The extension package curve2e

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Abstract

This file documents the <code>curve2e</code> extension package to the <code>pict2e</code> bundle implementation; the latter was described by Lamport himself in the 1994 second edition of his \LaTeX handbook.

Please take notice that on April 2011 a new updated version of the package pict2e has been released that incorporates some of the commands defined in early versions of this package; apparently there are no conflicts, but only the advanced features of curve2e remain available for extending the above package.

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This extension redefines some commands and introduces some more drawing facilities that allow to draw circular arcs and arbitrary curves with the minimum of user intervention. This version is open to the contribution of other users as well as it may be incorporated in other people's packages. Please cite the original author and the chain of contributors.

1 The configuration file

This package curve2e is distributed with a ltxdoc.cfg configuration file that contains, besides the preamble and the postamble comment lines, the following line of code:

\AtBeginDocument{\OnlyDescription}

If you want to type the whole documentation, comment out that code line in the ltxdoc.cfg file. This is the only modification allowed by the LPPL licence that does not require to change the file name.

For your information, the initial part is about 20 pages long; the whole documentation is about 80 pages long.

2 Package pict2e and this extension curve2e

Package pict2e was announced in issue 15 of latexnews around December 2003; it was declared that the new package would replace the dummy one that has been accompanying every release of $\text{IAT}_{E}X\ 2_{\varepsilon}$ since its beginnings in 1994. The dummy package was just issuing an info message that simply announced the temporary unavailability of the real package.

Eventually Gäßlein and Niepraschk implemented what Lamport himself had already documented in the second edition of his LATEX handbook, that is a LATEX package that contained the macros capable of removing all the limitations contained in the standard commands of the original picture environment; specifically what follows.

- 1. The line and vector slopes were limited to the ratios of relative prime onedigit integers of magnitude not exceeding 6 for lines and 4 for vectors.
- 2. Filled and unfilled full circles were limited by the necessarily limited number of specific glyphs contained in the special LATEX picture fonts.
- 3. Quarter circles were also limited in their radii for the same reason.
- 4. Ovals (rectangles with rounded corners) could not be too small because of the unavailability of small radius quarter circles, nor could be too large, in the sense that after a certain radius the rounded corners remained the same and would not increase proportionally to the oval size.
- 5. Vector arrows had only one possible shape and matched the limited number of vector slopes.

6. For circles and inclined lines and vectors just two possible thicknesses were available.

The package pict2e removes most if not all the above limitations.

- 1. Line and vector slopes are virtually unlimited; the only remaining limitation is that the direction coefficients must be three-digit integer numbers; they need not be relatively prime; with the 2009 upgrade even this limitation was removed and now slope coefficients can be any fractional number whose magnitude does not exceed 16 384, the maximum dimension in points that T_FX can handle.
- 2. Filled and unfilled circles can be of any size.
- 3. Ovals can be designed with any specified corner curvature and there is virtually no limitation to such curvatures; of course corner radii should not exceed half the lower value between the base and the height of the oval.
- 4. There are two shapes for the arrow tips; the triangular one traditional with LATEX vectors, or the arrow tip with PostScript style.
- 5. The \linethickness command changes the thickness of all lines, straight, curved, vertical, horizontal, arrow tipped, et cetera.

This specific extension package curve2e adds the following features.

1. Point coordinates my be specified in both cartesian and polar form: internally they are handled as cartesian coordinates, but the user can specify his/her points also in polar form. In order to avoid confusion with other graphic packages, curve2e uses the usual comma separated couple $\langle x,y\rangle$ of integer or fractional numbers for cartesian coordinates, and the couple $\langle \theta \rangle : \langle \rho \rangle$ for polar coordinates (the angle preceding the radius). All graphic object commands accept polar or cartesian coordinates at the choice of the user who may use for each object the formalism s/he prefers. Also the put and \multiput commands have been redefined so as to accept cartesian or polar coordinates.

Of course the user must pay attention to the meaning of cartesian vs. polar coordinates. Both imply a displacement with respect the actual origin of the axes. So when a circle is placed at coordinates a, b with a normal \put command, the circle is placed exactly in that point; with a normal \put command the same happens if coordinates $\alpha:\rho$ are specified. But if the \put command is nested into another \put command, the current origin of the axes is displaced — this is obvious and the purpose of nesting \put commands is exactly that. But if a segment is specified so that its ending point is at a specific distance and in specific direction form its starting point, polar coordinates appear to be the most convenient to use; in this case, though, the origin of the axes become the stating point of the segment, therefore the segment might be drawn in a strange way. Attention has been

- paid to avoid such misinterpretation, but maybe some unusual situation may not have come to my mind; feedback is very welcome. Meanwhile pay attention when you use polar coordinates.
- 2. Most if not all cartesian coordinate pairs and slope pairs are treated as ordered pairs, that is complex numbers; in practice the user does not notice any difference from what s/he was used to, but all the mathematical treatment to be applied to these entities is coded as complex number operations, since complex numbers may be viewed non only as ordered pairs, but also as vectors or as roto-amplification operators.
- 3. Commands for setting the line terminations were introduced; the user can chose between square or rounded caps; the default is set to rounded caps; now this feature is directly available with pict2e.
- 4. Commands for specifying the way two lines or curves join to one another.
- 5. Originally the \line macro was redefined so as to allow large (up to three digits) integer direction coefficients, but maintaining the same syntax as in the original picture environment; now pict2e removes the integer number limitations and allows fractional values, initially implemented by curve2e.
- 6. A new macro \Line was originally by curve2e defined so as to avoid the need to specify the horizontal projection of inclined lines; now this functionality is available directly with pict2e; but this curve2e macro name now conflicts with pict2e 2009 version; therefore its name is changed to \Line and supposedly it will not be used very often, if ever, by the end user (but it is used within this package macros).
- 7. A new macro \LINE was defined in order to join two points specified with their coordinates; this is now the normal behaviour of the \Line macro of pict2e so that in this package \LINE is now renamed \segment; there is no need to use the \put command with this line specification.
- 8. A new macro \DashLine (alias: \Dline) is defined in order to draw dashed lines joining any two given points; the dash length and gap (equal to one another) get specified through one of the macro arguments. The stating point mai be specified in cartesiano or polar form; the end point in cartesian format specifies the desired end point; while if the second point is in polar form it is meant relative to the starting point, not as an absolute end point. See the examples further on.
- 9. A similar new macro \Dotline is defined in order to draw dotted straight lines as a sequence of equally spaced dots, where the gap can be specified by the user; such straight line may have any inclination, as well as the above dashed lines. Polar coordinates for the second point have the same relative meaning as specified for the \Dashline macro.

- 10. Similar macros are redefined for vectors; \vector redefines the original macro but with the vector slope limitations removed; \Vector gets specified with its two horizontal and vertical components in analogy with \LIne; \VECTOR joins two specified points (without using the \put command) with the arrow pointing to the second point.
- 11. A new macro \polyline for drawing polygonal lines is defined that accepts from two vertices up to an arbitrary (reasonably limited) number of them (available now also in pict2e); here it is redefined so as to allow an optional specification of the way segments for the polyline are joined to one another. Vertices may be specified with polar coordinates and are always relative to the preceding point.
- 12. The pict2e polygon macro to draw closed polylines (in practice general polygons) has been redefined in such a way that it can accept the various vertices specified with (relative) polar coordinates. The polygon* macro produces a color filled polygon; the default color is black, but a different color may be specified with the usual \color command given within the same group where \polygon* is enclosed.
- 13. A new macro \arc is defined in order to draw an arc with arbitrary radius and arbitrary aperture (angle amplitude); this amplitude is specified in sexagesimal degrees, not in radians; a similar functionality is now achieved with the \arc macro of pict2e, which provides also the starred version \arc* that fills up the interior of the generated circular arc with the current color. It must be noticed that the syntax is slightly different, so that it's reasonable that these commands, in spite of producing identical arcs, might be more comfortable with this or that syntax.
- 14. Two new macros \VectorArc and \VectorARC are defined in order to draw circular arcs with an arrow at one or both ends.
- 15. A new macro \Curve is defined so as to draw arbitrary curved lines by means of cubic Bézier splines; the \Curve macro requires only the curve nodes and the directions of the tangents at each node. The starred version fills up the interior of the curve with the current color.
- 16. the above \Curve macro is a recursive macro that can draw an unlimited (reasonably limited) number of connected Bézier spline arcs with continuous tangents except for cusps; these arcs require only the specification of the tangent direction at the interpolation nodes. It is possible to use a lower level macro \CbezierTo that does the same but lets the user specify the control points of each arc; it is more difficult to use but it is more performant.
- 17. The basic macros used within the cumulative \Curve macro can be used individually in order to draw any curve, one cubic arc at the time; but they are intended for internal use, even if it is not prohibited to use them; by themselves such arcs are not different form those used by Curve, but the

final command, \FillCurve, should be used in place of \CurveFinish, so as to fill up the closed path with the locally specified color; see figure 10. It is much more convenient to use the starred version of the \Curve macro.

The pict2e package already defines macros such as \moveto, \lineto, \curveto, \closepath, \fillpath, and \strokepath; of course these macros can be used by the end user, and sometimes they perform better than the macros defined in this package, because the user has a better control on the position of the Bézier splines control points, while here the control points are sort of rigid. It would be very useful to resort to the hobby package, but its macros are compatible with those of the tikz and pgf packages, not with curve2e; an interface should be created in order to deal with the hobby package, but this has not been done yet. In any case they are redefined so as to accept symbolic names for the point coordinates in both the cartesian and polar form.

In order to make the necessary calculations many macros have been defined so as to use complex number arithmetics to manipulate point coordinates, directions (unit vectors, also known as 'versors'), rotations and the like. In the first versions of this package the trigonometric functions were also defined in a way that the author believed to be more efficient than those defined by the trig package; in any case the macro names were sufficiently different to accommodate both definition sets in the same IATEX run. With the progress of the IATEX 3 language, the xfp has recently become available, by which any sort of calculations can be done with floating point decimal numbers; therefore the most common algebraic, irrational and transcendental functions can be computed in the background with the stable internal floating point facilities. We maintain some computation with complex number algebra, but use the xfp functionalities for other computations.

Many aspects of this extension could be fine tuned for better performance; many new commands could be defined in order to further extend this extension. If the new service macros are accepted by other TEX and LATEX programmers, this version could become the start for a real extension of the pict2e package or even become a part of it. Actually some macros have already been included in the pict2e package. The \Curve algorithm, as I said before, might be redefined so as to use the macros introduced in the hobby package, that implements for the tikz and pgf packages the same functionalities that John Hobby implemented for the METAFONT and METAPOST programs.

For these reasons I suppose that every enhancement should be submitted to Gäßlein, Niepraschk, and Tkadlec who are the prime maintainers of pict2e; they are the only ones who can decide whether or not to incorporate new macros in their package.

3 Summary and examples of new commands

This package curve2e extends the power of pict2e with the following modifications and the following new commands.

1. This package curve2e calls directly the LATEX packages color and pict2e

to which it passes any possible option that the latter can receive; actually the only options that make sense for pict2e are those concerning the arrow tips, either LaTeX or PostScript styled, because it is assumed that if you use this package you are not interested in using the original LaTeX commands. See the pict2e documentation in order to see the correct options pict2e can receive. If the user wants to use the xcolor package, it has to load this one before curve2e.

- 2. The user is offered new commands in order to control the line terminators and the line joins; specifically:
 - \roundcap: the line is terminated with a semicircle;
 - \squarecap: the line is terminated with a half square;
 - \roundjoin: two lines are joined with a rounded join;
 - \beveljoin: two lines are joined with a bevel join;
 - \miterjoin: two lines are joined with a miter join.

All the above commands should respect the intended range; but since they act at the PostScript or PDF level, not at TEX level, it might be necessary to issue the necessary command in order to restore the previous terminator or join.

3. The commands \linethickness, \thicklines, \thinlines together with \defaultlinethickness always redefine the internal \@wholewidth and \@halfwidth so that the latter always refer to a full width and to a half of it in this way: if you issue the command \defaultlinewidth{2pt} all thin lines will be drawn with a thickness of 1 pt while, if a drawing command directly refers to the internal value \@wholewidth, its line will be drawn with a thickness of 2 pt. If one issues the declaration \thinlines all lines will be drawn with a 1 pt width, but if a command refers to the internal value \@halfwidth the line will be drawn with a thickness of 0.5 pt. The command \linethickness redefines the above internals but does not change the default width value; all these width specifications apply to all lines, straight ones, curved ones, circles, ovals, vectors, dashed, et cetera. It's better to recall that \thinlines and \thicklines are declarations that do not take arguments; on the opposite the other two commands follow the standard syntax:

```
\label{linethickness} $$ \left( dimensioned \ value \right) $$ \defaultlinewidth $$ \left( dimensioned \ value \right) $$
```

where $\langle dimensioned\ value \rangle$ means a length specification complete of its units, or a dimensional expression.

4. Straight lines and vectors are redefined in such a way that fractional slope coefficients may be specified; the zero length line does not produce errors and is ignored; the zero length vectors draw only the arrow tips.

```
\unitlength=.5mm
\begin{picture}(60,20)
\GraphGrid(80,20)
\put(0,0){\vector(1.5,2.3){10}}
\put(20,0){\Vector(10,15.33333)}
\VECTOR(40,0)(50,15.33333)
\ifdefined\VVECTOR(60,0)(80,10)\fi
\end{picture}
```



Figure 1: Three (displaced) identical vectors obtained with the three vector macros; a double tipped vector is also shown.

5. New line and vector macros are defined that avoid the necessity of specifying the horizontal component; \put(3,4){\LIne(25,15)} specifies a segment that starts at point (3,4) and goes to point (3+25,4+15); the command \segment(3,4)(28,19) achieves the same result without the need of using command \put.

The same applies to the vector commands \Vector and \VECTOR and \VVECTOR; the latter command behaves as \VECTOR but draws a vector with arrow tips at both ends; furthermore this command is available only with this new release of the curve2e package. Experience has shown that the commands intended to join two specified points are particularly useful.

6. The \polyline command has been introduced: it accepts an unlimited list of point coordinates enclosed within round parentheses; the command draws a sequence of connected segments that join in order the specified points; the syntax is:

```
\polyline[\langle optional\ join\ style \rangle](\langle P_1 \rangle)(\langle P_2 \rangle)\dots(\langle P_n \rangle)
```

See figure 2 where a regular pentagon is drawn; usage of polar coordinates is also shown; please notice how relative polar coordinates act in this figure.

```
\unitlength=.5mm
\begin{picture}(40,32)(-20,-20)
\polyline(90:20)(162:20)(234:20)(306:20)(378:20)(90:20)
\end{picture}
```

Figure 2: Polygonal line obtained by means of the **\polyline** command; vertex coordinates are in polar form.

Examples of using polar and cartesian coordinates are shown in figure 3.

A similar example may be obtained with the \polygon macro that does not require to terminate the polyline at the starting point. Figure 4 shows how to get a coloured filled pentagon.

7. The new command \Dashline (alias: \Dline for backwards compatibility)

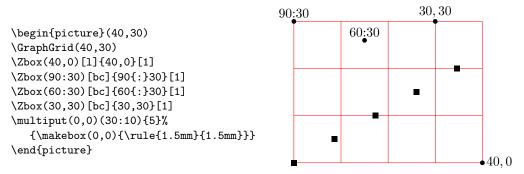


Figure 3: Use of cartesian and absolute polar coordinates. The \Zbox macro is just a shortcut to set a small dot with a (math) legend close to it; its definition by means of the xparse functionalities is straightforward.

```
\unitlength=.5mm
\begin{picture}(40,32)(-20,-20)
\color{magenta}
\polygon*(90:20)(162:20)(234:20)(306:20)(378:20)
\end{picture}
```

Figure 4: A pentagon obtained by means of the \polygon* command; vertex coordinates are in relative polar form.

```
\Delta (\langle first\ point \rangle) (\langle second\ point \rangle) \{\langle dash\ length \rangle\}
```

draws a dashed line containing as many dashes as possible, just as long as specified, and separated by a gap exactly the same size; actually, in order to make an even gap-dash sequence, the desired dash length is used to do some computations in order to find a suitable length, close to the one specified, such that the distance of the end points is evenly divided in equally sized dashes and gaps. The end points may be anywhere in the drawing area, without any constraint on the slope of the joining segment. The desired dash length is specified as a fractional multiple of \unitlength; see figure 5.

Another example of usage of cartesian and polar coordinates usage is shown in figure 3 together with its code.

Another

8. Analogous to \Dashline, a new command \Dotline draws a dotted line with the syntax:

```
\Dotline(\langle first\ point \rangle)(\langle end\ point \rangle){\langle dot\ gap \rangle}
```

See figures 5 and 7 for examples.

9. \GraphGrid is a command that draws a red grid under the drawing with lines separated 10\unitlengths apart; it is described only with a comma

```
\unitlength=1mm
\begin{picture}(40,40)
\GraphGrid(40,40)
\Dashline(0,0)(40,10){4}
\put(0,0){\circle*{2}}
\Dashline(40,10)(0,25){4}
\put(40,10){\circle*{2}}
\Dashline(0,25)(20,40){4}
\put(0,25){\circle*{2}}
\put(20,40){\circle*{2}}
\Dotline(0,0)(40,40){2}
\put(40,40){\circle*{2}}
\end{picture}
```

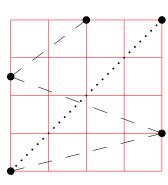


Figure 5: Dashed lines and graph grid

\begin{picture}(40,30) \GraphGrid(40,30) \Dashline(0,0)(40,10){2}\Dashline(0,0)(40,20){2} \Dashline(0,0)(40,30){2}\Dashline(0,0)(30,30){2} \Dashline(0,0)(20,30){2}\Dashline(0,0)(10,30){2} {\color{blue}% \Dashline(40,0)(108:30){2} \Dashline(40,0)(126:30){2} \Dashline(40,0)(144:30){2} \Dashline(40,0)(162:30){2}} \end{picture}

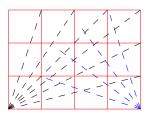


Figure 6: Different length dashed lines with the same nominal dash length; notice the relative polar coordinates used for the dashed lines starting at the grid lower right vertex.

separated couple of numbers, representing the base and the height of the grid, see figure 5; it's better to specify multiples of ten and the grid can be placed anywhere in the drawing canvas by means of \put, whose cartesian coordinates are multiples of 10; nevertheless the grid line distance is rounded to the nearest multiple of 10, while the point coordinates specified to \put are not rounded at all; therefore some care should be used to place the working grid on the drawing canvas. This grid is intended as an aid while drawing; even if you sketch your drawing on millimetre paper, the drawing grid turns out to be very useful; one must only delete or comment out the command when the drawing is finished. Several examples of usage of such grid are shown in several figures.

10. New trigonometric function macros have been computed by means of the functionalities of the xfp package. The difference woth the other existing macros is that angles are specified in sexagesimal degrees, so that the users need not transform to radians. The computations are done taking into

Figure 7: Different length dotted lines with the same nominal dot gap; again notice the relative polar coordinates for the dotted lines starting at the grid lower right vertex.

account that "abnormal" values can occasionally be avoided, for example $\tan 90^\circ$ must be avoided and replaced with a suitably large number, because the TeX system does not handle "infinity".

These trigonometric functions are used within the complex number macros; but if the user wants to use them the syntax is the following:

```
\SinOf\langle angle \rangle to\langle control\ sequence\rangle \CosOf\langle angle \rangle to\langle control\ sequence\rangle \TanOf\langle angle \rangle to\langle control\ sequence\rangle
```

The $\langle control\ sequence \rangle$ may then be used, for example, as a multiplying factor of a length.

11. Arcs can be drawn as simple circular arcs, or with one or two arrows at their ends (curved vectors); the syntax is:

```
\label{eq:localization} $\operatorname{\conter}(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter}))(\operatorname{\conter})(\operatorname{\conter}))(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter}))(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter}))(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(\operatorname{\conter})(
```

If the angle is specified numerically it must be enclosed in braces, while if it is specified with a control sequence the braces (curly brackets) are not necessary. The above macro \Arc draws a simple circular arc without arrows; \VectorArc draws an arc with an arrow tip at the ending point; \VectorARC draws an arc with arrow tips at both ends; see figure 8.

12. A multitude of commands have been defined in order to manage complex numbers; actually complex numbers are represented as a comma separated pair of fractional numbers (here we use only cartesian coordinates). They are used to address specific points in the drawing plane, but also as operators so as to scale and rotate other objects. In the following $\langle vector \rangle$ means a comma separated pair of fractional numbers, $\langle vector macro \rangle$ means a macro

```
\unitlength=0.5mm
\begin{picture}(60,40)
\GraphGrid(60,40)
\Arc(0,20)(30,0)\{60}
\VECTOR(0,20)(30,0)\VECTOR(0,20)(32.5,36)
\VectorArc(0,20)(15,10)\{60}
\put(20,20)\\makebox(0,0)[1]\{\$60^\circ\}\}
\VectorARC(60,20)(60,0)\{-180}
\end{picture}
```

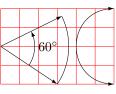


Figure 8: Arcs and curved vectors

that contains a comma separated pair of fractional numbers; $\langle angle\ macro\rangle$ means a macro that contains the angle of a vector in sexagesimal degrees; $\langle argument\rangle$ means a brace delimited numeric value, even a macro; $\langle numeric\ macro\rangle$ means a macro that contains a fractional number; macro is a valid macro name, i.e. a backslash followed by letters, or anything else that can receive a definition. A direction of a vector is its versor; the angle of a vector is the angle between the vector and the positive x axis in counterclockwise direction, as it is used in the Euler formula $\vec{v} = Me^{j\varphi}$.

- \MakeVectorFrom\(\lambda numeric macro\)\(\lambda numeric macro\)\(\tau \)
- \CopyVect\(first vector\)\to\(second vector macro\)
- $\bullet \quad \backslash \texttt{ModOfVect} \langle vector \rangle \texttt{to} \langle modulus\ macro \rangle$
- $\bullet \quad \verb|\DirOfvect| \langle vector \rangle \verb| to | \langle versor \ macro \rangle|$
- $\ModAndDirOfVect \langle vector \rangle to \langle modulus\ macro \rangle and \langle versor\ macro \rangle$
- \ModAndAngleOfVect(vector)to(modulus macro)and(angle macro)
- \DistanceAndDirOfVect\(\lambda\)1st vector\(\rangle\)minus\(\lambda\)2nd vector\(\rangle\)to\(\lambda\)istance macro\(\rangle\)and\(\lambda\)versor macro\(\rangle\)
- $\Text{Vector}\$ to $\Text{vector}\$
- \YpartOfVect\(\langle vector \rangle \to \langle macro \rangle \)
- $\DirFromAngle\langle angle\rangle to\langle versor\ macro\rangle$
- $\bullet \quad \texttt{\normalflet} \ \, \texttt{\normalflet} \ \,$
- $\ScaleVect(vector)$ by $\langle scaling\ factor \rangle$ to $\langle vector\ macro \rangle$
- $\ConjVect(vector)$ to $(conjugate\ vector\ macro)$
- \SubVect\subtrahend vector\from\minuend vector\to\vector macro\
- $\AddVect\langle first\ vector\rangle and\langle second\ vector\rangle to\langle vector\ macro\rangle$
- $\Mathemath{\mathsf{Multvect}}{\langle first\ vector\rangle}^*{\langle second\ vector\rangle}^*{\langle vector\ macro\rangle}$ (the asterisks are optional; either one changes the second vector into its complex conjugate)
- \MultVect\(first vector\)\by\(\second vector\)\to\(\second vector macro\)\) (discouraged; maintained for backwards compatibility)
- \MultVect\(first vector\)\by*\(second vector\)\to\(vector macro\)\ (discouraged; maintained for backwards compatibility)
- $\Divvect{\langle dividend\ vector\rangle}{\langle divisor\ vector\rangle}{\langle\ vector\ macro\rangle}$
- \DivVect\dividend vector\by\divisor vector\to\vector macro\ (maintained for backwards compatibility)

- 13. General curves can be drawn with the pict2e macro \curve but it requires the specification of the third-order Bézier-spline control points; sometimes it's better to be very specific with the control points and there is no other means to do a decent graph; sometimes the curves to be drawn are not so tricky and a general set of macros can be defined so as to compute the control points, while letting the user specify only the nodes through which the curve must pass, and the tangent direction of the curve in such nodes. Such commands are the following:
 - \Curve to draw a sequence of arcs as explained above, using third order (cubic) Bézier splines. The starred version of this command fills the internal part of the curve with the current color; if the last arc finishes where the fist arc starts, it is clear what is the interior; if it does not, the driver (not the code of this package, but the driver between this code and the physical representation on paper or screen) assumes a straight line closure of the whole path.
 - \Qurve similar to \Curve, but with second order (quadratic) Bézier splines. The starred version fills the interior with the current color.
 - \CurveBetween draws a single cubic Bézier spline between two given nodes and with two given directions vectors.
 - \CBezierBetween draws a single cubic Bézier spline between two given nodes, with two given directions versors along which the control node distances are specified. This is the most general macro (rather difficult to use) with which not only the arc end points are specified but also the control nodes coordinates are given.

