &= (0, R(1-\cos\theta)) \end{aligned}$$

The Bézier cubic spline interpolating the end and mid points is given by the parametric equation:

$$P = P_1(1-t)^3 + C_1 3(1-t)^2 t + C_2 3(1-t)t^2 + P_2 t^3$$

where the mid point is obtained for $t = 0.5$; the four coefficients then become $1/8, 3/8, 3/8, 1/8$ and the only unknown remains K. Solving for K we obtain the formula

$$K = \frac{4}{3}\,\frac{1-\cos\theta}{\sin\theta}R = \frac{4}{3}\,\frac{1-\cos\theta}{\sin^2\theta}s \tag{1}$$

where θ is half the arc aperture, R is its radius, and s is half the arc chord.

```
\ifdim\@tdA>\z@
  \DirFromAngle\@gradi to\@Dir \if\Segno-\ConjVect\@Dir to\@Dir \fi
  \SubVect\@Cent from\@pPun to\@V
  \Multvect{\@V}{\@Dir}\@V
  \AddVect\@Cent and\@V to\@sPun
  \@tdA=.5\@tdA \Numero\@gradi\@tdA
  \DirFromAngle\@gradi to\@Phimezzi
  \GetCoord(\@Phimezzi)\@cosphimezzi\@sinphimezzi
  \@tdB=1.3333333\p@ \@tdB=\@Raggio\@tdB
  \@tdC=\p@ \advance\@tdC -\@cosphimezzi\p@ \Numero\@tempa\@tdC
  \@tdB=\@tempa\@tdB
  \DividE\@tdB by\@sinphimezzi\p@ to\@cZ
  \ScaleVect\@Phimezzi by\@cZ to\@Phimezzi
  \ConjVect\@Phimezzi to\@mPhimezzi
  \if\Segno-%
    \let\@tempa\@Phimezzi
    \let\@Phimezzi\@mPhimezzi
    \let\@mPhimezzi\@tempa
  \fi
  \SubVect\@sPun from\@pPun to\@V
  \DirOfVect\@V to\@V
  \Multvect{\@Phimezzi}{\@V}\@Phimezzi
  \AddVect\@sPun and\@Phimezzi to\@scPun
  \ScaleVect\@V by-1to\@V
  \Multvect{\@mPhimezzi}{\@V}\@mPhimezzi
  \AddVect\@pPun and\@mPhimezzi to\@pcPun
  \GetCoord(\@pcPun)\@pcPunX\@pcPunY
  \GetCoord(\@scPun)\@scPunX\@scPunY
  \GetCoord(\@sPun)\@sPunX\@sPunY
  \pIIe@curveto{\@pcPunX\unitlength}{\@pcPunY\unitlength}%
               {\@scPunX\unitlength}{\@scPunY\unitlength}%
               {\@sPunX\unitlength}{\@sPunY\unitlength}%
\fi}
```

7.5.2 Arc vectors

We exploit much of the above definitions for the `\Arc` macro for drawing circular arcs with an arrow at one or both ends; the first macro `\VerctorArc` draws an arrow at the ending point of the arc; the second macro `\VectorARC` draws arrows at both ends; the arrows have the same shape as those for vectors; actually they are drawn by putting a vector of zero length at the proper arc end(s), therefore they are styled as traditional LaTeX or PostScript arrows according to the specific option to the `pict2e` package.

But the arc drawing done here shortens it so as not to overlap on the arrow(s); the only arrow (or both ones) are also lightly tilted in order to avoid the impression of a corner where the arc enters the arrow tip.

All these operations require a lot of "playing" with vector directions, but even

if the operations are numerous, they do not do anything else but: (a) determining the end point and its direction; (b) determining the arrow length as an angular quantity, i.e. the arc amplitude that must be subtracted from the total arc to be drawn; (c) the direction of the arrow should be corresponding to the tangent to the arc at the point where the arrow tip is attached; (d) tilting the arrow tip by half its angular amplitude; (e) determining the resulting position and direction of the arrow tip so as to draw a zero length vector; (f) possibly repeating the same procedure for the other end of the arc; (g) shortening the total arc angular amplitude by the amount of the arrow tip(s) already set, and finally (h) drawing the circular arc that joins the starting point to the final arrow or one arrow to the other one.

The calling macros are very similar to the `\Arc` macro initial one:

```
\def\VectorArc(#1)(#2)#3{\begingroup
\@tdA=#3\p@ \ifdim\@tdA=\z@\else
  \@VArc(#1)(#2)%
\fi
\endgroup\ignorespaces}%
%
\def\VectorARC(#1)(#2)#3{\begingroup
\@tdA=#3\p@
\ifdim\@tdA=\z@\else
  \@VARC(#1)(#2)%
\fi
\endgroup\ignorespaces}%
```

The single arrowed arc is defined with the following long macro where all the described operations are performed more or less in the described succession; probably the macro requires a little cleaning, but since it works fine we did not try to optimise it for time or number of tokens. The final part of the macro is almost identical to that of the plain arc; the beginning also is quite similar. The central part is dedicated to the positioning of the arrow tip and to the necessary calculations for determining the tip tilt and the reduction of the total arc length; pay attention that the arrow length, stored in `\@tdE` is a real length, while the radius stored in `\@Raggio` is just a multiple of the `\unitlength`, so that the division (that yields a good angular approximation to the arrow length as seen from the center of the arc) must be done with real lengths. The already defined `\@@Arc` macro actually draws the curved vector stem without stroking it.

```
\def\@VArc(#1)(#2){%
\ifdim\@tdA>\z@
  \let\Segno+%
\else
  \@tdA=-\@tdA \let\Segno-%
\fi \Numero\@gradi\@tdA
\ifdim\@tdA>360\p@
 \PackageWarning{curve2e}{The arc aperture is \@gradi\space degrees
      and gets reduced\MessageBreak%
      to the range 0--360 taking the sign into consideration}%
 \@whiledim\@tdA>360\p@\do{\advance\@tdA-360\p@}%
```

```
\fi
\SubVect#1from#2to\@V \ModOfVect\@V to\@Raggio \CopyVect#2to\@pPun
\@tdE=\pIIe@FAW\@wholewidth \@tdE=\pIIe@FAL\@tdE
\DividE\@tdE by \@Raggio\unitlength to\DeltaGradi
\@tdD=\DeltaGradi\p@
\@tdD=57.29578\@tdD \Numero\DeltaGradi\@tdD
\@tdD=\ifx\Segno--\fi\@gradi\p@ \Numero\@tempa\@tdD
\DirFromAngle\@tempa to\@Dir
\Multvect{\@V}{\@Dir}\@sPun
\edef\@tempA{\ifx\Segno-\m@ne\else\@ne\fi}%
\Multvect{\@sPun}{0,\@tempA}\@vPun
\DirOfVect\@vPun to\@Dir
\AddVect\@sPun and #1 to \@sPun
\GetCoord(\@sPun)\@tdX\@tdY
\@tdD\ifx\Segno--\fi\DeltaGradi\p@
\@tdD=.5\@tdD \Numero\DeltaGradi\@tdD
\DirFromAngle\DeltaGradi to\@Dird
\Multvect{\@Dir}*{\@Dird}\@Dir%
\GetCoord(\@Dir)\@xnum\@ynum
\put(\@tdX,\@tdY){\vector(\@xnum,\@ynum){0}}%
\@tdE =\ifx\Segno--\fi\DeltaGradi\p@
\advance\@tdA -\@tdE \Numero\@gradi\@tdA
\CopyVect#1to\@Cent \GetCoord(\@pPun)\@pPunX\@pPunY
\@@Arc
\strokepath\ignorespaces}%
```

The macro for the arc terminated with arrow tips at both ends is again very similar, except it is necessary to repeat the arrow tip positioning also at the starting point. The `\@@Arc` macro draws the curved stem.

```
\def\@VARC(#1)(#2){%
\ifdim\@tdA>\z@
  \let\Segno+%
\else
  \@tdA=-\@tdA \let\Segno-%
\fi \Numero\@gradi\@tdA
\ifdim\@tdA>360\p@
 \PackageWarning{curve2e}{The arc aperture is \@gradi\space degrees
      and gets reduced\MessageBreak%
      to the range 0--360 taking the sign into consideration}%
 \@whiledim\@tdA>360\p@\do{\advance\@tdA-360\p@}%
\fi
\SubVect#1from#2to\@V \ModOfVect\@V to\@Raggio \CopyVect#2to\@pPun
\@tdE=\pIIe@FAW\@wholewidth \@tdE=0.8\@tdE
\DividE\@tdE by \@Raggio\unitlength to\DeltaGradi
\@tdD=\DeltaGradi\p@ \@tdD=57.29578\@tdD \Numero\DeltaGradi\@tdD
\@tdD=\if\Segno--\fi\@gradi\p@ \Numero\@tempa\@tdD
\DirFromAngle\@tempa to\@Dir
\Multvect{\@V}{\@Dir}\@sPun% corrects the end point
\edef\@tempA{\if\Segno--\fi1}%
\Multvect{\@sPun}{0,\@tempA}\@vPun
```

```
\DirOfVect\@vPun to\@Dir
\AddVect\@sPun and #1 to \@sPun
\GetCoord(\@sPun)\@tdX\@tdY
\@tdD\if\Segno--\fi\DeltaGradi\p@
\@tdD=.5\@tdD \Numero\@tempB\@tdD
\DirFromAngle\@tempB to\@Dird
\Multvect{\@Dir}*{\@Dird}\@Dir
\GetCoord(\@Dir)\@xnum\@ynum
\put(\@tdX,\@tdY){\vector(\@xnum,\@ynum){0}}% end point arrow tip
\@tdE =\DeltaGradi\p@
\advance\@tdA -2\@tdE \Numero\@gradi\@tdA
\CopyVect#1to\@Cent \GetCoord(\@pPun)\@pPunX\@pPunY
\SubVect\@Cent from\@pPun to \@V
\edef\@tempa{\if\Segno-\else-\fi\@ne}%
\Multvect{\@V}{0,\@tempa}\@vPun
\@tdE\if\Segno--\fi\DeltaGradi\p@
\Numero\@tempB{0.5\@tdE}%
\DirFromAngle\@tempB to\@Dird
\Multvect{\@vPun}{\@Dird}\@vPun% corrects the starting point
\DirOfVect\@vPun to\@Dir\GetCoord(\@Dir)\@xnum\@ynum
\put(\@pPunX,\@pPunY){\vector(\@xnum,\@ynum){0}}% starting point arrow tip
\edef\@tempa{\if\Segno--\fi\DeltaGradi}%
\DirFromAngle\@tempa to \@Dir
\SubVect\@Cent from\@pPun to\@V
\Multvect{\@V}{\@Dir}\@V
\AddVect\@Cent and\@V to\@pPun
\GetCoord(\@pPun)\@pPunX\@pPunY
\@@Arc
\strokepath\ignorespaces}%
```

It must be understood that the curved vectors, the above circular arcs terminated with an arrow tip at one or both ends, have a nice appearance only if the arc radius is not too small, or, said in a different way, if the arrow tip angular width does not exceed a maximum of a dozen degrees (and this is probably already too much); the tip does not get curved as the arc is, therefore there is not a smooth transition from the curved stem and the straight arrow tip if this one is large in comparison to the arc radius.

7.6 General curves

The most used method to draw curved lines with computer programs is to connect several simple curved lines, general "arcs", one to another generally maintaining the same tangent at the junction. If the direction changes we are dealing with a cusp.

The simple general arcs that are directly implemented in every program that display typeset documents, are those drawn with the parametric curves called *Bézier splines*; given a sequence of points in the x, y plane, say $P_0, P_1, P_2, p_3, \ldots$ (represented as coordinate pairs, i.e. by complex numbers), the most common

Bézier splines are the following ones:

$$\mathcal{B}_1 = P_0(1-t) + P_1 t \tag{2}$$

$$\mathcal{B}_2 = P_0(1-t)^2 + P_1 2(1-t)t + P_2 t^2 \tag{3}$$

$$\mathcal{B}_3 = P_0(1-t)^3 + P_1 3(1-t)^2 t + P_2 3(1-t)t^2 + P_3 t^3 \tag{4}$$

All these splines depend on parameter t; they have the property that for $t = 0$ each line starts at the first point, while for $t = 1$ they reach the last point; in each case the generic point P on each curve takes off with a direction that points to the next point, while it lands on the destination point with a direction coming from the penultimate point; moreover, when t varies from 0 to 1, the curve arc is completely contained within the convex hull formed by the polygon that has the spline points as vertices.