The main macro is **\Curve** and must be followed by an "unlimited" sequence of node-direction coordinates as a quadruple defined as

$$(\langle node\ coordinates \rangle) < \langle direction\ vector \rangle >$$

Possibly if a sudden change of direction has to be performed (cusp) another item can be inserted after one of those quadruples in the form

$$\ldots (\langle \ldots \rangle) \langle \ldots \rangle > [\langle new \ direction \ vector \rangle] (\langle \ldots \rangle) \langle \ldots \rangle > \ldots$$

Possibly it is necessary to specify the "tension" or the "looseness" of a specific Bézier arc; such tension parameters range from 0 (zero) to 4; the zero value implies a very stiff arc, as if it was a string subject to a high tension (i.e. with zero looseness); a value of 4 implies a very low tension (very high looseness), almost as if the string was not subject to any tension. In METAFONT or METAPOST language such a concept is used very often; in this package, where the Hobby algorithms are not used, the parameter value appears to mean the opposite of tension. A couple of comma separated tension values may be optionally used, they are separated with a semicolon form the direction vector, and they apply to the arc terminating with the

last node; their specification must precede any possible change of tangent according to this $syntax^1$:

```
\ldots (\langle \ldots \rangle)<\langle direction vector; start tension, end tension\rangle>(\langle \ldots \rangle)<\langle \ldots \rangle>...
```

The \Curve macro does not (still) have facilities for cycling the path, that is to close the path from the last specified node-direction to the first specified node-direction; but, as already mentioned, if the ending node of the last arc does not coincide with the starting node of the first arc, a straight line is assumed to join such nodes; this line does not get drawn, but with starred commands no lines are drawn because only the interior is coloured. The tangent direction need not be specified with a unit vector, although only its direction is relevant; the scaling of the specified direction vector to a unit vector is performed by the macro itself. Therefore one cannot specify the fine tuning of the curve convexity as it can be done with other programs or commands, as, for example, with METAFONT or the pgf/tikz package and environment. See figure 9 for an example.

```
\unitlength=8mm\relax
\begin{picture}(5,5)
\put(0,0){\framebox(5,5){}}\thicklines\roundcap
\Curve(2.5,0)<1,1>(5,3.5)<0,1>%
(4,5)<-1,0>(2.5,3.5)<-.5,-1.2>[-.5,1.2]%
(1,5)<-1,0>(0,3.5)<0,-1>(2.5,0)<1,-1>
\end{picture}
```

Figure 9: A heart shaped curve with cusps drawn with \Curve

```
\unitlength=8mm\relax
\begin{picture}(5,5)
\put(0,0){\framebox(5,5){}}\thicklines\roundcap
\color{green}\relax
\Curve*(2.5,0)<1,1>(5,3.5)<0,1>%
(4,5)<-1,0>(2.5,3.5)<-.5,-1.2>[-.5,1.2]%
(1,5)<-1,0>(0,3.5)<0,-1>(2.5,0)<1,-1>
\end{picture}
```



Figure 10: Coloring the inside of a closed path drawn with \Curve*

¹The tension may be specified only for cubic splines, because the quadratic ones do not use enough parameters to control the tension; not all commands for drawing cubic splines accept this optional tension specification.

With the starred version of \Curve, instead of stroking the contour, the macro fills up the contour with the selected current color, figure 10.

Figure 11 shows a geometric construction that contains the geometric elements and symbols used to determine the parameters of a cubic spline required to draw a quarter circle. This construction contains many of the commands described so far.

To show what you can do with \CurveBetween see the code and result shown in figure 12. Notice the effect of changing the directions at both or a the end nodes of a single cubic spline. The directions are conveniently expressed with unit vectors described by polar coordinates.

A little more complicated is the use of the \CBezierBetween macro, figure 13. The directions are specified with unit vectors in polar form; the control points are specified by adding their distances from their neighbouring nodes; actually the right distance is maintained to the value 1, while the left one increases from 4 to 10. The black line corresponds to the standard \CurveBetween where the default distance is computed to trace an arc of a circle and is approximately 3.5.

In figure 14 the effect of tension specification is shown. The red line corresponds to the default tension, since the tension values are not specified. The black lines correspond to the various values used in the various commands to the \Curve macro. With a tension of zero, the spline is almost coincident with the horizontal base line of the frame. Increasing the parameter value to 4.5, the curved becomes taller and taller, until it wraps itself displaying an evident loop. We would say that the value of 2 is a reasonable maximum and increasing that value is just to obtain special effects.

Figure 15 displays two approximations of a sine wave; Bézier splines can approximate transcendental curves, but the approximation may be a poor one, depending on the approximated curve, when few arcs are used to draw it. With arcs specified with more complicated macros the approximation is better even with a lower number of arcs. With many arcs it is possible to approximate almost anything. On the left side a modest approximation is obtained with just three standard arcs obtained with \Curve and four node specifications; on the right we have just two arcs created with CBezierBetween with tension specification and control point distances; this drawing is almost undistinguishable from a real sinusoid.

In figure 16 some lines drawn are shown; they are drawn with quadratic splines by means of the \Qurve macro. In the left there are some open and closed curves inscribed within a square. On the right a "real" circle is compared to a quadratic spline circle; the word "real" is emphasised because it actually is an approximation with four quarter-circle cubic splines that, in spite of being drawn with third degree parametric polynomials, approximate very well a real circle; on the opposite the quadratic spline circle is clearly a poor approximation even if the maximum radial error amounts just to about 6% of the radius.

```
\unitlength=0.007\textwidth
\begin{picture}(100,90)(-50,-50)
\ \t(-50,0){\vector(1,0){100}}\put(50,1){\makebox(0,0)[br]{$x$}}
\put(20,-1){\makebox(0,0)[t]{$s$}}%
\t(0,0) {\circle*{2}}\put(-1,-1) {\makebox(0,0) [tr]{$M$}}%
\left(12,-45\right)\left(s=\left(MP_2\right)=R\right)
\begin{array}{l} \begin{array}{l} \begin{array}{l} \begin{array}{l} \begin{array}{l} \begin{array}{l} \\ \end{array} \end{array} \end{array} \end{array} \end{array} 
\put(1,40){\makebox(0,0)[t1]{$y$}}%
\t(0,-40){\circle*{2}}\put(1,-41){\makebox(0,0)[lt]{$C$}}
\ \cdots
\t(41,1){\mathbf (41,1)}{\mathbf (41,1)}
\t(0,0){\t(0,0){90}}%
\ensuremath{$\operatorname{\text{Nom}}(-20,20)} \to (-20,20) {\circle*{2}} 
\t(-20,21.5){\mathbf b}_{5}
\segment(40,0)(20,20)\put(20,20){\circle*{2}}%
\put(20,21.5){\mathbf b}_{5}%
\t(0,-40){\t(0,56.5685){\circle*{2}}}
\put(1,58){\makebox(0,0)[bl]{$P$}}}%
\t(-20,-18) {\mathbf x}(0,0) [b1] {R$}}
\put(-32,13){\makebox(0,0)[b1]{$K$}}%
\put(32,13){\makebox(0,0)[br]{$K$}}%
\end{picture}
```

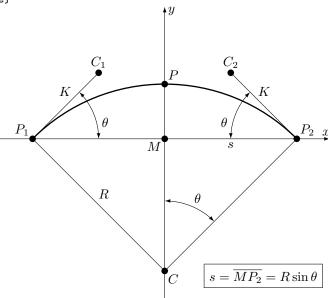


Figure 11: The code to display the nodes and control points for an arc to be approximated with a cubic Bézier spline

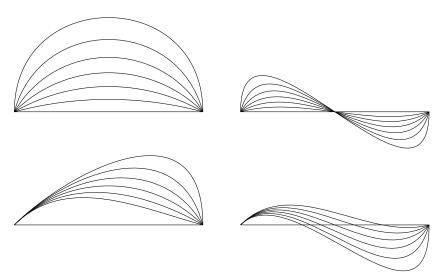


Figure 12: Curves between two points with different start and end slopes

```
\unitlength=0.1\textwidth
\begin{picture}(10,3)
\CurveBetween0,0and10,0WithDirs1,1and{1,-1}
\color{red}%
\CbezierBetween0,0And10,0 WithDirs45:1And-45:1UsingDists4And{1}
\CbezierBetween0,0And10,0 WithDirs45:1And-45:1UsingDists6And{1}
\CbezierBetween0,0And10,0 WithDirs45:1And-45:1UsingDists8And{1}
\CbezierBetween0,0And10,0 WithDirs45:1And-45:1UsingDists10And{1}
\CbezierBetween0,0And10,0 WithDirs45:1And-45:1UsingDists12And{1}
\character \text{ChezierBetween0,0And10,0 WithDirs45:1And-45:1UsingDists12And{1}}
\end{\text{picture}}
```

Figure 13: Comparison between similar arcs drawn with \CurveBetween (black) and \CbezierTo (red)

```
\raggedleft\unitlength=0.01\textwidth
\begin{picture}(70,70)
\put(0,0){\color{blue}\frame(70,70){}}
\put(0,0){\color{red}\Curve(0,0)<1,1>(70,0)<1,-1>}
\Curve(0,0)<1,1>(70,0)<1,-1;0,0>
\Curve(0,0)<1,1>(70,0)<1,-1;0.2,0.2>
\Curve(0,0)<1,1>(70,0)<1,-1;2,2>
\Curve(0,0)<1,1>(70,0)<1,-1;4.5,4.5>
\Curve(0,0)<1,1>(70,0)<1,-1;0,3>
\Curve(0,0)<1,1>(70,0)<1,-1;3,0>
\end{picture}
```

Figure 14: The effects of tension factors ${\cal L}$

```
\unitlength=0.01\textwidth
\begin{picture}(100,50)(0,-25)
\put(0,0){\VECTOR(0,0)(45,0)\VECTOR(0,-25)(0,25)}
\Zbox(45,0)[br]{x}\Zbox(0,26)[t1]{y}
\Curve(0,0)<77:1>(10,20)<1,0;2,0.4>(30,-20)<1,0;0.4,0.4>(40,0)<77:1;0.4,2>
}
\put(55,0){\VECTOR(0,0)(45,0)\VECTOR(0,-25)(0,25)}
\Zbox(45,0)[br]{x}\Zbox(0,26)[t1]{y}
\CbezierBetween0,0And20,0WithDirs77:1And-77:1UsingDists28And{28}
\CbezierBetween20,0And40,0WithDirs-77:1And77:1UsingDists28And{28}
\end{picture}
```

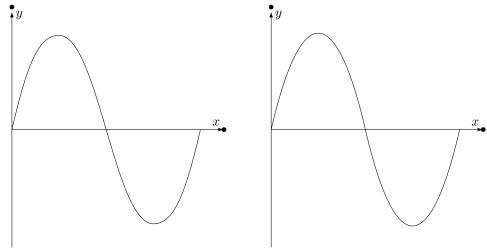


Figure 15: A sequence of arcs; the left figure has been drawn with the \Curve command with a sequence of four couples of point-direction arguments; the right figure has been drawn with two commands \CbezierBetween that include also the specification of the control points

Notice that the previous version of curve2e contained an error and would color the outside of the green four-pointed star. The curve2e-v161, attached to this bundle, has been corrected; therefore it is not actually identical to the previous version, although the latter one performed correctly for everything else except for color-filled quadratic paths.

14. The new version of \multiput is backwards compatibile with the original version contained in the LATEX kernel. The new macro adds the handling of the coordinate increments from one position to the next for the \langle object \rangle to include in the drawing.

On page 20 we show the code for the figure shown there. The red grid is nothing new, except that it displays the traditional \multiput used in this code, shown in a previous example, produces exactly the same result. But the for "graphs" on the grid, it display an alignment of black dots along

```
\ unitlength=0.0045\textwidth
%\begin{picture}(100,100)
%\put(0,0){\framebox(100,100){}}
%\put(50,50){%
       \Qurve(0,-50)<1,0>(50,0)<0,1>(0,50)<-1,0>(-50,0)<0,-1>(0,-50)<1,0>
%\color{green}
% \Qurve*(0,-50)<0,1>(50,0)<1,0>[-1,0](0,50)<0,1>[0,-1](-50,0)<-1,0>[1,0](0,-50)<0,-1>
%\Qurve(0,0)<1,4>(50,50)<1,0>(100,100)<1,4>
%\put(5,50){\Qurve(0,0)<1,1.5>(22.5,20)<1,0>(45,0)<1,-1.5>%
%(67.5,-20)<1,0>(90,0)<1,1.5>}
\Delta(0,0) [tc] \{0,0\} \Delta(100,0) [tc] \{100,0\}
%\Zbox(100,100)[bc]{100,100}\Zbox(0,100)[bc]{0,100}
%\Pall[2](0,0)\Pall[2](100,0)\Pall[2](100,100)\Pall[2](0,100)
%\end{picture}
%\hfill
%\begin{picture}(100,100)
%\put(0,0){\framebox(100,100){}}
%\put(50,50){%
%\Qurve(0,-50)<1,0>(50,0)<0,1>(0,50)<-1,0>(-50,0)<0,-1>(0,-50)<1,0>
%\Curve(0,-50)<1,0>(50,0)<0,1>(0,50)<-1,0>(-50,0)<0,-1>(0,-50)<1,0>}
\lower (50,50) [t] {0} \path{0} \path
%\end{picture}
```

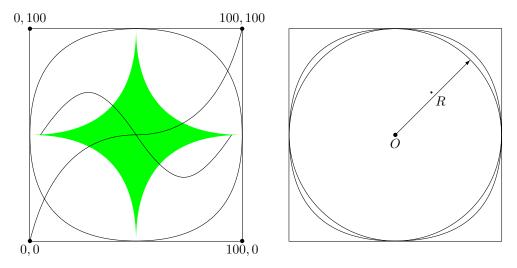


Figure 16: Several graphs drawn with quadratic Bézier splines. On the right a quadratic spline circle is compared with a cubic line circle.

the diagonal of the grid (again traditional \multiput rendered with the new version); a number of blue dots along a parabola; another number of magenta dots alined along a half sine wave; a number of little green squares aligned along a -15 o line starting from the center of the grid; notice the polar values that are used as polar relative coordinate increments.

A new command \multiput (not available with the previous versions of

```
\ \unitlength=0.01\linewidth
%\begin{picture}(100,100)
%\GraphGrid(100,100)
%\multiput(0,0)(10,10){11}{\circle*{2}}
%\color{blue!70!white}
\ \multiput(0,0)(10,0){11}{\circle*{2}}\%
  [\GetCoord(\R)\X\Y
  \left(X^{\left(X+10\right)}\right)
  \end{Y{\left( \X/10\right) **2}}
% \CopyVect\X,\Y to\R]
%\color{magenta}
%\multiput(0,0)(10,1){11}{\circle*{2}}%
% [\GetCoord(\R)\X\Y
    \left(X_{\xspace{10}}\right)
   \left( X*1.8 \right) 
   \CopyVect\X\Y to\R]
%\color{green!60!black}
%\multiput(50,50)(-15:5){11}}{%
%\polygon*(-1,-1)(1,-1)(1,1)(-1,1)}
%\end{picture}
```

Figure 17: Some examples of the $\langle handler \rangle$ optional argument

curve2e) extended with respect to the original \multiput is defined by using some L3 functions; in particular the cycling counter is accessible to the LATEX commands and it is stepped forward from 1 to the value specified in the proper command argument (in the original command it starts from that value and is stepped down to zero). See the figure on page 18 to inspect its usage. It is important to notice that if the command\rotatebox has to be used, as in the example of figure 18, the package graphics should be also loaded, because curve2e does not do it.

```
\unitlength=0.0095\linewidth
\begin{picture}(100,100)
\GraphGrid(100,100)
\put(50,50){\thicklines\circle{100}}
\xmultiput[50,50](60:40)(-30:1){12}%
{\makebox(0,0){\circle*{2}}}%
[\MultVect\R by\D to\R]%
\xmultiput[50,50](60:46)(-30:1){12}%
{\ArgOfVect\R to\Ang
\rotatebox{\fpeval{\Ang-90}}%
{\makebox(0,0)[b]{\Roman{multicnt}}}}%
[\Multvect{\R}{\D}\R]
\end{picture}
```

Figure 18: Usage example of the \multiput command

In spite of the relative simplicity of the macros contained in this package, the

described macros, as well as the original ones included in the pict2e package, allow to produce fine drawings that were unconceivable with the original LATEX picture environment. Leslie Lamport himself announced an extension to his environment when LATEX 2ε was first released in 1994; in the latexnews news-letter of December 2003; the first implementation was announced; the first version of this package curve2e was issued in 2006. It was time to have a better drawing environment; this package is a simple attempt to follow the initial path while extending the drawing facilities; but Till Tantau's pgf package has gone much farther.

4 Remark

There are other packages in the CTAN archives that deal with tracing curves of various kinds. PSTricks and tikz/pgf are the most powerful ones. But there is also the package curves that is intended to draw almost anything by using little dots or other symbols partially superimposed to one another. It uses only quadratic Bézier curves and the curve tracing is eased by specifying only the curve nodes, without specifying the control nodes; with a suitable option to the package call it is possible to reduce the memory usage by using short straight segments drawn with the PostScript facilities offered by the dvips driver.

Another package ebezier performs about the same as curve2e but draws its Bézier curves by using little dots partially superimposed to one another. The documentation is quite interesting since it explains very clearly what exactly are the Bézier splines. Apparently ebezier should be used only for DVI output without recourse to PostScript or PDF machinery.

The picture package extends the performance of the picture environment (extended with pict2e) by accepting coordinates and lengths in real absolute dimensions, not only as multiples of \unitlength; it provides commands to extend that functionality to other packages. In certain circumstances it is very useful.

Package xpicture builds over the picture LATEX environment so as to allow to draw the usual curves that are part of an introductory analytic geometry course; lines, circles, parabolas, ellipses, hyperbolas, and polynomials; the syntax is very comfortable; for all these curves it uses the quadratic Bézier splines.

Package hobby extends the cubic Bézier spline handling with the algorithms John Hobby created for METAFONT and METAPOST. But by now this package interfaces very well with tikz; it has not (yet) been adapted to the common picture environment, even when extended with pict2e, and, why not, with curve2e.

5 Acknowledgements

I wish to express my deepest thanks to Michel Goosens who spotted some errors and very kindly submitted them to me so that I was able to correct them.

Josef Tkadlec and the author collaborated extensively in order to make a better real long division so as to get correctly the quotient fractional part and to avoid as much as possible any numeric overflow; many Josef's ideas are incorporated in the macro that was implemented in the previous version of this package, although the macro used by Josef was slightly different. Both versions aim/aimed at a better accuracy and at widening the operand ranges. In this version we abandoned the long division macro, and substituted it with the floating point division provided by the xfp package.

Daniele Degiorgi spotted a fault in the kernel definition of \linethickness that heavily influenced also curve2e; see below in the code documentation part.

Thanks also to Jin-Hwan Cho and Juho Lee who suggested a small but crucial modification in order to have curve2e work smoothly also with XeTeX (XeLaTeX). Actually if pict2e, version 0.2x or later, dated 2009/08/05 or later, is being used, such modification is not necessary any more, but it's true that it becomes imperative if older versions are used.

6 Source code

6.1 Some preliminary extensions to the pict2e package

The necessary preliminary code has already been introduced. Here we require the color package and the pict2e one; for the latter one we make sure that a sufficiently recent version is used. If you want to use package xcolor, load it after curve2e.

Here we load also the xparse and xfp packages because we use their functionalities; but we do load them only if they are not already loaded with or without options; nevertheless we warn the user who wants to load them explicitly, to do this action before loading curve2e. The xfp package is absolutely required; if this package is not found in the TEX system installation, the loading of this new curve2e is aborted, and the previous version 1.61 is loaded in its place; the overall functionalities should non change much, but the functionalities of xfp are not available.

```
1 \IfFileExists{xfp.sty}{%
   \RequirePackage{color}
   \RequirePackageWithOptions{pict2e}[2014/01/01]
   \@ifl@aded{sty}{xparse}{}{\RequirePackage{xparse}}
   \@ifl@aded{sty}{xfp}{}{\RequirePackage{xfp}}%
6 }{%
     \RequirePackage{curve2e-v161}%
     \PackageWarningNoLine{curve2e}{%
8
       Package xfp is required, but apparently\MessageBreak%
9
       such package cannot be found in this \MessageBreak%
10
       TeX system installation\MessageBreak%
11
       Either your installation is not complete \MessageBreak%
12
       or it is older than 2018-10-17.\MessageBreak%
13
       \MessageBreak%
14
       15
       Version 1.61 of curve2e has been loaded\MessageBreak%
16
17
       instead of the current version\MessageBreak%
```

```
19 \endinput
20 }
```

Since we already loaded packagexfp or at least we explicitly load it in our preamble, we add, if not already defined by the package, the two new commands that allow to make floating point tests, and to implement a "while" cycle²

```
21 %
22 \ExplSyntaxOn
23 \AtBeginDocument{%
24 \ProvideExpandableDocumentCommand\fptest{m m m}{%
25 \fp_compare:nTF{#1}{#2}{#3}}
26 \ProvideExpandableDocumentCommand\fpdowhile{m m}{%
27 \fp_do_while:nn{#1}{#2}}
28 }
29 \ExplSyntaxOff
```

The next macros are just for debugging. With the trace package it would probably be better to define other macros, but this is not for the users, but for the developers.

```
31 \def\TRON{\tracingcommands\tw@ \tracingmacros\tw@}% 32 \def\TROF{\tracingcommands\z@ \tracingmacros\z@}%
```

Next we define some new dimension registers that will be used by the subsequent macros; should they be already defined, there will not be any redefinition; nevertheless the macros should be sufficiently protected so as to avoid overwriting register values loaded by other macro packages.

```
33 \ifx\undefined\@tdA \newdimen\@tdA \fi
34 \ifx\undefined\@tdB \newdimen\@tdB \fi
35 \ifx\undefined\@tdC \newdimen\@tdC \fi
36 \ifx\undefined\@tdD \newdimen\@tdD \fi
37 \ifx\undefined\@tdE \newdimen\@tdE \fi
38 \ifx\undefined\@tdF \newdimen\@tdF \fi
39 \ifx\undefined\defaultlinewidth \newdimen\defaultlinewidth \fi
```

6.2 Line thickness macros

It is better to define a macro for setting a different value for the line and curve thicknesses; the '\defaultlinewidth should contain the equivalent of \@wholewidth, that is the thickness of thick lines; thin lines are half as thick; so when the default line thickness is specified to, say, 1pt, thick lines will be 1pt thick and thin lines will be 0.5pt thick. The default whole width of thick lines is 0,8pt, but this is specified in the kernel of LATEX and/or in pict2e. On the opposite it is necessary to redefine \linethickness because the LATEX kernel global definition does not hide the space after the closed brace when you enter something such as \linethickness{1mm} followed by a space or a new line.³

²Thanks to Brian Dunn who spotted a bug in the previous version definitions.

³Thanks to Daniele Degiorgi degiorgi@inf.ethz.ch).

```
40 \gdef\linethickness#1{%
41 \@wholewidth#1\@halfwidth.5\@wholewidth\ignorespaces}%
42 \newcommand\defaultlinethickness[1]{\defaultlinewidth=#1\relax
43 \def\thicklines{\linethickness{\defaultlinewidth}}%
44 \def\thinlines{\linethickness{.5\defaultlinewidth}}\thinlines
45 \ignorespaces}%
```

The \ignorespaces at the end of these macros is for avoiding spurious spaces to get into the picture that is being drawn, because these spaces introduce picture deformities often difficult to spot and eliminate.

6.3 Improved line and vector macros

The macro \LIne allows to draw a line with arbitrary inclination as if it was a polygonal with just two vertices; actually it joins the canvas coordinate origin with the specified relative coordinate; therefore this object must be set in place by means of a \put command. Since its starting point is always at a relative 0,0 coordinate point inside the box created with \put, the two arguments define the horizontal and the vertical component respectively.

```
46 \def\LIne(#1){{\GetCoord(#1)\@tX\@tY  
47 \moveto(0,0)  
48 \pIIe@lineto{\@tX\unitlength}{\@tY\unitlength}\strokepath}\ignorespaces  
49 }%
```

A similar macro \segment operates between two explicit points with absolute coordinates, instead of relative to the position specified by a \put command; it resorts to the \polyline macro that shall be defined in a while. The \@killglue command might be unnecessary, but it does not harm; it eliminates any explicit or implicit spacing that might precede this command.

```
50 \def\segment(#1)(#2){\@killglue\polyline(#1)(#2)}%
```

By passing its ending points coordinates to the \polyline macro, both macro arguments are a pair of coordinates, not their components; in other words, if $P_1 = (x_1, y_2)$ and $P_2 = (x_2, y_2)$, then the first argument is the couple x_1, y_1 and likewise the second argument is x_2, y_2 . Notice that since \polyline accepts also the vertex coordinates in polar form, also\segment accepts the polar form. Please remember that the decimal separator is the decimal point, while the comma acts as coordinate separator. This recommendation is particularly important for non-English speaking users, since in all other languages the comma is or must be used as the decimal separator.

The \line macro is redefined by making use of a division routine performed in floating point arithmetics; for this reason the LATEX kernel and the overall TEX system installation must be as recent as the release date of the xfp package, i.e. 2018-10-17. The floating point division macro receives in input two fractional numbers and yields on output their fractional ratio. Notice that this command \line should follow the same syntax as the original pre 1994 LATEX version; but the new definition accepts the direction coefficients in polar mode; that is, instead of specifying a slope of 30° with the actual sine and cosine (or values proportional

to such functions), for example (0.5,0.866025), you may specify it as (30:1), i.e. as a unit vector with the required slope of 30° .

The beginning of the macro definition is the same as that of pict2e:

```
51 \def\line(#1)#2{\begingroup
52 \@linelen #2\unitlength
53 \ifdim\@linelen<\z@\@badlinearg\else</pre>
```

but as soon as it is verified that the line length is not negative, things change remarkably; in facts the machinery for complex numbers is invoked. This makes the code much simpler, not necessarily more efficient; nevertheless \DirOfVect takes the only macro argument (that actually contains a comma separated pair of fractional numbers) and copies it to \DirOline (an arbitrarily named control sequence) after re-normalizing to unit magnitude; this is passed to GetCoord that separates the two components into the control sequences \d@mX and \d@mY; these in turn are the values that are actually operated upon by the subsequent commands.

```
54 \expandafter\DirOfVect#1to\DirCline
55 \GetCoord(\DirCline)\d@mX\d@mY
```

The normalised vector direction is actually formed with the directing cosines of the line direction; since the line length is actually the horizontal component for non vertical lines, it is necessary to compute the actual line length for non vertical lines by dividing the given length by the magnitude of the horizontal cosine \d@mX, and the line length is accordingly scaled:

```
56 \ifdim\d@mX\p@=\z@\else
57 \edef\sc@lelen{\fpeval{1 / abs(\d@mX)}}\relax
58 \@linelen=\sc@lelen\@linelen
59 \fi
```

Of course, if the line is vertical this division must not take place. Finally the moveto, lineto and stroke language keywords are invoked by means of the internal pict2e commands in order to draw the line. Notice that even vertical lines are drawn with the PDF language commands instead of resorting to the DVI low level language that was used in both pict2e and the original (pre 1994) picture commands; it had a meaning in the old times, but it certainly does not have any nowadays, since lines are drawn by the driver that produces the output in a human visible document form, not by TeX the program.

```
60 \moveto(0,0)\pIIe@lineto{\d@mX\@linelen}{\d@mY\@linelen}%
61 \strokepath
62 \fi
63 \endgroup\ignorespaces}%
```

The new definition of the command \line, besides the ease with which is readable, does not do different things from the definition of pict2e 2009, even if it did perform in a better way compared to the 2004 version that was limited to integer direction coefficients up to 999 in magnitude. Moreover this curve2e version accepts polar coordinates as slope pairs, making it much simpler to draw lines with specific slopes.

It is necessary to redefine the low level macros \moveto, \lineto, and \curveto, because their original definitions accept only cartesian coordinates. We

```
proceed the same as for the \put command.
64 \let\originalmoveto\moveto
65 \let\originallineto\lineto
66 \let\originalcurveto\curveto
67
68 \def\moveto(#1){\GetCoord(#1)\MTx\MTy
69 \originalmoveto(\MTx,\MTy)\ignorespaces}
70 \def\lineto(#1){\GetCoord(#1)\LTx\LTy
71 \originallineto(\LTx,\LTy)\ignorespaces}
72 \def\curveto(#1)(#2)(#3){\GetCoord(#1)\CTpx\CTpy
73 \GetCoord(#2)\CTsx\CTsy\GetCoord(#3)\CTx\CTy
74 \originalcurveto(\CTpx,\CTpy)(\CTsx,\CTsy)(\CTx,\CTy)\ignorespaces}
```

6.4 Dashed and dotted lines

Dashed and dotted lines are very useful in technical drawings; here we introduce two macros that help drawing them in the proper way; besides the obvious difference between the use of dashes or dots, they may refer in a different way to the end points that must be specified to the various macros.

The coordinates of the first point P_1 , where le line starts, are always referred to the origin of the coordinate axes; the end point P_2 coordinates are referred to the origin of the axes if in cartesian form, while with the polar form they are referred to P_1 ; both coordinate types have their usefulness and figures 6 on page 10 and 7 on page 11 show how to use such coordinate types.

The above mentioned macros create dashed lines between two given points, with a dash length that must be specified, or dotted lines, with a dot gap that must be specified; actually the specified dash length or dot gap is a desired one; the actual length or gap is computed by integer division between the distance of the given points and the desired dash length or dot gap; when dashes are involved, this integer is tested in order to see if it is an odd number; if it's not, it is increased by unity. Then the actual dash length or dot gap is obtained by dividing the above distance by this number.

Another vector $P_2 - P_1$ is created by dividing it by this number; then, when dashes are involved, it is multiplied by two in order to have the increment from one dash to the next; finally the number of patterns is obtained by integer division of this number by 2 and increasing it by 1. Since the whole dashed or dotted line is put in position by an internal \put command, is is not necessary to enclose the definitions within groups, because they remain interna to the \put argument box.

Figure 6 on page 10 shows the effect of the slight changing of the dash length in order to maintain approximately the same dash-space pattern along the line, irrespective of the line length. The syntax is the following:

```
\Delta(second\ point) ((second\ point)) \{(dash\ length)\}
```

where $\langle first\ point \rangle$ contains the coordinates of the starting point and $\langle second\ point \rangle$ the absolute (cartesian) or relative (polar) coordinates of the ending point; of course the $\langle dash\ length \rangle$, which equals the dash gap, is mandatory. An optional

asterisk used to play a specific role with previous implementations; it is maintained for backwards compatibility, but its use is now superfluous; with the previous implementation of the code, in facts, if coordinates were specified in polar form, without the optional asterisk the dashed line was misplaced, while if the asterisk was specified, the whole object was put in the proper position. With this new implementation, both the cartesian and polar coordinates always play the role they are supposed to play independently from the asterisk. The \IsPolar macro is introduced to analyse the coordinate type used for the second argument, and uses such second argument accordingly.

```
75 \def\IsPolar#1:#2?{\def\QTempOne{#2}}\unless\ifx\QTempOne\empty
 76
      \expandafter\@firstoftwo\else
 77
      \expandafter\@secondoftwo\fi}
 78
 79 \ifx\Dashline\undefined
     \def\Dashline(\@ifstar{\Dashline@}}% bckwd compatibility
 80
     \let\Dline\Dashline
 81
 82
     \def\Dashline@(#1)(#2)#3{\put(#1){%
 83
 84
      \GetCoord(#1)\@tA\@tB \MakeVectorFrom\@tA\@tB to\V@ttA
      \GetCoord(#2)\@tA\@tB \MakeVectorFrom\@tA\@tB to\V@ttB
 85
      \IsPolar#2:?{%
                                              Polar
 86
         \Dashline@@(0,0)(\V@ttB){#3}}%
 87
      {%
                                              Cartesian
 88
 89
        \SubVect\V@ttA from\V@ttB to\V@ttC
 90
        \Dashline@@(0,0)(\V@ttC){#3}%
 91
 92 }}
 93
     \def\Dashline@@(#1)(#2)#3{%
 94
      \countdef\NumA3254\countdef\NumB3252\relax
 95
 96
      \GetCoord(#1)\@tA\@tB \MakeVectorFrom\@tA\@tB to\V@ttA
 97
      \GetCoord(#2)\@tA\@tB \MakeVectorFrom\@tA\@tB to\V@ttB
 98
      \SubVect\V@ttA from\V@ttB to\V@ttC
      \ModOfVect\V@ttC to\DlineMod
 99
      \DivideFN\DlineMod by#3 to\NumD
100
      \NumA=\fpeval{trunc(\NumD,0)}\relax
101
102
      \unless\ifodd\NumA\advance\NumA\@ne\fi
      \NumB=\NumA \divide\NumB\tw@
103
      \DividE\DlineMod\p@ by\NumA\p@ to\D@shMod
104
      \DividE\p@ by\NumA\p@ to \@tempa
105
106
      \Multvect{\V@ttC}{\@tempa,0}\V@ttB
      \Multvect{\V@ttB}{2,0}\V@ttC
107
      \advance\NumB\@ne
108
109
      \displaystyle \v(VOttA)_{\mathcal{O},0}(\VOttC)_{\mathcal{N}}_{\mathcal{O},0}}
110
      \ignorespaces}
111 \fi
```

A simpler \Dotline macro can draw a dotted line between two given points; the dots are rather small, therefore the inter dot distance is computed in such

a way as to have the first and the last dot at the exact position of the dottedline end-points; again the specified dot distance is nominal in the sense that it is recalculated in such a way that the first and last dots coincide with the line end points. Again if the second point coordinates are in polar form they are considered as relative to the first point. The syntax is as follows:

 $\Dotline(\langle start\ point \rangle)(\langle end\ point \rangle)\{\langle dot\ distance \rangle\}$

```
112 \ifx\Dotline\undefined
     \def\Dotline{\@ifstar{\Dotline@}{\Dotline@}}% backwards compatibility
     \def\Dotline@(#1)(#2)#3{\put(#1){%
114
     \IsPolar#2:?{%
                                         Polar
115
        \Dotline@@(0,0)(#2){#3}}
116
117
                                         Cartesian
      \CopyVect#1to\V@ttA
118
119
      \CopyVect#2to\V@ttB
120
      \SubVect\V@ttA from\V@ttB to\V@ttC
121
      \Dotline@@(0,0)(\V@ttC){#3}}%
122
123
124
     \def\Dotline@@(#1)(#2)#3{%
      \countdef\NumA 3254\relax
125
126
      \countdef\NumB 3255\relax
127
      \CopyVect#1to\V@ttA
      \CopyVect#2to\V@ttB
128
      \SubVect\V@ttA from\V@ttB to\V@ttC
129
130
      \ModOfVect\V@ttC to\DotlineMod
131
      \DivideFN\DotlineMod by#3 to\NumD
132
      \NumA=\fpeval{trunc(\NumD,0)}\relax
133
      \Divvect{\V@ttC}{\NumA,0}\V@ttB
134
      \advance\NumA\@ne
      \t(\V0ttA)_{\undergoonuput(0,0)(\V0ttB)_{\undergoonuput(0,0)%}
135
                   {\circle*{0.5}}}}
136
137
      \ignorespaces
138
     }%
139 \fi
```

Notice that vectors as complex numbers in their cartesian and polar forms always represent a point position referred to a local origin of the axes; this is why in figures 6 on page 10 and 7 on page 11 the dashed and dotted lines that start from the lower right corner of the graph grid, and that use polar coordinates, are correctly put in their correct position thanks to the different behaviour obtained with the \IsPolar macro.

6.5 Coordinate handling

The new macro \GetCoord splits a vector (or complex number) specification into its components; in particular it distinguishes the polar from the cartesian form of the coordinates. The latter have the usual syntax $\langle x, y \rangle$, while the former have

the syntax $\langle angle:radius \rangle$. The \put and \multiput commands are redefined to accept the same syntax; the whole work is done by \SplitNod@ and its subsidiaries.

Notice that package eso-pic uses picture macros in its definitions, but its original macro \LenToUnit is incompatible with this \GetCoord macro; its function is to translate real lengths into coefficients to be used as multipliers of the current \unitlength; in case that the eso-pic had been loaded, at the \begin{document} execution, the eso-pic macro is redefined using the e-TEX commands so as to make it compatible with these local macros.⁴

```
140 \AtBeginDocument{\@ifpackageloaded{eso-pic}{%
141 \renewcommand\LenToUnit[1]{\strip@pt\dimexpr#1*\p@/\unitlength}}{}}%
```

The above redefinition is delayed at \AtBeginDocument in order to have the possibility to check the the eso-pic package had actually been loaded. Nevertheless the code is defined here just because the original eso-pic macro was interfering with the algorithms of coordinate handling.

But let us come to the real subject of this section. We define a **\GettCoord** macro that passes control to the service macro with the expanded arguments; expanding arguments allows to use macros to named points, instead of explicit coordinates; with this version of **curve2e** this facility is not fully exploited, but a creative user can use this feature. Notice the usual trick to pass through a dummy macro that is defined within a group with expanded arguments, but where the group is closed by the macro itself, so that no traces remain behind after its expansion.

```
142 \def\GetCoord(#1)#2#3{\bgroup\edef\x{\egroup\noexpand\IsPolar#1:?}\x
143 {% Polar
144 \bgroup\edef\x{\egroup\noexpand\SplitPolar(#1)}\x\SCt@X\SCt@Y}\%
145 {% Cartesian
146 \bgroup\edef\x{\egroup\noexpand\SplitCartesian(#1)}\x\SCt@X\SCt@Y}\%
147 \edef#2{\SCt@X}\edef#3{\SCt@Y}\ignorespaces}
148
149 \def\SplitPolar(#1:#2)#3#4{%
150 \edef#3{\fpeval{#2 * cosd#1}}\edef#4{\fpeval{#2 * sind#1}}}
151
152 \def\SplitCartesian(#1,#2)#3#4{\edef#3{#1}\edef#4{#2}}
153
```

The macro that detects the form of the coordinates is \IsPolar; it examines the parameter syntax in order to see if it contains a colon; it has already been used with the definition of dashed and dotted lines.

In order to accept polar coordinates with \put and \multiput we resort to using \GetCoord; therefore the redefinition of \put is very simple because it suffices to save the original meaning of that macro and redefine the new one in terms of the old one.

⁴Thanks to Franz-Joseph Berthold who was so kind to spot the bug.

For \multiput it is more complicated, because the increments from one position to the next cannot be done efficiently because the increments in the original definition are executed within boxes, therefore any macro instruction inside these boxes is lost. It is a good occasion to modify the \multiput definition by means of the advanced macro definitions provided by package xparse; we can add also some error messages for avoiding doing anything when some mandatory parameters are missing ore are empty, or do not contain anything different from an ordered pair or a polar form. We add also an optional argument to handle the increments outside the boxes. The new macro has the following syntax:

```
\mbox{\mbox{\tt multiput}[$\langle displacement$\rangle$] ($\langle initial$\rangle$) ($\langle increment$\rangle$) {$\langle number$\rangle$} {$\langle object$\rangle$} {$[\langle handler$\rangle$]}
```

where the optional $\langle displacement \rangle$ is used to displace to whole set of $\langle object \rangle$ s from their original position; $\langle initial \rangle$ contains the cartesian or polar coordinates of the initial point; $\langle increment \rangle$ contains the cartesian or polar increment for the coordinates to be used from the second position to the last; $\langle number \rangle$ il the total number of $\langle object \rangle$ s to be drawn; $\langle object \rangle$ is the object to be put in position at each cycle repetition; the optional $\langle handler \rangle$ may be used to control the current values of the horizontal and vertical increments. The new definition contains two \put commands where the second is nested within a while-loop which in turn is within the argument of the first \put command. Basically it is the same idea that the original macros, but now the increments are computed within the while loop, but outside the argument of the inner \put command. If the optional $\langle handler \rangle$ is specified the increments are computed from the macros specified by the user.

The two increments components inside the optional argument may be set by means of mathematical expressions operated upon by the \fpeval function given by the \xfp package already loaded by curve2e. Of course it is the user responsibility to pay attention to the scales of the two axes and to write meaningful expressions; the figure and code shown in the first part of this documentation show some examples: see pages 20 and 20.

```
157 \RenewDocumentCommand{\multiput}{0{0,0} d() d() m m o }{%
     \IfNoValueTF{#2}{\PackageError{curve2e}%
158
         {\string\multiput\space initial point coordinates missing}%
159
160
         {Nothing done}
161
       }%
       {\IfNoValueTF{#3}{\PackageError{curve2e}
162
         {\string\multiput\space Increment components missing}%
163
         {Nothing done}
164
       }
165
       {\put(#1){\let\c@multicnt\@multicnt}
166
         \CopyVect #2 to \R
167
         \CopyVect#3 to\D
168
169
            \@multicnt=#4\relax
            \@whilenum \@multicnt > \z@\do{%
170
              \put(\R){#5}%
171
             \IfValueTF{#6}{\#6}{\AddVect#3 and\R to \R}
172
              \advance\@multicnt\m@ne
173
```

```
174 }
175 }
176 }
177 }\ightriangleighter
178 }
```

And here it is the new \multiput command; remember: the internal cycling TeX counter \@multicn is now accessible as it was a IaTeX counter, in particular the user can access its contents with a command such as \value{multicnt}. Such counter is stepped up by one at each cycle, instead of being stepped down as in the original \multiput command. The code is not so different from the one used for the new version of \multiput, but it appears more efficient and its code logically more readable.

```
179 \NewDocumentCommand{\xmultiput}\{0\{0,0\}\ d()\ d()\ m\ m\ o\ \}\{\%\}
180 \IfNoValueTF{#2}{\PackageError{curve2e}{%
     \string\Xmultiput\space initial point coordinates missing}%
181
     {Nothing done}}%
182
     {\IfNoValueTF{#3}{\PackageError{curve2e}{%
183
        \string\Xmultiput\space Increment components missing}%
184
185
        {Nothing done}}%
     {\put(#1)%
186
     {\let\c@multicnt\@multicnt
187
     \CopyVect #2 to \R
188
     \CopyVect #3 to \D
189
        \@multicnt=\@ne
190
          \fpdowhile{\value{multicnt} < \inteval{#4+1}}% Test
191
             {%
192
193
             \put(\R){#5}
             \IfValueTF{#6}{#6}{%
194
               \AddVect#3 and R to R
195
               \advance\@multicnt\@ne
196
          }
197
       }
198
199
     }}\ignorespaces
200 }
```

Notice that the internal macros \R and \D , (respectively the current point coordinates, in form of a complex number, where to put the $\langle object \rangle$, and the current displacement to find the next point) are accessible to the user both in the $\langle object \rangle$ argument field and the $\langle handler \rangle$ argument field. The code used in page 20 shows how to create the hour marks of a clock together with the rotated hour roman numerals.

6.6 Vectors

The redefinitions and the new definitions for vectors are a little more complicated than with segments, because each vector is drawn as a filled contour; the original pict2e 2004 macro checks if the slopes are corresponding to the limitations specified by Lamport (integer three digit signed numbers) and sets up a transformation

in order to make it possible to draw each vector as an horizontal left-to-right arrow and then to rotate it by its angle about its tail point; with pict2e 2009, possibly this redefinition of \vector is not necessary, but we do it as well and for the same reasons we had for redefining \line; actually there are two macros for tracing the contours that are eventually filled by the principal macro; each contour macro draws the vector with a LATEX or a PostScript styled arrow tip whose parameters are specified by default or may be taken from the parameters taken from the PSTricks package if this one is loaded before pict2e; in any case we did not change the contour drawing macros because if they are modified the same modification is passed on to the arrows drawn with the curve2e package redefinitions.

Because of these features the new macros are different from those used for straight lines.

We start with the redefinition of \vector and we use the machinery for vectors (as complex numbers) we used for \line.

```
201 \def\vector(#1)#2{%

202 \begingroup

203 \GetCoord(#1)\d@mX\d@mY

204 \@linelen#2\unitlength
```

As in pict2e we avoid tracing vectors if the slope parameters are both zero.

```
lifdim\d@mX\p@=\z@\ifdim\d@mY\p@=\z@\@badlinearg\fi\fi
```

But we check only for the positive nature of the l_x component; if it is negative, we simply change sign instead of blocking the typesetting process. This is useful also for macros \Vector, \VECTOR, and \VVECTOR to be defined in a while.

```
206 \ifdim\@linelen<\z@ \@linelen=-\@linelen\fi
```

We now make a vector with the given slope coefficients even if one or the other is zero and we determine its direction; the real and imaginary parts of the direction vector are also the values we need for the subsequent rotation.

```
207 \MakeVectorFrom\d@mX\d@mY to\@Vect
208 \DirOfVect\@Vect to\Dir@Vect
```

In order to be compatible with the original pict2e we need to transform the components of the vector direction in lengths with the specific names $\$ and $\$ dydim

```
209 \YpartOfVect\Dir@Vect to\@ynum \@ydim=\@ynum\p@
210 \XpartOfVect\Dir@Vect to\@xnum \@xdim=\@xnum\p@
```

If the vector is really sloping we need to scale the l_x component in order to get the vector total length; we have to divide by the cosine of the vector inclination which is the real part of the vector direction. We use the floating point division function; since it yields a "factor" We directly use it to scale the length of the vector. We finally memorise the true vector length in the internal dimension QtdB

```
211     \ifdim\d@mX\p@=\z@
212     \else\ifdim\d@mY\p@=\z@
213     \else
214     \edef\sc@lelen{\fpeval{1 / abs(\@xnum)}}\relax
```

```
215 \@linelen=\sc@lelen\@linelen
216 \fi
217 \fi
218 \@tdB=\@linelen
```

The remaining code is definitely similar to that of pict2e; the real difference consists in the fact that the arrow is designed by itself without the stem; but it is placed at the vector end; therefore the first statement is just the transformation matrix used by the output driver to rotate the arrow tip and to displace it the right amount. But in order to draw only the arrow tip we have to set the \@linelen length to zero.

```
219 \pIIe@concat\@xdim\@ydim{-\@ydim{\@xnum\@linelen}{\@ynum\@linelen}%
220 \@linelen\z@
221 \pIIe@vector
222 \fillpath
```

Now we can restore the stem length that must be shortened by the dimension of the arrow; by examining the documentation of pict2e we discover that we have to shorten it by an approximate amount of AL (with the notations of pict2e, figs 10 and 11); the arrow tip parameters are stored in certain variables with which we can determine the amount of the stem shortening; if the stem was too short and the new length is negative, we avoid designing such a stem.

```
223
          \@linelen=\@tdB
          \@tdA=\pIIe@FAW\@wholewidth
224
          \@tdA=\pIIe@FAL\@tdA
225
          \advance\@linelen-\@tdA
226
          \ifdim\@linelen>\z@
227
228
             \mbox{\mbox{moveto}(0,0)}
            \pIIe@lineto{\@xnum\@linelen}{\@ynum\@linelen}%
229
230
            \strokepath\fi
231
        \endgroup}
```

We define the macro that does not require the specification of the length or the l_x length component; the way the new \vector macro works does not actually require this specification, because T_EX can compute the vector length, provided the two direction components are exactly the horizontal and vertical vector components. If the horizontal component is zero, the actual length must be specified as the vertical component. The object defined with \Vector, as well as \vector, must be put in place by means of a \put command.

```
232 \def\Vector(#1){{%

233 \GetCoord(#1)\@tX\@tY

234 \ifdim\@tX\p@=\z@

235 \vector(\@tX,\@tY){\@tY}%

236 \else

237 \vector(\@tX,\@tY){\@tX}%

238 \fi}}
```

On the opposite the next macro specifies a vector by means of the coordinates of its end points; the first point is where the vector starts, and the second point is the arrow tip side. We need the difference of these two coordinates, because it represents the actual vector.

```
239 \def\VECTOR(#1)(#2){\begingroup
240 \SubVect#1from#2to\@tempa
241 \expandafter\put\expandafter(#1){\expandafter\Vector\expandafter(\@tempa)}%
242 \endgroup\ignorespaces}
```

The double tipped vector is built on the **\VECTOR** macro by simply drawing two vectors from the middle point of the double tipped vector.

```
243 \def\VVECTOR(#1) (#2) {{\SubVect#1from#2to\@tempb 244 \ScaleVect\@tempb by0.5to\@tempb 245 \AddVect\@tempb and#1to\@tempb 246 \VECTOR(\@tempb) (#2) \VECTOR(\@tempb) (#1) \ignorespaces}}
```

The pict2e documentation says that if the vector length is zero the macro draws only the arrow tip; this may work with macro \vector, certainly not with \Vector and \VECTOR. This might be useful for adding an arrow tip to a circular arc. See examples in figure 1 on page 8.

6.7 Polylines and polygons

We now define the polygonal line macro; its syntax is very simple:

```
\polygonal[\langle join \rangle](\langle P_0 \rangle)(\langle P_1 \rangle)(\langle P_2 \rangle) \dots (\langle P_n \rangle)
```

Remember: \polyline has been incorporated into pict2e 2009, but we redefine it so as to allow an optional argument to specify the line join type.

In order to write a recursive macro we need aliases for the parentheses; actually we need only the left parenthesis, but some editors complain about unmatched delimiters, so we define an alias also for the right parenthesis.

```
247 \left( \right) ( \left( \right)^{0})
```

The first call to \polyline, besides setting the line joints, examines the first point coordinates and moves the drawing position to this point; afterwards it looks for the second point coordinates; they start with a left parenthesis; if this is found the coordinates should be there, but if the left parenthesis is missing (possibly preceded by spaces that are ignored by the \@ifnextchar macro) then a warning message is output together with the line number where the missing parenthesis causes the warning: beware, this line number might point to several lines further on along the source file! In any case it's necessary to insert a \@killgluecommand, because \polyline refers to absolute coordinates, and not necessarily is put in position through a \put command that provides to eliminate any spurious spaces preceding this command.

In order to allow a specification for the joints of the various segments of a polyline it is necessary to allow for an optional parameter; the default is the bevel join.

```
251 \pIIe@moveto{\d@mX\unitlength}{\d@mY\unitlength}%
252 \@ifnextchar\lp@r{\p@lyline}{%
253 \PackageWarning{curve2e}%
254 {Polylines require at least two vertices!\MessageBreak
255 Control your polyline specification\MessageBreak}%
256 \ignorespaces}}
```

But if there is a second or further point coordinate, the recursive macro \p@lyline is called; it works on the next point and checks for a further point; if such a point exists the macro calls itself, otherwise it terminates the polygonal line by stroking it.

```
258 \def\p@lyline(#1){\GetCoord(#1)\d@mX\d@mY\
259 \pIIe@lineto{\d@mX\unitlength}{\d@mY\unitlength}%
260 \@ifnextchar\lp@r{\p@lyline}{\strokepath\ignorespaces}}
```

The same treatment must be done for the \polygon macros; we use the defining commands of package xparse, in order to use an optional asterisk; as it is usual with picture convex lines, the command with asterisk does not trace the contour, but fills the contour with the current color. The asterisk is tested at the beginning and, depending on its presence, a temporary switch is set to true; this being the case the contour is filled, otherwise it is simply stroked.

```
261 \providecommand\polygon{}
262 \RenewDocumentCommand\polygon{s O{\beveljoin} }{\@killglue\begingroup
263 \IfBooleanTF{#1}{\@tempswatrue}{\@tempswafalse}%
264 \@polygon[#2]}
265
266 \def\@polygon[#1](#2){\@killglue#1\GetCoord(#2)\d@mX\d@mY
       \pIIe@moveto{\d@mX\unitlength}{\d@mY\unitlength}%
267
       \@ifnextchar\lp@r{\@@polygon}{%
268
       \PackageWarning{curve2e}%
269
270
       {Polygons require at least two vertices!\MessageBreak
271
        Control your polygon specification\MessageBreak}%
272
       \ignorespaces}}
273
     \def\@@polygon(#1){\GetCoord(#1)\d@mX\d@mY
274
       \pIIe@lineto{\d@mX\unitlength}{\d@mY\unitlength}%
275
276
       \@ifnextchar\lp@r{\@@polygon}{\pIIe@closepath
```

```
\unitlength=0.07\hsize
\begin{picture}(8,8)(-4,-4)\color{red}
\polygon*(45:4)(135:4)(-135:4)(-45:4)
\end{picture}
```



Figure 19: The code and the result of defining a polygon with its vertex polar coordinates

```
277 \if@tempswa\pIIe@fillGraph\else\pIIe@strokeGraph\fi
278 \endgroup
279 \ignorespaces}}
```

Now, for example, a filled polygon can be drawn using polar coordinates for its vertices; see figure 19 on page 35.

Remember; the polygon polar coordinates are relative to the origin of the local axes; therefore in order to put a polygon in a different position, it is necessary to do it through a \put command.

6.8 The red service grid

The next command is handy for debugging while editing one's drawing; it draws a red grid with square meshes that are ten drawing units apart; there is no graduation along the grid, since it is supposed to be a debugging aid and the user should know what he/she is doing; nevertheless it is advisable to displace the grid by means of a \put command so that its grid lines coincide with graph coordinates that are multiples of 10. Missing to do so the readings become cumbersome. The \RoundUp macro provides to increase the grid dimensions to integer multiples of ten. Actually the new definition of this command does not need a put command (although it is not prohibited to use it) because its syntax now is the following one

```
\GraphGrid(\langle ll\ corner\ offset \rangle)(\langle grid\ dimensions \rangle)
```

where the first argument is optional: if it is missing, the lower left corner is put at the origin of the canvas coordinates. Of course also the lower left corner offset is recommended to be specified with coordinates that are integer multiples of 10; this is particularly important when the picture environment offset is specified wit non integer multiple of 10 values. Actually, since both arguments are delimited with round parentheses, a single argument is assumed to contain the grid dimensions, while if both arguments are given, the first one is the lower left corner offset, and the second one the grid dimensions.