Last but not least first order splines implement just straight lines and they are out of question for what concerns maxima, minima, inflection points and the like. Quadratic splines draw just parabolas, therefore they draw arcs that have the concavity just on one side of the path; therefore no inflection points. Cubic splines are extremely versatile and can draw lines with maxima, minima and inflection points. Virtually a multi-arc curve may be drawn by a set of cubic splines as well as a set of quadratic splines (fonts are a good example: Adobe Type 1 fonts have their contours described by cubic splines, while TrueType fonts have their contours described with quadratic splines; at naked eye it is impossible to notice the difference).

Each program that processes the file to be displayed is capable of drawing first order Bézier splines (segments) and third order Bézier splines, for no other reason, at least, because they have to draw vector fonts whose contours are described by Bézier splines; sometimes they have also the program commands to draw second order Bézier splines, but not always these machine code routines are available to the user for general use. For what concerns `pdftex`, `xetex` and `luatex`, they have the user commands for straight lines and cubic arcs. At least with `pdftex`, quadratic arcs must be simulated with a clever use of third order Bézier splines.

Notice that LaTeX 2_ε environment `picture` by itself is capable of drawing both cubic and quadratic Bézier splines as single arcs; but it resorts to "poor man" solutions. The `pict2e` package removes all the old limitations and implements the interface macros for sending the driver the necessary drawing information, including the transformation from typographical points (72.27 pt/inch) to PostScript big points (72 bp/inch). But for what concerns the quadratic spline it resorts to the clever use of a cubic spline.

Therefore here we treat first the drawings that can be made with cubic splines; then we describe the approach to quadratic splines.

7.7 Cubic splines

Now we define a macro for tracing a general, not necessarily circular, arc. This macro resorts to a general triplet of macros with which it is possible to draw almost anything. It traces a single Bézier spline from a first point where the

tangent direction is specified to a second point where again it is specified the tangent direction. Actually this is a special (possibly useless) case where the general `\curve` macro of `pict2e` could do the same or a better job. In any case...

```
\def\CurveBetween#1and#2WithDirs#3and#4{%
  \StartCurveAt#1WithDir{#3}\relax
  \CurveTo#2WithDir{#4}\CurveFinish\ignorespaces
}%
```

Actually the above macro is a special case of concatenation of the triplet formed by macros `\StartCurve`, `\CurveTo` and`\CurveFinish`; the second macro can be repeated an arbitrary number of times. In any case the directions specified with the direction arguments, both here and with the more general macro`\Curve`, the angle between the indicated tangent and the arc chord may give raise to some little problems when they are very close to 90° in absolute value. Some control is exercised on these values, but some tests might fail if the angle derives from computations; this is a good place to use polar forms for the direction vectors.

The first macro initialises the drawing and the third one strokes it; the real work is done by the second macro. The first macro initialises the drawing but also memorises the starting direction; the second macro traces the current Bézier arc reaching the destination point with the specified direction, but memorises this direction as the one with which to start the next arc. The overall curve is then always smooth because the various Bézier arcs join with continuous tangents. If a cusp is desired it is necessary to change the memorised direction at the end of the arc before the cusp and before the start of the next arc; this is better than stroking the curve before the cusp and then starting another curve, because the curve joining point at the cusp is not stroked with the same command, therefore we get two superimposed curve terminations. We therefore need another small macro `\ChangeDir` to perform this task.

It is necessary to recall that the direction vectors point to the control points, but they do not define the control points themselves; they are just directions, or, even better, they are simply vectors with the desired direction; the macros themselves provide to the normalisation and memorisation.

The next desirable feature would be to design a macro that accepts optional node directions and computes the missing ones according to a suitable strategy. We can think of many such strategies, but none seems to be generally applicable, in the sense that one strategy might give good results, say, with sinusoids and another one, say, with cardioids, but neither one is suitable for both cases.

For the moment we refrain from automatic direction computation, but we design the general macro as if directions were optional.

Here we begin with the first initialising macro that receives with the first argument the starting point and with the second argument the direction of the tangent (not necessarily normalised to a unit vector)

```
\def\StartCurveAt#1WithDir#2{%
\begingroup
\GetCoord(#1)\@tempa\@tempb
\CopyVect\@tempa,\@tempb to\@Pzero
\pIIe@moveto{\@tempa\unitlength}{\@tempb\unitlength}%
```

```
\GetCoord(#2)\@tempa\@tempb
\CopyVect\@tempa,\@tempb to\@Dzero
\DirOfVect\@Dzero to\@Dzero
\ignorespaces}
```

And this re-initialises the direction to create a cusp:

```
\def\ChangeDir<#1>{%
\GetCoord(#1)\@tempa\@tempb
\CopyVect\@tempa,\@tempb to\@Dzero
\DirOfVect\@Dzero to\@Dzero
\ignorespaces}
```

The next macros are the finishing ones; the first strokes the whole curve, while the second fills the (closed) curve with the default color; both close the group that was opened with `\StartCurve`. The third macro is explained in a while; we anticipate it is functional to chose between the first two macros when a star is possibly used to switch between stroking and filling.

```
\def\CurveFinish{\strokepath\endgroup\ignorespaces}%
\def\FillCurve{\fillpath\endgroup\ignorespaces}
\def\CurveEnd{\fillstroke\endgroup\ignorespaces}
```

In order to draw the internal arcs it would be desirable to have a single macro that, given the destination point, computes the control points that produce a cubic Bézier spline that joins the starting point with the destination point in the best possible way. The problem is strongly ill defined and has an infinity of solutions; here we give two solutions: (*a*) a supposedly smart one that resorts to osculating circles and requires only the direction at the destination point; and (*b*) a less smart solution that requires the control points to be specified in a certain format.

We start with solution (*b*), `\CbezierTo`, the code of which is simpler than that of solution (*a*); then we will produce the solution (*a*), `\CurveTo`, that will become the main building block for a general path construction macro, `\Curve`.

The "naïve" macro `\CBezierTo` simply uses the previous point direction saved in `\@Dzero` as a unit vector by the starting macro; specifies a destination point, the distance of the first control point from the starting point, the destination point direction that will save also for the next arc-drawing macro as a unit vector, and the distance of the second control point from the destination point along this last direction. Both distances must be positive possibly fractional numbers. The syntax therefore is the following:

`\CbezierTo`⟨*end point*⟩`WithDir`⟨*direction*⟩`AndDists`⟨K_0⟩`And`⟨K_1⟩

where ⟨*end point*⟩ is a vector macro or a comma separated pair of values; again ⟨*direction*⟩ is another vector macro or a comma separated pair of values, that not necessarily indicate a unit vector, since the macro provides to normalise it to unity; ⟨K_0⟩ and ⟨K_1⟩ are the distances of the control points from their respective node points; they must be positive integers or fractional numbers. If ⟨K_1⟩ is a number must be enclosed in curly braces, while if it is a macro name (containing the desired fractional or integer value) there is no need for braces.

This macro uses the input information to use the internal `pict2e` macro `\pIIe@curveto` with the proper arguments, and to save the final direction into the same `\@Dzero` macro for successive use of other arc-drawing macros.

```
\def\CbezierTo#1WithDir#2AndDists#3And#4{%
\GetCoord(#1)\@tX\@tY \MakeVectorFrom\@tX\@tY to\@Puno
\GetCoord(#2)\@tX\@tY \MakeVectorFrom\@tX\@tY to \@Duno
\DirOfVect\@Duno to\@Duno
\ScaleVect\@Dzero by#3to\@Czero \AddVect\@Pzero and\@Czero to\@Czero
\ScaleVect\@Duno by-#4to \@Cuno \AddVect\@Puno and\@Cuno to \@Cuno
\GetCoord(\@Czero)\@XCzero\@YCzero
\GetCoord(\@Cuno)\@XCuno\@YCuno
\GetCoord(\@Puno)\@XPuno\@YPuno
\pIIe@curveto{\@XCzero\unitlength}{\@YCzero\unitlength}%
              {\@XCuno\unitlength}{\@YCuno\unitlength}%
              {\@XPuno\unitlength}{\@YPuno\unitlength}%
\CopyVect\@Puno to\@Pzero
\CopyVect\@Duno to\@Dzero
\ignorespaces}%
```

With this building block it is not difficult to set up a macro that draws a Bézier arc between two given points, similarly to the other macro `\CurveBetween` previously described and defined here:

```
\def\CbezierBetween#1And#2WithDirs#3And#4UsingDists#5And#6{%
\StartCurveAt#1WithDir{#3}\relax
\CbezierTo#2WithDir#4AndDists#5And{#6}\CurveFinish}
```

An example of use is shown in figure **??** on page **??**; notice that the tangents at the end points are the same for the black curve drawn with `\CurveBetween` and the five red curves drawn with `\CbezierBetween`; the five red curves differ only for the distance of their control point C_0 from the starting point; the differences are remarkable and the topmost curve even presents a slight inflection close to the end point. These effects cannot be obtained with the "smarter" macro `\CurveBetween`. But certainly this simpler macro is more difficult to use because the distances of the control points are difficult to estimate and require a number of cut-and-try experiments.

The "smarter" curve macro comes next; it is supposed to determine the control points for joining the previous point (initial node) with the specified direction to the next point with another specified direction (final node). Since the control points are along the specified directions, it is necessary to determine the distances from the adjacent curve nodes. This must work correctly even if nodes and directions imply an inflection point somewhere along the arc.

The strategy we devised consists in determining each control point as if it were the control point of a circular arc, precisely an arc of an osculating circle, i.e. a circle tangent to the curve at that node. The ambiguity of the stated problem may be solved by establishing that the chord of the osculating circle has the same direction as the chord of the arc being drawn, and that the curve chord is divided into two equal parts each of which should be interpreted as half the chord of the osculating circle.

This makes the algorithm a little rigid; sometimes the path drawn is very pleasant, while in other circumstances the determined curvatures are too large or too small. We therefore add some optional information that lets us have some control over the curvatures; the idea is based on the concept of *tension*, similar but not identical to the one used in the drawing programs METAFONT and METAPOST. We add to the direction information, with which the control nodes of the osculating circle arcs are determined, a scaling factor that should be intuitively related to the tension of the arc (actually, since the tension of the 'rope' is high when this parameter is low, probably a name such as 'looseness' would be better suited): the smaller this number, the closer the arc resembles a straight line as a rope subjected to a high tension; value zero is allowed, while a value of 4 is close to "infinity" and turns a quarter circle into a line with an unusual loop; a value of 2 turns a quarter circle almost into a polygonal line with rounded vertices . Therefore these tension factors should be used only for fine tuning the arcs, not when a path is drawn for the first time.

We devised a syntax for specifying direction and tensions:

⟨*direction*; *tension factors*⟩

where *direction* contains a pair of fractional number that not necessarily refer to the components of a unit vector direction, but simply to a vector with the desired orientation (polar form is OK); the information contained from the semicolon (included) to the rest of the specification is optional; if it is present, the *tension factors* is simply a comma separated pair of fractional or integer numbers that represent respectively the tension at the starting or the ending node of a path arc.

We therefore need a macro to extract the mandatory and optional parts:

```
\def\@isTension#1;#2!!{\def\@tempA{#1}%
\def\@tempB{#2}\unless\ifx\@tempB\empty\strip@semicolon#2\fi}

\def\strip@semicolon#1;{\def\@tempB{#1}}
```

By changing the tension values we can achieve different results: see figure **??** on page **??**.

We use the formula we got for arcs (**??**), where the half chord is indicated with s, and we derive the necessary distances:

$$K_0 = \frac{4}{3} s \frac{1 - \cos\theta_0}{\sin^2\theta_0} \tag{5a}$$

$$K_1 = \frac{4}{3} s \frac{1 - \cos\theta_1}{\sin^2\theta_1} \tag{5b}$$

We therefore start with getting the points and directions and calculating the chord and its direction:

```
\def\CurveTo#1WithDir#2{%
\def\@Tuno{1}\def\@Tzero{1}\relax
\edef\@Puno{#1}\@isTension#2;!!%
\expandafter\DirOfVect\@tempA to\@Duno
\bgroup\unless\ifx\@tempB\empty\GetCoord(\@tempB)\@Tzero\@Tuno\fi
\DistanceAndDirOfVect\@Puno minus\@Pzero to\@Chord and\@DirChord
```

Then we rotate everything about the starting point so as to bring the chord on the real axis

```
\Multvect{\@Dzero}*{\@DirChord}\@Dpzero
\Multvect{\@Duno}*{\@DirChord}\@Dpuno
\GetCoord(\@Dpzero)\@DXpzero\@DYpzero
\GetCoord(\@Dpuno)\@DXpuno\@DYpuno
\DivideFN\@Chord by2 to\@semichord
```

The chord needs not be actually rotated because it suffices its length along the real axis; the chord length is memorised in `\@Chord` and its half is saved in `\@semichord`.

We now examine the various degenerate cases, when either tangent is perpendicular or parallel to the chord. Notice that we are calculating the distances of the control points from the adjacent nodes using the half chord length, not the full length. We also distinguish between the computations relative to the arc starting point and those relative to the end point.

```
\ifdim\@DXpzero\p@=\z@
   \@tdA=1.333333\p@
   \Numero\@KCzero{\@semichord\@tdA}%
\fi
\ifdim\@DYpzero\p@=\z@
  \@tdA=1.333333\p@
  \Numero\@Kpzero{\@semichord\@tdA}%
\fi
```

The distances we are looking for are positive generally fractional numbers; so if the components are negative, we take the absolute values. Eventually we determine the absolute control point coordinates.

```
\unless\ifdim\@DXpzero\p@=\z@
  \unless\ifdim\@DYpzero\p@=\z@
    \edef\@CosDzero{\ifdim\@DXpzero\p@<\z@ -\fi\@DXpzero}%
    \edef\@SinDzero{\ifdim\@DYpzero\p@<\z@ -\fi\@DYpzero}%
    \@tdA=\@semichord\p@ \@tdA=1.333333\@tdA
    \DividE\@tdA by\@SinDzero\p@ to \@KCzero
    \@tdA=\dimexpr(\p@-\@CosDzero\p@)\relax
    \DividE\@KCzero\@tdA by\@SinDzero\p@ to \@KCzero
  \fi
\fi
\MultiplyFN\@KCzero by \@Tzero to \@KCzero
\ScaleVect\@Dzero by\@KCzero to\@CPzero
\AddVect\@Pzero and\@CPzero to\@CPzero
```

We now repeat the calculations for the arc end point, taking into consideration that the end point direction points outwards, so that in computing the end point control point we have to take this fact into consideration by using a negative sign for the distance; in this way the displacement of the control point from the end point takes place in a backwards direction.

```
\ifdim\@DXpuno\p@=\z@
   \@tdA=-1.333333\p@
```

```
   \Numero\@KCuno{\@semichord\@tdA}%
\fi
\ifdim\@DYpuno\p@=\z@
  \@tdA=-1.333333\p@
  \Numero\@KCuno{\@semichord\@tdA}%
\fi
\unless\ifdim\@DXpuno\p@=\z@
    \unless\ifdim\@DYpuno\p@=\z@
    \edef\@CosDuno{\ifdim\@DXpuno\p@<\z@ -\fi\@DXpuno}%
    \edef\@SinDuno{\ifdim\@DYpuno\p@<\z@ -\fi\@DYpuno}%
    \@tdA=\@semichord\p@ \@tdA=-1.333333\@tdA
    \DividE\@tdA by \@SinDuno\p@ to \@KCuno
    \@tdA=\dimexpr(\p@-\@CosDuno\p@)\relax
    \DividE\@KCuno\@tdA by\@SinDuno\p@ to \@KCuno
  \fi
\fi
\MultiplyFN\@KCuno by \@Tuno to \@KCuno
\ScaleVect\@Duno by\@KCuno to\@CPuno
\AddVect\@Puno and\@CPuno to\@CPuno
```

Now we have the four points and we can instruct the internal `pict2e` macros to do the path drawing.

```
\GetCoord(\@Puno)\@XPuno\@YPuno
\GetCoord(\@CPzero)\@XCPzero\@YCPzero
\GetCoord(\@CPuno)\@XCPuno\@YCPuno
\pIIe@curveto{\@XCPzero\unitlength}{\@YCPzero\unitlength}%
             {\@XCPuno\unitlength}{\@YCPuno\unitlength}%
             {\@XPuno\unitlength}{\@YPuno\unitlength}\egroup
```

It does not have to stroke the curve because other Bézier splines might still be added to the path. On the opposite it memorises the final point as the initial point of the next spline

```
\CopyVect\@Puno to\@Pzero
\CopyVect\@Duno to\@Dzero
\ignorespaces}%
```

We finally define the overall `\Curve` macro that has two flavours: starred and unstarred; the former fills the curve path with the locally selected color, while the latter just strokes the path. Both recursively examine an arbitrary list of nodes and directions; node coordinates are grouped within regular parentheses while direction components are grouped within angle brackets. The first call of the macro initialises the drawing process and checks for the next node and direction; if a second node is missing, it issues a warning message and does not draw anything. It does not check for a change in direction, because it would be meaningless at the beginning of a curve. The second macro defines the path to the next point and checks for another node; if the next list item is a square bracket delimited argument, it interprets it as a change of direction, while if it is another parenthesis delimited argument it interprets it as a new node-direction specification; if the node and direction list is terminated, it issues the stroking or filling command through `\CurveEnd`, and exits the recursive process. The `\CurveEnd` control se-

quence has a different meaning depending on the fact that the main macro was starred or unstarred. The `@ChangeDir` macro is just an interface to execute the regular `\ChangeDir` macro, but also for recursing again by recalling `\@Curve`.

```
\def\Curve{\@ifstar{\let\fillstroke\fillpath\Curve@}%
{\let\fillstroke\strokepath\Curve@}}

\def\Curve@(#1)<#2>{%
    \StartCurveAt#1WithDir{#2}%
    \@ifnextchar\lp@r\@Curve{%
    \PackageWarning{curve2e}{%
    Curve specifications must contain at least two nodes!\Messagebreak
    Please, control your \string\Curve\space specifications\MessageBreak}}}
\def\@Curve(#1)<#2>{%
    \CurveTo#1WithDir{#2}%
    \@ifnextchar\lp@r\@Curve{%
    \@ifnextchar[\@ChangeDir\CurveEnd}}
\def\@ChangeDir[#1]{\ChangeDir<#1>\@Curve}
```

As a concluding remark, please notice that the `\Curve` macro is certainly the most comfortable to use, but it is sort of frozen in its possibilities. The user may certainly use the `\StartCurve`, `\CurveTo`, `\ChangeDir`, and `\CurveFinish` or `\FillCurve` for a more versatile set of drawing macros; evidently nobody forbids to exploit the full power of the `\cbezier` original macro for cubic splines; we made available macros `\CbezierTo` and the isolated arc macro `\CbezierBetween` in order to use the general internal cubic Bézier splines in a more comfortable way.

As it can be seen in figure **??** on page **??** the two diagrams should approximately represent a sine wave. With Bézier curves, that resort on polynomials, it is impossible to represent a transcendental function, but it is only possible to approximate it. It is evident that the approximation obtained with full control on the control points requires less arcs and it is more accurate than the approximation obtained with the recursive `\Curve` macro; this macro requires almost two times as many pieces of information in order to minimise the effects of the lack of control on the control points, and even with this added information the macro approaches the sine wave with less accuracy. At the same time for many applications the `\Curve` recursive macro proves to be much easier to use than with single arcs drawn with the `\CbezierBetween` macro.

7.8 Quadratic splines

We want to create a recursive macro with the same properties as the above described `\Curve` macro, but that uses quadratic splines; we call it `\Qurve` so that the initial macro name letter reminds us of the nature of the splines being used. For the rest they have an almost identical syntax; with quadratic splines it is not possible to specify the distance of the control points from the extrema, since quadratic splines have just one control point that must lay at the intersection of the two tangent directions; therefore with quadratic splines the tangents at each point cannot have the optional part that starts with a semicolon. The syntax, therefore, is just:

`\Qurve(`⟨*first point*⟩`)<`⟨*direction*⟩`>...(`⟨*any point*⟩`)<`⟨*direction*⟩`>...(`⟨*last point*⟩`)<`⟨*direction*⟩`>`

As with `\Curve`, also with `\Qurve` there is no limitation on the number of points, except for the computer memory size; it is advisable not to use many arcs otherwise it might become very difficult to find errors.

The first macros that set up the recursion are very similar to those we wrote for `\Curve`:

```
\def\Qurve{\@ifstar{\let\fillstroke\fillpath\Qurve@}%
{\let\fillstroke\strokepath\Qurve@}}

\def\Qurve@(#1)<#2>{%
    \StartCurveAt#1WithDir{#2}%
    \@ifnextchar\lp@r\@Qurve{%
    \PackageWarning{curve2e}{%
      Quadratic curve specifications must contain at least
      two nodes!\Messagebreak
      Please, control your Qurve specifications\MessageBreak}}%

\def\@Qurve(#1)<#2>{\QurveTo#1WithDir{#2}%
    \@ifnextchar\lp@r\@Qurve{%
    \@ifnextchar[\@ChangeQDir\CurveEnd}}%

\def\@ChangeQDir[#1]{\ChangeDir<#1>\@Qurve}%
```

Notice that in case of long paths it might be better to use the single macros `\StartCurveAt`, `\QurveTo`, `\ChangeDir` and `\CurveFinish` (or `\FillCurve`), with their respective syntax, in such a way that a long list of node-direction specifications passed to `\Qurve` may be split into shorter input lines in order to edit the input data in a more comfortable way.

The macro that does everything is `\QurveTo`. it starts with reading its arguments received through the calling macro `\@Qurve`

```
\def\QurveTo#1WithDir#2{%
\edef\@Puno{#1}\DirOfVect#2to\@Duno\bgroup
\DistanceAndDirOfVect\@Puno minus\@Pzero to\@Chord and\@DirChord
```

It verifies if `\@Dpzero` and `\@Dpuno`, the directions at the two extrema of the arc, are parallel or anti-parallel by taking their "scalar" product (`\@Dpzero` times `\@Dpuno*`); if the imaginary component of the scalar product vanishes the two directions are parallel; in this case we produce an error message, but we continue by skipping this arc destination point; evidently the drawing will not be the desired one, but the job should not abort.

```
\Multvect{\@Dzero}*{\@Duno}\@Scalar
\YpartOfVect\@Scalar to \@YScalar
\ifdim\@YScalar\p@=\z@
\PackageWarning{curve2e}%
   {Quadratic Bezier arcs cannot have their starting\MessageBreak
   and ending directions parallel or antiparallel with\MessageBreak
   each other. This arc is skipped and replaced with
```

```
    a dotted line.\MessageBreak}%
    \Dotline(\@Pzero)(\@Puno){2}\relax
\else
```

Otherwise we rotate everything about the starting point so as to bring the chord on the real axis; we get also the components of the two directions that, we should remember, are unit vectors, not generic vectors, although the user can use the vector specifications that are more understandable to him/her:

```
\Multvect{\@Dzero}*{\@DirChord}\@Dpzero
\Multvect{\@Duno}*{\@DirChord}\@Dpuno
\GetCoord(\@Dpzero)\@DXpzero\@DYpzero
\GetCoord(\@Dpuno)\@DXpuno\@DYpuno
```