Rounding up is useful because also the grid margins fall on coordinates multiples of 10. It resorts to the \Integer macro that will be described in a while.

```
291 \def\RoundUp#1modulo#2to#3{\expandafter\@tempcnta\Integer#1.??% 292 \count254\@tempcnta\divide\count254by#2\relax
```

```
293 \multiply\count254by#2\relax
```

- 294 \count252\@tempcnta\advance\count252-\count254
- 295 \ifnum\count252>0\advance\count252-#2\relax
- $296 \advance \ensuremath{\tt 0tempcnta-\count252\fi\edef\#3{\tt number\0tempcnta}\higher} \\$

The \Integer macro takes a possibly fractional number whose decimal separator, if present, must be the decimal point and uses the point as an argument delimiter. If one has the doubt that the number being passed to \Integer might be an integer, he/she should call the macro with a further point; if the argument is truly integer this point works as the delimiter of the integer part; if the argument being passed is fractional this extra point gets discarded as well as the fractional part of the number.

297 \def\Integer#1.#2??{#1}%

7 Math operations on fractional operands

This is not the place to complain about the fact that all programs of the TEX system used only integer arithmetics; now, with the 2018 distribution of the modern TEX system, package xfp is available: this package resorts in the background to language LATEX 3; this language now can compute fractional number operations coded in decimal, not in binary, and accepts also numbers written in the usual way in computer science, that is as a fractional, possibly signed, number followed by an expression that contains the exponent to 10 necessary to (ideally) move the fractional separator in one or the other direction according to the sign of the exponent of 10; in other words the L3 library for floating point calculations accepts such expressions as 123.456, 0.12345e3, and 12345e-3, and any other equivalent expression. If the first number is integer, it assumes that the decimal separator is to the right of the rightmost digit of the digit string.

Floating point calculations may be done through the \fpeval L3 function with a very simple syntax:

 $\footnotemath{\texttt{fpeval}}{\langle mathematical\ expression \rangle}$

where $\langle mathematical\ exression \rangle$ can contain the usual algebraic operation sings, = - * / ** ^ and the function names of the most common algebraic, trigonometric, and transcendental functions; for direct and inverse trigonometric functions it accepts arguments in radians and in sexagesimal degrees; it accepts the group of rounding/truncating operators; it can perform several kinds of comparisons; as to now (Nov. 2019) the todo list includes the direct and inverse hyperbolic functions. The mantissa length of the floating point operands amounts to 16 decimal digits. Further details may be read in the documentations of the xfp and interface3 packages, just by typing into a command line window the command texdoc $\langle document \rangle$, where $\langle document \rangle$ is just the name of the above named files without extension.

Furthermore we added a couple of more interface macros with the internal L3 floating point functions; \fptest and \fpdowhile. They have the following syntax:

```
\footnote{\code} {\code} {\c
```

The $\langle logical\ expression \rangle$ compares the values of any kind by means of the usual >, =, and < operators that may be negated with the "not' operator!; furthermore the logical results of these comparisons may be acted upon with the "and" operator && and the "or" operator. The $\langle true\ code \rangle$, and $\langle code \rangle$ are executed if or while the $\langle logical\ expression \rangle$ is true, while the $\langle false\ code \rangle$ is executed if the $\langle logical\ expression \rangle$ is false

Before the availability of the xfp package, it was necessary to fake fractional number computations by means of the native e-TeX commands \dimexpr, i.e. to multiply each fractional number by the unit \p@ (1 pt) so as to get a length; operate un such lengths, and then stripping off the 'pt' component from the result; very error prone and with less precision as the one that the modern decimal floating point calculations can do. Of course it is not so important to use fractional numbers with more that 5 or 6 fractional digits, because the other TeX and LaTeX macros cannot handle them, but it is very convenient to have simpler and more readable code. We therefore switched to the new floating point functionality, even if this maintains the curve2e functionality, but renders this package unusable with older LaTeX kernel installations. It has already been explained that the input of this up to date version of curve2e is aborted if the xfp package is not available, but the previous version 1.61 version is loaded; very little functionality is lost, but, evidently, this new version performs in a better way.

7.1 The division macro

The most important macro is the division of two fractional numbers; we seek a macro that gets dividend and divisor as fractional numbers and saves their ratio in a macro; this is done in a simple way with the following code.

```
298 \det DividE#1by#2to#3{\det #3{fpeval{#1 / #2}}}
```

In order to avoid problems with divisions by zero, or with numbers that yield results to large to be used as multipliers of lengths, it would be preferable that the above code be preceded or followed by some tests and possible messages. Actually we decided to avoid such tests and messages, because the internal L3 functions already provide some. This was done in the previous versions of this package, when the \fpeval L3 function was not available.

Notice that operands #1 and #2 may be integer numbers or fractional, or mixed numbers. They may be also dimensions, but while dimensions in printer points (72.27pt=1in) are handled as assumed, when different units are used, the length must be enclosed in parentheses:

```
\DividE(1mm)by(3mm) to\result
```

yields correctly \result=0.33333333. Without parentheses the result is unpredictable.

For backwards compatibility we need an alias. 299 \let\DivideFN\DividE

We do the same in order to multiply two integer o fractional numbers held in the first two arguments and the third argument is a definable token that will hold the result of multiplication in the form of a fractional number, possibly with a non null fractional part; a null fractional part is stripped away

```
300 \def\MultiplY#1by#2to#3{\edef#3{\fpeval{#1 * #2}}}\relax 301 \let\MultiplyFN\MultiplY
```

but with multiplication it is better to avoid computations with lengths.

The next macro uses the \fpeval macro to get the numerical value of a measure in points. One has to call \Numero with a control sequence and a dimension; the dimension value in points is assigned to the control sequence.

```
302 \unless\ifdefined\Numero  
303 \def\Numero#1#2{\edef#1{\fpeval{round(#2,6)}}\ignorespaces}%  
304 \fi
```

The numerical value is rounded to 6 fractional digits that are more than sufficient for the graphical actions performed by curve2e.

The \ifdefined primitive command is provided by the e-TEX extension of the typesetting engine; the test does not create any hash table entry; it is a different way than the \ifx\csname...\endcsname test, because the latter first possibly creates a macro meaning \relax then executes the test; therefore an undefined macro name is always defined to mean \relax.

7.2 Trigonometric functions

We now start with trigonometric functions. In previous versions of this package we defined the macros \SinOf, \CosOf and \TanOf (\CotOf does not appear so essential) by means of the parametric formulas that require the knowledge of the tangent of the half angle. We wanted, and still want, to specify the angles in sexagesimal degrees, not in radians, so that accurate reductions to the main quadrants are possible. The formulas are

$$\sin \theta = \frac{2}{\cot x + \tan x}$$

$$\cos \theta = \frac{\cot x - \tan x}{\cot x + \tan x}$$

$$\tan \theta = \frac{2}{\cot x - \tan x}$$

where

 $x = \theta/114.591559$

is the half angle in degrees converted to radians.

But now, in this new version, the availability of the floating point computations with the specific L3 library makes all the above superfluous; actually the above approach gave good results but it was cumbersome and limited by the fixed radix computations of the TeX system programs.

Matter of facts, we compared the results (with 6 fractional digits) the computations executed with the sind function name, in order to use the angles in degrees, and a table of trigonometric functions with the same number of fractional digits, and we di not find any difference, not even one unit on the sixth decimal digit. Probably the \fpeval computations, without rounding before the sixteenth significant digit, are much more accurate, but it is useless to have a better accuracy when the other TeX and LATeX macros would not be able to exploit them.

Having available such powerful instrument, even the tangent appears to be of little use for the kind of computations that are supposed to be required in this package.

The codes for the computation of \SinOf and \CosOf of the angle in degrees is now therefore the following

```
305 \def\SinOf#1to#2{\edef#2{\fpeval{round(sind#1,6)}}}\relax 306 \def\CosOf#1to#2{\edef#2{\fpeval{round(cosd#1,6)}}}\relax
```

As of today the anomaly (angle) of a complex number may not be necessary, but it might become useful in the future; therefore with macro \ArgOfVect we calculate the four quadrant arctangent (in degrees) of the given vector taking into account the sings of the vector components. We use the xfp atand with two arguments, so that it automatically takes into account all the signs for determining the argument of vector x, y by giving the values x and y in the proper order to the function atan:

```
if x + iy = Me^{i\varphi} then \varphi = \{\text{atand}(y, x)\}
```

The \ArgOfVect macro receives on input a vector; from the signs of the horizontal and vertical components it determines the ratio and from this ratio the arctangent; but before doing this it tests the components in order to determine the quadrant of the vector tip; depending on signs it possibly adds what is necessary to determine the angle in the range $-180^{\circ} < \varphi \le +180^{\circ}$. If both components are zero, the angle is undefined, but for what concerns curve2e it is assigned the angle $0^{c}irc$.

```
307 \def\ArgOfVect#1to#2{\GetCoord(#1){\t0X}}{\308 \fptest{\t0X=\z0 && \t0Y=\z0}{\def#2{0}}{%}\\ 309 \edef#2{\fpeval{round(atand(\t0Y,\t0X),6)}}\ignorespaces}
```

The anomaly of a null vector is meaningless, but we set it to zero in case that input data are wrong. Computations go on anyway, but the results my be worthless; such strange results are an indications that some controls on the code should be done.

It is worth examining the following table, where the angles of nine vectors 45° degrees apart from one another are computed from this macro.

Real computations with the \ArgOfVect macro produce those very numbers without the need of rounding; \fpeval produces all trimming of lagging zeros and rounding by itself.

7.3 Arcs and curves preliminary information

We would like to define now a macro for drawing circular arcs of any radius and any angular aperture; the macro should require the arc center, the arc starting point and the angular aperture. The arc has its reference point in its center, therefore it does not need to be put in place by the command \put; nevertheless if \put is used, it may displace the arc into another position.

The command should have the following syntax:

If the $\langle angle \rangle$, i.e. the arc angular aperture, is positive the arc runs counterclockwise from the starting point; clockwise if it is negative. Notice that since the $\langle starting\ point \rangle$ is relative to the $\langle center \rangle$ point, its polar coordinates are very convenient, since they become ($\langle start\ angle \rangle : \langle radius \rangle \rangle$), where the $\langle start\ angle \rangle$ is relative to the arc center. Therefore you can think about a syntax such as this one:

```
\Arc(\langle\langle center\rangle\rangle)(\langle start\ angle: radius\rangle)\{\langle angle\rangle\}
```

The difference between the pict2e \arc definition consists in a very different syntax:

```
\arc[\langle start\ angle \rangle, \langle end\ angle \rangle] \{\langle radius \rangle\}
```

and the center is assumed to be at the coordinate established with a required \put command; moreover the difference in specifying angles is that $\langle end \; angle \rangle$ equals the sum of $\langle start \; angle \rangle$ and $\langle angle \rangle$. With the definition of this curve2e package use of a \put command is not prohibited, but it may be used for fine tuning the arc position by means of a simple displacement; moreover the $\langle starting \; point \rangle$ may be specified with polar coordinates (that are relative to the arc center).

It's necessary to determine the end point and the control points of the Bézier spline(s) that make up the circular arc.

The end point is obtained from the rotation of the starting point around the center; but the pict2e command \pIIe@rotate is such that the pivoting point appears to be non relocatable. It is therefore necessary to resort to low level TEX commands and the defined trigonometric functions and a set of macros that operate on complex numbers used as vector roto-amplification operators.

7.4 Complex number macros

In this package *complex number* is a vague phrase; it may be used in the mathematical sense of an ordered pair of real numbers; it can be viewed as a vector joining the origin of the coordinate axes to the coordinates indicated by the ordered pair; it can be interpreted as a roto-amplification operator that scales its

operand and rotates it about a pivot point; besides the usual conventional representation used by the mathematicians where the ordered pair is enclosed in round parentheses (which is in perfect agreement with the standard code used by the picture environment) there is the other conventional representation used by the engineers that stresses the roto-amplification nature of a complex number:

$$(x,y) = x + iy = Me^{j\theta}$$

Even the imaginary unit is indicated with i by the mathematicians and with j by the engineers. In spite of these differences, these objects, the *complex numbers*, are used without any problem by both mathematicians and engineers.

The important point is that these objects can be summed, subtracted, multiplied, divided, raised to any power (integer, fractional, positive or negative), be the argument of transcendental functions according to rules that are agreed upon by everybody. We do not need all these properties, but we need some and we must create the suitable macros for doing some of these operations.

In facts we need macros for summing, subtracting, multiplying, dividing complex numbers, for determining their directions (unit vectors or versors); a unit vector is the complex number divided by its magnitude so that the result is the cartesian or polar form of the Euler's formula

$$e^{j\phi} = \cos\phi + j\sin\phi$$

The magnitude of a vector is determined by taking the square root of a function of the real and the imaginary parts; see further on.

It's better to represent each complex number with one control sequence; this implies frequent assembling and disassembling the pair of real numbers that make up a complex number. These real components are assembled into the defining control sequence as a couple of coordinates, i.e. two comma separated integer or fractional signed decimal numbers.

For assembling two real numbers into a complex number we use the following elementary macro:

310 \def\MakeVectorFrom#1#2to#3{\edef#3{#1,#2}\ignorespaces}%

Another elementary macro copies a complex number into another one:

311 \def\CopyVect#1to#2{\edef#2{#1}\ignorespaces}%

The magnitude is determined with the macro \ModOfVect with delimited arguments; as usual it is assumed that the results are retrieved by means of control sequences, not used directly.

In the preceding version of package curve2e the magnitude M was determined by taking the moduli of the real and imaginary parts, by changing their signs if necessary; the larger component was then taken as the reference one, so that, if a is larger than b, the square root of the sum of their squares is computed as such:

$$M = \sqrt{a^2 + b^2} = |a|\sqrt{1 + (b/a)^2}$$

In this way the radicand never exceeds 2 and it was quite easy to get its square root by means of the Newton iterative process; due to the quadratic convergence,

five iterations were more than sufficient. When one of the components was zero, the Newton iterative process was skipped. With the availability of the xfp package and its floating point algorithms it is much easier to compute the magnitude of a complex number; since these algorithms allow to use very large numbers, it is not necessary to normalise the complex number components to the largest one; therefore the code is much simpler than the one used for implementing the Newton method in the previous versions of this package.

```
312 \def\ModOfVect#1to#2{\GetCoord(#1)\t@X\t@Y
313 \edef#2{\fpeval{round(sqrt(\t@X*\t@X + \t@Y*\t@Y),6)}}%
314 \ignorespaces}%
```

Since the macro for determining the magnitude of a vector is available, we can now normalise the vector to its magnitude, therefore getting the Cartesian form of the direction vector. If by any chance the direction of the null vector is requested, the output is again the null vector, without normalisation.

```
315 \def\DirOfVect#1to#2{\GetCoord(#1)\t@X\t@Y
316 \ModOfVect#1to\@tempa
317 \fptest{\@tempa=\z@}{}{%
318 \edef\t@X{\fpeval{round(\t@X/\@tempa,6)}}%
319 \edef\t@Y{\fpeval{round(\t@Y/\@tempa,6)}}%
320 }\MakeVectorFrom\t@X\t@Y to#2\ignorespaces}%
```

A cumulative macro uses the above ones to determine with one call both the magnitude and the direction of a complex number. The first argument is the input complex number, the second its magnitude, and the third is again a complex number normalised to unit magnitude (unless the input was the null complex number); remember always that output quantities must be specified with control sequences to be used at a later time.

```
321 \def\ModAndDirOfVect#1to#2and#3{% 322 \ModOfVect#1to#2% 323 \DirOfVect#1to#3\ignorespaces}%
```

The next macro computes the magnitude and the direction of the difference of two complex numbers; the first input argument is the minuend, the second is the subtrahend; the output quantities are the third argument containing the magnitude of the difference and the fourth is the direction of the difference. The service macro \SubVect executes the difference of two complex numbers and is described further on.

```
324 \def\DistanceAndDirOfVect#1minus#2to#3and#4{% 325 \SubVect#2from#1to\@tempa 326 \ModAndDirOfVect\@tempa to#3and#4\ignorespaces}%
```

We now have two macros intended to fetch just the real or, respectively, the imaginary part of the input complex number.

```
327 \def\XpartOfVect#1to#2{%
328 \GetCoord(#1)#2\@tempa\ignorespaces}%
329 %
330 \def\YpartOfVect#1to#2{%
331 \GetCoord(#1)\@tempa#2\ignorespaces}%
```

With the next macro we create a direction vector (second argument) from a given angle (first argument, in degrees).

```
332 \def\DirFromAngle#1to#2{%

333 \edef\t@X{\fpeval{round(cosd#1,6)}}%

334 \edef\t@Y{\fpeval{round(sind#1,6)}}%

335 \MakeVectorFrom\t@X\t@Y to#2\ignorespaces}%
```

Sometimes it is necessary to scale a vector by an arbitrary real factor; this implies scaling both the real and imaginary part of the input given vector.

```
336 \def\ScaleVect#1by#2to#3{\GetCoord(#1)\t@X\t@Y
337 \edef\t@X{\fpeval{#2 * \t@X}}%
338 \edef\t@Y{\fpeval{#2 * \t@Y}}%
339 \MakeVectorFrom\t@X\t@Y to#3\ignorespaces}%
```

Again, sometimes it is necessary to reverse the direction of rotation; this implies changing the sign of the imaginary part of a given complex number; this operation produces the complex conjugate of the given number.

```
340 \def\ConjVect#1to#2{\GetCoord(#1)\t@X\t@Y} \\ 341 \edef\t@Y{-\t@Y}% \\ 342 \MakeVectorFrom\t@X\t@Y to#2\ignorespaces}%
```

With all the low level elementary operations we can now proceed to the definitions of the binary operations on complex numbers. We start with the addition:

```
343 \def\AddVect#1and#2to#3{\GetCoord(#1)\tu@X\tu@Y
344 \GetCoord(#2)\td@X\td@Y
345 \edef\t@X{\fpeval{\tu@X + \td@X}}%
346 \edef\t@Y{\fpeval{\tu@Y + \td@Y}}%
347 \MakeVectorFrom\t@X\t@Y to#3\ignorespaces}%

Then the subtraction:
348 \def\SubVect#1from#2to#3{\GetCoord(#1)\tu@X\tu@Y
349 \GetCoord(#2)\td@X\td@Y
350 \edef\t@X{\fpeval{\td@X - \tu@X}}%
351 \edef\t@Y{\fpeval{\td@Y - \tu@Y}}%
352 \MakeVectorFrom\t@X\t@Y to#3\ignorespaces}%
```

For the multiplication we need to split the operation according to the fact that we want to multiply by the second operand or by the complex conjugate of the second operand; it would be nice if we could use the usual postfixed asterisk notation for the complex conjugate, but in the previous versions of this package we could not find a simple means for doing so. Therefore the previous version contained e definition of the \MultVect macro that followed a simple syntax with an optional asterisk prefixed to the second operand. Its syntax, therefore, allowed the following two forms:

```
\label{eq:local_second_factor} $$\operatorname{MultVect}\langle first\ factor\rangle$ by $$\langle second\ factor\rangle$ to $\langle output\ macro\rangle$ $$\operatorname{MultVect}\langle first\ factor\rangle$ by $**$ $\langle second\ factor\rangle$ to $\langle output\ macro\rangle$ $$
```

With the availability of the **xparse** package and its special argument descriptors for the arguments, we were able to define a different macro, **\Multvect**, with both optional positions for the asterisk: *after* and *before*; its syntax allows the following four forms:

Nevertheless we maintain a sort of interface between the old syntax and the new one, so that the two old forms can be mapped to two suitable forms of the new syntax. Old documents are still compilable; users who got used to the old syntax can maintain their habits.

First we define the new macro: it receives the three arguments, the first two as balanced texts; the last one must always be a macro, therefore a single (complex) token and doe not require braces, even if it is not forbidden to use them. Asterisks are optional. The input arguments are transformed into couples of anomaly and modulus; this makes multiplication much simpler as the output modulus is just the product of the input moduli, while the output anomaly is just the sum of input anomalies; eventually it is necessary to transform this polar version of the result into an ordered couple of cartesian values to be assigned to the output macro. In order to maintain the single macros pretty simple we need a couple of service macros and a named counter. We use \ModOfVect previously defined, and a new macro \ModAndAngleOfVect with the following syntax:

The output quantities are always macros, so they do not need balanced bracing; angles in degrees are always preferred because, on case of necessity, they are easy to reduce to the range $-180^{\circ} < \alpha < +180^{\circ}$.

```
353 \def\ModAndAngleOfVect#1to#2and#3{\ModOfVect#1to#2\relax 354 \ArgOfVect#1to#3\ignorespaces}
```

We name a counter in the upper range accessible with all the modern three typesetting engines, pdfLaTeX, LuaLaTeX and XeLaTeX.

```
355 \countdef\MV@C=2560\relax
```

The user is warned; The counter register number is sort of casual, but it is not excluded that its name or number get in conflict with other macros from other packages. I would be grateful is such an event takes place.

Nov comes the real macro⁵:

```
356 \NewDocumentCommand\Multvect{m s m s m}{% 357 \MV@C=0 358 \ModAndAngleOfVect#1to\MV@uM and\MV@uA 359 \ModAndAngleOfVect#3to\MV@dM and\MV@dA 360 \IfBooleanT{#2}{\MV@C=1}\relax 361 \IfBooleanT{#4}{\MV@C=1}\relax
```

⁵A warm thank-you to Enrico Gregorio, who kindly attracted my attention on the necessity of braces when using this kind of macro; being used to the syntax with delimited arguments I had taken the bad habit of avoiding braces. Braces are very important, but the syntax of the original TEX language, that did not have available the L3 one, spoiled me with the abuse of delimited arguments.

```
362 \unless\ifnum\MV@C=O\edef\MV@dA{-\MV@dA}\fi
363 \edef\MV@rM{\fpeval{round((\MV@uM * \MV@dM),6)}}%
364 \edef\MV@rA{\fpeval{round((\MV@uA + \MV@dA),6)}}%
365 \GetCoord(\MV@rA:\MV@rM)\t@X\t@Y
366 \MakeVectorFrom\t@X\t@Y to#5}
```

The macro to remain backward compatible, reduce to two simple macros that take the input delimited arguments and passes them in braced form to the above general macro:

```
367 \def\MultVect#1by{\@ifstar{\let\MV@c\@ne\@MultVect#1by}\%
368 {\let\MV@c\empty\@MultVect#1by}}
369
370 \def\@MultVect#1by#2to#3{%
371 \unless\ifx\MV@c\empty\Multvect{#1}{#2}*{#3}\else
372 \Multvect{#1}{#2}{#3}\fi}
```

Testing of both the new and the old macros show that they behave as expected, although, using real numbers for trigonometric functions, some small rounding unit on the sixth decimal digit still remain; nothing to worry about with a package used for drawing.

The division of two complex numbers implies scaling down the dividend by the magnitude of the divisor and by rotating the dividend scaled vector by the conjugate versor of the divisor:

$$\frac{\vec{N}}{\vec{D}} = \frac{\vec{N}}{M\vec{u}} = \frac{\vec{N}}{M}\vec{u}^*$$

therefore:

```
373 \def\DivVect#1by#2to#3{\Divvect{#1}{#2}{#3}}
374
375 \NewDocumentCommand\Divvect{ m m m }{%
376 \ModAndDirOfVect#2to\@Mod and\@Dir
377 \edef\@Mod{\fpeval{1 / \@Mod}}%
378 \ConjVect\@Dir to\@Dir
379 \ScaleVect#1by\@Mod to\@tempa
380 \Multvect{\@tempa}{\@Dir}#3\ignorespaces}%
```

Macros \DivVect and \Divvect are almost equivalent; the second is possibly slightly more robust. They match the corresponding macros for multiplying two vectors.

7.5 Arcs and curved vectors

We are now in the position of really doing graphic work.

7.5.1 Arcs

We start with tracing a circular arc of arbitrary center, arbitrary starting point and arbitrary aperture; the first macro checks the aperture; if this is not zero it actually proceeds with the necessary computations, otherwise it does nothing.

```
381 \def\Arc(#1)(#2)#3{\begingroup
382 \@tdA=#3\p@
383 \unless\ifdim\@tdA=\z@
384 \@Arc(#1)(#2)%
385 \fi
386 \endgroup\ignorespaces}%
```

The aperture is already memorised in \@tdA; the \@Arc macro receives the center coordinates in the first argument and the coordinates of the starting point in the second argument.

```
387 \def\@Arc(#1)(#2){%
388 \ifdim\@tdA>\z@
389 \let\Segno+%
390 \else
391 \@tdA=-\@tdA \let\Segno-%
392 \fi
```

The rotation angle sign is memorised in \Segno and \QtdA now contains the absolute value of the arc aperture.

If the rotation angle is larger than 360° a message is issued that informs the user that the angle will be reduced modulo 360° ; this operation is performed by successive subtractions rather than with modular arithmetics on the assumption that in general one subtraction suffices.

```
393 \Numero\@gradi\@tdA
394 \ifdim\@tdA>360\p@
395 \PackageWarning{curve2e}{The arc aperture is \@gradi\space degrees
396 and gets reduced\MessageBreak%
397 to the range 0--360 taking the sign into consideration}%
398 \@whiledim\@tdA>360\p@\do{\advance\@tdA-360\p@}%
399 \fi
```

Now the radius is determined and the drawing point is moved to the starting point.

```
400 \SubVect#2from#1to\@V \ModOfVect\@V to\@Raggio
401 \CopyVect#2to\@pPun
402 \CopyVect#1to\@Cent \GetCoord(\@pPun)\@pPunX\@pPunY
```

From now on it's better to define a new macro that will be used also in the subsequent macros that draw arcs; here we already have the starting point coordinates and the angle to draw the arc, therefore we just call the new macro, stroke the line and exit.

```
403 \@@Arc\strokepath\ignorespaces}%
```

And the new macro \@@Arc starts with moving the drawing point to the first point and does everything needed for drawing the requested arc, except stroking it; we leave the stroke command to the completion of the calling macro and nobody forbids to use the \@@Arc macro for other purposes.

```
404 \ensuremath{\def\ensuremath{\def\ensuremath{\def\ensuremath{\def\ensuremath{\def}}}} \
```

If the aperture is larger than 180° it traces a semicircle in the right direction and correspondingly reduces the overall aperture.

```
406 \ifdim\@tdA>180\p@
     \advance\@tdA-180\p@
407
     \Numero\@gradi\@tdA
408
     \SubVect\@pPun from\@Cent to\@V
409
     \AddVect\@V and\@Cent to\@sPun
410
     Multvect{\0V}{0,-1.333333330}\0V
411
412
     \if\Segno-\ScaleVect\@V by-1to\@V\fi
413
     \AddVect\@pPun and\@V to\@pcPun
     \AddVect\@sPun and\@V to\@scPun
414
     \GetCoord(\@pcPun)\@pcPunX\@pcPunY
415
     \GetCoord(\@scPun)\@scPunX\@scPunY
416
     \GetCoord(\@sPun)\@sPunX\@sPunY
417
     \pIIe@curveto{\@pcPunX\unitlength}{\@pcPunY\unitlength}%
418
                   {\@scPunX\unitlength}{\@scPunY\unitlength}%
419
                   {\@sPunX\unitlength}{\@sPunY\unitlength}%
420
     \CopyVect\@sPun to\@pPun
421
422 \fi
```

If the remaining aperture is not zero it continues tracing the rest of the arc. Here we need the extrema of the arc and the coordinates of the control points of the Bézier cubic spline that traces the arc. The control points lay on the perpendicular to the vectors that join the arc center to the starting and end points respectively.

With reference to figure 11 on page 16, the points P_1 and P_2 are the arc endpoints; C_1 and C_2 are the Bézier-spline control-points; P is the arc mid-point, that should be distant from the center of the arc the same as P_1 and P_2 . Choosing a convenient orientation of the arc relative to the coordinate axes, the coordinates of these five points are:

$$P_1 = (-R\sin\theta, 0)$$

$$P_2 = (R\sin\theta, 0)$$

$$C_1 = (-R\sin\theta + K\cos\theta, K\sin\theta)$$

$$C_2 = (R\sin\theta - K\cos\theta, K\sin\theta)$$

$$P = (0, R(1 - \cos\theta))$$

The Bézier cubic spline interpolating the end and mid points is given by the parametric equation:

$$P = P_1(1-t)^3 + C_13(1-t)^2t + C_23(1-t)t^2 + P_2t^3$$

where the mid point is obtained for t=0.5; the four coefficients then become 1/8, 3/8, 3/8, 1/8 and the only unknown remains K. Solving for K we obtain the formula

$$K = \frac{4}{3} \frac{1 - \cos \theta}{\sin \theta} R = \frac{4}{3} \frac{1 - \cos \theta}{\sin^2 \theta} s \tag{1}$$

where θ is half the arc aperture, R is its radius, and s is half the arc chord.

 $423 \d \d \d \A > \z \end{dense}$

424 \DirFromAngle\@gradi to\@Dir \if\Segno-\ConjVect\@Dir to\@Dir \fi 425 \SubVect\@Cent from\@pPun to\@V

```
\Multvect{\0V}{\0Dir}\0V
426
     \AddVect\@Cent and\@V to\@sPun
427
     \@tdA=.5\@tdA \Numero\@gradi\@tdA
428
     \DirFromAngle\@gradi to\@Phimezzi
429
     \GetCoord(\@Phimezzi)\@cosphimezzi\@sinphimezzi
430
     \@tdB=1.3333333\p@ \@tdB=\@Raggio\@tdB
431
432
     \@tdC=\p@ \advance\@tdC -\@cosphimezzi\p@ \Numero\@tempa\@tdC
433
     \DividE\@tdB by\@sinphimezzi\p@ to\@cZ
434
     \ScaleVect\@Phimezzi by\@cZ to\@Phimezzi
435
     \ConjVect\@Phimezzi to\@mPhimezzi
436
     \if\Segno-%
437
       \let\@tempa\@Phimezzi
438
       \let\@Phimezzi\@mPhimezzi
439
       \let\@mPhimezzi\@tempa
440
441
     \SubVect\@sPun from\@pPun to\@V
442
     \DirOfVect\@V to\@V
443
     \Multvect{\@Phimezzi}{\@V}\@Phimezzi
444
445
     \AddVect\@sPun and\@Phimezzi to\@scPun
446
     \ScaleVect\@V by-1to\@V
     \Multvect{\@mPhimezzi}{\@V}\@mPhimezzi
447
     \AddVect\@pPun and\@mPhimezzi to\@pcPun
448
     \GetCoord(\@pcPun)\@pcPunX\@pcPunY
449
     \GetCoord(\@scPun)\@scPunX\@scPunY
450
     \GetCoord(\@sPun)\@sPunX\@sPunY
451
     \pIIe@curveto{\@pcPunX\unitlength}{\@pcPunY\unitlength}%
452
453
                  {\@scPunX\unitlength}{\@scPunY\unitlength}%
454
                  {\@sPunX\unitlength}{\@sPunY\unitlength}%
455 \fi}
```

7.5.2 Arc vectors

We exploit much of the above definitions for the \Arc macro for drawing circular arcs with an arrow at one or both ends; the first macro \VectorArc draws an arrow at the ending point of the arc; the second macro \VectorARC draws arrows at both ends; the arrows have the same shape as those for vectors; actually they are drawn by putting a vector of zero length at the proper arc end(s), therefore they are styled as traditional IATEX or PostScript arrows according to the specific option to the pict2e package.

But the arc drawing done here shortens it so as not to overlap on the arrow(s); the only arrow (or both ones) are also lightly tilted in order to avoid the impression of a corner where the arc enters the arrow tip.

All these operations require a lot of "playing" with vector directions, but even if the operations are numerous, they do not do anything else but: (a) determining the end point and its direction; (b) determining the arrow length as an angular quantity, i.e. the arc amplitude that must be subtracted from the total arc to be drawn; (c) the direction of the arrow should be corresponding to the tangent to

the arc at the point where the arrow tip is attached; (d) tilting the arrow tip by half its angular amplitude; (e) determining the resulting position and direction of the arrow tip so as to draw a zero length vector; (f) possibly repeating the same procedure for the other end of the arc; (g) shortening the total arc angular amplitude by the amount of the arrow tip(s) already set, and finally (h) drawing the circular arc that joins the starting point to the final arrow or one arrow to the other one.

The calling macros are very similar to the \Arc macro initial one:

```
456 \def\VectorArc(#1)(#2)#3{\begingroup
457 \@tdA=#3\p@ \ifdim\@tdA=\z@\else
458 \@VArc(#1)(#2)%
459 \fi
460 \endgroup\ignorespaces}%
461 %
462 \def\VectorARC(#1)(#2)#3{\begingroup
463 \@tdA=#3\p@
464 \ifdim\@tdA=\z@\else
465 \@VARC(#1)(#2)%
466 \fi
467 \endgroup\ignorespaces}%
```

The single arrowed arc is defined with the following long macro where all the described operations are performed more or less in the described succession; probably the macro requires a little cleaning, but since it works fine we did not try to optimise it for time or number of tokens. The final part of the macro is almost identical to that of the plain arc; the beginning also is quite similar. The central part is dedicated to the positioning of the arrow tip and to the necessary calculations for determining the tip tilt and the reduction of the total arc length; pay attention that the arrow length, stored in \@tdE is a real length, while the radius stored in \@Raggio is just a multiple of the \unitlength, so that the division (that yields a good angular approximation to the arrow length as seen from the center of the arc) must be done with real lengths. The already defined \@CArc macro actually draws the curved vector stem without stroking it.

```
468 \def\@VArc(#1)(#2){%
469 \left(\frac{0tdA}{z}\right)
470
    \let\Segno+%
471 \else
     \@tdA=-\@tdA \let\Segno-%
473 \fi \Numero\@gradi\@tdA
474 \ifdim\@tdA>360\p@
    \PackageWarning{curve2e}{The arc aperture is \@gradi\space degrees
475
476
        and gets reduced\MessageBreak%
        to the range 0--360 taking the sign into consideration}%
477
    478
480 \SubVect#1from#2to\@V \ModOfVect\@V to\@Raggio \CopyVect#2to\@pPun
481 \@tdE=\pIIe@FAU\@wholewidth \@tdE=\pIIe@FAL\@tdE
482 \DividE\@tdE by \@Raggio\unitlength to\DeltaGradi
```

```
483 \@tdD=\DeltaGradi\p@
484 \d \D=57.29578\d \Mumero\DeltaGradi\d \d \
485 \ensuremath{\tt 0}{\tt tdD=\ifx\Segno--fi\@gradi\p@\Numero\@tempa\0}{\tt tdD}
486 \DirFromAngle\@tempa to\@Dir
487 \Multvect{\QV}{\QDir}\QsPun
488 \edef\@tempA{\ifx\Segno-\m@ne\else\@ne\fi}%
489 \Multvect{\@sPun}{0,\@tempA}\@vPun
490 \DirOfVect\@vPun to\@Dir
491 \AddVect\@sPun and #1 to \@sPun
492 \GetCoord(\@sPun)\@tdX\@tdY
493 \@tdD\ifx\Segno--\fi\DeltaGradi\p@
494 \@tdD=.5\@tdD \Numero\DeltaGradi\@tdD
495 \DirFromAngle\DeltaGradi to\@Dird
496 \Multvect{\@Dir}*{\@Dird}\@Dir%
497 \GetCoord(\@Dir)\@xnum\@ynum
498 \neq (\dX,\dY) {\vector(\dxnum,\dynum){0}}%
499 \@tdE =\ifx\Segno--\fi\DeltaGradi\p@
500 \advance\@tdA -\@tdE \Numero\@gradi\@tdA
501 \verb|\CopyVect#1to\Cent \GetCoord(\QpPun)\QpPunX\QpPunY|
502 \@@Arc
503 \strokepath\ignorespaces}%
```

The macro for the arc terminated with arrow tips at both ends is again very similar, except it is necessary to repeat the arrow tip positioning also at the starting point. The \@@Arc macro draws the curved stem.

```
504 \def\@VARC(#1)(#2){%
505 \ifdim\@tdA>\z@
506 \let\Segno+%
507 \else
    \@tdA=-\@tdA \let\Segno-%
509 \fi \Numero\@gradi\@tdA
510 \ifdim\@tdA>360\p@
    \PackageWarning{curve2e}{The arc aperture is \@gradi\space degrees
511
         and gets reduced\MessageBreak%
512
         to the range 0--360 taking the sign into consideration}%
513
   514
516 \SubVect#1from#2to\@V \ModOfVect\@V to\@Raggio \CopyVect#2to\@pPun
517 \@tdE=\pIIe@FAW\@wholewidth \@tdE=0.8\@tdE
518 \DividE\@tdE by \@Raggio\unitlength to\DeltaGradi
519 \verb|\dD=\DeltaGradi\\p@ \ddD=57.29578\\@tdD \Numero\\DeltaGradi\\@tdD=519 \end{picture}
520 \@tdD=\if\Segno--\fi\@gradi\p@ \Numero\@tempa\@tdD
521 \DirFromAngle\@tempa to\@Dir
522 \Multvect{\@V}{\@Dir}\@sPun% corrects the end point
523 \edef\@tempA{\if\Segno--\fi1}%
524 \Multvect{\@sPun}{0,\@tempA}\@vPun
525 \DirOfVect\@vPun to\@Dir
526 \AddVect\@sPun and #1 to \@sPun
527 \GetCoord(\@sPun)\@tdX\@tdY
528 \@tdD\if\Segno--\fi\DeltaGradi\p@
```

```
529 \@tdD=.5\@tdD \Numero\@tempB\@tdD
530 \DirFromAngle\@tempB to\@Dird
531 \Multvect{\QDir}*{\QDird}\QDir
532 \GetCoord(\@Dir)\@xnum\@ynum
533 \neq (\dX,\dY){\vector(\exnum,\dynum){0}}\% end point arrow tip
534 \@tdE =\DeltaGradi\p@
535 \advance\@tdA -2\@tdE \Numero\@gradi\@tdA
536 \CopyVect#1to\@Cent \GetCoord(\@pPun)\@pPunX\@pPunY
537 \SubVect\@Cent from\@pPun to \@V
538 \edef\@tempa{\if\Segno-\else-\fi\@ne}%
539 \Multvect{\QV}{0,\Qtempa}\QvPun
540 \@tdE\if\Segno--\fi\DeltaGradi\p@
541 \mbox{Numero}\ensuremath{\mbox{@tempB}{0.5}\ensuremath{\mbox{@tdE}}\%}
542 \DirFromAngle\@tempB to\@Dird
543 \Multvect{\QvPun}{\QDird}\QvPun\% corrects the starting point
544 \DirOfVect\@vPun to\@Dir\GetCoord(\@Dir)\@xnum\@ynum
545 \put(\@pPunX,\@pPunY){\vector(\@xnum,\@ynum){0}}% starting point arrow tip
546 \edef\@tempa{\if\Segno--\fi\DeltaGradi}%
547 \DirFromAngle\@tempa to \@Dir
548 \SubVect\@Cent from\@pPun to\@V
549 \Multvect{\QV}{\QDir}\QV
550 \AddVect\@Cent and\@V to\@pPun
551 \GetCoord(\@pPun)\@pPunX\@pPunY
552 \@@Arc
553 \strokepath\ignorespaces}%
```

It must be understood that the curved vectors, the above circular arcs terminated with an arrow tip at one or both ends, have a nice appearance only if the arc radius is not too small, or, said in a different way, if the arrow tip angular width does not exceed a maximum of a dozen degrees (and this is probably already too much); the tip does not get curved as the arc is, therefore there is not a smooth transition from the curved stem and the straight arrow tip if this one is large in comparison to the arc radius.

7.6 General curves

The most used method to draw curved lines with computer programs is to connect several simple curved lines, general "arcs", one to another generally maintaining the same tangent at the junction. If the direction changes we are dealing with a cusp.

The simple general arcs that are directly implemented in every program that display typeset documents, are those drawn with the parametric curves called $B\acute{e}zier\ splines$; given a sequence of points in the x,y plane, say P_0,P_1,P_2,p_3,\ldots (represented as coordinate pairs, i.e. by complex numbers), the most common

Bézier splines are the following ones:

$$\mathcal{B}_1 = P_0(1-t) + P_1 t \tag{2}$$

$$\mathcal{B}_2 = P_0(1-t)^2 + P_1 2(1-t)t + P_2 t^2 \tag{3}$$

$$\mathcal{B}_3 = P_0(1-t)^3 + P_13(1-t)^2t + P_23(1-t)t^2 + P_3t^3 \tag{4}$$

All these splines depend on parameter t; they have the property that for t=0 each line starts at the first point, while for t=1 they reach the last point; in each case the generic point P on each curve takes off with a direction that points to the next point, while it lands on the destination point with a direction coming from the penultimate point; moreover, when t varies from 0 to 1, the curve arc is completely contained within the convex hull formed by the polygon that has the spline points as vertices.

Last but not least first order splines implement just straight lines and they are out of question for what concerns maxima, minima, inflection points and the like. Quadratic splines draw just parabolas, therefore they draw arcs that have the concavity just on one side of the path; therefore no inflection points. Cubic splines are extremely versatile and can draw lines with maxima, minima and inflection points. Virtually a multi-arc curve may be drawn by a set of cubic splines as well as a set of quadratic splines (fonts are a good example: Adobe Type 1 fonts have their contours described by cubic splines, while TrueType fonts have their contours described with quadratic splines; at naked eye it is impossible to notice the difference).

Each program that processes the file to be displayed is capable of drawing first order Bézier splines (segments) and third order Bézier splines, for no other reason, at least, because they have to draw vector fonts whose contours are described by Bézier splines; sometimes they have also the program commands to draw second order Bézier splines, but not always these machine code routines are available to the user for general use. For what concerns pdftex, xetex and luatex, they have the user commands for straight lines and cubic arcs. At least with pdftex, quadratic arcs must be simulated with a clever use of third order Bézier splines.

Notice that $\text{IAT}_{\text{E}}X \ 2_{\varepsilon}$ environment picture by itself is capable of drawing both cubic and quadratic Bézier splines as single arcs; but it resorts to "poor man" solutions. The pict2e package removes all the old limitations and implements the interface macros for sending the driver the necessary drawing information, including the transformation from typographical points (72.27 pt/inch) to PostScript big points (72 bp/inch). But for what concerns the quadratic spline it resorts to the clever use of a cubic spline.

Therefore here we treat first the drawings that can be made with cubic splines; then we describe the approach to quadratic splines.

7.7 Cubic splines

Now we define a macro for tracing a general, not necessarily circular, arc. This macro resorts to a general triplet of macros with which it is possible to draw almost anything. It traces a single Bézier spline from a first point where the

tangent direction is specified to a second point where again it is specified the tangent direction. Actually this is a special (possibly useless) case where the general \curve macro of pict2e could do the same or a better job. In any case...

```
554 \def\CurveBetween#1and#2WithDirs#3and#4{%
555 \StartCurveAt#1WithDir{#3}\relax
556 \CurveTo#2WithDir{#4}\CurveFinish\ignorespaces
557 }%
```

Actually the above macro is a special case of concatenation of the triplet formed by macros \StartCurve, \CurveTo and\CurveFinish; the second macro can be repeated an arbitrary number of times. In any case the directions specified with the direction arguments, both here and with the more general macro\Curve, the angle between the indicated tangent and the arc chord may give raise to some little problems when they are very close to 90° in absolute value. Some control is exercised on these values, but some tests might fail if the angle derives from computations; this is a good place to use polar forms for the direction vectors.

The first macro initialises the drawing and the third one strokes it; the real work is done by the second macro. The first macro initialises the drawing but also memorises the starting direction; the second macro traces the current Bézier arc reaching the destination point with the specified direction, but memorises this direction as the one with which to start the next arc. The overall curve is then always smooth because the various Bézier arcs join with continuous tangents. If a cusp is desired it is necessary to change the memorised direction at the end of the arc before the cusp and before the start of the next arc; this is better than stroking the curve before the cusp and then starting another curve, because the curve joining point at the cusp is not stroked with the same command, therefore we get two superimposed curve terminations. We therefore need another small macro \ChangeDir to perform this task.

It is necessary to recall that the direction vectors point to the control points, but they do not define the control points themselves; they are just directions, or, even better, they are simply vectors with the desired direction; the macros themselves provide to the normalisation and memorisation.

The next desirable feature would be to design a macro that accepts optional node directions and computes the missing ones according to a suitable strategy. We can think of many such strategies, but none seems to be generally applicable, in the sense that one strategy might give good results, say, with sinusoids and another one, say, with cardioids, but neither one is suitable for both cases.

For the moment we refrain from automatic direction computation, but we design the general macro as if directions were optional.

Here we begin with the first initialising macro that receives with the first argument the starting point and with the second argument the direction of the tangent (not necessarily normalised to a unit vector)

```
558 \def\StartCurveAt#1WithDir#2{%
559 \begingroup
560 \GetCoord(#1)\@tempa\@tempb
561 \CopyVect\@tempa,\@tempb to\@Pzero
562 \pIIe@moveto{\@tempa\unitlength}{\@tempb\unitlength}%
```

```
563 \GetCoord(#2)\@tempa\@tempb
564 \CopyVect\@tempa,\@tempb to\@Dzero
565 \DirOfVect\@Dzero to\@Dzero
566 \ignorespaces}

And this re-initialises the direction to create a cusp:
567 \def\ChangeDir<#1>{%
568 \GetCoord(#1)\@tempa\@tempb
569 \CopyVect\@tempa,\@tempb to\@Dzero
570 \DirOfVect\@Dzero to\@Dzero
571 \ignorespaces}
```

The next macros are the finishing ones; the first strokes the whole curve, while the second fills the (closed) curve with the default color; both close the group that was opened with \StartCurve. The third macro is explained in a while; we anticipate it is functional to chose between the first two macros when a star is possibly used to switch between stroking and filling.

```
572 \def\CurveFinish{\strokepath\endgroup\ignorespaces}%
573 \def\FillCurve{\fillpath\endgroup\ignorespaces}
574 \def\CurveEnd{\fillstroke\endgroup\ignorespaces}
```

In order to draw the internal arcs it would be desirable to have a single macro that, given the destination point, computes the control points that produce a cubic Bézier spline that joins the starting point with the destination point in the best possible way. The problem is strongly ill defined and has an infinity of solutions; here we give two solutions: (a) a supposedly smart one that resorts to osculating circles and requires only the direction at the destination point; and (b) a less smart solution that requires the control points to be specified in a certain format.

We start with solution (b), \C bezierTo, the code of which is simpler than that of solution (a); then we will produce the solution (a), \C urveTo, that will become the main building block for a general path construction macro, \C urve.

The "naïve" macro \CBezierTo simply uses the previous point direction saved in \@Dzero as a unit vector by the starting macro; specifies a destination point, the distance of the first control point from the starting point, the destination point direction that will save also for the next arc-drawing macro as a unit vector, and the distance of the second control point from the destination point along this last direction. Both distances must be positive possibly fractional numbers. The syntax therefore is the following:

```
\CbezierTo\langle end\ point\rangle WithDir\langle direction\rangle AndDists\langle K_0\rangle And\langle K_1\rangle
```

where $\langle end\ point \rangle$ is a vector macro or a comma separated pair of values; again $\langle direction \rangle$ is another vector macro or a comma separated pair of values, that not necessarily indicate a unit vector, since the macro provides to normalise it to unity; $\langle K_0 \rangle$ and $\langle K_1 \rangle$ are the distances of the control points from their respective node points; they must be positive integers or fractional numbers. If $\langle K_1 \rangle$ is a number must be enclosed in curly braces, while if it is a macro name (containing the desired fractional or integer value) there is no need for braces.

This macro uses the input information to use the internal pict2e macro \pIIe@curveto with the proper arguments, and to save the final direction into the same \@Dzero macro for successive use of other arc-drawing macros.

```
575 \def\CbezierTo#1WithDir#2AndDists#3And#4{%
576 \GetCoord(#1)\@tX\@tY \MakeVectorFrom\@tX\@tY to\@Puno
577 \GetCoord(#2)\@tX\@tY \MakeVectorFrom\@tX\@tY to \@Duno
578 \DirOfVect\@Duno to\@Duno
579 \ScaleVect\@Dzero by#3to\@Czero \AddVect\@Pzero and\@Czero to\@Czero
580 \ScaleVect\@Duno by-#4to \@Cuno \AddVect\@Puno and\@Cuno to \@Cuno
581 \GetCoord(\@Czero)\@XCzero\@YCzero
582 \GetCoord(\@Cuno)\@XCuno\@YCuno
583 \GetCoord(\@Puno)\@XPuno\@YPuno
584 \pIIe@curveto{\@XCzero\unitlength}{\@YCzero\unitlength}%
                {\@XCuno\unitlength}{\@YCuno\unitlength}%
585
586
                {\@XPuno\unitlength}{\@YPuno\unitlength}%
587 \CopyVect\@Puno to\@Pzero
588 \CopyVect\@Duno to\@Dzero
589 \ignorespaces}%
```

With this building block it is not difficult to set up a macro that draws a Bézier arc between two given points, similarly to the other macro \CurveBetween previously described and defined here:

```
590 \def\CbezierBetween#1And#2WithDirs#3And#4UsingDists#5And#6{% 591 \StartCurveAt#1WithDir{#3}\relax 592 \CbezierTo#2WithDir#4AndDists#5And{#6}\CurveFinish}
```

An example of use is shown in figure 13 on page 17; notice that the tangents at the end points are the same for the black curve drawn with $\colon CurveBetween$ and the five red curves drawn with $\colon CurveBetween$; the five red curves differ only for the distance of their control point C_0 from the starting point; the differences are remarkable and the topmost curve even presents a slight inflection close to the end point. These effects cannot be obtained with the "smarter" macro $\colon CurveBetween$. But certainly this simpler macro is more difficult to use because the distances of the control points are difficult to estimate and require a number of cut-and-try experiments.

The "smarter" curve macro comes next; it is supposed to determine the control points for joining the previous point (initial node) with the specified direction to the next point with another specified direction (final node). Since the control points are along the specified directions, it is necessary to determine the distances from the adjacent curve nodes. This must work correctly even if nodes and directions imply an inflection point somewhere along the arc.

The strategy we devised consists in determining each control point as if it were the control point of a circular arc, precisely an arc of an osculating circle, i.e. a circle tangent to the curve at that node. The ambiguity of the stated problem may be solved by establishing that the chord of the osculating circle has the same direction as the chord of the arc being drawn, and that the curve chord is divided into two equal parts each of which should be interpreted as half the chord of the osculating circle.

This makes the algorithm a little rigid; sometimes the path drawn is very pleasant, while in other circumstances the determined curvatures are too large or too small. We therefore add some optional information that lets us have some control over the curvatures; the idea is based on the concept of tension, similar but not identical to the one used in the drawing programs METAFONT and META-POST. We add to the direction information, with which the control nodes of the osculating circle arcs are determined, a scaling factor that should be intuitively related to the tension of the arc (actually, since the tension of the 'rope' is high when this parameter is low, probably a name such as 'looseness' would be better suited): the smaller this number, the closer the arc resembles a straight line as a rope subjected to a high tension; value zero is allowed, while a value of 4 is close to "infinity" and turns a quarter circle into a line with an unusual loop; a value of 2 turns a quarter circle almost into a polygonal line with rounded vertices. Therefore these tension factors should be used only for fine tuning the arcs, not when a path is drawn for the first time.

We devised a syntax for specifying direction and tensions:

 $\langle direction; tension factors \rangle$

where direction contains a pair of fractional number that not necessarily refer to the components of a unit vector direction, but simply to a vector with the desired orientation (polar form is OK); the information contained from the semicolon (included) to the rest of the specification is optional; if it is present, the tension factors is simply a comma separated pair of fractional or integer numbers that represent respectively the tension at the starting or the ending node of a path arc.

We therefore need a macro to extract the mandatory and optional parts:

```
593 \def\@isTension#1;#2!!{\def\@tempA{#1}%
594 \def\@tempB{#2}\unless\ifx\@tempB\empty\strip@semicolon#2\fi}
596 \def\strip@semicolon#1;{\def\@tempB{#1}}
```

By changing the tension values we can achieve different results: see figure 14 on page 17.

We use the formula we got for arcs (1), where the half chord is indicated with s, and we derive the necessary distances:

$$K_0 = \frac{4}{3}s \frac{1 - \cos\theta_0}{\sin^2\theta_0} \tag{5a}$$

$$K_{0} = \frac{4}{3}s \frac{1 - \cos \theta_{0}}{\sin^{2} \theta_{0}}$$

$$K_{1} = \frac{4}{3}s \frac{1 - \cos \theta_{1}}{\sin^{2} \theta_{1}}$$
(5a)

We therefore start with getting the points and directions and calculating the chord and its direction:

```
597 \def\CurveTo#1WithDir#2{%
```

- 598 \def\@Tuno{1}\def\@Tzero{1}\relax
- 599 \edef\@Puno{#1}\@isTension#2;!!%
- 600 \expandafter\DirOfVect\@tempA to\@Duno
- 601 \bgroup\unless\ifx\@tempB\empty\GetCoord(\@tempB)\@Tzero\@Tuno\fi
- 602 \DistanceAndDirOfVect\@Puno minus\@Pzero to\@Chord and\@DirChord

Then we rotate everything about the starting point so as to bring the chord on the real axis

```
603 \Multvect{\@Dzero}*{\@DirChord}\@Dpzero
604 \Multvect{\@Duno}*{\@DirChord}\@Dpuno
605 \GetCoord(\@Dpzero)\@DXpzero\@DYpzero
606 \GetCoord(\@Dpuno)\@DXpuno\@DYpuno
607 \DivideFN\@Chord by2 to\@semichord
```

The chord needs not be actually rotated because it suffices its length along the real axis; the chord length is memorised in \@Chord and its half is saved in \@semichord.