We check if the two directions point to the same half plane; this implies that these rotated directions point to different sides of the chord vector; all this is equivalent that the two direction Y components have opposite signs, so that their product is strictly negative, while the two X components product is not negative.

```
\MultiplyFN\@DXpzero by\@DXpuno to\@XXD
\MultiplyFN\@DYpzero by\@DYpuno to\@YYD
\unless\ifdim\@YYD\p@<\z@\ifdim\@XXD\p@<\z@
\PackageWarning{curve2e}%
    {Quadratic Bezier arcs cannot have inflection points\MessageBreak
    Therefore the tangents to the starting and ending arc\MessageBreak
    points cannot be directed to the same half plane.\MessageBreak
    This arc is skipped and replaced by a dotted line\MessageBreak}%
    \Dotline(\@Pzero)(\@Puno){2}\fi
\else
```

After these tests we should be in a "normal" situation. We first copy the expanded input information into new macros that have more explicit names: macros stating wit 'S' denote the sine of the direction angle, while those starting with 'C' denote the cosine of that angle. We will use these expanded definitions as we know we are working with the actual values. These directions are those relative to the arc chord.

```
\edef\@CDzero{\@DXpzero}\relax
\edef\@SDzero{\@DYpzero}\relax
\edef\@CDuno{\@DXpuno}\relax
\edef\@SDuno{\@DYpuno}\relax
```

Suppose we write the parametric equations of a straight line that departs from the beginning of the chord with direction angle ϕ_0 and the corresponding equation of the straight line departing from the end of the chord (of length c) with direction angle ϕ_1. We have to find the coordinates of the intersection point of these two straight lines.

$$t\cos\phi_0 - s\cos\phi_1 = c \tag{6a}$$

$$t\sin\phi_0 - s\sin\phi_1 = 0 \tag{6b}$$

The parameters t and s are just the running parameters; we have to solve those simultaneous equations in the unknown variables t and s; these values let us

compute the coordinates of the intersection point:

$$X_C = \frac{c\cos\phi_0\sin\phi_1}{\sin\phi_0\cos\phi_1 - \cos\phi_0\sin\phi_1} \tag{7a}$$

$$Y_C = \frac{c\sin\phi_0\sin\phi_1}{\sin\phi_0\cos\phi_1 - \cos\phi_0\sin\phi_1} \tag{7b}$$

Having performed the previous tests we are sure that the denominator is not vanishing (direction are not parallel or anti-parallel) and that it lays at the same side as the direction with angle ϕ_0 with respect to the chord.

The coding then goes on like this:

```
\MultiplyFN\@SDzero by\@CDuno to\@tempA
\MultiplyFN\@SDuno by\@CDzero to\@tempB
\edef\@tempA{\strip@pt\dimexpr\@tempA\p@-\@tempB\p@}\relax
\@tdA=\@SDuno\p@ \@tdB=\@Chord\p@ \@tdC=\@tempA\p@
\edef\@tempC{\strip@pt\dimexpr \@tdA*\@tdB/\@tdC}\relax
\MultiplyFN\@tempC by\@CDzero to \@XC
\MultiplyFN\@tempC by\@SDzero to \@YC
\ModOfVect\@XC,\@YC to\@KC
```

Now we have the coordinates and the module of the intersection point vector taking into account the rotation of the real axis; getting back to the original coordinates before rotation, we get:

```
\ScaleVect\@Dzero by\@KC to\@CP
\AddVect\@Pzero and\@CP to\@CP
\GetCoord(\@Pzero)\@XPzero\@YPzero
\GetCoord(\@Puno)\@XPuno\@YPuno
\GetCoord(\@CP)\@XCP\@YCP
```

We have now the coordinates of the two extrema point of the quadratic arc and of the control point. Keeping in mind that the symbols P_0, P_1 and C denote geometrical points but also their coordinates as ordered pairs of real numbers (i.e. they are complex numbers) we have to determine the parameters of a cubic spline that with suitable values get simplifications in its parametric equation so that it becomes a second degree function instead of a third degree one. It is possible, in spite of the fact the it appears impossible that e cubic form becomes a quadratic one; we should determine the values of P_a and P_b such that:

$$P_0(1-t)^3 + 3P_a(1-t)^2t + 3P_b(1-t)t^2 + P_1t^3$$

is equivalent to

$$P_0(1-t)^2 + 2C(1-t)t + P_1t^2$$

It turns out that the solution is given by

$$P_a = C + (P_0 - C)/3 \qquad \text{and} \qquad P_b = C + (P_1 - C)/3 \tag{8}$$

The transformations implied by equations (**??**) are performed by the following macros already available from the `pict2e` package; we use them here with the actual arguments used for this task:

```
\@ovxx=\@XPzero\unitlength \@ovyy=\@YPzero\unitlength
\@ovdx=\@XCP\unitlength \@ovdy=\@YCP\unitlength
\@xdim=\@XPuno\unitlength \@ydim=\@YPuno\unitlength
      \pIIe@bezier@QtoC\@ovxx\@ovdx\@ovro
      \pIIe@bezier@QtoC\@ovyy\@ovdy\@ovri
      \pIIe@bezier@QtoC\@xdim\@ovdx\@clnwd
      \pIIe@bezier@QtoC\@ydim\@ovdy\@clnht
```

We call the basic `pict2e` macro to draw a cubic spline and we finish the conditional statements with which we started these calculations; eventually we close the group we opened at the beginning and we copy the terminal node information (position and direction) into the 0-labelled macros that indicate the starting point of the next arc.

```
      \pIIe@curveto\@ovro\@ovri\@clnwd\@clnht\@xdim\@ydim
\fi\fi\egroup
\CopyVect\@Puno to\@Pzero
\CopyVect\@Duno to\@Dzero
\ignorespaces}
```

An example of usage is shown at the left in figure **??**[6] on page **??** created with the code shown in the same page.

Notice also that the inflexed line is made with two arcs that meet at the inflection point; the same is true for the line that resembles a sine wave. The cusps of the inner border of the green area are obtained with the usual optional star already used also with the `\Curve` recursive macro.

The "circle" inside the square frame is visibly different from a real circle, in spite of the fact that the maximum deviation from the true circle is just about 6% relative to the radius; a quarter circle obtained with a single parabola is definitely a poor approximation of a real quarter circle; possibly by splitting each quarter circle in three or four partial arcs the approximation of a real quarter circle would be much better. On the right of figure **??** on page **??** it is possible to compare a "circle" obtained with quadratic arcs with the the internal circle obtained with cubic arcs; the difference is easily seen even without using measuring instruments.

With quadratic arcs we decided to avoid defining specific macros similar to `\CurveBetween` and `\CbezierBetween`; the first macro would not save any typing to the operator; furthermore it may be questionable if it was really useful even with cubic splines; the second macro with quadratic arcs is meaningless, since with quadratic arcs there is just one control point and there is no choice on its position.

[6] The commands `\legenda`, `\Pall` and `\Zbox` are specifically defined in the preamble of this document; they must be used within a `picture` environment. `\legenda` draws a framed legend made up of a single (short) math formula; `\Pall` is just a shorthand to put a filled small circle at a specified position' `\Zbox` puts a symbol in math mode a little displaced in the proper direction relative to a specified position. They are just handy to label certain objects in a `picture` diagram, but they are not part of the `curve2e` package.

8 Conclusion

I believe that the set of new macros provided by this package can really help the user to draw his/her diagrams with more agility; it will be the accumulated experience to decide if this is true.

As a personal experience we found very comfortable to draw ellipses and to define macros to draw not only such shapes or filled elliptical areas, but also to create "legends" with coloured backgrounds and borders. But this is an application of the functionality implemented in this package.

9 The `README.txt` file

The following is the text that forms the contents of the `README.txt` file that accompanies the package. We found it handy to have it in the documented source, because in this way certain pieces of information don't need to be repeated again and again in different files.

```
741 The package bundle curve2e is composed of the following files
742
743 curve2e.dtx
744 curve2e.pdf
745 README.txt
746 ltxdoc.cfg
747
748 curve2e.dtx is the documented TeX source file of file curve2e.sty; you
749 get curve2e.sty, curve2e.pdf, and curve2e-v161.sty by running pdflatex
750 on curve2e.dtx. The ltxdoc.cfg file customises the way the documentation
751 file is typeset. This specific .cfg file is part of the ltxdoc package functionality and it is
752
753 README.txt, this file, contains general information.
754
755 Curve2e-v161.sty contains a previous version of the package; see below
756 why the older version might become necessary for the end user.
757
758 Curve2e.sty is an extension of the package pict2e.sty which extends the
759 standard picture LaTeX environment according to what Leslie Lamport
760 specified in the second edition of his LaTeX manual (1994).
761
762 This further extension curve2e.sty allows to draw lines and vectors
763 with any non integer slope parameters, to draw dashed lines of any
764 slope, to draw arcs and curved vectors, to draw curves where just
765 the interpolating nodes are specified together with the slopes at
766 the nodes; closed paths of any shape can be filled with color; all
767 coordinates are treated as ordered pairs, i.e. 'complex numbers';
768 coordinates may be expressed also in polar form.
769 Some of these features have been incorporated in the 2011 version of
770 pict2e; therefore this package avoids any modification to the original
771 pict2e commands.
772
```

Curve2e now accepts polar coordinates in addition to the usual cartesian ones; several macros have been upgraded and a new macro for tracing cubic Bezier splines with their control nodes specified in polar form is available. The same applies to quadratic Bezier splines. The \multiput command has been completely modified in a backwards compatible way, as to manipulate the increment components in a configurable way.

This version solves a conflict with package eso-pic.

This version of curve2e is almost fully compatible with pict2e dated 2014/01/12 version 0.2z and later.

If you specify

\usepackage[<pict2e options>]{curve2e}

the package pict2e is automatically invoked with the specified options.

The -almost compatible- frase is necessary to explain that this version of curve2e uses some 'functions' of the LaTeX3 language that were made available to the LaTeX developers by mid October 2018. Should the user have an older or a basic/incomplete installation of the TeX system, such L3 functions might not be available. This is why this package checks the presence of the developer interface; in case such interface is not available it falls back to the previous version renamed curve2e-v161.sty, which is part of this bundle, and that must not be renamed in any way. The compatibility mentioned above implies that the user macros remain the same, but their implementation requires the L3 interface.

The package has the LPPL status of author maintained.

According to the LPPL licence, you are entitled to modify this package, as long as you fulfil the few conditions set forth by the Licence.

Nevertheless this package is an extension to the standard LaTeX package pict2e (2014). Therefore any change must be controlled on the parent package pict2e, so as to avoid redefining or interfering with what is already contained in the official package.

If you prefer sending me your modifications, as long as I will maintain this package, I will possibly include every (documented) suggestion or modification into this package and, of course, I will acknowledge your contribution.