We now examine the various degenerate cases, when either tangent is perpendicular or parallel to the chord. Notice that we are calculating the distances of the control points from the adjacent nodes using the half chord length, not the full length. We also distinguish between the computations relative to the arc starting point and those relative to the end point.

```
608 \ifdim\@DXpzero\p@=\z@
609 \@tdA=1.333333\p@
610 \Numero\@KCzero{\@semichord\@tdA}%
611 \fi
612 \ifdim\@DYpzero\p@=\z@
613 \@tdA=1.333333\p@
614 \Numero\@Kpzero{\@semichord\@tdA}%
615 \fi
```

The distances we are looking for are positive generally fractional numbers; so if the components are negative, we take the absolute values. Eventually we determine the absolute control point coordinates.

```
616 \unless\ifdim\@DXpzero\p@=\z@
     \unless\ifdim\@DYpzero\p@=\z@
617
       \edef\@CosDzero{\ifdim\@DXpzero\p@<\z@ -\fi\@DXpzero}%
618
       \edef\@SinDzero{\ifdim\@DYpzero\p@<\z@ -\fi\@DYpzero}%
619
       \@tdA=\@semichord\p@ \@tdA=1.333333\@tdA
620
       \DividE\@tdA by\@SinDzero\p@ to \@KCzero
621
       \@tdA=\dimexpr(\p@-\@CosDzero\p@)\relax
622
       \DividE\@KCzero\@tdA by\@SinDzero\p@ to \@KCzero
623
624
625 \fi
626 \MultiplyFN\@KCzero by \@Tzero to \@KCzero
627 \ScaleVect\@Dzero by\@KCzero to\@CPzero
628 \AddVect\@Pzero and\@CPzero to\@CPzero
```

We now repeat the calculations for the arc end point, taking into consideration that the end point direction points outwards, so that in computing the end point control point we have to take this fact into consideration by using a negative sign for the distance; in this way the displacement of the control point from the end point takes place in a backwards direction.

```
629 \ifdim\@DXpuno\p@=\z@
630 \@tdA=-1.333333\p@
```

```
\Numero\@KCuno{\@semichord\@tdA}%
631
632 \fi
633 \ifdim\@DYpuno\p@=\z@
                \@tdA=-1.333333\p@
634
                \Numero\@KCuno{\@semichord\@tdA}%
635
636 \fi
637 \unless\ifdim\@DXpuno\p@=\z@
638
                        \unless\ifdim\@DYpuno\p@=\z@
                        \edef\@CosDuno{\ifdim\@DXpuno\p@<\z@ -\fi\@DXpuno}%
639
                        \edef\@SinDuno{\ifdim\@DYpuno\p@<\z@ -\fi\@DYpuno}%
640
                        \d = \ensuremath{\d} \ensuremath{\d} \d = -1.333333\ensuremath{\d} \d = -1.3333333\ensuremath{\d} \d = -1.33333333\ensuremath{\d} \d = -1.3333333\ensuremath{\d} \d = -1.333333\ensuremath{\d} \d = -1.3333333\ensuremath{\d} \d = -1.3333333\ensuremath{\d} \d = -1.33333333\ensuremath{\d} \d = -1.3333333\ensuremath{\d} \d = -1.333333333\ensuremath{\d} \d = -1.3333333\ensuremath{\d} \d = -1.333333333\ensuremath{\d} \d = -1.33333333\ensuremath{\d} \d = -1.33333333\ensuremath{\d} \d = -1.33333333\ensuremath{\d} \d = -1.333333333\ensuremath{\d} \d = -1.33333333\ensuremath{\d} \d = -1.333333333\ensuremath{\d} \d = -1.333333333\ensuremath{\d} \d = -1.333
641
                        \DividE\@tdA by \@SinDuno\p@ to \@KCuno
642
                        \@tdA=\dimexpr(\p@-\@CosDuno\p@)\relax
643
                        \DividE\@KCuno\@tdA by\@SinDuno\p@ to \@KCuno
644
645
646 \fi
647 \MultiplyFN\@KCuno by \@Tuno to \@KCuno
648 \ScaleVect\@Duno by\@KCuno to\@CPuno
649 \AddVect\@Puno and\@CPuno to\@CPuno
   Now we have the four points and we can instruct the internal pict2e macros to
  do the path drawing.
650 \GetCoord(\@Puno)\@XPuno\@YPuno
651 \GetCoord(\@CPzero)\@XCPzero\@YCPzero
652 \GetCoord(\@CPuno)\@XCPuno\@YCPuno
653 \pIIe@curveto{\@XCPzero\unitlength}{\@YCPzero\unitlength}%
                                                    {\@XCPuno\unitlength}{\@YCPuno\unitlength}%
654
```

It does not have to stroke the curve because other Bézier splines might still be added to the path. On the opposite it memorises the final point as the initial point of the next spline

{\@XPuno\unitlength}{\@YPuno\unitlength}\egroup

```
656 \CopyVect\@Puno to\@Pzero
657 \CopyVect\@Duno to\@Dzero
658 \ignorespaces}%
```

655

We finally define the overall \Curve macro that has two flavours: starred and unstarred; the former fills the curve path with the locally selected color, while the latter just strokes the path. Both recursively examine an arbitrary list of nodes and directions; node coordinates are grouped within regular parentheses while direction components are grouped within angle brackets. The first call of the macro initialises the drawing process and checks for the next node and direction; if a second node is missing, it issues a warning message and does not draw anything. It does not check for a change in direction, because it would be meaningless at the beginning of a curve. The second macro defines the path to the next point and checks for another node; if the next list item is a square bracket delimited argument, it interprets it as a change of direction, while if it is another parenthesis delimited argument it interprets it as a new node-direction specification; if the node and direction list is terminated, it issues the stroking or filling command through \CurveEnd, and exits the recursive process. The \CurveEnd control se-

quence has a different meaning depending on the fact that the main macro was starred or unstarred. The @ChangeDir macro is just an interface to execute the regular \ChangeDir macro, but also for recursing again by recalling \@Curve.

```
659 \def\Curve{\@ifstar{\let\fillstroke\fillpath\Curve@}%
660 {\let\fillstroke\strokepath\Curve@}}
661
662 \def\Curve@(#1)<#2>{%
663
       \StartCurveAt#1WithDir{#2}%
664
       \@ifnextchar\lp@r\@Curve{%
665
       \PackageWarning{curve2e}{%
666
       Curve specifications must contain at least two nodes!\Messagebreak
667
       Please, control your \string\Curve\space specifications\MessageBreak}}}
668 \def\@Curve(#1)<#2>{%
       \CurveTo#1WithDir{#2}%
669
       \@ifnextchar\lp@r\@Curve{%
670
       \@ifnextchar[\@ChangeDir\CurveEnd}}
672 \def\@ChangeDir[#1] {\ChangeDir<#1>\@Curve}
```

As a concluding remark, please notice that the \Curve macro is certainly the most comfortable to use, but it is sort of frozen in its possibilities. The user may certainly use the \StartCurve, \CurveTo, \ChangeDir, and \CurveFinish or \FillCurve for a more versatile set of drawing macros; evidently nobody forbids to exploit the full power of the \cbezier original macro for cubic splines; we made available macros \CbezierTo and the isolated arc macro \CbezierBetween in order to use the general internal cubic Bézier splines in a more comfortable way.

As it can be seen in figure 15 on page 18 the two diagrams should approximately represent a sine wave. With Bézier curves, that resort on polynomials, it is impossible to represent a transcendental function, but it is only possible to approximate it. It is evident that the approximation obtained with full control on the control points requires less arcs and it is more accurate than the approximation obtained with the recursive \Curve macro; this macro requires almost two times as many pieces of information in order to minimise the effects of the lack of control on the control points, and even with this added information the macro approaches the sine wave with less accuracy. At the same time for many applications the \Curve recursive macro proves to be much easier to use than with single arcs drawn with the \CbezierBetween macro.

7.8 Quadratic splines

We want to create a recursive macro with the same properties as the above described \Curve macro, but that uses quadratic splines; we call it \Qurve so that the initial macro name letter reminds us of the nature of the splines being used. For the rest they have an almost identical syntax; with quadratic splines it is not possible to specify the distance of the control points from the extrema, since quadratic splines have just one control point that must lay at the intersection of the two tangent directions; therefore with quadratic splines the tangents at each point cannot have the optional part that starts with a semicolon. The syntax, therefore, is just:

```
\label{eq:qurve} $$\operatorname{Qurve}(\langle first\ point\rangle) < \langle direction\rangle > ... (\langle any\ point\rangle) < \langle direction\rangle > ... (\langle last\ point\rangle) < \langle direction\rangle > ... (\langle
```

As with \Curve, also with \Qurve there is no limitation on the number of points, except for the computer memory size; it is advisable not to use many arcs otherwise it might become very difficult to find errors.

The first macros that set up the recursion are very similar to those we wrote for **\Curve**:

```
673 \def\Qurve{\@ifstar{\let\fillstroke\fillpath\Qurve@}%
674 {\let\fillstroke\strokepath\Qurve@}}
676 \def\Qurve@(#1)<#2>{%
677
       \StartCurveAt#1WithDir{#2}%
678
       \@ifnextchar\lp@r\@Qurve{%
       \PackageWarning{curve2e}{%
679
680
         Quadratic curve specifications must contain at least
681
         two nodes!\Messagebreak
         Please, control your Qurve specifications\MessageBreak}}}%
682
683
684 \def\@Qurve(#1)<#2>{\QurveTo#1WithDir{#2}%
       \@ifnextchar\lp@r\@Qurve{%
685
       \@ifnextchar[\@ChangeQDir\CurveEnd}}%
686
687
688 \def\@ChangeQDir[#1]{\ChangeDir<#1>\@Qurve}%
```

Notice that in case of long paths it might be better to use the single macros \StartCurveAt, \QurveTo, \ChangeDir and \CurveFinish (or \FillCurve), with their respective syntax, in such a way that a long list of node-direction specifications passed to \Qurve may be split into shorter input lines in order to edit the input data in a more comfortable way.

The macro that does everything is \QurveTo. it starts with reading its arguments received through the calling macro \Qurve

```
689 \def\QurveTo#1WithDir#2{%
690 \edef\QPuno{#1}\DirOfVect#2to\@Duno\bgroup
691 \DistanceAndDirOfVect\@Puno minus\@Pzero to\@Chord and\@DirChord
```

It verifies if \@Dpzero and \@Dpuno, the directions at the two extrema of the arc, are parallel or anti-parallel by taking their "scalar" product (\@Dpzero times \@Dpuno*); if the imaginary component of the scalar product vanishes the two directions are parallel; in this case we produce an error message, but we continue by skipping this arc destination point; evidently the drawing will not be the desired one, but the job should not abort.

```
692 \Multvect{\@Dzero}*{\@Duno}\@Scalar
693 \YpartOfVect\@Scalar to \@YScalar
694 \ifdim\@YScalar\p@=\z@
695 \PackageWarning{curve2e}%
696 {Quadratic Bezier arcs cannot have their starting\MessageBreak
697 and ending directions parallel or antiparallel with\MessageBreak
698 each other. This arc is skipped and replaced with
```

```
699 a dotted line.\MessageBreak}%
700 \Dotline(\@Pzero)(\@Puno){2}\relax
701 \else
```

Otherwise we rotate everything about the starting point so as to bring the chord on the real axis; we get also the components of the two directions that, we should remember, are unit vectors, not generic vectors, although the user can use the vector specifications that are more understandable to him/her:

```
702 \Multvect{\@Dzero}*{\@DirChord}\@Dpzero 703 \Multvect{\@Duno}*{\@DirChord}\@Dpuno 704 \GetCoord(\@Dpzero)\@DXpzero\@DYpzero 705 \GetCoord(\@Dpuno)\@DXpuno\@DYpuno
```

We check if the two directions point to the same half plane; this implies that these rotated directions point to different sides of the chord vector; all this is equivalent that the two direction Y components have opposite signs, so that their product is strictly negative, while the two X components product is not negative.

```
706 \MultiplyFN\@DXpzero by\@DXpuno to\@XXD
707 \MultiplyFN\@DYpzero by\@DYpuno to\@YYD
708 \unless\ifdim\@YYD\p@<\z@\ifdim\@XXD\p@<\z@
709 \PackageWarning{curve2e}%
710 {Quadratic Bezier arcs cannot have inflection points\MessageBreak
711 Therefore the tangents to the starting and ending arc\MessageBreak
712 points cannot be directed to the same half plane.\MessageBreak
713 This arc is skipped and replaced by a dotted line\MessageBreak}%
714 \Dotline(\@Pzero)(\@Puno){2}\fi
715 \else
```

After these tests we should be in a "normal" situation. We first copy the expanded input information into new macros that have more explicit names: macros stating wit 'S' denote the sine of the direction angle, while those starting with 'C' denote the cosine of that angle. We will use these expanded definitions as we know we are working with the actual values. These directions are those relative to the arc chord.

```
716 \edef\@CDzero{\@DXpzero}\relax
717 \edef\@SDzero{\@DYpzero}\relax
718 \edef\@CDuno{\@DXpuno}\relax
719 \edef\@SDuno{\@DYpuno}\relax
```

Suppose we write the parametric equations of a straight line that departs from the beginning of the chord with direction angle ϕ_0 and the corresponding equation of the straight line departing from the end of the chord (of length c) with direction angle ϕ_1 . We have to find the coordinates of the intersection point of these two straight lines.

$$t\cos\phi_0 - s\cos\phi_1 = c \tag{6a}$$

$$t\sin\phi_0 - s\sin\phi_1 = 0\tag{6b}$$

The parameters t and s are just the running parameters; we have to solve those simultaneous equations in the unknown variables t and s; these values let us

compute the coordinates of the intersection point:

$$X_C = \frac{c\cos\phi_0\sin\phi_1}{\sin\phi_0\cos\phi_1 - \cos\phi_0\sin\phi_1}$$
 (7a)

$$Y_C = \frac{c \sin \phi_0 \sin \phi_1}{\sin \phi_0 \cos \phi_1 - \cos \phi_0 \sin \phi_1}$$
 (7b)

Having performed the previous tests we are sure that the denominator is not vanishing (direction are not parallel or anti-parallel) and that it lays at the same side as the direction with angle ϕ_0 with respect to the chord.

The coding then goes on like this:

```
720 \MultiplyFN\@SDzero by\@CDuno to\@tempA
```

721 \MultiplyFN\@SDuno by\@CDzero to\@tempB

 $722 \edef\@tempA{\strip@pt\dimexpr\@tempA\pQ-\@tempB\pQ}\relax$

 $723 \end{p@ \endp} \end{p@ \end{p@ \endp}} \end{p@ \end{p@ \endp}} \end{p@ \end{p@ \endp}} \end{p@ \end{p@ \end{p@ \endp}} \end{p@ \en$

724 \edef\@tempC{\strip@pt\dimexpr \@tdA*\@tdB/\@tdC}\relax

725 \MultiplyFN\@tempC by\@CDzero to \@XC

726 \MultiplyFN\@tempC by\@SDzero to \@YC

727 \ModOfVect\@XC,\@YC to\@KC

Now we have the coordinates and the module of the intersection point vector taking into account the rotation of the real axis; getting back to the original coordinates before rotation, we get:

728 \ScaleVect\@Dzero by\@KC to\@CP

729 \AddVect\@Pzero and\@CP to\@CP

730 \GetCoord(\@Pzero)\@XPzero\@YPzero

731 \GetCoord(\@Puno)\@XPuno\@YPuno

732 \GetCoord(\@CP)\@XCP\@YCP

We have now the coordinates of the two extrema point of the quadratic arc and of the control point. Keeping in mind that the symbols P_0 , P_1 and C denote geometrical points but also their coordinates as ordered pairs of real numbers (i.e. they are complex numbers) we have to determine the parameters of a cubic spline that with suitable values get simplifications in its parametric equation so that it becomes a second degree function instead of a third degree one. It is possible, in spite of the fact the it appears impossible that e cubic form becomes a quadratic one; we should determine the values of P_a and P_b such that:

$$P_0(1-t)^3 + 3P_a(1-t)^2t + 3P_b(1-t)t^2 + P_1t^3$$

is equivalent to

$$P_0(1-t)^2 + 2C(1-t)t + P_1t^2$$

It turns out that the solution is given by

$$P_a = C + (P_0 - C)/3$$
 and $P_b = C + (P_1 - C)/3$ (8)

The transformations implied by equations (8) are performed by the following macros already available from the pict2e package; we use them here with the actual arguments used for this task:

```
733 \@ovxx=\@XPzero\unitlength \@ovyy=\@YPzero\unitlength
734 \@ovdx=\@XCP\unitlength \@ovdy=\@YCP\unitlength
735 \@xdim=\@XPuno\unitlength \@ydim=\@YPuno\unitlength
736 \pIIe@bezier@QtoC\@ovxx\@ovdx\@ovro
737 \pIIe@bezier@QtoC\@ovyy\@ovdy\@ovri
738 \pIIe@bezier@QtoC\@xdim\@ovdx\@clnwd
739 \pIIe@bezier@QtoC\@ydim\@ovdy\@clnht
```

We call the basic pict2e macro to draw a cubic spline and we finish the conditional statements with which we started these calculations; eventually we close the group we opened at the beginning and we copy the terminal node information (position and direction) into the 0-labelled macros that indicate the starting point of the next arc.

```
740 \pIIe@curveto\@ovro\@ovri\@clnwd\@clnht\@xdim\@ydim
741 \fi\fi\egroup
742 \CopyVect\@Puno to\@Pzero
743 \CopyVect\@Duno to\@Dzero
744 \ignorespaces}
```

An example of usage is shown at the left in figure 16^6 on page 19 created with the code shown in the same page.

Notice also that the inflexed line is made with two arcs that meet at the inflection point; the same is true for the line that resembles a sine wave. The cusps of the inner border of the green area are obtained with the usual optional star already used also with the \Curve recursive macro.

The "circle" inside the square frame is visibly different from a real circle, in spite of the fact that the maximum deviation from the true circle is just about 6% relative to the radius; a quarter circle obtained with a single parabola is definitely a poor approximation of a real quarter circle; possibly by splitting each quarter circle in three or four partial arcs the approximation of a real quarter circle would be much better. On the right of figure 16 on page 19 it is possible to compare a "circle" obtained with quadratic arcs with the the internal circle obtained with cubic arcs; the difference is easily seen even without using measuring instruments.

With quadratic arcs we decided to avoid defining specific macros similar to \CurveBetween and \CbezierBetween; the first macro would not save any typing to the operator; furthermore it may be questionable if it was really useful even with cubic splines; the second macro with quadratic arcs is meaningless, since with quadratic arcs there is just one control point and there is no choice on its position.

⁶The commands \legenda, \Pall and \Zbox are specifically defined in the preamble of this document; they must be used within a picture environment. \legenda draws a framed legend made up of a single (short) math formula; \Pall is just a shorthand to put a filled small circle at a specified position' \Zbox puts a symbol in math mode a little displaced in the proper direction relative to a specified position. They are just handy to label certain objects in a picture diagram, but they are not part of the curve2e package.

8 Conclusion

I believe that the set of new macros provided by this package can really help the user to draw his/her diagrams with more agility; it will be the accumulated experience to decide if this is true.

As a personal experience we found very comfortable to draw ellipses and to define macros to draw not only such shapes or filled elliptical areas, but also to create "legends" with coloured backgrounds and borders. But this is an application of the functionality implemented in this package.

9 The README.txt file

The following is the text that forms the contents of the README.txt file that accompanies the package. We found it handy to have it in the documented source, because in this way certain pieces of information don't need to be repeated again and again in different files.

```
745 The package bundle curve2e is composed of the following files
746
747 curve2e.dtx
748 curve2e.pdf
749 README.txt
750 ltxdoc.cfg
752 curve2e.dtx is the documented TeX source file of file curve2e.sty; you
753 get curve2e.sty, curve2e.pdf, and curve2e-v161.sty by running pdflatex
754 on curve2e.dtx. The ltxdoc.cfg file customises the way the documentation
755 file is typeset. This specific .cfg file is part of the ltxdoc package
756 functionality and it is supposed to be configured for each specific
757 other bundle.
758
759 README.txt, this file, contains general information.
760
761 Curve2e-v161.sty contains a previous version of the package; see below
762 why the older version might become necessary for the end user.
764 Curve2e.sty is an extension of the package pict2e.sty which extends the
765 standard picture LaTeX environment according to what Leslie Lamport
766 specified in the second edition of his LaTeX manual (1994).
767
768 This further extension curve2e.sty allows to draw lines and vectors
769 with any non integer slope parameters, to draw dashed lines of any
770 slope, to draw arcs and curved vectors, to draw curves where just
771 the interpolating nodes are specified together with the slopes at
772 the nodes; closed paths of any shape can be filled with color; all
773 coordinates are treated as ordered pairs, i.e. 'complex numbers';
774 coordinates may be expressed also in polar form. Coordinates may be specified with macros, so t
775 Some of these features have been incorporated in the 2011 version of
776 pict2e; therefore this package avoids any modification to the original
```

```
777 pict2e commands.
778
779 Curve2e now accepts polar coordinates in addition to the usual cartesian
780 ones; several macros have been upgraded and a new macro for tracing cubic
781 Bezier splines with their control nodes specified in polar form is
782 available. The same applies to quadratic Bezier splines. The multiput
783 command has been completely modified in a backwards compatible way, as
784 to manipulate the increment components in a configurable way. A new
785 xmultiput command has been defined that is more configurable than the
786 original one; both \multiput and \xmultiput are backwards compatible with
787 the original picture environment definition.
789 This version solves a conflict with package eso-pic.
791 This version of curve2e is almost fully compatible with pict2e dated
792\ 2014/01/12 version 0.2z and later.
794 If you specify
795
796 \usepackage[<pict2e options>]{curve2e}
798 the package pict2e is automatically invoked with the specified options.
800\,\mathrm{The} -almost compatible- frase is necessary to explain that this version
801 of curve2e uses some 'functions' of the LaTeX3 language that were made
802 available to the LaTeX developers by mid October 2018. Should the user
803 have an older or a basic/incomplete installation of the TeX system,
804 such L3 functions might not be available. This is why this
805\,\mathrm{package} checks the presence of the developer interface; in case
806 \; \mathrm{such} interface is not available it falls back to the previous version
807 renamed curve2e-v161.sty, which is part of this bundle; this order
808 package name must not be modified in any way. The compatibility mentioned
809 above implies that the user macros remain the same, but their
810 implementation requires the L3 interface.
811
812 The package has the LPPL status of author maintained.
814 According to the LPPL licence, you are entitled to modify this package,
815 as long as you fulfil the few conditions set forth by the Licence.
817 Nevertheless this package is an extension to the standard LaTeX package
818 \; \text{pict2e} (2014). Therefore any change must be controlled on the
819 parent package pict2e, so as to avoid redefining or interfering with
820 what is already contained in the official package.
821
822 If you prefer sending me your modifications, as long as I will maintain
823 this package, I will possibly include every (documented) suggestion or
824 modification into this package and, of course, I will acknowledge your
825 contribution.
```

826

```
827 Claudio Beccari
828
829 claudio dot beccari at gmail dot com
```

10 The fall-back package version curve2e-v161

this is the fall-back version of curve2e-v161.sty to which the main file curve2e.sty falls back in case the interface package xfp is not available.

```
830 \NeedsTeXFormat{LaTeX2e} [2016/01/01]
831 \ProvidesPackage{curve2e-v161}%
832
            [2019/02/07 v.1.61 Extension package for pict2e]
833
834 \RequirePackage{color}
835 \RequirePackageWithOptions{pict2e}[2014/01/01]
836 \RequirePackage{xparse}
837 \def\TRON{\tracingcommands\tw@ \tracingmacros\tw@}%
838 \def\TROF{\tracingcommands\z@ \tracingmacros\z@}%
839 \ifx\undefined\@tdA \newdimen\@tdA \fi
840 \ifx\undefined\@tdB \newdimen\@tdB \fi
841 \ifx\undefined\@tdC \newdimen\@tdC \fi
842 \ifx\undefined\@tdD \newdimen\@tdD \fi
843 \ifx\undefined\@tdE \newdimen\@tdE \fi
844 \ifx\undefined\@tdF \newdimen\@tdF \fi
845 \ifx\undefined\defaultlinewidth \newdimen\defaultlinewidth \fi
846 \gdef\linethickness#1{\@wholewidth#1\@halfwidth.5\@wholewidth\ignorespaces}%
847 \newcommand\defaultlinethickness[1] {\defaultlinewidth=#1\relax
848 \def\thicklines{\linethickness{\defaultlinewidth}}%
849 \def\thinlines{\linethickness{.5\defaultlinewidth}}%
850 \thinlines\ignorespaces}
851 \def\LIne(#1){{\GetCoord(#1)\@tX\@tY
         \mbox{moveto}(0.0)
852
         \pIIe@lineto{\@tX\unitlength}{\@tY\unitlength}\strokepath}\ignorespaces}%
853
854 \def\segment(#1)(#2){\@killglue\polyline(#1)(#2)}%
855 \def\line(#1)#2{\begingroup
     \@linelen #2\unitlength
     \ifdim\@linelen<\z@\@badlinearg\else
857
        \expandafter\DirOfVect#1to\Dir@line
858
        \GetCoord(\Dir@line)\d@mX\d@mY
859
       \left( \frac{d^mX}{p0} \right) = \z \cdot \left( \frac{d^m X}{p0} \right)
860
         \DividE\ifdim\d@mX\p@<\z@-\fi\p@ by\d@mX\p@ to\sc@lelen
861
862
         \@linelen=\sc@lelen\@linelen
        \fi
863
        \mbox{moveto}(0,0)
864
        \pIIe@lineto{\d@mX\@linelen}{\d@mY\@linelen}%
865
        \strokepath
866
     \fi
867
868 \endgroup\ignorespaces}%
869 \ifx\Dashline\undefined
```

```
870 \def\Dashline{\@ifstar{\Dashline@@}{\Dashline@}}
871 \def\Dashline@(#1)(#2)#3{%
872 \bgroup
      \verb|\countdef|NumA3254| countdef|NumB3252| relax|
873
      \GetCoord(#1)\@tA\@tB \MakeVectorFrom\@tA\@tB to\V@ttA
874
875
      \GetCoord(#2)\@tA\@tB \MakeVectorFrom\@tA\@tB to\V@ttB
876
      \SubVect\V@ttA from\V@ttB to\V@ttC
      \ModOfVect\V@ttC to\DlineMod
877
      \DivideFN\DlineMod by#3 to\NumD
878
      \NumA\expandafter\Integer\NumD.??
879
      \ifodd\NumA\else\advance\NumA\@ne\fi
880
      \NumB=\NumA \divide\NumB\tw@
881
      \DividE\DlineMod\p@ by\NumA\p@ to\D@shMod
882
      \DividE\p@ by\NumA\p@ to \@tempa
      \MultVect\V@ttC by\@tempa,0 to\V@ttB
884
      \MultVect\V@ttB by 2,0 to\V@ttC
885
      \advance\NumB\@ne
886
      \edef\@mpt{\noexpand\egroup
887
888
      \noexpand\multiput(\V@ttA)(\V@ttC){\number\NumB}%
889
         {\noexpand\LIne(\V@ttB)}}%
890
      \@mpt\ignorespaces}%
891 \let\Dline\Dashline
892
893 \def\Dashline@@(#1)(#2)#3{\put(#1){\Dashline@(0,0)(#2){#3}}}
894\fi
895 \ifx\Dotline\undefined
896 \def\Dotline{\@ifstar{\Dotline@@}{\Dotline@}}
897 \def\Dotline@(#1)(#2)#3{%
898 \bgroup
      \countdef\NumA 3254\relax \countdef\NumB 3255\relax
899
      \GetCoord(#1)\@tA\@tB \MakeVectorFrom\@tA\@tB to\V@ttA
900
901
      \GetCoord(#2)\@tA\@tB \MakeVectorFrom\@tA\@tB to\V@ttB
902
      \SubVect\V@ttA from\V@ttB to\V@ttC
      \ModOfVect\V@ttC to\DotlineMod
903
904
      \DivideFN\DotlineMod by#3 to\NumD
905
      \NumA=\expandafter\Integer\NumD.??
      \DivVect\V@ttC by\NumA,O to\V@ttB
906
      \advance\NumA\@ne
907
908
      \edef\@mpt{\noexpand\egroup
      \noexpand\multiput(\V@ttA)(\V@ttB){\number\NumA}%
909
910
         {\noexpand\makebox(0,0){\noexpand\circle*{0.5}}}}%
911
      \@mpt\ignorespaces}%
912
913 \def\Dotline@@(#1)(#2)#3{\put(#1){\Dotline@(0,0)(#2){#3}}}
914 \fi
915 \AtBeginDocument{\@ifpackageloaded{eso-pic}{%
916 \renewcommand\LenToUnit[1] {\strip@pt\dimexpr#1*\p@/\unitlength}}{}}
918 \def\GetCoord(#1)#2#3{%
919 \expandafter\SplitNod@\expandafter(#1)#2#3\ignorespaces}
```

```
920 \def\isnot@polar#1:#2!!{\def\@tempOne{#2}\ifx\@tempOne\empty
921 \expandafter\@firstoftwo\else
922 \expandafter\@secondoftwo\fi
923 {\SplitNod@@}{\SplitPolar@@}}
924
925 \def\SplitNod@(#1)#2#3{\isnot@polar#1:!!(#1)#2#3}%
926 \def\SplitNod@@(#1,#2)#3#4{\edef#3{#1}\edef#4{#2}}%
927 \def\SplitPolar@@(#1:#2)#3#4{\DirFromAngle#1to\@DirA
928 \ScaleVect\@DirA by#2to\@DirA
929 \expandafter\SplitNod@@\expandafter(\@DirA)#3#4}
930
931 \let\originalput\put
932 \def\put(#1) {\bgroup\GetCoord(#1) \@tX\@tY
933 \edef\x{\noexpand\egroup\noexpand\originalput(\@tX,\@tY)}\x}
935 \let\originalmultiput\multiput
936 \let\original@multiput\@multiput
937
938 \long\def\@multiput(#1)#2#3{\bgroup\GetCoord(#1)\@mptX\@mptY
939 \edef\x{\noexpand\egroup\noexpand\original@multiput(\@mptX,\@mptY)}%
940 \x{#2}{#3}\ignorespaces}
941
942 \gdef\multiput(#1)#2{\bgroup\GetCoord(#1)\@mptX\@mptY
943 \edef\x{\noexpand\egroup\noexpand\originalmultiput(\@mptX,\@mptY)}\x(}%)
     \def\vector(#1)#2{%
945
       \begingroup
         \GetCoord(#1)\d@mX\d@mY
946
947
         \@linelen#2\unitlength
         948
         \ifdim\@linelen<\z@ \@linelen=-\@linelen\fi
949
         \MakeVectorFrom\d@mX\d@mY to\@Vect
950
951
         \DirOfVect\@Vect to\Dir@Vect
952
           \YpartOfVect\Dir@Vect to\@ynum \@ydim=\@ynum\p@
           \XpartOfVect\Dir@Vect to\@xnum \@xdim=\@xnum\p@
953
           \left( \frac{dm}{dmX} \right) = 20
954
           \left(\frac{d^mY}{p^e}\right)
955
             \else
956
               \DividE\ifdim\@xnum\p@<\z@-\fi\p@ by\@xnum\p@ to\sc@lelen
957
               \@linelen=\sc@lelen\@linelen
958
             \fi
959
960
           \fi
         \@tdB=\@linelen
961
962 \pIIe@concat\@xdim\@ydim{-\@ydim}\@xnum\@linelen}{\@ynum\@linelen}%
           \@linelen\z@
963
964
           \pIIe@vector
965
           \fillpath
966
         \@linelen=\@tdB
967
         \@tdA=\pIIe@FAW\@wholewidth
         \@tdA=\pIIe@FAL\@tdA
968
         \advance\@linelen-\@tdA
969
```

```
\ifdim\@linelen>\z@
970
            \mbox{moveto}(0,0)
971
            \pIIe@lineto{\@xnum\@linelen}{\@ynum\@linelen}%
972
            \strokepath\fi
973
        \endgroup}
974
975 \def\Vector(#1){{%
976 \GetCoord(#1)\@tX\@tY
977 \ifdim\@tX\p@=\z@\vector(\@tX,\@tY)\{\@tY\}
978 \else
979 \c(\CtX,\CtY){\ctX}\fi}
980 \def\VECTOR(#1)(#2){\begingroup
981 \SubVect#1from#2to\@tempa
982 \expandafter\put\expandafter(#1){\expandafter\Vector\expandafter(\@tempa)}%
983 \endgroup\ignorespaces}
984 \let\lp@r( \let\rp@r)
985 \renewcommand*\polyline[1][\beveljoin]{\p@lylin@[#1]}
986
987 \def\p@lylin@[#1](#2){\@killglue#1\GetCoord(#2)\d@mX\d@mY
988
        \pIIe@moveto{\d@mX\unitlength}{\d@mY\unitlength}%
989
        \@ifnextchar\lp@r{\p@lyline}{%
990
        \PackageWarning{curve2e}%
        {Polylines require at least two vertices!\MessageBreak
991
         Control your polyline specification\MessageBreak}%
992
        \ignorespaces}}
993
994
995 \def\p@lyline(#1){\GetCoord(#1)\d@mX\d@mY
        \pIIe@lineto{\d@mX\unitlength}{\d@mY\unitlength}%
996
997
        \@ifnextchar\lp@r{\p@lyline}{\strokepath\ignorespaces}}
998 \providecommand\polygon{}
999 \RenewDocumentCommand\polygon{s O{\beveljoin} }{\@killglue\begingroup
1000 \IfBooleanTF{#1}{\@tempswatrue}{\@tempswafalse}%
1001 \@polygon[#2]}
1002
1003 \def\@polygon[#1](#2){\@killglue#1\GetCoord(#2)\d@mX\d@mY
        \pIIe@moveto{\d@mX\unitlength}{\d@mY\unitlength}%
1004
        \@ifnextchar\lp@r{\@@polygon}{%
1005
        \PackageWarning{curve2e}%
1006
1007
        {Polygons require at least two vertices!\MessageBreak
         Control your polygon specification\MessageBreak}%
1008
        \ignorespaces}}
1009
1010
1011
      \def\@@polygon(#1){\GetCoord(#1)\d@mX\d@mY
        \pIIe@lineto{\d@mX\unitlength}{\d@mY\unitlength}%
1012
        \@ifnextchar\lp@r{\@@polygon}{\pIIe@closepath
1013
1014
          \if@tempswa\pIIe@fillGraph\else\pIIe@strokeGraph\fi
1015
          \endgroup
1016
          \ignorespaces}}
1017 \def\GraphGrid(#1,#2) {\bgroup\textcolor{red} {\linethickness{.1\p0}%
1018 \RoundUp#1modulo10to\@GridWd \RoundUp#2modulo10to\@GridHt
1019 \@tempcnta=\@GridWd \divide\@tempcnta10\relax \advance\@tempcnta\@ne
```

```
1020 \text{ } \text{ } \{100,0\} \
1021 \verb|\dtempcnta=\QGridHt \divide\Qtempcnta10\advance\Qtempcnta\Qne
\label{line(1,0)(0,10)(0,10)(0)(0,10)(0)(0,10)(0)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,10)(0,
1023 \egroup\ignorespaces}
1024 \end{ter} {\bf 1024 \end{ter} empcnta\end{ter} integer $\tt 1.??\% }
1025 \count254\@tempcnta\divide\count254by#2\relax
1026 \text{\mbox{multiply}}\count254by#2\relax
1027 \count252\@tempcnta\advance\count252-\count254
1028 \ifnum\count252>0\advance\count252-#2\relax
1029 \advance \dtempcnta-\count 252 fi\edef \#3{\number\dtempcnta}\ignorespaces \}\%
1030 \def\Integer#1.#2??{#1}%
1031 \ifdefined\dimexpr
                 \unless\ifdefined\DividE
1032
1033 \def\DividE#1by#2to#3{\bgroup
1034 \dimendef\Num2254\relax \dimendef\Den2252\relax
1035 \dimendef\@DimA 2250
1036 \Num=\p@ \Den=#2\relax
1037 \left| \text{ifdim}\right| = \z@
          \edef\x{\noexpand\endgroup\noexpand\def\noexpand#3{\strip@pt\maxdimen}}%
1039 \else
1040
            \@DimA=#1\relax
            \left( x_{x}\right) 
1041
                  \noexpand\egroup\noexpand\def\noexpand#3{%
1042
                        \strip@pt\dimexpr\@DimA*\Num/\Den\relax}}%
1043
1044 \fi
1045 \x\ignorespaces}%
1046 \fi
                 \unless\ifdefined\DivideFN
1047
                            1048
                 \fi
1049
                 \unless\ifdefined\MultiplY
1050
1051
                         \def\MultiplY#1by#2to#3{\bgroup
1052
                          \dimendef\@DimA 2254 \dimendef\@DimB2255
                          \@DimA=#1\p@\relax \@DimB=#2\p@\relax
1053
                          \left( x_{x}\right) 
1054
                                \noexpand\egroup\noexpand\def\noexpand#3{%
1055
                                     \strip@pt\dimexpr\@DimA*\@DimB/\p@\relax}}%
1056
1057
                          \x\ignorespaces}%
                          \let\MultiplyFN\MultiplY
1058
                 \fi
1059
1060 \fi
1061
1062 \unless\ifdefined\Numero
            \def\Numero#1#2{\bgroup\dimen3254=#2\relax
1063
1064
                  \edef\x{\noexpand\egroup\noexpand\edef\noexpand#1{%
1065
                     \strip@pt\dimen3254}}\x\ignorespaces}%
1066 \fi
1067 \def\g@tTanCotanFrom#1to#2and#3{%
1068 \DividE 114.591559\p@ by#1to\X@ \@tdB=\X@\p@
1069 \countdef\I=2546\def\Tan{0}\I=11\relax
```

```
1070 \@whilenum\I>\z@\do{%
1071
                            \C \Tan\p@ \C \L \B
                            1072
1073
                            \advance\I-2\relax}%
1074 \ensuremath{\mbox{$1074$}}\begin{tabular}{l} 1074 \ensuremath{\mbox{$1074$}}\begin{tabula
1075 \def\SinOf#1to#2{\bgroup%
1076 \@tdA=#1\p@%
1079 \else%
                        1080
1081 fi \left(\frac{0}{4}\right) = 1081 c
                        \def\@tempA{0}\%
1083 \else
                        \left(\frac{dx}{dx}\right)
1084
1085
                                \def\Segno{+}%
                        \else
1086
                                 \def\Segno{-}%
1087
1088
                                 \d=-\dA
1089
                        \ifdim\@tdA>90\p@
1090
                                 1091
                        \fi
1092
                        \footnote{Model} \begin{tabular}{ll} $$ \left( A=90 \right) $$ (a) $$ (b) $$ (b) $$ (c) $$ (c
1093
1094
                                 \def\@tempA{\Segno1}%
1095
                                 \left(\frac{0}{4}=180\right)
1096
                                         \def\0\text{tempA}{0}%
1097
                                 \else
1098
                                        \ifdim\@tdA<\p@
1099
                                                 \@tdA=\Segno0.0174533\@tdA
1100
1101
                                                 \DividE\@tdA by\p@ to \@tempA%
1102
                                                 \g@tTanCotanFrom\@tdA to\T and\Tp
1103
                                                 \@tdA=\T\p@ \advance\@tdA \Tp\p@
1104
                                                 \DividE \Segno2\p@ by\@tdA to \@tempA%
1105
                                         \fi
1106
                                 \fi
1107
1108
                        \fi
1109 \fi
1110 \edef\endSinOf{\noexpand\egroup
                        1112 \endSinOf}%
1113 \def\CosOf#1to#2{\bgroup%
1114 \@tdA=#1\p@%
1115 \ifdim\@tdA>\z@%
                   1117 \else%
1119 \fi
```

```
1120 \ifdim\@tdA>180\p@
1121 \d \@tdA=-\@tdA \advance\@tdA 360\p@
1122 \fi
1123 \left( A<90 \right)
1124 \ \def\Segno{+}%
1125 \else
1126
                \def\Segno{-}%
                1127
1128 \fi
1129 \left( A=\z \right)
1130 \def\@tempA{\Segno1}\%
1131 \else
1132
                \ifdim\@tdA<\p@
                      \d \d \Mumero\d \d \Mumero\d \d \A
1133
                      1134
                      \advance\@tdA \p@
1135
                      \DividE\@tdA by\p@ to\@tempA%
1136
1137
                \else
1138
                      \footnote{Model} \foo
1139
                            \def\@tempA{0}\%
1140
                            \g@tTanCotanFrom\@tdA to\T and\Tp
1141
                            \label{lem:condition} $$ \operatorname{A-Tp}_0 \ \advance_0tdA-Tp_0 $$
1142
                            1143
                            \DividE\Segno\@tdA by\@tdB to\@tempA%
1144
1145
                      \fi
1146
                \fi
1147 \fi
1148 \edef\edcosOf{\noexpand\egroup}
                      \noexpand\def\noexpand#2{\@tempA}\noexpand\ignorespaces}%
1149
1150 \endCosOf}%
1151 \def\TanOf#1to#2{\bgroup%
1152 \@tdA=#1\p@%
1153 \ifdim\@tdA>90\p@%
             \@whiledim\@tdA>90\p@\do{\advance\@tdA -180\p@}%
1155 \else%
1156 \@whiledim\@tdA<-90\p@\do{\advance\@tdA 180\p@}%
1157 \fi%
1158 \ifdim\@tdA=\z@%
              \def\@tempA{0}%
1159
1160 \else
1161
              \ifdim\@tdA>\z@
1162
                     \def\Segno{+}%
               \else
1163
1164
                      \def\Segno{-}%
1165
                      \d=-\dA
1166
                \ifdim\@tdA=90\p@
1167
                      \label{lem:lempA} $$ \left( Segno 16383.99999 \right) % $$
1168
                \else
1169
```

```
\index(0) \
1170
                                         \@tdA=\Segno0.0174533\@tdA
1171
                                         \DividE\@tdA by\p@ to\@tempA%
1172
                           \else
1173
                                  \g@tTanCotanFrom\@tdA to\T and\Tp
1174
1175
                                  \@tdA\Tp\p@ \advance\@tdA -\T\p@
1176
                                  \DividE\Segno2\p@ by\@tdA to\@tempA%
1177
                           \fi
                   \fi
1178
1179 \fi
1180 \edef\endTanOf{\noexpand\egroup
                       \noexpand\def\noexpand#2{\@tempA}\noexpand\ignorespaces}%
1182 \endTanOf}%
1183 \def\ArcTanOf#1to#2{\bgroup
1184 \countdef\Inverti 4444\Inverti=0
1185 \def\Segno{}
1188 \ifdim\@tdD>\p@
1189 \Inverti=\One
1192 \unless\ifdim\@tdD>0.02\p@
                           \def\@tX{\strip@pt\dimexpr57.295778\@tdD\relax}%
1193
1194 \else
1195
                           \ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath{\def}\ensuremath}\ensuremath{\def}\ensurema
                            \countdef\I 2523 \I=9\relax
1196
1197
                            \@whilenum\I>0\do{\TanOf\@tX to\@tG
                           \edef\@tG{\strip@pt\dimexpr\@tG\p@-\@tdD\relax}\relax
1198
                           \MultiplY\@tG by57.295778to\@tG
1199
                           \CosOf\@tX to\@tH
1200
1201
                           \MultiplY\@tH by\@tH to\@tH
1202
                           \MultiplY\@tH by\@tG to \@tH
                           \edef\@tX{\strip@pt\dimexpr\@tX\p@ - \@tH\p@\relax}\relax
1203
1204
                            \advance\I\m@ne}%
1205 \fi
1206 \ifnum\Inverti=\@ne
1207 \edef\@tX{\strip@pt\dimexpr90\p@-\@tX\p@\relax}
1208 \fi
1209 \edef\x{\egroup\noexpand\edef\noexpand#2{\Segno\@tX}}\x\ignorespaces}%
1210 \def\MakeVectorFrom#1#2to#3{\edef#3{#1,#2}\ignorespaces}%
1211 \def\CopyVect#1to#2{\edef#2{#1}\ignorespaces}%
1212 \end{figure} $$1212 \end{figure} $$1212
1213 \@tempdima=\t@X\p@ \ifdim\@tempdima<\z@ \@tempdima=-\@tempdima\fi
1214 \@tempdimb=\t@Y\p@ \ifdim\@tempdimb<\z@ \@tempdimb=-\@tempdimb\fi
1215 \left( \frac{0}{2} \right)
1216
                           \ifdim\@tempdimb=\z@
1217
                                        \def\0T{0}\\def\z0
1218
                           \else
                                        1219
```

```
\fi
1220
1221 \else
                   \ifdim\@tempdima>\@tempdimb
1222
                          \DividE\@tempdimb by\@tempdima to\@T
1223
                          \@tempdimc=\@tempdima
1224
1225
                   \else
1226
                                  \DividE\@tempdima by\@tempdimb to\@T
                                  \@tempdimc=\@tempdimb
1227
1228
                   \fi
1229 \fi
1230 \unless\ifdim\@tempdimc=\z@
1231
                   \unless\ifdim\0T\p0=\z0
1232
                          \@tempdima=\@T\p@ \@tempdima=\@T\@tempdima
                          \advance\@tempdima\p@%
1233
                          \@tempdimb=\p@%
1234
                          \@tempcnta=5\relax
1235
                          \@whilenum\@tempcnta>\z@\do{\DividE\@tempdima by\@tempdimb to\@T
1236
                          \advance\@tempdimb \@T\p@ \@tempdimb=.5\@tempdimb
1237
1238
                          \advance\@tempcnta\m@ne}%
1239
                           \@tempdimc=\@T\@tempdimc
                   \fi
1240
1241 \fi
1242 \Numero#2\@tempdimc
1243 \ignorespaces}%
1244 \ensuremath{\mbox{\mbox{$1$}}\ensuremath{\mbox{$2$}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremath{\mbox{$4$}}\ensuremat
1245 \ModOfVect#1to\@tempa
1246 \unless\ifdim\@tempdimc=\z@
1247
                 \DividE\t@X\p@ by\@tempdimc to\t@X
                1248
1249 \fi
1250 \MakeVectorFrom\t@X\t@Y to#2\ignorespaces}%
1251 \def\ModAndDirOfVect#1to#2and#3{%
1252 \GetCoord(#1)\t@X\t@Y
1253 \ModOfVect#1to#2%
1254 \left( \frac{c}{c} \right) = 1254 
              \DividE\t@X\p@ by\@tempdimc to\t@X
              \DividE\t@Y\p@ by\@tempdimc to\t@Y
1256
1257 \fi
1258 \MakeVectorFrom\t@X\t@Y to#3\ignorespaces}%
1259 \def\DistanceAndDirOfVect#1minus#2to#3and#4{%
1260 \SubVect#2from#1to\@tempa
1261 \ModAndDirOfVect\@tempa to#3and#4\ignorespaces}%
1262 \def\XpartOfVect#1to#2{%}
1263 \GetCoord(#1)#2\@tempa\ignorespaces}%
1264 \def\YpartOfVect#1to#2{%
1265 \GetCoord(#1)\@tempa#2\ignorespaces}%
1266 \def\DirFromAngle#1to#2{%
1267 \CosOf#1to\t@X
1268 \SinOf#1to\t@Y
1269 \MakeVectorFrom\t@X\t@Y to#2\ignorespaces}%
```

```
1270 \end{ArgOfVect} $$1270 \end{ArgOfVect}
1271 \def\s@gno{}\def\addflatt@ngle{0}
1272 \left( \frac{x}{p0} \right) = 20
                      \left( \frac{Y}{p} = \frac{20}{z} \right)
1273
                               \def\ArcTan{0}%
1274
1275
                      \else
1276
                               \def\ArcTan{90}%
1277
                               \left(\frac{y}{p0}\right)^{0}\def\som_{-}\pi
                      \fi
1278
1279 \else
                      \left( \frac{Y}{p0} \right) = 20
1280
1281
                              \left( \frac{x}{p} \right) 
1282
                                     \def\ArcTan{180}%
1283
                                     \def\ArcTan{0}%
1284
                              \fi
1285
                      \else
1286
                              \left( \frac{x}{p} \right) 
1287
1288
                                     \def\addflatt@ngle{180}%
1289
                                     \edef\t@X{\strip@pt\dimexpr-\t@X\p@}%
                                     \edef\t@Y{\strip@pt\dimexpr-\t@Y\p@}%
1290
                                     1291
                                             \def\s@gno{-}%
1292
                                             \ensuremath{\texttt{def}\t@Y{-\t@Y}}\%
1293
1294
                                     \fi
1295
                              \fi
                       \DivideFN\t@Y by\t@X to \t@A
1296
                       \ArcTanOf\t@A to\ArcTan
1297
1298
                      \fi
1299 \fi
1300 \edef\ArcTan{\unless\ifx\s@gno\empty\s@gno\fi\ArcTan}%
1301 \unless\ifnum\addflatt@ngle=0\relax
1302