Claudio Beccari

claudio dot beccari at gmail dot com

10 The fall-back package version curve2e-v161

this is the fall-back version of curve2e-v161.sty to which the main file curve2e.sty falls back in case the interface package xfp is not available.

```
\NeedsTeXFormat{LaTeX2e}[2016/01/01]
\ProvidesPackage{curve2e-v161}%
        [2019/02/07 v.1.61 Extension package for pict2e]

\RequirePackage{color}
\RequirePackageWithOptions{pict2e}[2014/01/01]
\RequirePackage{xparse}
\def\TRON{\tracingcommands\tw@ \tracingmacros\tw@}%
\def\TROF{\tracingcommands\z@ \tracingmacros\z@}%
\ifx\undefined\@tdA \newdimen\@tdA \fi
\ifx\undefined\@tdB \newdimen\@tdB \fi
\ifx\undefined\@tdC \newdimen\@tdC \fi
\ifx\undefined\@tdD \newdimen\@tdD \fi
\ifx\undefined\@tdE \newdimen\@tdE \fi
\ifx\undefined\@tdF \newdimen\@tdF \fi
\ifx\undefined\defaultlinewidth \newdimen\defaultlinewidth \fi
\gdef\linethickness#1{\@wholewidth#1\@halfwidth.5\@wholewidth\ignorespaces}%
\newcommand\defaultlinethickness[1]{\defaultlinewidth=#1\relax
\def\thicklines{\linethickness{\defaultlinewidth}}%
\def\thinlines{\linethickness{.5\defaultlinewidth}}%
\thinlines\ignorespaces}
\def\LIne(#1){{\GetCoord(#1)\@tX\@tY
      \moveto(0,0)
      \pIIe@lineto{\@tX\unitlength}{\@tY\unitlength}\strokepath}\ignorespaces}%
\def\segment(#1)(#2){\@killglue\polyline(#1)(#2)}%
\def\line(#1)#2{\begingroup
  \@linelen #2\unitlength
  \ifdim\@linelen<\z@\@badlinearg\else
    \expandafter\DirOfVect#1to\Dir@line
    \GetCoord(\Dir@line)\d@mX\d@mY
    \ifdim\d@mX\p@=\z@\else
      \DividE\ifdim\d@mX\p@<\z@-\fi\p@ by\d@mX\p@ to\sc@lelen
      \@linelen=\sc@lelen\@linelen
    \fi
    \moveto(0,0)
    \pIIe@lineto{\d@mX\@linelen}{\d@mY\@linelen}%
    \strokepath
  \fi
\endgroup\ignorespaces}%
\ifx\Dashline\undefined
\def\Dashline{\@ifstar{\Dashline@@}{\Dashline@}}
\def\Dashline@(#1)(#2)#3{%
\bgroup
   \countdef\NumA3254\countdef\NumB3252\relax
   \GetCoord(#1)\@tA\@tB \MakeVectorFrom\@tA\@tB to\V@ttA
```

```
   \GetCoord(#2)\@tA\@tB \MakeVectorFrom\@tA\@tB to\V@ttB
   \SubVect\V@ttA from\V@ttB to\V@ttC
   \ModOfVect\V@ttC to\DlineMod
   \DivideFN\DlineMod by#3 to\NumD
   \NumA\expandafter\Integer\NumD.??
   \ifodd\NumA\else\advance\NumA\@ne\fi
   \NumB=\NumA \divide\NumB\tw@
   \DividE\DlineMod\p@ by\NumA\p@  to\D@shMod
   \DividE\p@ by\NumA\p@ to \@tempa
   \MultVect\V@ttC by\@tempa,0 to\V@ttB
   \MultVect\V@ttB by 2,0 to\V@ttC
   \advance\NumB\@ne
   \edef\@mpt{\noexpand\egroup
   \noexpand\multiput(\V@ttA)(\V@ttC){\number\NumB}%
      {\noexpand\LIne(\V@ttB)}}%
   \@mpt\ignorespaces}%
\let\Dline\Dashline

\def\Dashline@@(#1)(#2)#3{\put(#1){\Dashline@(0,0)(#2){#3}}}
\fi
\ifx\Dotline\undefined
\def\Dotline{\@ifstar{\Dotline@@}{\Dotline@}}
\def\Dotline@(#1)(#2)#3{%
\bgroup
   \countdef\NumA 3254\relax \countdef\NumB 3255\relax
   \GetCoord(#1)\@tA\@tB \MakeVectorFrom\@tA\@tB to\V@ttA
   \GetCoord(#2)\@tA\@tB \MakeVectorFrom\@tA\@tB to\V@ttB
   \SubVect\V@ttA from\V@ttB to\V@ttC
   \ModOfVect\V@ttC to\DotlineMod
   \DivideFN\DotlineMod by#3 to\NumD
   \NumA=\expandafter\Integer\NumD.??
   \DivVect\V@ttC by\NumA,0 to\V@ttB
   \advance\NumA\@ne
   \edef\@mpt{\noexpand\egroup
   \noexpand\multiput(\V@ttA)(\V@ttB){\number\NumA}%
      {\noexpand\makebox(0,0){\noexpand\circle*{0.5}}}}%
   \@mpt\ignorespaces}%

\def\Dotline@@(#1)(#2)#3{\put(#1){\Dotline@(0,0)(#2){#3}}}
\fi
\AtBeginDocument{\@ifpackageloaded{eso-pic}{%
\renewcommand\LenToUnit[1]{\strip@pt\dimexpr#1*\p@/\unitlength}}{}}

\def\GetCoord(#1)#2#3{%
\expandafter\SplitNod@\expandafter(#1)#2#3\ignorespaces}
\def\isnot@polar#1:#2!!{\def\@tempOne{#2}\ifx\@tempOne\empty
\expandafter\@firstoftwo\else
\expandafter\@secondoftwo\fi
{\SplitNod@@}{\SplitPolar@@}}

```

```
\def\SplitNod@(#1)#2#3{\isnot@polar#1:!!(#1)#2#3}%
\def\SplitNod@@(#1,#2)#3#4{\edef#3{#1}\edef#4{#2}}%
\def\SplitPolar@@(#1:#2)#3#4{\DirFromAngle#1to\@DirA
\ScaleVect\@DirA by#2to\@DirA
\expandafter\SplitNod@@\expandafter(\@DirA)#3#4}

\let\originalput\put
\def\put(#1){\bgroup\GetCoord(#1)\@tX\@tY
\edef\x{\noexpand\egroup\noexpand\originalput(\@tX,\@tY)}\x}

\let\originalmultiput\multiput
\let\original@multiput\@multiput

\long\def\@multiput(#1)#2#3{\bgroup\GetCoord(#1)\@mptX\@mptY
\edef\x{\noexpand\egroup\noexpand\original@multiput(\@mptX,\@mptY)}%
\x{#2}{#3}\ignorespaces}

\gdef\multiput(#1)#2{\bgroup\GetCoord(#1)\@mptX\@mptY
\edef\x{\noexpand\egroup\noexpand\originalmultiput(\@mptX,\@mptY)}\x(}%)
  \def\vector(#1)#2{%
    \begingroup
      \GetCoord(#1)\d@mX\d@mY
      \@linelen#2\unitlength
      \ifdim\d@mX\p@=\z@\ifdim\d@mY\p@=\z@\@badlinearg\fi\fi
      \ifdim\@linelen<\z@ \@linelen=-\@linelen\fi
      \MakeVectorFrom\d@mX\d@mY to\@Vect
      \DirOfVect\@Vect to\Dir@Vect
        \YpartOfVect\Dir@Vect to\@ynum \@ydim=\@ynum\p@
        \XpartOfVect\Dir@Vect to\@xnum \@xdim=\@xnum\p@
        \ifdim\d@mX\p@=\z@
        \else\ifdim\d@mY\p@=\z@
          \else
            \DividE\ifdim\@xnum\p@<\z@-\fi\p@ by\@xnum\p@ to\sc@lelen
            \@linelen=\sc@lelen\@linelen
          \fi
        \fi
      \@tdB=\@linelen
\pIIe@concat\@xdim\@ydim{-\@ydim}\@xdim{\@xnum\@linelen}{\@ynum\@linelen}%
        \@linelen\z@
        \pIIe@vector
        \fillpath
      \@linelen=\@tdB
      \@tdA=\pIIe@FAW\@wholewidth
      \@tdA=\pIIe@FAL\@tdA
      \advance\@linelen-\@tdA
      \ifdim\@linelen>\z@
        \moveto(0,0)
        \pIIe@lineto{\@xnum\@linelen}{\@ynum\@linelen}%
        \strokepath\fi
    \endgroup}
```

```
\def\Vector(#1){{%
\GetCoord(#1)\@tX\@tY
\ifdim\@tX\p@=\z@\vector(\@tX,\@tY){\@tY}
\else
\vector(\@tX,\@tY){\@tX}\fi}}
\def\VECTOR(#1)(#2){\begingroup
\SubVect#1from#2to\@tempa
\expandafter\put\expandafter(#1){\expandafter\Vector\expandafter(\@tempa)}%
\endgroup\ignorespaces}
\let\lp@r( \let\rp@r)
\renewcommand*\polyline[1][\beveljoin]{\p@lylin@[#1]}

\def\p@lylin@[#1](#2){\@killglue#1\GetCoord(#2)\d@mX\d@mY
    \pIIe@moveto{\d@mX\unitlength}{\d@mY\unitlength}%
    \@ifnextchar\lp@r{\p@lyline}{%
    \PackageWarning{curve2e}%
    {Polylines require at least two vertices!\MessageBreak
     Control your polyline specification\MessageBreak}%
    \ignorespaces}}

\def\p@lyline(#1){\GetCoord(#1)\d@mX\d@mY
    \pIIe@lineto{\d@mX\unitlength}{\d@mY\unitlength}%
    \@ifnextchar\lp@r{\p@lyline}{\strokepath\ignorespaces}}
\providecommand\polygon{}
\RenewDocumentCommand\polygon{s O{\beveljoin} }{\@killglue\begingroup
\IfBooleanTF{#1}{\@tempswatrue}{\@tempswafalse}%
\@polygon[#2]}

\def\@polygon[#1](#2){\@killglue#1\GetCoord(#2)\d@mX\d@mY
    \pIIe@moveto{\d@mX\unitlength}{\d@mY\unitlength}%
    \@ifnextchar\lp@r{\@@polygon}{%
    \PackageWarning{curve2e}%
    {Polygons require at least two vertices!\MessageBreak
     Control your polygon specification\MessageBreak}%
    \ignorespaces}}

  \def\@@polygon(#1){\GetCoord(#1)\d@mX\d@mY
    \pIIe@lineto{\d@mX\unitlength}{\d@mY\unitlength}%
    \@ifnextchar\lp@r{\@@polygon}{\pIIe@closepath
      \if@tempswa\pIIe@fillGraph\else\pIIe@strokeGraph\fi
      \endgroup
      \ignorespaces}}
\def\GraphGrid(#1,#2){\bgroup\textcolor{red}{\linethickness{.1\p@}%
\RoundUp#1modulo10to\@GridWd \RoundUp#2modulo10to\@GridHt
\@tempcnta=\@GridWd \divide\@tempcnta10\relax \advance\@tempcnta\@ne
\multiput(0,0)(10,0){\@tempcnta}{\line(0,1){\@GridHt}}%
\@tempcnta=\@GridHt \divide\@tempcnta10\advance\@tempcnta\@ne
\multiput(0,0)(0,10){\@tempcnta}{\line(1,0){\@GridWd}}\thinlines}%
\egroup\ignorespaces}
\def\RoundUp#1modulo#2to#3{\expandafter\@tempcnta\Integer#1.??%
```

```
\count254\@tempcnta\divide\count254by#2\relax
\multiply\count254by#2\relax
\count252\@tempcnta\advance\count252-\count254
\ifnum\count252>0\advance\count252-#2\relax
\advance\@tempcnta-\count252\fi\edef#3{\number\@tempcnta}\ignorespaces}%
\def\Integer#1.#2??{#1}%
\ifdefined\dimexpr
    \unless\ifdefined\DividE
\def\DividE#1by#2to#3{\bgroup
\dimendef\Num2254\relax \dimendef\Den2252\relax
\dimendef\@DimA 2250
\Num=\p@ \Den=#2\relax
\ifdim\Den=\z@
  \edef\x{\noexpand\endgroup\noexpand\def\noexpand#3{\strip@pt\maxdimen}}%
\else
  \@DimA=#1\relax
  \edef\x{%
    \noexpand\egroup\noexpand\def\noexpand#3{%
       \strip@pt\dimexpr\@DimA*\Num/\Den\relax}}%
\fi
\x\ignorespaces}%
\fi
    \unless\ifdefined\DivideFN
         \def\DivideFN#1by#2to#3{\DividE#1\p@ by#2\p@ to{#3}}%
    \fi
    \unless\ifdefined\MultiplY
        \def\MultiplY#1by#2to#3{\bgroup
        \dimendef\@DimA 2254 \dimendef\@DimB2255
        \@DimA=#1\p@\relax \@DimB=#2\p@\relax
        \edef\x{%
           \noexpand\egroup\noexpand\def\noexpand#3{%
             \strip@pt\dimexpr\@DimA*\@DimB/\p@\relax}}%
        \x\ignorespaces}%
        \let\MultiplyFN\MultiplY
    \fi
\fi

\unless\ifdefined\Numero
  \def\Numero#1#2{\bgroup\dimen3254=#2\relax
    \edef\x{\noexpand\egroup\noexpand\edef\noexpand#1{%
      \strip@pt\dimen3254}}\x\ignorespaces}%
\fi
\def\g@tTanCotanFrom#1to#2and#3{%
\DividE 114.591559\p@ by#1to\X@ \@tdB=\X@\p@
\countdef\I=2546\def\Tan{0}\I=11\relax
\@whilenum\I>\z@\do{%
   \@tdC=\Tan\p@ \@tdD=\I\@tdB
   \advance\@tdD-\@tdC \DividE\p@ by\@tdD to\Tan
   \advance\I-2\relax}%
\def#2{\Tan}\DividE\p@ by\Tan\p@ to\Cot \def#3{\Cot}\ignorespaces}%
```

```
\def\SinOf#1to#2{\bgroup%
\@tdA=#1\p@%
\ifdim\@tdA>\z@%
  \@whiledim\@tdA>180\p@\do{\advance\@tdA -360\p@}%
\else%
  \@whiledim\@tdA<-180\p@\do{\advance\@tdA 360\p@}%
\fi \ifdim\@tdA=\z@
  \def\@tempA{0}%
\else
  \ifdim\@tdA>\z@
    \def\Segno{+}%
  \else
    \def\Segno{-}%
    \@tdA=-\@tdA
  \fi
  \ifdim\@tdA>90\p@
    \@tdA=-\@tdA \advance\@tdA 180\p@
  \fi
  \ifdim\@tdA=90\p@
    \def\@tempA{\Segno1}%
  \else
    \ifdim\@tdA=180\p@
      \def\@tempA{0}%
    \else
      \ifdim\@tdA<\p@
        \@tdA=\Segno0.0174533\@tdA
        \DividE\@tdA by\p@ to \@tempA%
      \else
        \g@tTanCotanFrom\@tdA to\T and\Tp
        \@tdA=\T\p@ \advance\@tdA \Tp\p@
        \DividE \Segno2\p@ by\@tdA to \@tempA%
      \fi
    \fi
  \fi
\fi
\edef\endSinOf{\noexpand\egroup
  \noexpand\def\noexpand#2{\@tempA}\noexpand\ignorespaces}%
\endSinOf}%
\def\CosOf#1to#2{\bgroup%
\@tdA=#1\p@%
\ifdim\@tdA>\z@%
  \@whiledim\@tdA>360\p@\do{\advance\@tdA -360\p@}%
\else%
  \@whiledim\@tdA<\z@\do{\advance\@tdA 360\p@}%
\fi
\ifdim\@tdA>180\p@
  \@tdA=-\@tdA \advance\@tdA 360\p@
\fi
\ifdim\@tdA<90\p@
  \def\Segno{+}%
```

```
\else
  \def\Segno{-}%
  \@tdA=-\@tdA \advance\@tdA 180\p@
\fi
\ifdim\@tdA=\z@
  \def\@tempA{\Segno1}%
\else
  \ifdim\@tdA<\p@
    \@tdA=0.0174533\@tdA \Numero\@tempA\@tdA
    \@tdA=\@tempA\@tdA \@tdA=-.5\@tdA
    \advance\@tdA \p@
    \DividE\@tdA by\p@ to\@tempA%
  \else
    \ifdim\@tdA=90\p@
      \def\@tempA{0}%
    \else
      \g@tTanCotanFrom\@tdA to\T and\Tp
      \@tdA=\Tp\p@ \advance\@tdA-\T\p@
      \@tdB=\Tp\p@ \advance\@tdB\T\p@
      \DividE\Segno\@tdA by\@tdB to\@tempA%
    \fi
  \fi
\fi
\edef\endCosOf{\noexpand\egroup
    \noexpand\def\noexpand#2{\@tempA}\noexpand\ignorespaces}%
\endCosOf}%
\def\TanOf#1to#2{\bgroup%
\@tdA=#1\p@%
\ifdim\@tdA>90\p@%
  \@whiledim\@tdA>90\p@\do{\advance\@tdA -180\p@}%
\else%
  \@whiledim\@tdA<-90\p@\do{\advance\@tdA 180\p@}%
\fi%
\ifdim\@tdA=\z@%
  \def\@tempA{0}%
\else
  \ifdim\@tdA>\z@
    \def\Segno{+}%
  \else
    \def\Segno{-}%
    \@tdA=-\@tdA
  \fi
  \ifdim\@tdA=90\p@
    \def\@tempA{\Segno16383.99999}%
  \else
    \ifdim\@tdA<\p@
        \@tdA=\Segno0.0174533\@tdA
        \DividE\@tdA by\p@ to\@tempA%
    \else
      \g@tTanCotanFrom\@tdA to\T and\Tp
```

```
      \@tdA\Tp\p@ \advance\@tdA -\T\p@
      \DividE\Segno2\p@ by\@tdA to\@tempA%
    \fi
  \fi
\fi
\edef\endTanOf{\noexpand\egroup
   \noexpand\def\noexpand#2{\@tempA}\noexpand\ignorespaces}%
\endTanOf}%
\def\ArcTanOf#1to#2{\bgroup
\countdef\Inverti 4444\Inverti=0
\def\Segno{}
\edef\@tF{#1}\@tdF=\@tF\p@ \@tdE=57.295778\p@
\@tdD=\ifdim\@tdF<\z@ -\@tdF\def\Segno{-}\else\@tdF\fi
\ifdim\@tdD>\p@
\Inverti=\@ne
\@tdD=\dimexpr\p@*\p@/\@tdD\relax
\fi
\unless\ifdim\@tdD>0.02\p@
    \def\@tX{\strip@pt\dimexpr57.295778\@tdD\relax}%
\else
    \edef\@tX{45}\relax
    \countdef\I 2523 \I=9\relax
    \@whilenum\I>0\do{\TanOf\@tX to\@tG
    \edef\@tG{\strip@pt\dimexpr\@tG\p@-\@tdD\relax}\relax
    \MultiplY\@tG by57.295778to\@tG
    \CosOf\@tX to\@tH
    \MultiplY\@tH by\@tH to\@tH
    \MultiplY\@tH by\@tG to \@tH
    \edef\@tX{\strip@pt\dimexpr\@tX\p@ - \@tH\p@\relax}\relax
    \advance\I\m@ne}%
\fi
\ifnum\Inverti=\@ne
\edef\@tX{\strip@pt\dimexpr90\p@-\@tX\p@\relax}
\fi
\edef\x{\egroup\noexpand\edef\noexpand#2{\Segno\@tX}}\x\ignorespaces}%
\def\MakeVectorFrom#1#2to#3{\edef#3{#1,#2}\ignorespaces}%
\def\CopyVect#1to#2{\edef#2{#1}\ignorespaces}%