                          \edef\ArcTan{%
                           \strip@pt\dimexpr\ArcTan\p@\ifx\s@gno\empty-\else+\fi
1303
1304
                          \addflatt@ngle\p@\relax}%
1305 \fi
1306 \edf\x{\noexpand\egroup\noexpand\edef\noexpand\#2{\ArcTan}}\%
1307 \x\ignorespaces}
1308 \def\ScaleVect#1by#2to#3{\GetCoord(#1)\t@X\t@Y
1309 \@tempdima=\t@X\p@ \@tempdima=#2\@tempdima\Numero\t@X\@tempdima
1310 \ensuremath{\mbox{\tt 01}}\ensuremath{\mbox{\tt 02}}\ensuremath{\mbox{\tt 02}
1311 \MakeVectorFrom\t@X\t@Y to#3\ignorespaces}%
1312 \def\ConjVect#1to#2{\GetCoord(#1)\t@X\t@Y
1313 \@tempdima=-\t@Y\p@\Numero\t@Y\@tempdima
 1314 \MakeVectorFrom\t@X\t@Y to#2\ignorespaces}%
1315 \def\AddVect#1and#2to#3{\GetCoord(#1)\tu@X\tu@Y
1316 \GetCoord(#2)\td@X\td@Y
1317 \@tempdima\tu@X\p@\advance\@tempdima\td@X\p@ \Numero\t@X\@tempdima
1318 \@tempdima\tu@Y\p@\advance\@tempdima\td@Y\p@ \Numero\t@Y\@tempdima
1319 \MakeVectorFrom\t@X\t@Y to#3\ignorespaces}%
```

```
1320 \def\SubVect#1from#2to#3{\GetCoord(#1)\tu@X\tu@Y
1321 \GetCoord(#2)\td@X\td@Y
1322 \verb|\dtempdima| td@X\p@\advance\\@tempdima-\tu@X\p@ \Numero\\t@X\@tempdima
1323 \verb|\dtempdima| td@Y\\p@\advance\\\dtempdima-\tu@Y\\p@\Numero\\\td@Y\\\dtempdima
1324 \MakeVectorFrom\t@X\t@Y to#3\ignorespaces}%
1325 \label{locality} $$1325 \def\MultVect#1by{\Qifstar{\QConjMultVect#1by}}, $$
1326 \def\@MultVect#1by#2to#3{\GetCoord(#1)\tu@X\tu@Y
1327 \GetCoord(#2)\td@X\td@Y
1329 \verb|\dempdimc=\td@X\dempdima\advance\dempdimc-\td@Y\dempdimb|\\
1330 \Numero\t@X\@tempdimc
1331 \verb|\dtempdimc=\td@Y\\| \texttt{Qtempdima} \\| \texttt{Qtempdimc} \\| \texttt{td@Y}\\| \texttt{Qtempdimb} \\| \texttt{Qtempdimc} \\| \texttt{Qtempd
1332 \Numero\t@Y\@tempdimc
1333 \MakeVectorFrom\t@X\t@Y to#3\ignorespaces}%
1334 \def\@ConjMultVect#1by#2to#3{\GetCoord(#1)\tu@X\tu@Y
1335 \GetCoord(#2)\td@X\td@Y \@tempdima\tu@X\p@ \@tempdimb\tu@Y\p@
1336 \verb|\dtempdimc=\td@X\\| @tempdima\advance\\| @tempdimc+\td@Y\\| @tempdimb
1337 \Numero\t@X\@tempdimc
1338 \end{are} $$1338 \end{are} \end{are} $$1338 \end{a
1339 \Numero\t@Y\@tempdimc
1340 \MakeVectorFrom\t@X\t@Y to#3\ignorespaces}
1341 \def\DivVect#1by#2to#3{\ModAndDirOfVect#2to\@Mod and\@Dir
1342\ \ensuremath{\mbox{\sc loss}}\ by
 \ensuremath{\mbox{\sc loss}}\ to
 \ensuremath{\mbox{\sc loss}}\ to
 \ensuremath{\mbox{\sc loss}}\
1343 \ScaleVect#1by\@Mod to\@tempa
1344 \MultVect\@tempa by\@Dir to#3\ignorespaces}%
1345 \def\Arc(#1)(#2)#3{\begingroup
1346 \@tdA=#3\p@
1347 \leq 
1348 \@Arc(#1)(#2)%
1349 \fi
1350 \endgroup\ignorespaces}%
1351 \def\@Arc(#1)(#2){%
1352 \ifdim\@tdA>\z@
1353 \let\Segno+%
1354 \else
1355 \@tdA=-\@tdA \let\Segno-%
1356 \fi
1357 \Numero\@gradi\@tdA
1358 \ifdim\@tdA>360\p@
1359 \PackageWarning{curve2e}{The arc aperture is \@gradi\space degrees
1360
                                                            and gets reduced\MessageBreak%
1361
                                                            to the range 0--360 taking the sign into consideration}%
1362 \end{align*} $$1362 \end{align*} $$1362
1364 \SubVect#2from#1to\@V \ModOfVect\@V to\@Raggio \CopyVect#2to\@pPun
1365 \CopyVect#1to\@Cent \GetCoord(\@pPun)\@pPunX\@pPunY
1366 \@@Arc
1367 \strokepath\ignorespaces}%
1368 \def\@@Arc{%
1369 \pIIe@moveto{\@pPunX\unitlength}{\@pPunY\unitlength}%
```

```
1370 \ifdim\@tdA>180\p@
      \advance\@tdA-180\p@
1371
      \Numero\@gradi\@tdA
1372
      \SubVect\@pPun from\@Cent to\@V
1373
1374
      \AddVect\@V and\@Cent to\@sPun
1375
      1376
      \AddVect\@pPun and\@V to\@pcPun
1377
      \AddVect\@sPun and\@V to\@scPun
      \GetCoord(\@pcPun)\@pcPunX\@pcPunY
1378
      \GetCoord(\@scPun)\@scPunX\@scPunY
1379
      \GetCoord(\@sPun)\@sPunX\@sPunY
1380
      \pIIe@curveto{\@pcPunX\unitlength}{\@pcPunY\unitlength}%
1381
1382
                   {\@scPunX\unitlength}{\@scPunY\unitlength}%
                   {\@sPunX\unitlength}{\@sPunY\unitlength}%
1383
1384
      \CopyVect\@sPun to\@pPun
1385 \fi
1386 \left(\frac{0}{4}\right)
      \DirFromAngle\@gradi to\@Dir \if\Segno-\ConjVect\@Dir to\@Dir \fi
1387
1388
      \SubVect\@Cent from\@pPun to\@V
1389
      \MultVect\@V by\@Dir to\@V
1390
      \AddVect\@Cent and\@V to\@sPun
      \@tdA=.5\@tdA \Numero\@gradi\@tdA
1391
      \DirFromAngle\@gradi to\@Phimezzi
1392
      \GetCoord(\@Phimezzi)\@cosphimezzi\@sinphimezzi
1393
1394
      \@tdB=1.3333333\p@ \@tdB=\@Raggio\@tdB
      \@tdC=\p@ \advance\@tdC -\@cosphimezzi\p@ \Numero\@tempa\@tdC
1395
      \del{dB}=\del{def} \del{dB}
1396
      \DividE\@tdB by\@sinphimezzi\p@ to\@cZ
1397
      \ScaleVect\@Phimezzi by\@cZ to\@Phimezzi
1398
      \ConjVect\@Phimezzi to\@mPhimezzi
1399
1400
      \if\Segno-%
1401
        \let\@tempa\@Phimezzi
1402
        \let\@Phimezzi\@mPhimezzi
        \let\@mPhimezzi\@tempa
1403
      \fi
1404
      \SubVect\@sPun from\@pPun to\@V
1405
      \DirOfVect\@V to\@V
1406
      \MultVect\@Phimezzi by\@V to\@Phimezzi
1407
1408
      \AddVect\@sPun and\@Phimezzi to\@scPun
      \ScaleVect\@V by-1to\@V
1409
1410
      \MultVect\@mPhimezzi by\@V to\@mPhimezzi
1411
      \AddVect\@pPun and\@mPhimezzi to\@pcPun
1412
      \GetCoord(\@pcPun)\@pcPunX\@pcPunY
      \GetCoord(\@scPun)\@scPunX\@scPunY
1413
1414
      \GetCoord(\@sPun)\@sPunX\@sPunY
1415
      \pIIe@curveto{\@pcPunX\unitlength}{\@pcPunY\unitlength}%
1416
                   {\@scPunX\unitlength}{\@scPunY\unitlength}%
1417
                   {\@sPunX\unitlength}{\@sPunY\unitlength}%
1418 \fi}
1419 \def\VectorArc(#1)(#2)#3{\begingroup
```

```
1421 \@VArc(#1)(#2)%
1422 \fi
1423 \endgroup\ignorespaces}%
1424 \def\VectorARC(#1)(#2)#3{\begingroup}
1425 \@tdA=#3\p@
1426 \left( A=\z@\leq \right)
1427 \@VARC(#1)(#2)%
1428 \fi
1429 \verb|\endgroup\ignorespaces|| \%
1430 \def\@VArc(#1)(#2){%
1431 \ifdim\@tdA>\z@
1432 \let\Segno+%
1433 \else
1434 \@tdA=-\@tdA \let\Segno-%
1435 \fi \Numero\@gradi\@tdA
1436 \left(\frac{0}{4}\right)
1437 \PackageWarning{curve2e}{The arc aperture is \@gradi\space degrees
1438
                                               and gets reduced\MessageBreak%
1439
                                               to the range 0--360 taking the sign into consideration}%
1440 \ensuremath{\verb|||} \ensuremath{\ensuremath{||||} \ensuremath{\ensuremath{|||} \ensuremath{\ensuremath{||||} \ensuremath{\ensuremath{|||} \ensuremath{\ensuremath{||||} \ensure
1441 \fi
1442 \SubVect#1from#2to\@V \ModOfVect\@V to\@Raggio \CopyVect#2to\@pPun
1443 \ensuremath{\tt 0tdE=\pIIe@FAL\ensuremath{\tt 0tdE=\pIIe@FAL\ensuremath{\tt 0tdE}}
1444 \DividE\@tdE by \@Raggio\unitlength to\DeltaGradi
1445 \@tdD=\DeltaGradi\p@
1446 \d \D=57.29578 \d \Mumero \DeltaGradi \d \D
1447 \ensuremake{$1447$ \ensuremake} Numero\ensuremake{$1447$ \ensuremake} Numero\ensuremake} Numero\ensuremake{$1447$ \ensuremake} Numero\ensuremake{$1447$ \ensuremake} Numero\ensuremake} Numero\ensuremake{$1447$ \ensuremake} Numero\ensuremake{$1447$ \ensuremake} Numero\ensuremake} Numero\ensuremake{$1447$ \ensuremake} Numero\ensuremake} Numero\ensuremake{$1447$ \ensuremake} Numero\ensuremake} Numero\ensur
1448 \DirFromAngle\@tempa to\@Dir
1449 \MultVect\@V by\@Dir to\@sPun
1450 \edgno-\mbox{\edgno-\mbox{\edgno-\mbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbox{\edgno-\hbo
1451 \MultVect\@sPun by 0,\@tempA to\@vPun
1452 \DirOfVect\@vPun to\@Dir
1453 \AddVect\@sPun and #1 to \@sPun
1454 \GetCoord(\@sPun)\@tdX\@tdY
1455 \@tdD\ifx\Segno--\fi\DeltaGradi\p@
1456 \@tdD=.5\@tdD \Numero\DeltaGradi\@tdD
1457 \DirFromAngle\DeltaGradi to\@Dird
1458 \MultVect\@Dir by*\@Dird to\@Dir
1459 \GetCoord(\@Dir)\@xnum\@ynum
1460 \put(\@tdX,\@tdY){\vector(\@xnum,\@ynum){0}}%
1461 \@tdE =\ifx\Segno--\fi\DeltaGradi\p@
1462 \advance\@tdA -\ \Numero\@gradi\@tdA
1463 \verb|\CopyVect#1to\\@Cent \\ $\GetCoord(\@pPun)\\@pPunX\\@pPunY
1464 \@@Arc
1465 \strokepath\ignorespaces}%
1466 \def\@VARC(#1)(#2){%
1467 \ifdim\@tdA>\z@
1468 \let\Segno+%
1469 \else
```

```
1470 \@tdA=-\@tdA \let\Segno-%
1471 \fi \Numero\@gradi\@tdA
1472 \left( A>360 \right)
1473 \PackageWarning{curve2e}{The arc aperture is \@gradi\space degrees
                   and gets reduced\MessageBreak%
1474
1475
                   to the range 0--360 taking the sign into consideration}%
1476 \end{advance} $$1476 \e
1477 \fi
1478 \SubVect#1from#2to\@V \ModOfVect\@V to\@Raggio \CopyVect#2to\@pPun
1479 \cdE=\pIIe@FAW\cwholewidth \cdE=0.8\cdE
1480 \DividE\@tdE by \@Raggio\unitlength to\DeltaGradi
1481 \OtdD=\DeltaGradi\pO \OtdD=57.29578\OtdD \Numero\DeltaGradi\OtdD
1482 \ensuremath{\tt 0tdD=\if\Segno--fi\Qgradi\p@\Numero\Qtempa\QtdD}
1483 \DirFromAngle\@tempa to\@Dir
1484 \MultVect\@V by\@Dir to\@sPun% corrects the end point
1485 \edgno--fi1}%
1486 \MultVect\@sPun by 0,\@tempA to\@vPun
1487 \DirOfVect\@vPun to\@Dir
1488 \AddVect\@sPun and #1 to \@sPun
1489 \GetCoord(\@sPun)\@tdX\@tdY
1490 \@tdD\if\Segno--\fi\DeltaGradi\p@
1491 \@tdD=.5\@tdD \Numero\@tempB\@tdD
1493 \MultVect\@Dir by*\@Dird to\@Dir
1494 \GetCoord(\@Dir)\@xnum\@ynum
1495 \put(\@tdX,\@tdY){\vector(\@xnum,\@ynum){0}}% end point arrowt ip
1496 \@tdE =\DeltaGradi\p@
1497 \advance\@tdA -2\@tdE \Numero\@gradi\@tdA
1498 \CopyVect#1to\@Cent \GetCoord(\@pPun)\@pPunX\@pPunY
1499 \SubVect\@Cent from\@pPun to \@V
1500 \edef\@tempa{\if\Segno-\else-\fi\@ne}%
1501 \MultVect\@V by0,\@tempa to\@vPun
1502 \@tdE\if\Segno--\fi\DeltaGradi\p@
1503 \Numero\00tempB{0.5\0tdE}%
1504 \DirFromAngle\@tempB to\@Dird
1505 \MultVect\@vPun by\@Dird to\@vPun% corrects the starting point
1506 \DirOfVect\@vPun to\@Dir\GetCoord(\@Dir)\@xnum\@ynum
1507 \put(\@pPunX,\@pPunY){\vector(\@xnum,\@ynum){0}}% starting point arrow tip
1508 \edef\@tempa{\if\Segno--\fi\DeltaGradi}%
1509 \DirFromAngle\@tempa to \@Dir
1510 \SubVect\@Cent from\@pPun to\@V
1511 \MultVect\@V by\@Dir to\@V
1512 \AddVect\@Cent and\@V to\@pPun
1513 \GetCoord(\@pPun)\@pPunX\@pPunY
1514 \@@Arc
1515 \strokepath\ignorespaces}%
1516 \def\CurveBetween#1and#2WithDirs#3and#4{%
1517 \StartCurveAt#1WithDir{#3}\relax
1518 \CurveTo#2WithDir{#4}\CurveFinish\ignorespaces}%
1519 \def\StartCurveAt#1WithDir#2{%
```

```
1520 \begingroup
1521 \GetCoord(#1)\@tempa\@tempb
1522 \CopyVect\@tempa,\@tempb to\@Pzero
1523 \pIIe@moveto{\@tempa\unitlength}{\@tempb\unitlength}%
1524 \GetCoord(#2)\@tempa\@tempb
1525 \CopyVect\@tempa,\@tempb to\@Dzero
1526 \DirOfVect\@Dzero to\@Dzero
1527 \ignorespaces}
1528 \def\ChangeDir<#1>{%
1529 \GetCoord(#1)\@tempa\@tempb
1530 \CopyVect\@tempa,\@tempb to\@Dzero
1531 \DirOfVect\@Dzero to\@Dzero
1532 \ignorespaces}
1533 \def\CurveFinish{\strokepath\endgroup\ignorespaces}%
1534 \def\FillCurve{\fillpath\endgroup\ignorespaces}
1535 \def\CurveEnd{\fillstroke\endgroup\ignorespaces}
1536 \ensuremath{\mbox{\sc 1536}} \ensuremath{\mbox{\sc 154}} And \ensuremath{\mbox{\sc 154}} Bir#2And \ensuremath{\mbox
1537 \GetCoord(#1)\@tX\@tY \MakeVectorFrom\@tX\@tY to\@Puno
1538 \GetCoord(#2)\@tX\@tY \MakeVectorFrom\@tX\@tY to \@Duno
1539 \DirOfVect\@Duno to\@Duno
1540 \ScaleVect\@Dzero by#3to\@Czero \AddVect\@Pzero and\@Czero to\@Czero
1541 \ScaleVect\@Duno by-#4to \@Cuno \AddVect\@Puno and\@Cuno to \@Cuno
1542 \GetCoord(\@Czero)\@XCzero\@YCzero
1543 \GetCoord(\@Cuno)\@XCuno\@YCuno
1544 \GetCoord(\@Puno)\@XPuno\@YPuno
1545 \pIIe@curveto{\@XCzero\unitlength}{\@YCzero\unitlength}%
                                                                                  {\@XCuno\unitlength}{\@YCuno\unitlength}%
1547
                                                                                  {\@XPuno\unitlength}{\@YPuno\unitlength}%
1548 \CopyVect\@Puno to\@Pzero
1549 \CopyVect\@Duno to\@Dzero
1550 \ignorespaces}%
1551 \def\CbezierBetween#1And#2WithDirs#3And#4UsingDists#5And#6{%
1552 \StartCurveAt#1WithDir{#3}\relax
1553 \CbezierTo#2WithDir#4AndDists#5And{#6}\CurveFinish}
1554
1555 \def\@isTension#1;#2!!{\def\@tempA{#1}%
1556 \end{area} $$1556 \end{area} \end{area} \end{area} $$1556 \end{area} \end{area} $$1556 \end{are
1557 \def\strip@semicolon#1;{\def\@tempB{#1}}
1558 \def\CurveTo#1WithDir#2{%
1559 \def\Cuno{1}\def\Cuno{1}\relax
1560 \edef\@Puno{#1}\@isTension#2;!!%
1561 \expandafter\DirOfVect\@tempA to\@Duno
1562 \verb|\documn| less \le \texttt{\documn} \end{\text{\tt Coord(\documn} B) \documn} if if it is a substitution of the property of the prop
1563 \verb|\DistanceAndDirOfVect\QPuno minus\QPzero to \verb|\QChord and\QDirChord and QDirChord and QDIrC
1564 \MultVect\@Dzero by*\@DirChord to \@Dpzero
1565 \MultVect\@Duno by*\@DirChord to \@Dpuno
1566 \GetCoord(\@Dpzero)\@DXpzero\@DYpzero
1567 \GetCoord(\@Dpuno)\@DXpuno\@DYpuno
1568 \DivideFN\@Chord by2 to\@semichord
1569 \left( DXpzero \right) = \z0
```

```
\@tdA=1.333333\p@
1570
               \Numero\@KCzero{\@semichord\@tdA}%
1571
1572 \fi
1573 \left( \frac{0}{2} \right) = 1573 
             \@tdA=1.333333\p@
             \Numero\@Kpzero{\@semichord\@tdA}%
1576 \fi
1577 \unless\ifdim\@DXpzero\p@=\z@
             \unless\ifdim\@DYpzero\p@=\z@
                  \edef\@CosDzero{\ifdim\@DXpzero\p@<\z@ -\fi\@DXpzero}%
1579
                  \edef\@SinDzero{\ifdim\@DYpzero\p@<\z@ -\fi\@DYpzero}%
1580
1581
                  \DividE\@tdA by\@SinDzero\p@ to \@KCzero
1582
                  \@tdA=\dimexpr(\p@-\@CosDzero\p@)\relax
1583
                 \DividE\@KCzero\@tdA by\@SinDzero\p@ to \@KCzero
1584
            \fi
1585
1586 \fi
1587 \MultiplyFN\@KCzero by \@Tzero to \@KCzero
1588 \ScaleVect\@Dzero by\@KCzero to\@CPzero
1589 \AddVect\@Pzero and\@CPzero to\@CPzero
1590 \ifdim\@DXpuno\p@=\z@
               \@tdA=-1.333333\p@
1591
               \Numero\@KCuno{\@semichord\@tdA}%
1592
1593 \fi
1594 \ifdim\@DYpuno\p@=\z@
             \@tdA=-1.333333\p@
             \Numero\@KCuno{\@semichord\@tdA}%
1596
1597 \fi
1598 \unless\ifdim\@DXpuno\p@=\z@
                  \ \ \unless\ifdim\@DYpuno\p@=\z@
1599
                  \edef\@CosDuno{\ifdim\@DXpuno\p@<\z@ -\fi\@DXpuno}%
1600
1601
                  \edef\@SinDuno{\ifdim\@DYpuno\p@<\z@ -\fi\@DYpuno}%
1602
                  \d = \ensuremath{\d} \ensuremath{\d} \d = -1.333333\ensuremath{\d} \d = -1.3333333\ensuremath{\d} \d = -1.33333333\ensuremath{\d} \d = -1.3333333\ensuremath{\d} \d = -1.3333333\ensuremath{\d} \d = -1.333333\ensuremath{\d} \d = -1.3333333\ensuremath{\d} \d = -1.333333\ensuremath{\d} \d = -1.333333\ensuremath{\d} \d = -1.333333\ensuremath{\d} \d = -1.333333\ensuremath{\d} \d = -1.3333333\ensuremath{\d} \d = -1.3333333\ensuremath{\d} \d = -1.3333333\ensuremath{\d} \d = -1.33333333\ensuremath{\d} \d = -1.33333333\ensuremath{\d} \d = -1.33333333\ensuremath{\d} \d = -1.3333333\ensuremath{\d} \d = -1.33333333\ensuremath{\d} \d = -1.33333333\ensuremath{\d} \d = -1.33333333\ensuremath{\d} \d = -1.33333333\ensuremath{\d} \d = -1.33333333\ensuremat
                  \DividE\@tdA by \@SinDuno\p@ to \@KCuno
1603
1604
                  \@tdA=\dimexpr(\p@-\@CosDuno\p@)\relax
1605
                  \DividE\@KCuno\@tdA by\@SinDuno\p@ to \@KCuno
            \fi
1606
1607 \fi
1608 \MultiplyFN\@KCuno by \@Tuno to \@KCuno
1609 \ScaleVect\@Duno by\@KCuno to\@CPuno
1610 \AddVect\@Puno and\@CPuno to\@CPuno
1611 \GetCoord(\@Puno)\@XPuno\@YPuno
1612 \GetCoord(\@CPzero)\@XCPzero\@YCPzero
1613 \GetCoord(\@CPuno)\@XCPuno\@YCPuno
1614 \pIIe@curveto{\@XCPzero\unitlength}{\@YCPzero\unitlength}%
1615
                                     {\@XCPuno\unitlength}{\@YCPuno\unitlength}%
1616
                                     {\@XPuno\unitlength}{\@YPuno\unitlength}\egroup
1617 \CopyVect\@Puno to\@Pzero
1618 \CopyVect\@Duno to\@Dzero
1619 \ignorespaces}%
```

```
1620 \end{Curve} \end{Curve} \label{lem:curve} \hspace{1620 \end{Curve}} \hspace{1620 \end{Cur
1621 {\let\fillstroke\strokepath\Curve0}}
1622 \def\Curve@(#1)<#2>{%
                                \StartCurveAt#1WithDir{#2}%
1623
1624
                                 \@ifnextchar\lp@r\@Curve{%
1625
                                \PackageWarning{curve2e}{%
1626
                                Curve specifications must contain at least two nodes!\Messagebreak
1627
                               Please, control your Curve specifications\MessageBreak}}}
1628 \def\@Curve(#1)<#2>{%
                                 \CurveTo#1WithDir{#2}%
1629
                                 \@ifnextchar\lp@r\@Curve{%
1630
                                 \@ifnextchar[\@ChangeDir\CurveEnd}}
1632 \def\@ChangeDir[#1] {\ChangeDir<#1>\@Curve}
1633 \end{are} \end{are} $$1633 \end{are} \end{are} $$1633 \end{are} $$1
1634 {\let\fillstroke\strokepath\Qurve@}}
1635
1636 \def\Qurve@(#1)<#2>{%
                                \StartCurveAt#1WithDir{#2}%
1637
1638
                                 \@ifnextchar\lp@r\@Qurve{%
1639
                                 \PackageWarning{curve2e}{%
1640
                                        Quadratic curve specifications must contain at least
                                        two nodes!\Messagebreak
1641
1642
                                       Please, control your Qurve specifications\MessageBreak}}}%
1643 \def\@Qurve(#1)<#2>{\QurveTo#1WithDir{#2}%
1644
                                \@ifnextchar\lp@r\@Qurve{%
                                 \@ifnextchar[\@ChangeQDir\CurveEnd}}%
1645
1646 \def\@ChangeQDir[#1] {\ChangeDir<#1>\@Qurve}%
1647 \def\QurveTo#1WithDir#2{%
1648 \edef\@Puno{#1}\DirOfVect#2to\@Duno\bgroup
1649 \verb|\DistanceAndDirOfVect\\|\\@Puno minus\\|\\@Pzero to\\|\\@Chord and\\|\\@DirChord and\\|\\@DirCh
1650 \MultVect\@Dzero by*\@Duno to \@Scalar
1651 \YpartOfVect\@Scalar to \@YScalar
1652 \left( \frac{9}{2} \right) = 1652 
1653 \PackageWarning{curve2e}%
                            {Quadratic Bezier arcs cannot have their starting\MessageBreak
1654
                           and ending directions parallel or antiparallel with \MessageBreak
1655
1656
                           each other. This arc is skipped and replaced with
1657
                           a dotted line.\MessageBreak}%
                           \Dotline(\@Pzero)(\@Puno){2}\relax
1658
1659 \else
1660 \MultVect\@Dzero by*\@DirChord to \@Dpzero
1661 \MultVect\@Duno by*\@DirChord to \@Dpuno
1662 \GetCoord(\@Dpzero)\@DXpzero\@DYpzero
1663 \GetCoord(\@Dpuno)\@DXpuno\@DYpuno
1664 \MultiplyFN\@DXpzero by\@DXpuno to\@XXD
1665 \MultiplyFN\@DYpzero by\@DYpuno to\@YYD
1666 \verb|\unless| ifdim|@YYD|p@<\\z@\\ifdim|@XXD|p@<\\z@
1667 \PackageWarning{curve2e}%
1668
                           {Quadratic Bezier arcs cannot have inflection points\MessageBreak
1669
                           Therefore the tangents to the starting and ending arc\MessageBreak
```

```
points cannot be directed to the same half plane.\MessageBreak
1670
1671
                 This arc is skipped and replaced by a dotted line\MessageBreak}%
                  \Dotline(\@Pzero)(\@Puno){2}\fi
1672
1673 \else
1674 \edef\@CDzero{\@DXpzero}\relax
1675 \edef\@SDzero{\@DYpzero}\relax
1676 \edef\@CDuno{\@DXpuno}\relax
1677 \edef\@SDuno{\@DYpuno}\relax
1678 \MultiplyFN\@SDzero by\@CDuno to\@tempA
1679 \MultiplyFN\@SDuno by\@CDzero to\@tempB
1680 \edgn{\color=0.0cm} $1680 \edgn{\color=0.0cm} \color=0.0cm\\ \colo
1681 \@tdA=\@SDuno\p@ \@tdB=\@Chord\p@ \@tdC=\@tempA\p@
1682 \edef\@tempC{\strip@pt\dimexpr \@tdA*\@tdB/\@tdC}\relax
1683 \MultiplyFN\@tempC by\@CDzero to \@XC
1684 \MultiplyFN\@tempC by\@SDzero to \@YC
1685 \ModOfVect\@XC,\@YC to\@KC
1686 \ScaleVect\@Dzero by\@KC to\@CP
1687 \AddVect\@Pzero and\@CP to\@CP
1688 \GetCoord(\@Pzero)\@XPzero\@YPzero
1689 \GetCoord(\@Puno)\@XPuno\@YPuno
1690 \GetCoord(\@CP)\@XCP\@YCP
1691 \@ovxx=\@XPzero\unitlength \@ovyy=\@YPzero\unitlength
1692 \@ovdx=\@XCP\unitlength \@ovdy=\@YCP\unitlength
1693 \@xdim=\@XPuno\unitlength \@ydim=\@YPuno\unitlength
1694
                          \pIIe@bezier@QtoC\@ovxx\@ovdx\@ovro
1695
                          \pIIe@bezier@QtoC\@ovyy\@ovdy\@ovri
                          \pIIe@bezier@QtoC\@xdim\@ovdx\@clnwd
1696
1697
                          \pIIe@bezier@QtoC\@ydim\@ovdy\@clnht
                          \pIIe@moveto\@ovxx\@ovyy
1698
                          \pIIe@curveto\@ovro\@ovri\@clnwd\@clnht\@xdim\@ydim
1699
1700 \fi\fi\egroup
1701 \CopyVect\@Puno to\@Pzero
1702 \CopyVect\@Duno to\@Dzero
1703 \ignorespaces}
1704
```

References

[1] Gäßlein H., Niepraschk R., and Tkadlec J. *The pict2e package*, 2019, PDF documentation of pict2e; this package is part of any modern complete distribution of the TEX system; it may be read by means of the line command texdoc pict2e. In case of a basic or partial system installation, the package may be installed by means of the specific facilities of the distribution.