\def\ModOfVect#1to#2{\GetCoord(#1)\t@X\t@Y
\@tempdima=\t@X\p@ \ifdim\@tempdima<\z@ \@tempdima=-\@tempdima\fi
\@tempdimb=\t@Y\p@ \ifdim\@tempdimb<\z@ \@tempdimb=-\@tempdimb\fi
\ifdim\@tempdima=\z@
    \ifdim\@tempdimb=\z@
        \def\@T{0}\@tempdimc=\z@
    \else
        \def\@T{0}\@tempdimc=\@tempdimb
    \fi
\else
    \ifdim\@tempdima>\@tempdimb
       \DividE\@tempdimb by\@tempdima to\@T
       \@tempdimc=\@tempdima
```

```
    \else
          \DividE\@tempdima by\@tempdimb to\@T
          \@tempdimc=\@tempdimb
    \fi
\fi
\unless\ifdim\@tempdimc=\z@
    \unless\ifdim\@T\p@=\z@
       \@tempdima=\@T\p@ \@tempdima=\@T\@tempdima
       \advance\@tempdima\p@%
       \@tempdimb=\p@%
       \@tempcnta=5\relax
       \@whilenum\@tempcnta>\z@\do{\DividE\@tempdima by\@tempdimb to\@T
       \advance\@tempdimb \@T\p@ \@tempdimb=.5\@tempdimb
       \advance\@tempcnta\m@ne}%
       \@tempdimc=\@T\@tempdimc
    \fi
\fi
\Numero#2\@tempdimc
\ignorespaces}%
\def\DirOfVect#1to#2{\GetCoord(#1)\t@X\t@Y
\ModOfVect#1to\@tempa
\unless\ifdim\@tempdimc=\z@
   \DividE\t@X\p@ by\@tempdimc to\t@X
   \DividE\t@Y\p@ by\@tempdimc to\t@Y
\fi
\MakeVectorFrom\t@X\t@Y to#2\ignorespaces}%
\def\ModAndDirOfVect#1to#2and#3{%
\GetCoord(#1)\t@X\t@Y
\ModOfVect#1to#2%
\ifdim\@tempdimc=\z@\else
  \DividE\t@X\p@ by\@tempdimc to\t@X
  \DividE\t@Y\p@ by\@tempdimc to\t@Y
\fi
\MakeVectorFrom\t@X\t@Y to#3\ignorespaces}%
\def\DistanceAndDirOfVect#1minus#2to#3and#4{%
\SubVect#2from#1to\@tempa
\ModAndDirOfVect\@tempa to#3and#4\ignorespaces}%
\def\XpartOfVect#1to#2{%
\GetCoord(#1)#2\@tempa\ignorespaces}%
\def\YpartOfVect#1to#2{%
\GetCoord(#1)\@tempa#2\ignorespaces}%
\def\DirFromAngle#1to#2{%
\CosOf#1to\t@X
\SinOf#1to\t@Y
\MakeVectorFrom\t@X\t@Y to#2\ignorespaces}%
\def\ArgOfVect#1to#2{\bgroup\GetCoord(#1){\t@X}{\t@Y}%
\def\s@gno{}\def\addflatt@ngle{0}
\ifdim\t@X\p@=\z@
  \ifdim\t@Y\p@=\z@
    \def\ArcTan{0}%
```

```
  \else
    \def\ArcTan{90}%
    \ifdim\t@Y\p@<\z@\def\s@gno{-}\fi
  \fi
\else
  \ifdim\t@Y\p@=\z@
    \ifdim\t@X\p@<\z@
      \def\ArcTan{180}%
    \else
      \def\ArcTan{0}%
    \fi
  \else
    \ifdim\t@X\p@<\z@%
      \def\addflatt@ngle{180}%
      \edef\t@X{\strip@pt\dimexpr-\t@X\p@}%
      \edef\t@Y{\strip@pt\dimexpr-\t@Y\p@}%
      \ifdim\t@Y\p@<\z@
        \def\s@gno{-}%
        \edef\t@Y{-\t@Y}%
      \fi
    \fi
  \DivideFN\t@Y by\t@X to \t@A
  \ArcTanOf\t@A to\ArcTan
  \fi
\fi
\edef\ArcTan{\unless\ifx\s@gno\empty\s@gno\fi\ArcTan}%
\unless\ifnum\addflatt@ngle=0\relax
   \edef\ArcTan{%
   \strip@pt\dimexpr\ArcTan\p@\ifx\s@gno\empty-\else+\fi
   \addflatt@ngle\p@\relax}%
\fi
\edef\x{\noexpand\egroup\noexpand\edef\noexpand#2{\ArcTan}}%
\x\ignorespaces}
\def\ScaleVect#1by#2to#3{\GetCoord(#1)\t@X\t@Y
\@tempdima=\t@X\p@ \@tempdima=#2\@tempdima\Numero\t@X\@tempdima
\@tempdima=\t@Y\p@ \@tempdima=#2\@tempdima\Numero\t@Y\@tempdima
\MakeVectorFrom\t@X\t@Y to#3\ignorespaces}%
\def\ConjVect#1to#2{\GetCoord(#1)\t@X\t@Y
\@tempdima=-\t@Y\p@\Numero\t@Y\@tempdima
\MakeVectorFrom\t@X\t@Y to#2\ignorespaces}%
\def\AddVect#1and#2to#3{\GetCoord(#1)\tu@X\tu@Y
\GetCoord(#2)\td@X\td@Y
\@tempdima\tu@X\p@\advance\@tempdima\td@X\p@ \Numero\t@X\@tempdima
\@tempdima\tu@Y\p@\advance\@tempdima\td@Y\p@ \Numero\t@Y\@tempdima
\MakeVectorFrom\t@X\t@Y to#3\ignorespaces}%
\def\SubVect#1from#2to#3{\GetCoord(#1)\tu@X\tu@Y
\GetCoord(#2)\td@X\td@Y
\@tempdima\td@X\p@\advance\@tempdima-\tu@X\p@ \Numero\t@X\@tempdima
\@tempdima\td@Y\p@\advance\@tempdima-\tu@Y\p@ \Numero\t@Y\@tempdima
\MakeVectorFrom\t@X\t@Y to#3\ignorespaces}%
```

```
\def\MultVect#1by{\@ifstar{\@ConjMultVect#1by}{\@MultVect#1by}}%
\def\@MultVect#1by#2to#3{\GetCoord(#1)\tu@X\tu@Y
\GetCoord(#2)\td@X\td@Y
\@tempdima\tu@X\p@ \@tempdimb\tu@Y\p@
\@tempdimc=\td@X\@tempdima\advance\@tempdimc-\td@Y\@tempdimb
\Numero\t@X\@tempdimc
\@tempdimc=\td@Y\@tempdima\advance\@tempdimc\td@X\@tempdimb
\Numero\t@Y\@tempdimc
\MakeVectorFrom\t@X\t@Y to#3\ignorespaces}%
\def\@ConjMultVect#1by#2to#3{\GetCoord(#1)\tu@X\tu@Y
\GetCoord(#2)\td@X\td@Y \@tempdima\tu@X\p@ \@tempdimb\tu@Y\p@
\@tempdimc=\td@X\@tempdima\advance\@tempdimc+\td@Y\@tempdimb
\Numero\t@X\@tempdimc
\@tempdimc=\td@X\@tempdimb\advance\@tempdimc-\td@Y\@tempdima
\Numero\t@Y\@tempdimc
\MakeVectorFrom\t@X\t@Y to#3\ignorespaces}
\def\DivVect#1by#2to#3{\ModAndDirOfVect#2to\@Mod and\@Dir
\DividE\p@ by\@Mod\p@ to\@Mod \ConjVect\@Dir to\@Dir
\ScaleVect#1by\@Mod to\@tempa
\MultVect\@tempa by\@Dir to#3\ignorespaces}%
\def\Arc(#1)(#2)#3{\begingroup
\@tdA=#3\p@
\unless\ifdim\@tdA=\z@
  \@Arc(#1)(#2)%
\fi
\endgroup\ignorespaces}%
\def\@Arc(#1)(#2){%
\ifdim\@tdA>\z@
  \let\Segno+%
\else
  \@tdA=-\@tdA \let\Segno-%
\fi
\Numero\@gradi\@tdA
\ifdim\@tdA>360\p@
 \PackageWarning{curve2e}{The arc aperture is \@gradi\space degrees
      and gets reduced\MessageBreak%
      to the range 0--360 taking the sign into consideration}%
 \@whiledim\@tdA>360\p@\do{\advance\@tdA-360\p@}%
\fi
\SubVect#2from#1to\@V \ModOfVect\@V to\@Raggio \CopyVect#2to\@pPun
\CopyVect#1to\@Cent \GetCoord(\@pPun)\@pPunX\@pPunY
\@@Arc
\strokepath\ignorespaces}%
\def\@@Arc{%
\pIIe@moveto{\@pPunX\unitlength}{\@pPunY\unitlength}%
\ifdim\@tdA>180\p@
  \advance\@tdA-180\p@
  \Numero\@gradi\@tdA
  \SubVect\@pPun from\@Cent to\@V
  \AddVect\@V and\@Cent to\@sPun
```

```
  \MultVect\@V by0,-1.3333333to\@V \if\Segno-\ScaleVect\@V by-1to\@V\fi
  \AddVect\@pPun and\@V to\@pcPun
  \AddVect\@sPun and\@V to\@scPun
  \GetCoord(\@pcPun)\@pcPunX\@pcPunY
  \GetCoord(\@scPun)\@scPunX\@scPunY
  \GetCoord(\@sPun)\@sPunX\@sPunY
  \pIIe@curveto{\@pcPunX\unitlength}{\@pcPunY\unitlength}%
               {\@scPunX\unitlength}{\@scPunY\unitlength}%
               {\@sPunX\unitlength}{\@sPunY\unitlength}%
  \CopyVect\@sPun to\@pPun
\fi
\ifdim\@tdA>\z@
  \DirFromAngle\@gradi to\@Dir \if\Segno-\ConjVect\@Dir to\@Dir \fi
  \SubVect\@Cent from\@pPun to\@V
  \MultVect\@V by\@Dir to\@V
  \AddVect\@Cent and\@V to\@sPun
  \@tdA=.5\@tdA \Numero\@gradi\@tdA
  \DirFromAngle\@gradi to\@Phimezzi
  \GetCoord(\@Phimezzi)\@cosphimezzi\@sinphimezzi
  \@tdB=1.3333333\p@ \@tdB=\@Raggio\@tdB
  \@tdC=\p@ \advance\@tdC -\@cosphimezzi\p@ \Numero\@tempa\@tdC
  \@tdB=\@tempa\@tdB
  \DividE\@tdB by\@sinphimezzi\p@ to\@cZ
  \ScaleVect\@Phimezzi by\@cZ to\@Phimezzi
  \ConjVect\@Phimezzi to\@mPhimezzi
  \if\Segno-%
    \let\@tempa\@Phimezzi
    \let\@Phimezzi\@mPhimezzi
    \let\@mPhimezzi\@tempa
  \fi
  \SubVect\@sPun from\@pPun to\@V
  \DirOfVect\@V to\@V
  \MultVect\@Phimezzi by\@V to\@Phimezzi
  \AddVect\@sPun and\@Phimezzi to\@scPun
  \ScaleVect\@V by-1to\@V
  \MultVect\@mPhimezzi by\@V to\@mPhimezzi
  \AddVect\@pPun and\@mPhimezzi to\@pcPun
  \GetCoord(\@pcPun)\@pcPunX\@pcPunY
  \GetCoord(\@scPun)\@scPunX\@scPunY
  \GetCoord(\@sPun)\@sPunX\@sPunY
  \pIIe@curveto{\@pcPunX\unitlength}{\@pcPunY\unitlength}%
               {\@scPunX\unitlength}{\@scPunY\unitlength}%
               {\@sPunX\unitlength}{\@sPunY\unitlength}%
\fi}
\def\VectorArc(#1)(#2)#3{\begingroup
\@tdA=#3\p@ \ifdim\@tdA=\z@\else
  \@VArc(#1)(#2)%
\fi
\endgroup\ignorespaces}%
\def\VectorARC(#1)(#2)#3{\begingroup
```

```
\@tdA=#3\p@
\ifdim\@tdA=\z@\else
  \@VARC(#1)(#2)%
\fi
\endgroup\ignorespaces}%
\def\@VArc(#1)(#2){%
\ifdim\@tdA>\z@
  \let\Segno+%
\else
  \@tdA=-\@tdA \let\Segno-%
\fi \Numero\@gradi\@tdA
\ifdim\@tdA>360\p@
 \PackageWarning{curve2e}{The arc aperture is \@gradi\space degrees
      and gets reduced\MessageBreak%
      to the range 0--360 taking the sign into consideration}%
 \@whiledim\@tdA>360\p@\do{\advance\@tdA-360\p@}%
\fi
\SubVect#1from#2to\@V \ModOfVect\@V to\@Raggio \CopyVect#2to\@pPun
\@tdE=\pIIe@FAW\@wholewidth \@tdE=\pIIe@FAL\@tdE
\DividE\@tdE by \@Raggio\unitlength to\DeltaGradi
\@tdD=\DeltaGradi\p@
\@tdD=57.29578\@tdD \Numero\DeltaGradi\@tdD
\@tdD=\ifx\Segno--\fi\@gradi\p@ \Numero\@tempa\@tdD
\DirFromAngle\@tempa to\@Dir
\MultVect\@V by\@Dir to\@sPun
\edef\@tempA{\ifx\Segno-\m@ne\else\@ne\fi}%
\MultVect\@sPun by 0,\@tempA to\@vPun
\DirOfVect\@vPun to\@Dir
\AddVect\@sPun and #1 to \@sPun
\GetCoord(\@sPun)\@tdX\@tdY
\@tdD\ifx\Segno--\fi\DeltaGradi\p@
\@tdD=.5\@tdD \Numero\DeltaGradi\@tdD
\DirFromAngle\DeltaGradi to\@Dird
\MultVect\@Dir by*\@Dird to\@Dir
\GetCoord(\@Dir)\@xnum\@ynum
\put(\@tdX,\@tdY){\vector(\@xnum,\@ynum){0}}%
\@tdE =\ifx\Segno--\fi\DeltaGradi\p@
\advance\@tdA -\@tdE \Numero\@gradi\@tdA
\CopyVect#1to\@Cent \GetCoord(\@pPun)\@pPunX\@pPunY
\@@Arc
\strokepath\ignorespaces}%
\def\@VARC(#1)(#2){%
\ifdim\@tdA>\z@
  \let\Segno+%
\else
  \@tdA=-\@tdA \let\Segno-%
\fi \Numero\@gradi\@tdA
\ifdim\@tdA>360\p@
 \PackageWarning{curve2e}{The arc aperture is \@gradi\space degrees
      and gets reduced\MessageBreak%
```

```
      to the range 0--360 taking the sign into consideration}%
 \@whiledim\@tdA>360\p@\do{\advance\@tdA-360\p@}%
\fi
\SubVect#1from#2to\@V \ModOfVect\@V to\@Raggio \CopyVect#2to\@pPun
\@tdE=\pIIe@FAW\@wholewidth \@tdE=0.8\@tdE
\DividE\@tdE by \@Raggio\unitlength to\DeltaGradi
\@tdD=\DeltaGradi\p@ \@tdD=57.29578\@tdD \Numero\DeltaGradi\@tdD
\@tdD=\if\Segno--\fi\@gradi\p@ \Numero\@tempa\@tdD
\DirFromAngle\@tempa to\@Dir
\MultVect\@V by\@Dir to\@sPun% corrects the end point
\edef\@tempA{\if\Segno--\fi1}%
\MultVect\@sPun by 0,\@tempA to\@vPun
\DirOfVect\@vPun to\@Dir
\AddVect\@sPun and #1 to \@sPun
\GetCoord(\@sPun)\@tdX\@tdY
\@tdD\if\Segno--\fi\DeltaGradi\p@
\@tdD=.5\@tdD \Numero\@tempB\@tdD
\DirFromAngle\@tempB to\@Dird
\MultVect\@Dir by*\@Dird to\@Dir
\GetCoord(\@Dir)\@xnum\@ynum
\put(\@tdX,\@tdY){\vector(\@xnum,\@ynum){0}}% end point arrowt ip
\@tdE =\DeltaGradi\p@
\advance\@tdA -2\@tdE \Numero\@gradi\@tdA
\CopyVect#1to\@Cent \GetCoord(\@pPun)\@pPunX\@pPunY
\SubVect\@Cent from\@pPun to \@V
\edef\@tempa{\if\Segno-\else-\fi\@ne}%
\MultVect\@V by0,\@tempa to\@vPun
\@tdE\if\Segno--\fi\DeltaGradi\p@
\Numero\@tempB{0.5\@tdE}%
\DirFromAngle\@tempB to\@Dird
\MultVect\@vPun by\@Dird to\@vPun% corrects the starting point
\DirOfVect\@vPun to\@Dir\GetCoord(\@Dir)\@xnum\@ynum
\put(\@pPunX,\@pPunY){\vector(\@xnum,\@ynum){0}}% starting point arrow tip
\edef\@tempa{\if\Segno--\fi\DeltaGradi}%
\DirFromAngle\@tempa to \@Dir
\SubVect\@Cent from\@pPun to\@V
\MultVect\@V by\@Dir to\@V
\AddVect\@Cent and\@V to\@pPun
\GetCoord(\@pPun)\@pPunX\@pPunY
\@@Arc
\strokepath\ignorespaces}%
\def\CurveBetween#1and#2WithDirs#3and#4{%
\StartCurveAt#1WithDir{#3}\relax
\CurveTo#2WithDir{#4}\CurveFinish\ignorespaces}%
\def\StartCurveAt#1WithDir#2{%
\begingroup
\GetCoord(#1)\@tempa\@tempb
\CopyVect\@tempa,\@tempb to\@Pzero
\pIIe@moveto{\@tempa\unitlength}{\@tempb\unitlength}%
\GetCoord(#2)\@tempa\@tempb
```

```
\CopyVect\@tempa,\@tempb to\@Dzero
\DirOfVect\@Dzero to\@Dzero
\ignorespaces}
\def\ChangeDir<#1>{%
\GetCoord(#1)\@tempa\@tempb
\CopyVect\@tempa,\@tempb to\@Dzero
\DirOfVect\@Dzero to\@Dzero
\ignorespaces}
\def\CurveFinish{\strokepath\endgroup\ignorespaces}%
\def\FillCurve{\fillpath\endgroup\ignorespaces}
\def\CurveEnd{\fillstroke\endgroup\ignorespaces}
\def\CbezierTo#1WithDir#2AndDists#3And#4{%
\GetCoord(#1)\@tX\@tY \MakeVectorFrom\@tX\@tY to\@Puno
\GetCoord(#2)\@tX\@tY \MakeVectorFrom\@tX\@tY to \@Duno
\DirOfVect\@Duno to\@Duno
\ScaleVect\@Dzero by#3to\@Czero \AddVect\@Pzero and\@Czero to\@Czero
\ScaleVect\@Duno by-#4to \@Cuno \AddVect\@Puno and\@Cuno to \@Cuno
\GetCoord(\@Czero)\@XCzero\@YCzero
\GetCoord(\@Cuno)\@XCuno\@YCuno
\GetCoord(\@Puno)\@XPuno\@YPuno
\pIIe@curveto{\@XCzero\unitlength}{\@YCzero\unitlength}%
             {\@XCuno\unitlength}{\@YCuno\unitlength}%
             {\@XPuno\unitlength}{\@YPuno\unitlength}%
\CopyVect\@Puno to\@Pzero
\CopyVect\@Duno to\@Dzero
\ignorespaces}%
\def\CbezierBetween#1And#2WithDirs#3And#4UsingDists#5And#6{%
\StartCurveAt#1WithDir{#3}\relax
\CbezierTo#2WithDir#4AndDists#5And{#6}\CurveFinish}

\def\@isTension#1;#2!!{\def\@tempA{#1}%
\def\@tempB{#2}\unless\ifx\@tempB\empty\strip@semicolon#2\fi}
\def\strip@semicolon#1;{\def\@tempB{#1}}
\def\CurveTo#1WithDir#2{%
\def\@Tuno{1}\def\@Tzero{1}\relax
\edef\@Puno{#1}\@isTension#2;!!%
\expandafter\DirOfVect\@tempA to\@Duno
\bgroup\unless\ifx\@tempB\empty\GetCoord(\@tempB)\@Tzero\@Tuno\fi
\DistanceAndDirOfVect\@Puno minus\@Pzero to\@Chord and\@DirChord
\MultVect\@Dzero by*\@DirChord to \@Dpzero
\MultVect\@Duno by*\@DirChord to \@Dpuno
\GetCoord(\@Dpzero)\@DXpzero\@DYpzero
\GetCoord(\@Dpuno)\@DXpuno\@DYpuno
\DivideFN\@Chord by2 to\@semichord
\ifdim\@DXpzero\p@=\z@
   \@tdA=1.333333\p@
   \Numero\@KCzero{\@semichord\@tdA}%
\fi
\ifdim\@DYpzero\p@=\z@
  \@tdA=1.333333\p@
```

```
  \Numero\@Kpzero{\@semichord\@tdA}%
\fi
\unless\ifdim\@DXpzero\p@=\z@
  \unless\ifdim\@DYpzero\p@=\z@
    \edef\@CosDzero{\ifdim\@DXpzero\p@<\z@ -\fi\@DXpzero}%
    \edef\@SinDzero{\ifdim\@DYpzero\p@<\z@ -\fi\@DYpzero}%
    \@tdA=\@semichord\p@ \@tdA=1.333333\@tdA
    \DividE\@tdA by\@SinDzero\p@ to \@KCzero
    \@tdA=\dimexpr(\p@-\@CosDzero\p@)\relax
    \DividE\@KCzero\@tdA by\@SinDzero\p@ to \@KCzero
  \fi
\fi
\MultiplyFN\@KCzero by \@Tzero to \@KCzero
\ScaleVect\@Dzero by\@KCzero to\@CPzero
\AddVect\@Pzero and\@CPzero to\@CPzero
\ifdim\@DXpuno\p@=\z@
   \@tdA=-1.333333\p@
   \Numero\@KCuno{\@semichord\@tdA}%
\fi
\ifdim\@DYpuno\p@=\z@
  \@tdA=-1.333333\p@
  \Numero\@KCuno{\@semichord\@tdA}%
\fi
\unless\ifdim\@DXpuno\p@=\z@
    \unless\ifdim\@DYpuno\p@=\z@
    \edef\@CosDuno{\ifdim\@DXpuno\p@<\z@ -\fi\@DXpuno}%
    \edef\@SinDuno{\ifdim\@DYpuno\p@<\z@ -\fi\@DYpuno}%
    \@tdA=\@semichord\p@ \@tdA=-1.333333\@tdA
    \DividE\@tdA by \@SinDuno\p@ to \@KCuno
    \@tdA=\dimexpr(\p@-\@CosDuno\p@)\relax
    \DividE\@KCuno\@tdA by\@SinDuno\p@ to \@KCuno
  \fi
\fi
\MultiplyFN\@KCuno by \@Tuno to \@KCuno
\ScaleVect\@Duno by\@KCuno to\@CPuno
\AddVect\@Puno and\@CPuno to\@CPuno
\GetCoord(\@Puno)\@XPuno\@YPuno
\GetCoord(\@CPzero)\@XCPzero\@YCPzero
\GetCoord(\@CPuno)\@XCPuno\@YCPuno
\pIIe@curveto{\@XCPzero\unitlength}{\@YCPzero\unitlength}%
             {\@XCPuno\unitlength}{\@YCPuno\unitlength}%
             {\@XPuno\unitlength}{\@YPuno\unitlength}\egroup
\CopyVect\@Puno to\@Pzero
\CopyVect\@Duno to\@Dzero
\ignorespaces}%
\def\Curve{\@ifstar{\let\fillstroke\fillpath\Curve@}%
{\let\fillstroke\strokepath\Curve@}}
\def\Curve@(#1)<#2>{%
    \StartCurveAt#1WithDir{#2}%
    \@ifnextchar\lp@r\@Curve{%
```

```
    \PackageWarning{curve2e}{%
    Curve specifications must contain at least two nodes!\Messagebreak
    Please, control your Curve specifications\MessageBreak}}}
\def\@Curve(#1)<#2>{%
    \CurveTo#1WithDir{#2}%
    \@ifnextchar\lp@r\@Curve{%
    \@ifnextchar[\@ChangeDir\CurveEnd}}
\def\@ChangeDir[#1]{\ChangeDir<#1>\@Curve}
\def\Qurve{\@ifstar{\let\fillstroke\fillpath\Qurve@}%
{\let\fillstroke\strokepath\Qurve@}}

\def\Qurve@(#1)<#2>{%
    \StartCurveAt#1WithDir{#2}%
    \@ifnextchar\lp@r\@Qurve{%
    \PackageWarning{curve2e}{%
      Quadratic curve specifications must contain at least
      two nodes!\Messagebreak
      Please, control your Qurve specifications\MessageBreak}}}%
\def\@Qurve(#1)<#2>{\QurveTo#1WithDir{#2}%
    \@ifnextchar\lp@r\@Qurve{%
    \@ifnextchar[\@ChangeQDir\CurveEnd}}%
\def\@ChangeQDir[#1]{\ChangeDir<#1>\@Qurve}%
\def\QurveTo#1WithDir#2{%
\edef\@Puno{#1}\DirOfVect#2to\@Duno\bgroup
\DistanceAndDirOfVect\@Puno minus\@Pzero to\@Chord and\@DirChord
\MultVect\@Dzero by*\@Duno to \@Scalar
\YpartOfVect\@Scalar to \@YScalar
\ifdim\@YScalar\p@=\z@
\PackageWarning{curve2e}%
   {Quadratic Bezier arcs cannot have their starting\MessageBreak
   and ending directions parallel or antiparallel with\MessageBreak
   each other. This arc is skipped and replaced with
   a dotted line.\MessageBreak}%
   \Dotline(\@Pzero)(\@Puno){2}\relax
\else
\MultVect\@Dzero by*\@DirChord to \@Dpzero
\MultVect\@Duno by*\@DirChord to \@Dpuno
\GetCoord(\@Dpzero)\@DXpzero\@DYpzero
\GetCoord(\@Dpuno)\@DXpuno\@DYpuno
\MultiplyFN\@DXpzero by\@DXpuno to\@XXD
\MultiplyFN\@DYpzero by\@DYpuno to\@YYD
\unless\ifdim\@YYD\p@<\z@\ifdim\@XXD\p@<\z@
\PackageWarning{curve2e}%
   {Quadratic Bezier arcs cannot have inflection points\MessageBreak
   Therefore the tangents to the starting and ending arc\MessageBreak
   points cannot be directed to the same half plane.\MessageBreak
   This arc is skipped and replaced by a dotted line\MessageBreak}%
   \Dotline(\@Pzero)(\@Puno){2}\fi
\else
\edef\@CDzero{\@DXpzero}\relax
```

```
\edef\@SDzero{\@DYpzero}\relax
\edef\@CDuno{\@DXpuno}\relax
\edef\@SDuno{\@DYpuno}\relax
\MultiplyFN\@SDzero by\@CDuno to\@tempA
\MultiplyFN\@SDuno by\@CDzero to\@tempB
\edef\@tempA{\strip@pt\dimexpr\@tempA\p@-\@tempB\p@}\relax
\@tdA=\@SDuno\p@ \@tdB=\@Chord\p@ \@tdC=\@tempA\p@
\edef\@tempC{\strip@pt\dimexpr \@tdA*\@tdB/\@tdC}\relax
\MultiplyFN\@tempC by\@CDzero to \@XC
\MultiplyFN\@tempC by\@SDzero to \@YC
\ModOfVect\@XC,\@YC to\@KC
\ScaleVect\@Dzero by\@KC to\@CP
\AddVect\@Pzero and\@CP to\@CP
\GetCoord(\@Pzero)\@XPzero\@YPzero
\GetCoord(\@Puno)\@XPuno\@YPuno
\GetCoord(\@CP)\@XCP\@YCP
\@ovxx=\@XPzero\unitlength \@ovyy=\@YPzero\unitlength
\@ovdx=\@XCP\unitlength \@ovdy=\@YCP\unitlength
\@xdim=\@XPuno\unitlength \@ydim=\@YPuno\unitlength
      \pIIe@bezier@QtoC\@ovxx\@ovdx\@ovro
      \pIIe@bezier@QtoC\@ovyy\@ovdy\@ovri
      \pIIe@bezier@QtoC\@xdim\@ovdx\@clnwd
      \pIIe@bezier@QtoC\@ydim\@ovdy\@clnht
      \pIIe@moveto\@ovxx\@ovyy
      \pIIe@curveto\@ovro\@ovri\@clnwd\@clnht\@xdim\@ydim
\fi\fi\egroup
\CopyVect\@Puno to\@Pzero
\CopyVect\@Duno to\@Dzero
\ignorespaces}

```

References

[1] Gäßlein H., Niepraschk R., and Tkadlec J. *The `pict2e` package*, 2019, PDF documentation of `pict2e`; this package is part of any modern complete distribution of the TEX system; it may be read by means of the line command `texdoc pict2e`. In case of a basic or partial system installation, the package may be installed by means of the specific facilities of the distribution.